$$\frac{d}{dt}\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} -\frac{1}{T} & B_0\gamma & 0 \\ -B_0\gamma & -\frac{1}{T} & 0 \\ 0 & 0 & -\frac{1}{T} \end{pmatrix}\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{M_0}{T} \end{pmatrix}$$

gives $\displaystyle \frac{dM_z}{dt} = -\frac{M_z}{T} + \frac{M_0}{T}$

$$\frac{dM_z}{(M_0 - M_z)} = \frac{dt}{T} \qquad u = M_0 - M_z$$

$$\frac{du}{dM_z} = -1$$

$$\int -\frac{1}{u}\,du = \int \frac{1}{T}\,dt \qquad du = -dM_z$$

$$-\ln u = \frac{t}{T} + c$$ constant of integration

$$\ln u = \frac{-t}{T} + c.$$

$$u = Ce^{-t/T}$$

$$M_0 - M_z = Ce^{-t/T}$$

$$M_z = M_0 - Ce^{-t/T}$$

initial condition: $M_z(0) = 0$.

$$0 = M_0 - e^0(C)$$

$$C = M_0$$

$$\Rightarrow M_z(t) = M_0\left(1 - e^{-t/T}\right)$$

as in report, eq. 4.

As for x, y:

$$\frac{d}{dt}\begin{pmatrix} M_x \\ M_y \end{pmatrix} = \begin{pmatrix} -\frac{1}{T} & B_0\gamma \\ -B_0\gamma & -\frac{1}{T} \end{pmatrix}\begin{pmatrix} M_x \\ M_y \end{pmatrix}$$

Let $\underline{M} = \begin{pmatrix} M_x \\ M_y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix} e^{\lambda t}$

$$\Rightarrow \frac{d\underline{M}}{dt} = \lambda\begin{pmatrix} u \\ v \end{pmatrix} e^{\lambda t}$$

$$\Rightarrow \lambda\begin{pmatrix} u \\ v \end{pmatrix}\cancelto{1}{e^{\lambda t}} = \begin{pmatrix} -\frac{1}{T} & B_0\gamma \\ -B_0\gamma & -\frac{1}{T} \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}\cancelto{1}{e^{\lambda t}}$$

$\Rightarrow$ eigenvalue-eigenvector equation.

$$\det\left(\begin{pmatrix}-\frac{1}{T} & \gamma B_0\\ -\gamma B_0 & -\frac{1}{T}\end{pmatrix}+\begin{pmatrix}-\lambda & 0\\ 0 & -\lambda\end{pmatrix}\right) = \begin{vmatrix}-\frac{1}{T}-\lambda & B_0\gamma\\ -B_0\gamma & -\frac{1}{T}-\lambda\end{vmatrix} = \left(-\frac{1}{T}-\lambda\right)^2 - (-B_0\gamma)(B_0\gamma) = 0$$

$$\lambda^2 + \frac{2\lambda}{T} + \frac{1}{T^2} + \gamma^2 B_0^2 = 0$$

$$\frac{-\frac{2}{T} \pm \sqrt{\frac{4}{T^2} - 4(1)\left[(\gamma^2 B_0^2) + \frac{1}{T^2}\right]}}{2} = \lambda$$

$$-\frac{1}{T} \pm \frac{1}{2}\sqrt{\cancel{\frac{4}{T^2}-\frac{4}{T^2}}^{\,0} - 4\gamma^2 B_0^2}$$

$$-\frac{1}{T} \pm i\gamma B_0 = \lambda \quad \begin{cases}\lambda_+ = -\frac{1}{T} + i\gamma B_0\\ \lambda_- = -\frac{1}{T} - i\gamma B_0\end{cases}$$

$$\lambda_+ : \begin{pmatrix}-\frac{1}{T}+\frac{1}{T}-i\gamma B_0 & \gamma B_0\\ -\gamma B_0 & -\frac{1}{T}+\frac{1}{T}-i\gamma B_0\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix} = 0$$

$$= \gamma B_0\begin{pmatrix}-i & 1\\ -1 & -i\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix} = 0 \Rightarrow ui = v \Rightarrow \text{eigenvector } \begin{pmatrix}i\\ 1\end{pmatrix}$$

$$\lambda_- : \begin{pmatrix}-\frac{1}{T}+\frac{1}{T}+i\gamma B_0 & \gamma B_0\\ -\gamma B_0 & -\frac{1}{T}+\frac{1}{T}+i\gamma B_0\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix} = 0$$

$$= \gamma B_0\begin{pmatrix}i & 1\\ -1 & i\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix} = 0 \Rightarrow ui = -v \Rightarrow \text{eigenvector } \begin{pmatrix}i\\ -1\end{pmatrix}$$

$$\underline{M} = C_1\begin{pmatrix}i\\ 1\end{pmatrix}e^{-t/T}e^{i\gamma B_0 t} + \begin{pmatrix}i\\ -1\end{pmatrix}C_2 e^{-t/T}e^{-i\gamma B_0 t}$$

superposition of two eigenvectors/eigenvalues.

$$M(0) = \begin{pmatrix}M_0\\ 0\end{pmatrix}.$$

$$C_1 i e^0 e^0 + C_2 i e^0 e^0 = M_0 \quad \text{and}$$

$$C_1 e^0 e^0 - C_2 e^0 e^0 = 0$$

for x, y components.

$\Rightarrow C_2 = C_1$ from y-component, thus.

$$M_0 = 2C_1 i$$

$$-i\frac{M_0}{2} = C_1$$

so, $M_x(t) = M_0 e^{-t/T}\cos B_0\gamma t$

$M_y(t) = -M_0 e^{-t/T}\sin B_0\gamma t$

after some simplification (Eq 4)