1 Basics

Multiset extensions

$$S_1 \succ_{mul} S_2 \iff \exists X, Y \in M(\forall y \in Y \exists x \in X : x \succ y)$$
$$S_1 \succ_{mul}^{HO} S_2 \iff S_1 \neq S_2 \land \forall m \in M : (S_2(m) > S_1(m) \implies \exists m' \in M : m' \succ m \land S_1(m') > S_2(m'))$$
$$S_1 \succ_{mul}^{1} S_2 \iff \exists x \in S_1, Y : S_2 = (S_1 - \{x\}) \cup Y \land \forall y \in Y : x \succ y$$

X, Y are multisets over base set M.
$\succ_{mul}^{*}$ is the transitive closure of $\succ_{mul}^{1}$ given by.

2 Propositional Logic

Polarities

$pol(F, \epsilon)$	$:=$	1
$pol(\neg F, 1p)$	$:=$	$-pol(F, p)$
$pol(F_1 \circ F_2)$	$:=$	$pol(F_i, p)$
$pol(F_1 \implies F_2)$	$:=$	$-pol(F_1, p)$
$pol(F_1 \implies F_2)$	$:=$	$pol(F_2, p)$
$pol(F_1 \iff F_2)$	$:=$	0

CDCL

Unit prop.	$M\|N \cup \{C \lor L\}$	$\Longrightarrow_{\text{CDCL}}$	$ML\|N \cup \{C \lor L\}$
Decide	$M\|N$	$\Longrightarrow_{\text{CDCL}}$	$ML^d\|N$
Fail	$M\|N \cup \{C\}$	$\Longrightarrow_{\text{CDCL}}$	fail
Backjump	$M'L^dM''\|N$ on backjump clause $C \lor L'$	$\Longrightarrow_{\text{CDCL}}$	$M'L'\|N$
Learn	$M\|N$	$\Longrightarrow_{\text{CDCL}}$	$M\|N \cup \{C\}$
Forget	$M\|N \cup \{C\}$	$\Longrightarrow_{\text{CDCL}}$	$M\|N$
Restart	$M\|N$	$\Longrightarrow_{\text{CDCL}}$	$\epsilon\|N$

CNF transformation

1. eliminate trivial subformulas
2. replace beneficial subformulas
   - $pol(H, p) = 1 : H[F]_p \Longrightarrow_{\text{OCNF}} H[Q]_p \land Q \implies F$
   - $pol(H, p) = -1 : H[F]_p \Longrightarrow_{\text{OCNF}} H[Q]_p \land F \implies Q$
   - $pol(H, p) = 0 : H[F]_p \Longrightarrow_{\text{OCNF}} H[Q]_p \land F \iff Q$
3. eliminate equivalences
   - $pol(H, p) \in \{0, 1\} : H[F \iff G]_p \Longrightarrow_{\text{OCNF}} H[(F \implies G) \land (G \implies F)]_p$
   - $pol(H, p) = -1 : H[F \iff G]_p \Longrightarrow_{\text{OCNF}} H[(F \land G) \lor (\neg F \land \neg G)]_p$
4. $H[F \iff G]_p \Longrightarrow_{\text{OCNF}} H[(F \implies G) \land (G \implies F)]_p$
5. $H[F \implies G]_p \Longrightarrow_{\text{OCNF}} H[\neg F \lor G]_p$
6. $H[\neg(F \lor G)]_p \Longrightarrow_{\text{OCNF}} H[\neg F \land \neg G]_p$
   $H[\neg(F \land G)]_p \Longrightarrow_{\text{OCNF}} H[\neg F \lor \neg G]_p$
7. $H[\neg\neg F]_p \Longrightarrow_{\text{OCNF}} H[F]_p$
8. $H[(F \land F') \lor G]_p \Longrightarrow_{\text{OCNF}} H[(F \lor G) \land (F' \lor G)]_p$

3 First-order logic

Substitutions

$f(s_1, ..., s_n)\sigma$	$=$	$f(s_1\sigma, ..., s_n\sigma)$	similarly for predicates
$\bot\sigma$	$=$	$\bot$	similarly for $\top$
$(u \approx v)\sigma$	$=$	$(u\sigma \approx v\sigma)$	
$\neg F\sigma$	$=$	$\neg(F\sigma)$	
$(F \circ G)\sigma$	$=$	$(F\sigma \circ G\sigma)$	
$(\mathbf{Q}xF)\sigma$	$=$	$\mathbf{Q}z(F\sigma[x \mapsto z])$	z is fresh

CNF transformation

1	eliminate trivial subformulas		
2	$if\ pol(H,p)=1:\ H[F]_p$	$\implies_{\text{OCNF}}$	$H[P(x_1,...,x_n)]_p \land \forall x_1,...,x_n(P(x_1,...,x_n) \implies F)$
	$if\ pol(H,p)=-1:\ H[F]_p$	$\implies_{\text{OCNF}}$	$H[P(x_1,...,x_n)]_p \land \forall x_1,...,x_n(F \implies P(x_1,...,x_n))$
	$if\ pol(H,p)=0:\ H[F]_p$	$\implies_{\text{OCNF}}$	$H[P(x_1,...,x_n)]_p \land \forall x_1,...,x_n(F \iff P(x_1,...,x_n))$
3	$if\ pol(H,p) \in \{0,1\}:\ H[F \iff G]_p$	$\implies_{\text{NNF}}$	$H[(F \implies G) \land (G \implies F)]_p$
	$if\ pol(H,p)=-1:\ H[F \iff G]_p$	$\implies_{\text{NNF}}$	$H[(F \land G) \lor (\neg F \land \neg G)]_p$
	$H[F \implies G]_p$	$\implies_{\text{NNF}}$	$H[\neg F \lor G]_p$
	$H[\neg\neg F]_p$	$\implies_{\text{NNF}}$	$H[F]_p$
	$H[\neg(F \lor G)]_p$	$\implies_{\text{NNF}}$	$H[\neg F \land \neg G]_p$
	$H[\neg(F \land G)]_p$	$\implies_{\text{NNF}}$	$H[\neg F \lor \neg G]_p$
	$H[\neg \mathbf{Q} x F]_p$	$\implies_{\text{NNF}}$	$H[\bar{\mathbf{Q}} x \neg F]_p$
4	Miniscoping: x free in F, F'; x not free in G		
	$H[\mathbf{Q}x(F \land G)]_p$	$\implies_{\text{MS}}$	$H[(\mathbf{Q}xF) \land G]_p$
	$H[\mathbf{Q}x(F \lor G)]_p$	$\implies_{\text{MS}}$	$H[(\mathbf{Q}xF) \lor G]_p$
	$H[\forall x(F \land F')]_p$	$\implies_{\text{MS}}$	$H[(\forall xF) \land (\forall xF')]_p$
	$H[\exists x(F \lor F')]_p$	$\implies_{\text{MS}}$	$H[(\exists xF) \lor (\exists xF')]_p$
	$H[\mathbf{Q}xG]_p$	$\implies_{\text{MS}}$	$H[G]_p$
5	Rename variables s.t. same variable isn't bound by two quantifiers		
6	$H[\exists xF]_p$	$\implies_{\text{SK}}$	$H[F\{x \mapsto f(y_1,...,y_n)\}]_p$
	for minimal length p, free variables y_i		
7	$H[(\forall xF) \circ G]_p$	$\implies_{\text{OCNF}}$	$H[\forall x(F \circ G)]_p$
	$H[(F \land F') \lor G]_p$	$\implies_{\text{OCNF}}$	$H[(F \lor G) \land (F' \lor G)]_p$