Task# 2

Language : python

Goal : The use of direct methods (one-dimensional methods of exhaustive search, dichotomy, golden section search; multidimensional methods of exhaustive search, Gauss (coordinate descent), Nelder-Mead) in the tasks of unconstrained nonlinear optimization

Problem : Use the one-dimensional methods of exhaustive search, dichotomy and golden section search to find an approximate (with precision x: $f(x) \rightarrow min$ for the following functions and domains:

I.

Theory:

exhaustive search : In this approach, we generate each element in the problem and then select the ones that satisfy all the constraints, and finally find a desired element

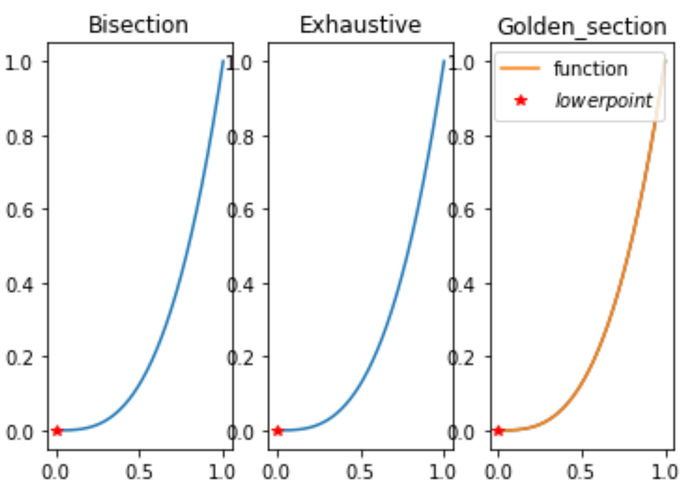
dichotomy search : use bound index to keep finding mid index , and keep trying find the smallest func(index)

golden section search : use golden rate (0.618) and two bound indexes to shrink the range and find the min

Gauss (coordinate descent) : is used to solve non-linear least squares problems, which is equivalent to minimizing a sum of squared function values

Nelder-Mead : is a numerical method used to find the minimum or maximum of an objective function by reflection, expansion, contraction and shrink coefficients in a multidimensional space

1. $f(x)=x^{\wedge 3}, x \in [0,1];$

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iteration is  : 10
The value of root is :  0.0010
evaluation of objective function is : 41


bounday is minimun
evaluation of objective function is : 2995
iteration is  : 998

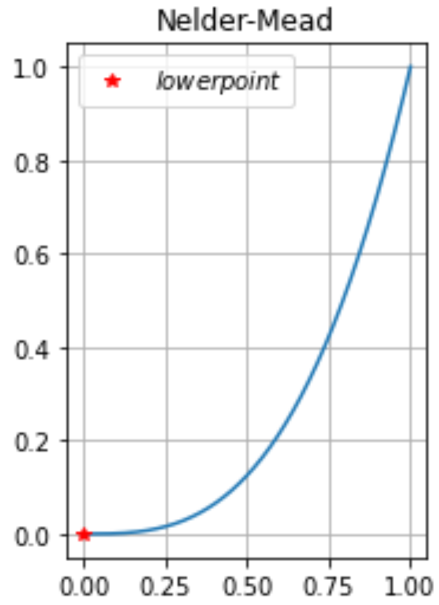

evaluation of objective function is : 32
iteration is  : 15
```

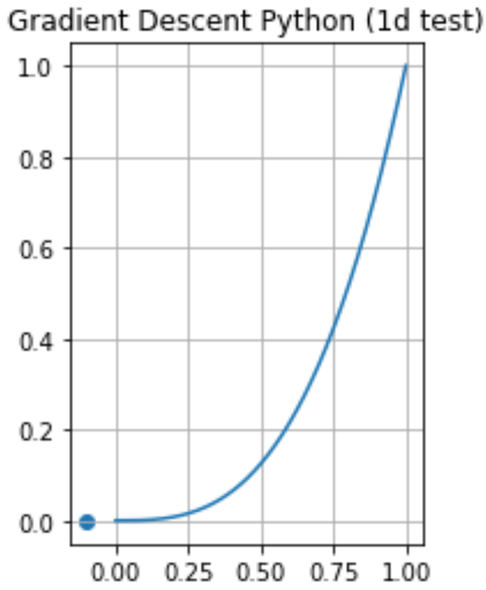
(iter , evalation)

Bisection (10,41)

Exhaustive (998,2995)

golden_sec(15,32)





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(iter , evalation)

Nelder:(1,2)

Gradient:(1,1)

```
  success: True
        x: array([1.e-06])
iteration is : 1
evaluation of objective function is : 2


iteration is  : 1
lowest point -0.099999,-0.001000
evaluation of objective function is : 1
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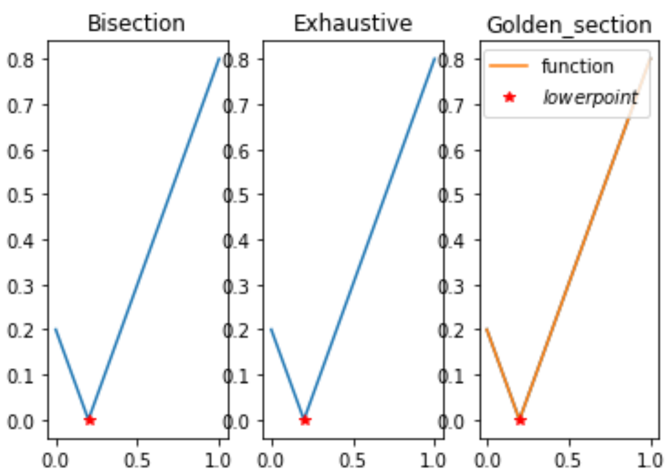
2. $f(x)=|x-0.2|, x\in[0,1]$;



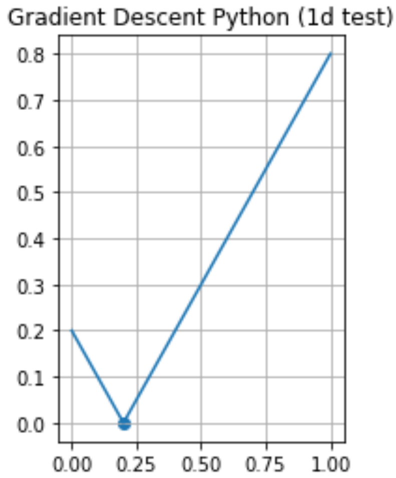
(iter , evalation)

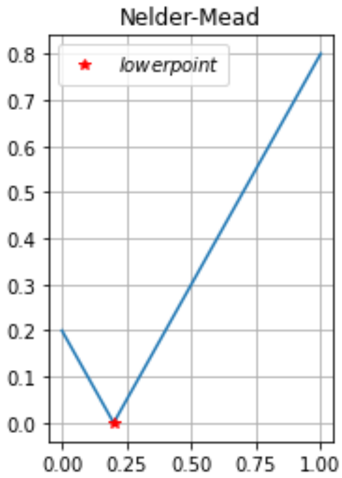
Bisection (10,25)

Exhaustive (200,601)

golden_sec(15,18)

```
iteration is  : 10
The value of root is :  0.2002
evaluation of objective function is : 25



The minimum point lies between  0.19900080100000025  &  0.20100079900000026
evaluation of objective function is : 601
iteration is  : 200



evaluation of objective function is : 18
iteration is  : 15
```






```
iteration is : 2
evaluation of objective function is : 72
nelder is called : 1


iteration is  : 52
lowest point 0.199165,0.000835
evaluation of objective function is : 52
```

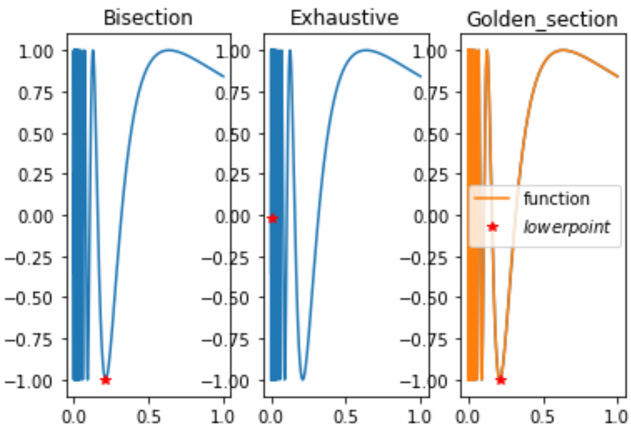
(iter , evalation)

Nelder:(1,2)

Gradient:(52,52)

3. $f(x)=x\sin(1/x), x\in[0.01,1]$. :

```
iteration is  : 10
The value of root is :  0.2119
evaluation of objective function is : 17


The minimum point lies between  0.0010009989999999998  &  0.0030009969999999999
evaluation of objective function is : 7
iteration is  : 2


evaluation of objective function is : 18
iteration is  : 15
```

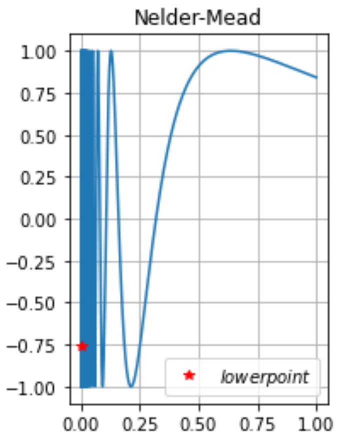


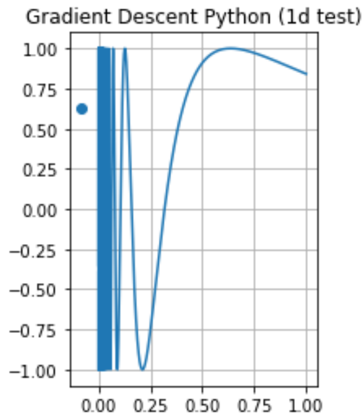
(iter , evalation)

Bisection (10,17)

Exhaustive (2,7)

golden_sec(18,15)






```
iteration is : 1
evaluation of objective function is : 97


iteration is  : 1
lowest point -0.084146,0.630563
evaluation of objective function is : 1
```

(iter , evalation)

Nelder:(1,97)

Gradient:(1,1)

II.

Generate random numbers $\alpha \in (0,1)$ *and* $\beta \in (0,1)$. *Furthermore, generate the noisy data* $\{x_>$, $y_>\}$, *where* $k = 0, \dots ,100$, *according to the following rule:*

$$y_k=\alpha x_k+\beta+\delta_k, \quad x_k= k/100$$

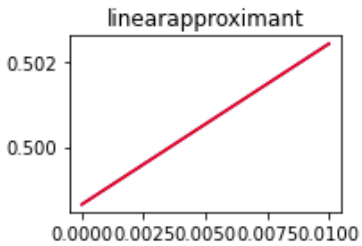
where $\delta_k \sim N(0,1)$ *are values of a random variable with standard normal distribution. Approximate the data by the following linear and rational functions:*

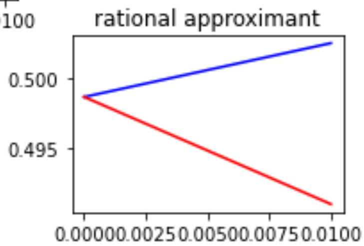
1. $F(x,a,b)=ax+b$ *(linear approximant)*

2. $F(x, a, b) = a/1+bx$ *(rational approximant),*

by means of least squares through the numerical minimization (with precision ε *=*

0.001*)*

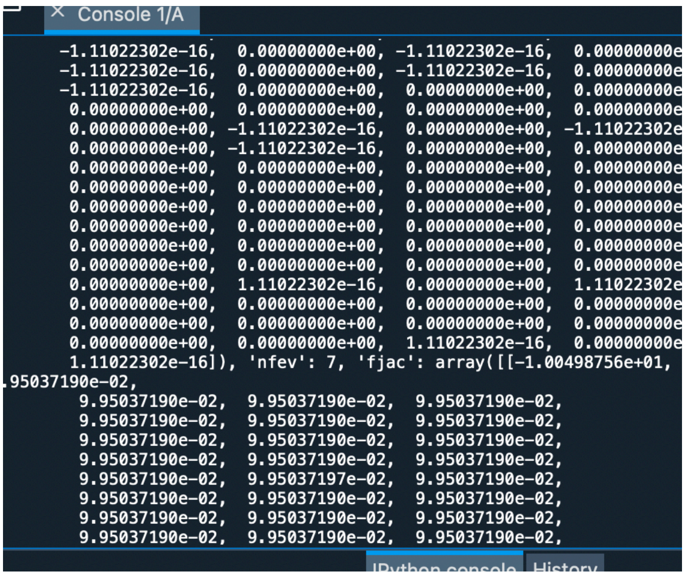


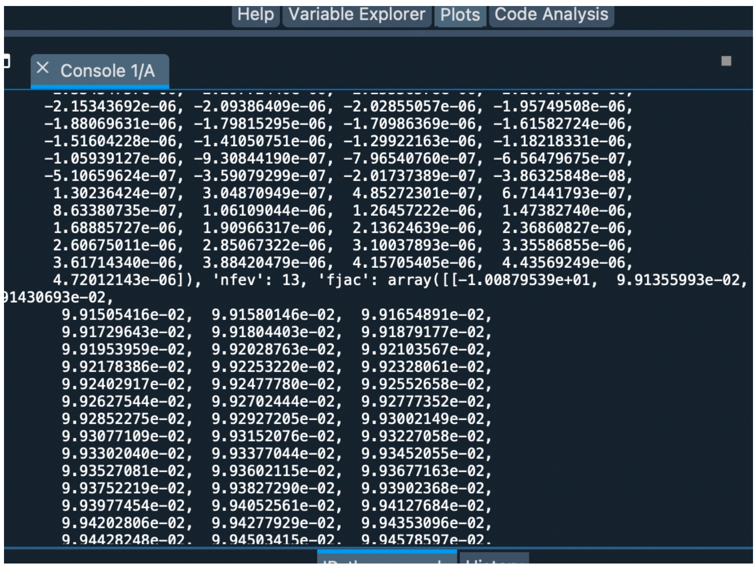


nfev is Number of evaluations of the objective functions

Leftside(linear) : nfev=7

right(rational) : nfev=13





Conclusion :

In first section , through graphs which I created , we can see that in most of formulas , we can achieve minimum values from each algorithm which I tested . But as we can see I found difficulty to use gauss and Nelder-Mead to find minimum from any function with trigonometric .

Most of time it cost a lot of iterations for exhaustive search to find a min , but somehow in third one with trigonometric , it only cost 2 times of iterations and 7 times of function , probably there is error in my algorithm . But we still can conclude that exhaustive search is not efficient to make calculation , it takes too much time to calculate each possibility .

On the contrary , we can see that Nelder-Mead and gauss show a very efficient way to get to result . But there is little mistake , so I can not get to desired answer .

In the second section , we used linear and rational approximant to try to approximate the data . As we can see that linear one complete with prettier way , and rational goes to wrong side . Maybe it is because the original data that I use to approximate is closer to a line , so rational approximant can not optimize it with better way . Even though we still can see that number of evaluation for rational approximant is higher .

Appendix

https://github.com/MaChengYuan/task2.git