

Examination Timetabling Problem

Course of Discrete Optimization

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Definition of the Problem

Problem

- Exams, $E = \{e_1, e_2, \dots, e_n\}$
- Students, $S = \{s_1, s_2, \dots, s_m\}$
- Time slots, $T_j = \{t_1 \leq t_2 \leq \dots \leq t_T\}$ (ordered)
- N_{e_1, e_2} = number of common student between two exams

Problem

- each exam is scheduled exactly once during the examination period;
- two conflicting exams are not scheduled in the same time-slot;
- the total penalty resulting from the created timetable is minimized

Problem

A penalty is assigned for each pair of conflicting exams scheduled up to a distance of 5 time-slots.

Given two exams e_1, e_2 scheduled at distance i of time-slots, with $1 \leq i \leq 5$, the relative penalty is $2^{(5-i)} \cdot \frac{n_{e_1, e_2}}{|S|}$.

Definition of the Basic Model

Data

- exams=[], list of exams
- Time_slots, list of time slots from 1 to T (ordered)
- enrollment=dict(), dict where students are keys and the list of the exams they belong is the value
- Conflicting exams={} dict where for each pair of conflicting exams we store the number of conflicts (n(e1,e2))

Variables and Constraints

$x_{t,e} \in \{0, 1\}$, binary variable declaring if exam e is scheduled in time slot t

$$x = \begin{cases} 1, & \text{if exam } e \text{ is scheduled in } t \\ 0, & \text{otherwise} \end{cases}$$

Variables and Constraints

1. Each exam is scheduled exactly once during the examination period.

$$\sum_{j=1}^T x_{t_j, e} = 1, \forall e \in E$$

2. Two conflicting exams are not scheduled in the same time-slot

$$x_{t, e_1} + x_{t, e_2} \leq 1, \forall t \in T_j, \forall (e_1, e_2) \in \text{conflicting_exams}$$

Variables and Constraints

1. $a[\textit{distance}, \textit{time_slot}, \textit{exam}_1, \textit{exam}_2] \in \{0, 1\}$, binary variable for scheduling \textit{exam}_1 before \textit{exam}_2
2. $b[\textit{distance}, \textit{time_slot}, \textit{exam}_1, \textit{exam}_2] \in \{0, 1\}$, binary variable for scheduling \textit{exam}_2 before \textit{exam}_1

1. $a[i, t, \textit{exam}_1, \textit{exam}_2] \geq x[t, \textit{exam}_1] + x[t + i, \textit{exam}_2] - 1$ exam 2 is scheduled after exam 1
2. $b[i, t, \textit{exam}_1, \textit{exam}_2] \geq x[t + i, \textit{exam}_1] + x[t, \textit{exam}_2] - 1$ exam 1 is scheduled after exam 2

Variables and Constraints

$p[i, t, exam_1, exam_2] = \text{sum}(a[i, t, e1, e2] + b[i, t, e1, e2])$ for t in $\text{range}(1, \text{len}(\text{time_slots}) - i + 1)$ exam 2 is scheduled after exam 1

Finally, the objective function:

$$\min \sum_{e_1, e_2 \in \text{conflicting_exams}} p_{e_1, e_2}$$

Results

Results

instance	penalty value
Test	3.375
instance 01	162.585
instance 02	53.359