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# Exercise 3: Graphical Models and Exact Inference

due **before** 2019-06-05

**Important information regarding the exercises:**

- The exercise is not mandatory and there will be no corrections. Nevertheless, we encourage you to work on the exercises and present your solutions in the exercise class.

## Question 1: Bayesian Networks . . . . . $(\Sigma = 0)$

The elections are coming up and you are trying to reason about them using Bayes networks. You know that whenever an election is about to occur, parties make promises to spend money on education and health care. Different parties have different agendas, so they promise to spend differently on these items. Promised spending can be low, moderate or high. When an election is not imminent, parties tend to promise moderate or high spending in any category only with probability 0.1 and 0.01 respectively. The probability of an election being imminent is 0.5. There are three parties in this election:

- Party A favors health care, and so their probability of promising lots of spending there is 0.8, and their probability of promising moderate spending is 0.15. In education, they are equally likely to promise spending in all three categories.
- Party B favors education, so their probability of promising lots of spending there is 0.7, and they have a 0.15 chance of promising a moderate amount. They have equal probability of promising in all three categories for health care.
- Party C likes to make lots of promises, so they have a 0.8 chance of promising high spending and 0.2 chance of promising low spending in both education and health care.

The parties have roughly equal following. Denote by $P$ the random variable defining the party, $SE$ the random variable describing the promised spending in education, $SH$ the random variable describing promised spending in health care and $E$ the random variable denoting whether an election is imminent. (Note that other encodings of the facts in random variables are possible, we are choosing this one for uniformity reasons).

(a) Draw the Bayesian network corresponding to this problem and specify all tables for prior and conditional probabilities.

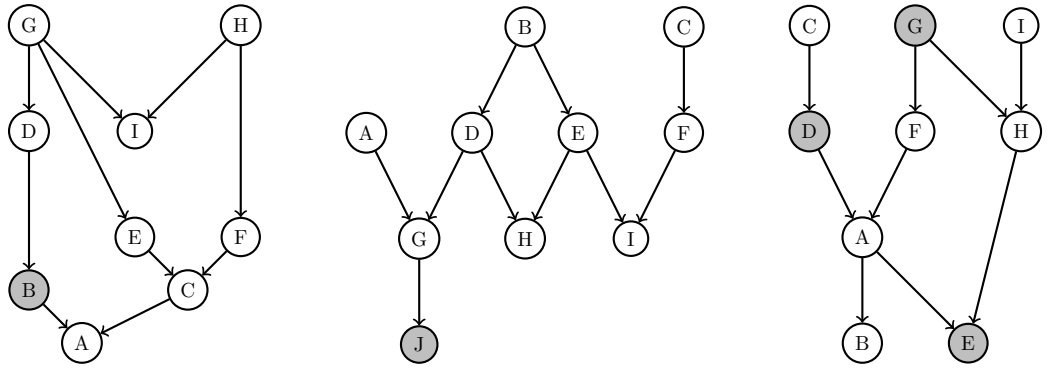
(b) Elections are coming up in two weeks. You pass by your local library and observe a candidate promising a high amount of spending on education and a moderate amount of spending on health care. What is the most likely party of this candidate? Show all your calculations.

(c) How does your answer change if you know that Party C is not fielding any candidates in your constituency?

## Question 2: Bayes Ball, D-Separation . . . . . $(\Sigma = 0)$

Here we compute some global independence statements from some directed graphical models using the Bayes ball algorithm:

(a) Consider the DAG in Figure 1a. List all variables $X$ such that, $X \perp\!\!\!\perp A|B$, i.e., variables that are independent of $A$ given evidence on $B$.




(a) $B$ is observed (b) $J$ is observed (c) $D, E, G$ is observed

Figure 1: Bayes Networks with observed variables.

(b) Consider the DAG in Figure 1b. List all variables $X$ such that, $X \perp\!\!\!\perp A|J$, i.e., variables that are independent of $A$ given evidence on $J$.

(c) Consider the DAG in Figure 1c. List all variables $X$ such that, $X \perp\!\!\!\perp A|D, G, E$, i.e., variables that are independent of $A$ given evidence on $D, G, E$.

## Question 3: Bayes Net Toolbox ..... $(\mathbf{\Sigma = 0})$

In this exercise you should implement Bayes networks in Matlab using the Bayes Net Toolbox provided by Kevin Murphy[1]. Installation instructions can be found in the Readme.

As next step read the section *Creating your first Bayes net*. You should read the instructions and test out the code in Matlab. You can skip the sections on *Random Parameters* and *Loading a network from a file*, and stop after the section on *Computing joint distributions*.

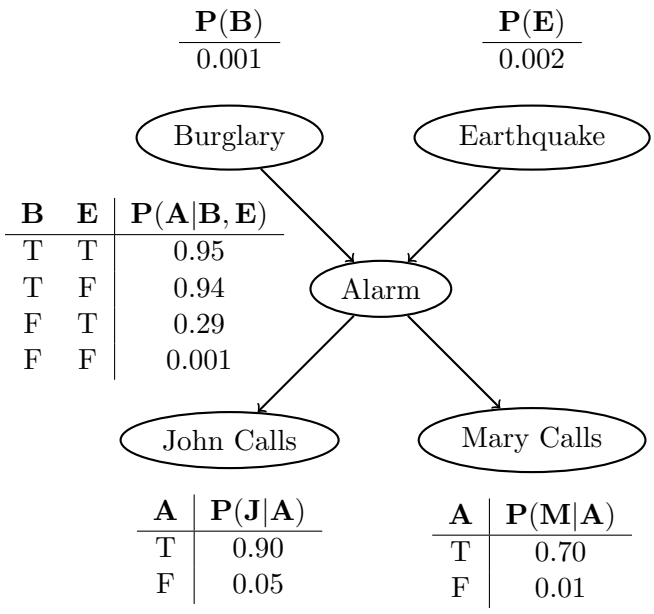



Figure 2: Burglar-Alarm Bayes Net

(a) As next step you should build a Bayes net for the burglar alarm example (Artificial Intelligence: A Modern Approach, S. Russel and P. Novig) shown in Figure 2 For

[1] https://github.com/bayesnet/bnt

implementing your function please use the provided function templates.

Please write your own code where it is specified in the code, and test your solution! Provide the visual representation of your Bayes net using the information that you can find in the *Graph visualization* section.

(b) Compute the answers to Question 1 (b) and (c) using the toolbox and print out the visual representation of the Bayes net.

## Question 4: Factor Graphs, Sum-Product Algorithm ..... $(\mathbf{\Sigma = 0})$

Consider the following Bayes Net (Figure 3) with random variables $\{W, S, D, T, J\}$:

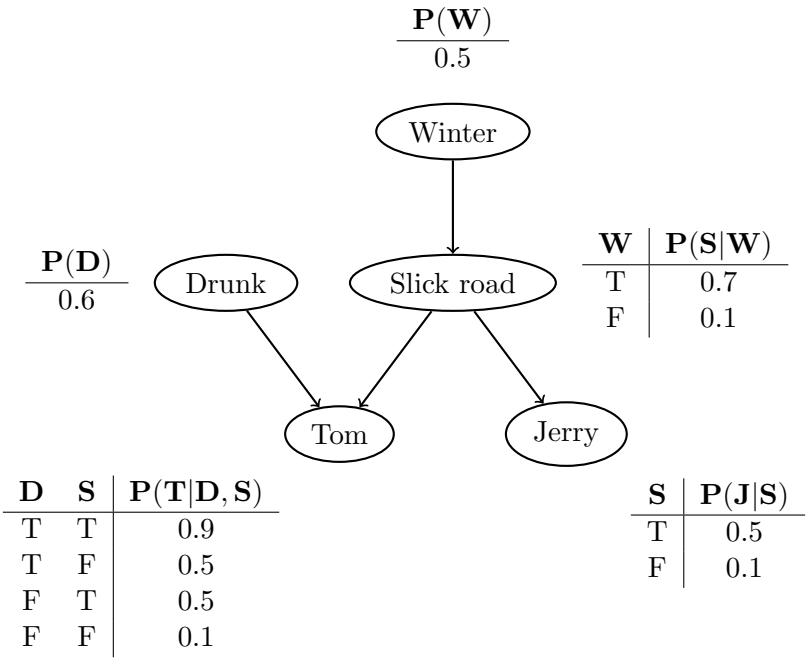



Figure 3: Bayes Net for Accident Scenario

As you know from the lecture, an evidence can have an impact on states of all random variables. E.g. the evidence (Jerry has an accident) increases the probability for slick roads and slick roads increases the probability for winter and Tom having an accident. There exist several algorithms for computing the exact inference in Bayesian belief networks.

(a) Assume we observe the following evidence: Jerry has an accident. Convert the graphical model into a factor graph and draw this factor graph. Perform the inference using the Sum-Product algorithm to compute all marginals symbolically.

(b) Now, use the factor graph that you have created in order to perform the Sum-Product algorithm using the Bayes Net Toolbox from Question 3. To implement this you can follow the example code given in the directory `bnt/BNT/examples/static/fgraph` (part of the BNT toolbox). What is the marginal distribution of the road to be slick?