1 Multiset extensions

Dershowitz-Manna

$$S_1 \succ_{mul} S_2 \iff \exists X, Y \in M(\forall y \in Y \exists x \in X : x \succ y)$$

Huet-Oppen

$$S_1 \succ_{mul}^{HO} S_2 \iff S_1 \neq S_2 \wedge \forall m \in M : (S_2(m) > S_1(m) \implies \exists m' \in M : m' \succ m \wedge S_1(m') > S_2(m'))$$

Transitive Closure $\succ_{mul}^{*}$ is the transitive closure of $\succ_{mul}^{1}$ given by

$$S_1 \succ_{mul}^{1} S_2 \iff \exists x \in S_1, Y : S_2 = (S_1 - \{x\}) \cup Y \wedge \forall y \in Y : x \succ y$$

Y is a multiset over M

2 Propositional Logic

Polarities

$$\begin{array}{lcl}
pol(F, \epsilon) & := & 1 \\
pol(\neg F, 1p) & := & -pol(F, p) \\
pol(F_1 \circ F_2) & := & pol F_i, p \\
pol(F_1 \implies F_2) & := & -pol(F_1, p) \\
pol(F_1 \implies F_2) & := & pol(F_2, p) \\
pol(F_1 \iff F_2) & := & 0
\end{array}$$

CNF transformation

1. eliminate trivial subformulas
2. replace beneficial subformulas
   - $pol(H, p) = 1 : H[F]_p \implies_{\text{OCNF}} H[Q]_p \wedge Q \implies F$
   - $pol(H, p) = -1 : H[F]_p \implies_{\text{OCNF}} H[Q]_p \wedge F \implies Q$
   - $pol(H, p) = 0 : H[F]_p \implies_{\text{OCNF}} H[Q]_p \wedge F \iff Q$
3. eliminate equivalences
   - $pol(H, p) \in \{0, 1\} : H[F \iff G]_p \implies_{\text{OCNF}} H[(F \implies G) \wedge (G \implies F)]_p$
   - $pol(H, p) = -1 : H[F \iff G]_p \implies_{\text{OCNF}} H[(F \wedge G) \vee (\neg F \wedge \neg G)]_p$
4. $H[F \iff G]_p \implies_{\text{OCNF}} H[(F \implies G) \wedge (G \implies F)]_p$
5. $H[F \implies G]_p \implies_{\text{OCNF}} H[\neg F \vee G]_p$
6. $H[\neg(F \vee G)]_p \implies_{\text{OCNF}} H[\neg F \wedge \neg G]_p$
   $H[\neg(F \wedge G)]_p \implies_{\text{OCNF}} H[\neg F \vee \neg G]_p$
7. $H[\neg\neg F]_p \implies_{\text{OCNF}} H[F]_p$
8. $H[(F \wedge F') \vee G]_p \implies_{\text{OCNF}} H[(F \vee G) \wedge (F' \vee G)]_p$

CDCL

Unit prop.	$M \| N \cup \{C \vee L\}$	$\implies_{\text{CDCL}}$	$ML \| N \cup \{C \vee L\}$
Decide	$M \| N$	$\implies_{\text{CDCL}}$	$ML^d \| N$
Fail	$M \| N \cup \{C\}$	$\implies_{\text{CDCL}}$	fail
Backjump	$M' L^d M'' \| N$	$\implies_{\text{CDCL}}$	$M' L' \| N$
	on backjump clause $C \vee L'$		
Learn	$M \| N$	$\implies_{\text{CDCL}}$	$M \| N \cup \{C\}$
Forget	$M \| N \cup \{C\}$	$\implies_{\text{CDCL}}$	$M \| N$
Restart	$M \| N$	$\implies_{\text{CDCL}}$	$\epsilon \| N$

Substitutions

$f(s_1, ..., s_n)\sigma$	$=$	$f(s_1\sigma, ..., s_n\sigma)$	similarly for predicates
$\bot\sigma$	$=$	$\bot$	similarly for $\top$
$(u \approx v)\sigma$	$=$	$(u\sigma \approx v\sigma)$	
$\neg F\sigma$	$=$	$\neg(F\sigma)$	
$(F \circ G)\sigma$	$=$	$(F\sigma \circ G\sigma)$	
$(\mathbf{Q}xF)\sigma$	$=$	$\mathbf{Q}z(F\sigma[x \mapsto z])$	z is fresh

CNF transformation

1	eliminate trivial subformulas		
2	$if\ pol(H,p) = 1:\ H[F]_p$	$\Longrightarrow_{\text{OCNF}}$	$H[P(x_1, ..., x_n)]_p \land \forall x_1, ..., x_n(P(x$
	$if\ pol(H,p) = -1:\ H[F]_p$	$\Longrightarrow_{\text{OCNF}}$	$H[P(x_1, ..., x_n)]_p \land \forall x_1, ..., x_n(F \ =$
	$if\ pol(H,p) = 0:\ H[F]_p$	$\Longrightarrow_{\text{OCNF}}$	$H[P(x_1, ..., x_n)]_p \land \forall x_1, ..., x_n(F \ \Leftarrow$
3	$if\ pol(H,p) \in \{0,1\}:\ H[F \iff G]_p$	$\Longrightarrow_{\text{NNF}}$	$H[(F \implies G) \land (G \implies F)]_p$
	$if\ pol(H,p) = -1:\ H[F \iff G]_p$	$\Longrightarrow_{\text{NNF}}$	$H[(F \land G) \lor (\neg F \land \neg G)]_p$
	$H[F \implies G]_p$	$\Longrightarrow_{\text{NNF}}$	$H[\neg F \lor G]_p$
	$H[\neg\neg F]_p$	$\Longrightarrow_{\text{NNF}}$	$H[F]_p$
	$H[\neg(F \lor G)]_p$	$\Longrightarrow_{\text{NNF}}$	$H[\neg F \land \neg G]_p$
	$H[\neg(F \land G)]_p$	$\Longrightarrow_{\text{NNF}}$	$H[\neg F \lor \neg G]_p$
	$H[\neg\mathbf{Q}xF]_p$	$\Longrightarrow_{\text{NNF}}$	$H[\bar{\mathbf{Q}}x\neg F]_p$
4	Miniscoping: x free in F, F'; x not free in G		
	$H[\mathbf{Q}x(F \land G)]_p$	$\Longrightarrow_{\text{MS}}$	$H[(\mathbf{Q}xF) \land G]_p$
	$H[\mathbf{Q}x(F \lor G)]_p$	$\Longrightarrow_{\text{MS}}$	$H[(\mathbf{Q}xF) \lor G]_p$
	$H[\forall(F \land F')]_p$	$\Longrightarrow_{\text{MS}}$	$H[(\forall xF) \land (\forall xF')]_p$
	$H[\exists(F \lor F')]_p$	$\Longrightarrow_{\text{MS}}$	$H[(\exists xF) \lor (\exists xF')]_p$
	$H[\mathbf{Q}xG]_p$	$\Longrightarrow_{\text{MS}}$	$H[G]_p$
5	Rename variables s.t. same variable isn't bound by two quantifiers		
6	$H[\exists xF]_p$	$\Longrightarrow_{\text{SK}}$	$H[F\{x \mapsto f(y_1, ..., y_n)\}]_p$
	for minimal length p, free variables y_i		
7	$H[(\forall xF) \circ G]_p$	$\Longrightarrow_{\text{OCNF}}$	$H[\forall x(F \circ G)]_p$
	$H[(F \land F') \lor G]_p$	$\Longrightarrow_{\text{OCNF}}$	$H[(F \lor G) \land (F' \lor G)]_p$