FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION

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ITMO UNIVERSITY

Report

on the practical task No. 6

“Algorithms on graphs. Path search algorithms on weighted graphs ”

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Language : python

Goal : The use of path search algorithms on weighted graphs (Dijkstra's, A* and Bellman- Ford algorithms)

Problem :

I. *Generate a random adjacency matrix for a simple undirected weighted graph of 100 vertices and 500 edges with assigned random positive integer weights (note that the matrix should be symmetric and contain only 0s and weights as elements). Use Dijkstra's and Bellman-Ford algorithms to find shortest paths between a random starting vertex and other vertices. Measure the time required to find the paths for each algorithm. Repeat the experiment 10 times for the same starting vertex and calculate the average time required for the paths search of each algorithm. Analyse the results obtained.*

II. *Generate a 10x20 cell grid with 40 obstacle cells. Choose two random non- obstacle cells and find a shortest path between them using A* algorithm. Repeat the experiment 5 times with different random pair of cells. Analyse the results obtained.*

III. *Describe the data structures and design techniques used within the algorithms.*

Theory:

Bellman Ford's algorithm and **Dijkstra's algorithm** both are single-source shortest path algorithm

Bellman-Ford algorithm is used to find the shortest path from the source vertex to every vertex in a weighted graph

Time Complexity: O(V * E), where V is the number of vertices in the graph and E is the number of edges in the graph

Auxiliary Space: O(E)

Dijkstra's algorithm is used to find the shortest path , which every time, take out the point of the minimum path from the starting point from the point where the shortest path has not been found, and use this point as a bridge to refresh the distance of the point where the shortest path is not found.

Time Complexity: O(E * logV), Where E is the number of edges and V is the number of vertices.

Auxiliary Space: O(V)

The only difference between the Dijkstra algorithm and the bellman ford algorithm is that Dijkstra's algorithm just visits the neighbour vertex in each iteration but the bellman ford algorithm visits each vertex through each edge in every iteration.

A* Search algorithm is one of the best and popular technique used in path-finding and graph traversals.

What A* Search Algorithm does is that at each step it picks the node according to a value-'f' which is a parameter equal to the sum of two other parameters – 'g' and 'h'. At each step it picks the node/ cell having the lowest 'f', and process that node/cell

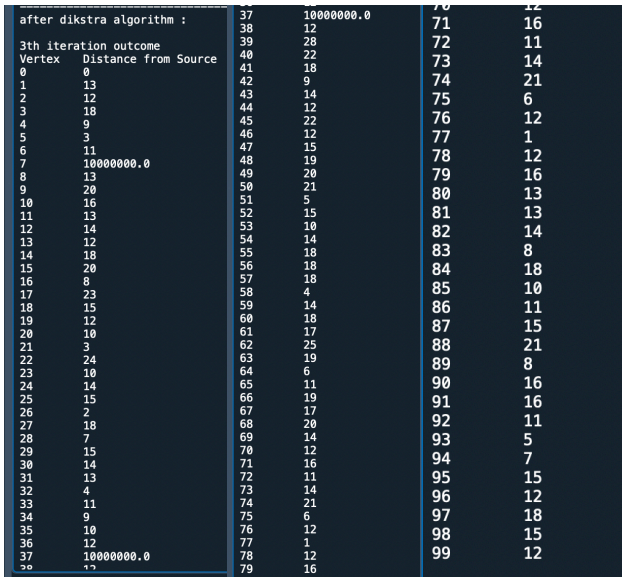
g = the movement cost to move from the starting point to a given square on the grid, following the path generated to get there.
h = the estimated movement cost to move from that given square on the grid to the final destination

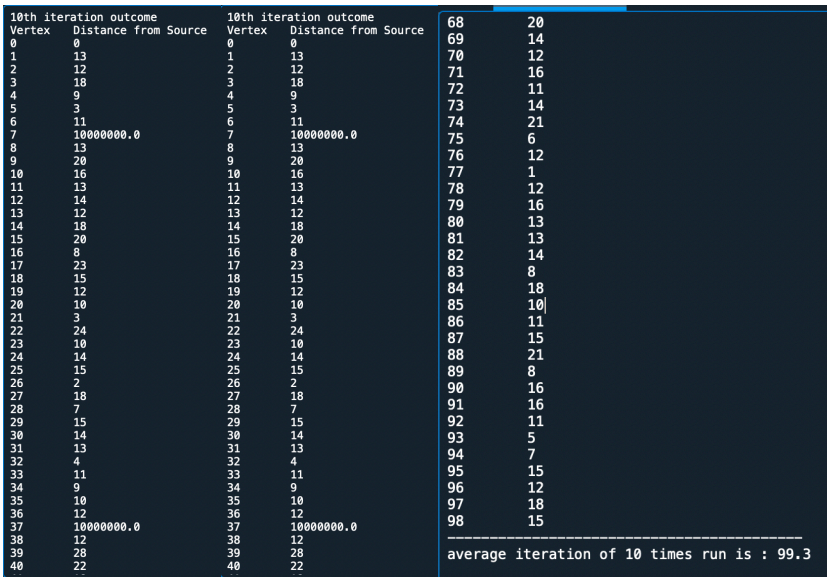
Results:

I

This is outcome after Dijkstra algorithm :

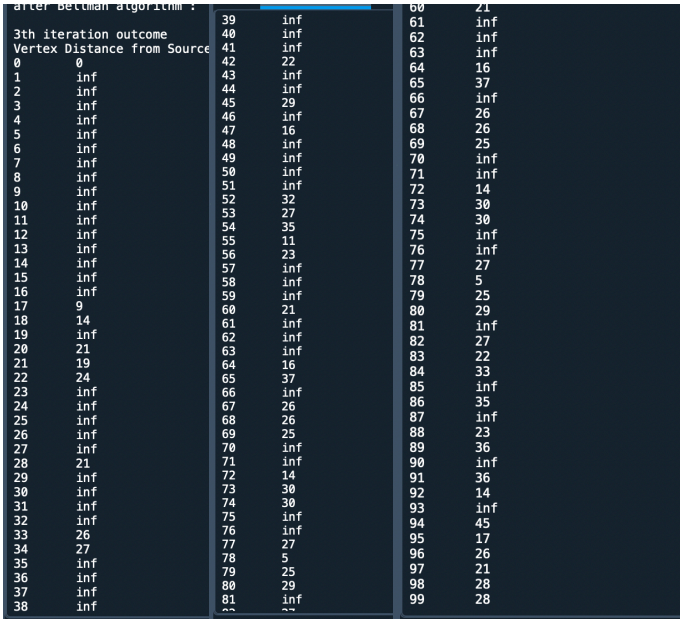
Randomly show two iteration of outcome



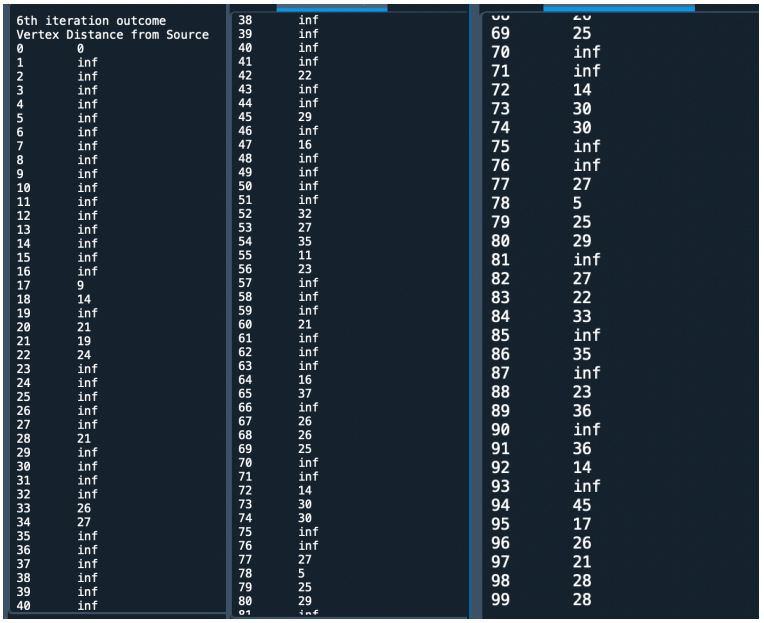


This is outcome after Bellman-ford algorithm :

Randomly show two iteration of outcome



```
3th iteration outcome
Vertex Distance from Source
0       0
1       inf
2       inf
3       inf
4       inf
5       inf
6       inf
7       inf
8       inf
9       inf
10      inf
11      inf
12      inf
13      inf
14      inf
15      inf
16      inf
17      9
18      14
19      inf
20      21
21      19
22      24
23      inf
24      inf
25      inf
26      inf
27      inf
28      21
29      inf
30      inf
31      inf
32      inf
33      26
34      27
35      inf
36      inf
37      inf
38      inf
39      inf
40      inf
41      inf
42      22
43      inf
44      inf
45      29
46      inf
47      16
48      inf
49      inf
50      inf
51      inf
52      32
53      27
54      35
55      11
56      23
57      inf
58      inf
59      inf
60      21
61      inf
62      inf
63      inf
64      16
65      37
66      inf
67      26
68      26
69      25
70      inf
71      inf
72      14
73      30
74      30
75      inf
76      inf
77      27
78      5
79      25
80      29
81      inf
82      27
83      22
84      33
85      inf
86      35
87      inf
88      23
89      36
90      inf
91      36
92      14
93      inf
94      45
95      17
96      26
97      21
98      28
99      28
```



```
6th iteration outcome
Vertex Distance from Source
0       0
1       inf
2       inf
3       inf
4       inf
5       inf
6       inf
7       inf
8       inf
9       inf
10      inf
11      inf
12      inf
13      inf
14      inf
15      inf
16      inf
17      9
18      14
19      inf
20      21
21      19
22      24
23      inf
24      inf
25      inf
26      inf
27      inf
28      21
29      inf
30      inf
31      inf
32      inf
33      26
34      27
35      inf
36      inf
37      inf
38      inf
39      inf
40      inf
41      inf
42      22
43      inf
44      inf
45      29
46      inf
47      16
48      inf
49      inf
50      inf
51      inf
52      32
53      27
54      35
55      11
56      23
57      inf
58      inf
59      inf
60      21
61      inf
62      inf
63      inf
64      16
65      37
66      inf
67      26
68      26
69      25
70      inf
71      inf
72      14
73      30
74      30
75      inf
76      inf
77      27
78      5
79      25
80      29
81      inf
82      27
83      22
84      33
85      inf
86      35
87      inf
88      23
89      36
90      inf
91      36
92      14
93      inf
94      45
95      17
96      26
97      21
98      28
99      28
```

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average iteration of 10 times run is : 19800.0
```

II. These are 3 out of 10 outcomes of a* algorithm to find shortest path

```
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2th iteration
[[1 1 1 0 0 0 0 1 0 0 0 1 0 1 0 1 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 1 1 0 0 0 1 0 1 0 0]
 [0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0]
 [1 0 1 0 0 0 1 0 0 1 1 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 0 0]
 [0 0 0 0 0 0 1 0 0 1 0 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0]
 [0 0 0 0 0 0 0 0 1 0 1 0 0 1 0 0 1 0 0 0]
 [0 1 0 0 0 0 1 0 1 0 0 0 0 1 0 0 0 0 0 1]
 [0 1 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]]

start from : [1, 16]
to : [8, 18]

best route is [(1, 16), (2, 17), (3, 18), (4, 18), (5, 18), (6, 18), (7, 18), (8, 18)]

[[1 1 1 0 0 0 0 1 0 0 0 1 0 1 0 1 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 1 1 0 0 0 1 7 1 0 0]
 [0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 7 0 0]
 [1 0 1 0 0 0 1 0 0 1 1 0 1 0 0 0 0 0 7 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 7 0]
 [0 0 0 0 0 0 1 0 0 1 0 0 1 0 0 0 0 0 7 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 7 0]
 [0 0 0 0 0 0 0 0 1 0 1 0 0 1 0 0 1 0 7 0]
 [0 1 0 0 0 0 1 0 1 0 0 0 0 1 0 0 0 0 7 1]
 [0 1 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0]]

----------------------------------------
```

```
10th iteration
[[1 1 1 0 0 0 0 1 0 0 0 1 0 1 0 1 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 1 1 0 0 0 1 0 1 0 0]
 [0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0]
 [1 0 1 0 0 0 1 0 0 1 1 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 0 0]
 [0 0 0 0 0 0 1 0 0 1 0 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0]
 [0 0 0 0 0 0 0 0 1 0 1 0 0 1 0 0 1 0 0 0]
 [0 1 0 0 0 0 1 0 1 0 0 0 1 0 0 0 0 0 0 1]
 [0 1 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]]

start from : [8, 9]
to : [7, 2]

best route is [(8, 9), (9, 8), (8, 7), (7, 6), (7, 5), (7, 4), (7, 3), (7, 2)]

[[1 1 1 0 0 0 0 1 0 0 0 1 0 1 0 1 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 1 1 0 0 0 1 0 1 0 0]
 [0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0]
 [1 0 1 0 0 0 1 0 0 1 1 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 0 0]
 [0 0 0 0 0 0 1 0 0 1 0 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0]
 [0 0 7 7 7 7 7 0 1 0 1 0 0 1 0 0 1 0 0 0]
 [0 1 0 0 0 0 1 7 1 7 0 0 0 1 0 0 0 0 0 1]
 [0 1 0 1 0 0 0 0 7 0 0 0 0 0 1 0 0 0 0 0]]
```

```
--------------------------------------
3th iteration
[[1 1 1 0 0 0 0 1 0 0 0 1 0 1 0 1 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 1 1 0 0 0 1 0 1 0 0]
 [0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0]
 [1 0 1 0 0 0 1 0 0 1 1 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 0 0]
 [0 0 0 0 0 0 1 0 0 1 0 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0]
 [0 0 0 0 0 0 0 0 1 0 1 0 0 1 0 0 1 0 0 0]
 [0 1 0 0 0 0 1 0 1 0 0 0 1 0 0 0 0 0 0 1]
 [0 1 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]]

start from : [7, 16]
to : [6, 13]

best route is [(7, 16), (7, 15), (6, 14), (6, 13)]

[[1 1 1 0 0 0 0 1 0 0 0 1 0 1 0 1 0 0 0 0]
 [0 1 0 0 0 0 0 0 0 0 1 1 0 0 0 1 0 1 0 0]
 [0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0]
 [1 0 1 0 0 0 1 0 0 1 1 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 0 0]
 [0 0 0 0 0 0 1 0 0 1 0 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 1 7 7 1 0 0 0 0]
 [0 0 0 0 0 0 0 0 1 0 1 0 0 1 0 7 7 0 0 0]
 [0 1 0 0 0 0 1 0 1 0 0 0 1 0 0 0 0 0 0 1]
 [0 1 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]]
```

III.

I used numpy array and list as mainly structure

Firstly I created random matrix using random implementation in python

And converted them into adjacency list to use it with **Dijkstra algorithm** and **Bellman-ford algorithm**

Dijkstra algorithm :

Step-01: Create a list of “distances” equal to the number of nodes and initialize each value to infinity

Step-02: Set the “distance” to the starting node equal to 0

Step-03: Create a list of “visited” nodes set to false for each node (since we haven’t visited any yet)

Step-04: Loop through all the nodes

1. Loop through all the nodes again, and pick the one that is the shortest distance away *and* not yet visited
2. Set that node to visited
3. Set the distance in the distance list to the distance to that node

Step-05: The original "distance" list should now contain the shortest distance to each node or infinity if a node is unreachable from the desired starting node

Bellman-ford algorithm :

Step-01: Find the number of iterations to be performed.

If Total nodes are 6 and the number of iterations will be 1 less than the number of nodes which is 6 – 1 = 5.

Step-02: Initialization Initialize the value of the source node with 0, and the rest of the nodes with infinity as shown below:

Step-03: What to do in each iteration?
For each iteration, visit every edge of the graph and update values accordingly.

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a* algorithm :

Initial condition - we create two lists - Open List and Closed List.

the following steps are

- The open list must be initialized.
- Put the starting node on the open list (leave its f at zero). Initialize the closed list.
- Follow the steps until the open list is non-empty:

1. Find the node with the least f on the open list and name it "q".
2. Remove Q from the open list.
3. Produce q's eight descendants and set q as their parent.
4. For every descendant:

i) If finding a successor is the goal, cease looking

ii) Else, calculate g and h for the successor.

successor.g = q.g + the calculated distance between the successor and the q.

successor.h = the calculated distance between the successor and the goal. We will cover three heuristics to do this: the Diagonal, the Euclidean, and the Manhattan heuristics.

successor.f = successor.g plus successor.h

iii) Skip this successor if a node in the OPEN list with the same location as it but a lower f value than the successor is present.

iv) Skip the successor if there is a node in the CLOSED list with the same position as the successor but a lower f value; otherwise, add the node to the open list end (for loop).

- Push Q into the closed list and end the while loop.

We will now discuss how to calculate the Heuristics for the nodes.

Conclusion:

From iterations of Bell-man and Dijkstra , we can that Dijkstra is much more efficient , and from outcome , we can see Dijkstra is more accurate , and most of distance can have a sharp reduction . But in Bell-man graph , we can see there are still bunch of numbers remaining the same

After 10 iterations , we can see that a* algorithm always can successfully got a shortest path from any random nodes . It is highly and common and efficient way to use in machine learning .

Appendix

https://raw.githubusercontent.com/MaChengYuan/task6/main/task6_dijkstra.py

https://raw.githubusercontent.com/MaChengYuan/task6/main/task6_bellman.py

https://raw.githubusercontent.com/MaChengYuan/task6/main/task6_a*algorithm.py