April 23, 2012

Math 70
Exam 2

1. The base 10 logarithm function is defined by the following equivalent statements:

$$\log(x) = y \quad \Leftrightarrow \quad 10^y = x$$

Use this definition to prove the following properties of logarithms:

a $\log(A \cdot B) = \log(A) + \log(B)$
b $\log(A/B) = \log(A) - \log(B)$
c $\log(A^p) = p\log(A)$

2. Solve for x:

a $10^{4x} = 100,000,000$

b $5 \cdot 10^{6-x} = 0.005$

c $\log(4/2) = 2$

d $\log(10^x) = 88.1$

Expand and simplify as much as possible:

a $\log\left(\frac{8x^2}{\sqrt[3]{y}}\right)$

b $\log(1000\frac{x}{y})$

Give a numerical answer :

a $2\log(2) + \log(5)$

b $\log(300) - \log(3)$

c $\log(\frac{3}{100}) - \log(3)$

3. After observing the mold growing on the wall of bathroom, you conclude that it seems to double in size every month.

a If it is measured to take up 3 square feet at the beginning of this month, write an exponential function that keeps track of the size of the mold given the number of months from now.

b The wall is 9 by 11 feet. How long until the mold completely covers the wall assuming that it doubles in size consistently. Give an estimate as well as an exact answer in terms of logarithms.

4. The half life of a radioactive element X is 8 months. That is after 8 months a sample of X will loose half its mass.
a If we start out with 1280 grams, how much will be left after 4 years?
b Write an exponential decay equation for the half life of this element where the dependent variable t is measured in 8 month intervals and given the above initial amount.
c Write an exponential decay equation where t is measured in **one** month intervals and give the same initial amount as above.

5. Let n be the number of times the interest is compounded each year, and r the annual interest rate (APR), and P_0 the principal investment. Then the amount you will have after t number of years is

$$P(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt}$$

a Write the equation that tracks the value of \$10,000 put into an investment with a 12% annual rate compounded once every other month.

b If that same investment's interest was compounded twice a year, what would the corresponding equation be? Find the **effective** interest rate. That is how much interest you actually earn after one year.

6. If a principal P_0 is invested at an annual rate of r (expressed as a decimal not as a percent) and is compounded continuously then its value for any given time t measured in years is

$$P(t) = P_0 e^{rt}$$

a \$100 is invested at a continuously compounded rate. After 5 years we have \$200. Find the original annual rate it was invested at. (You can express your answer in terms of the natural log, ln, or base-e logarithm.)