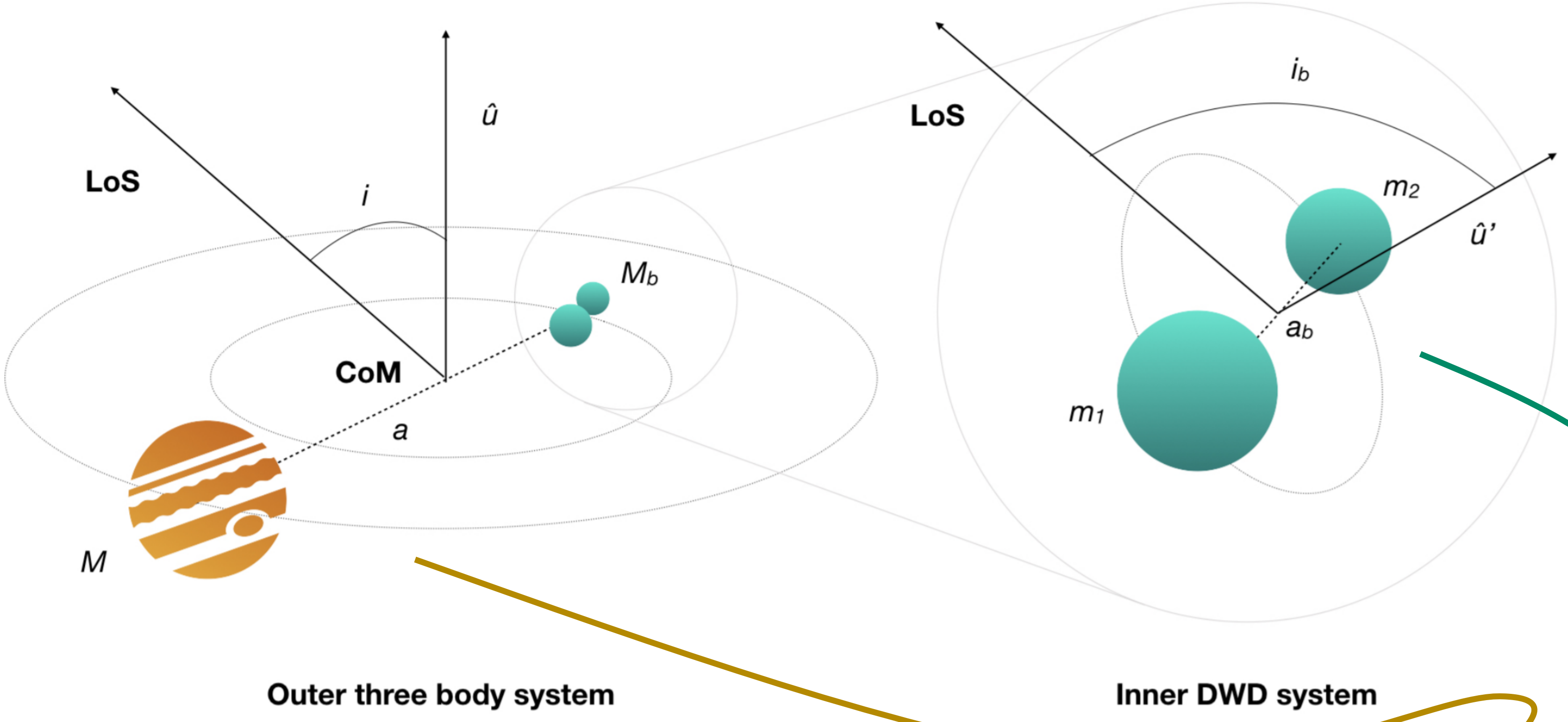
Detecting Exoplanets and recovering Parameters

What results could I arrive at over the last couple weeks?

Points of interest

1. Short Recap of the project
2. Ice Giant bugfixes
3. LISA improvements
4. Parameter space for verification binaries
5. LISA x IceGiant (x Taiji?)

1. Short Recap: Exoplanets



Induced Doppler velocity by exoplanet/brown dwarf: $f_{obs}(t) = \left(1 + \frac{v_\parallel(t)}{c}\right) f_{GW}(t)$

Where newtonian calculation gives $v_\parallel(t) = -K \cos \frac{2\pi}{P} t$ with $K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M}{(M_b+M)^{2/3}} \sin i$

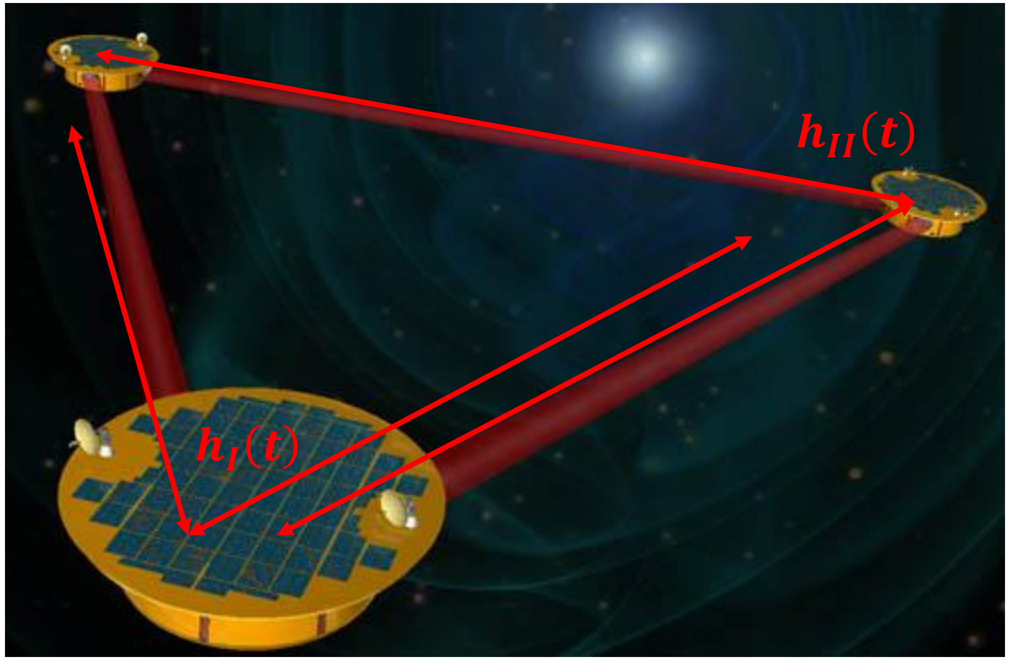
1. Short Recap: LISA

Cutler based analysis:

In LISA two independant signals I, II from three arms and their length differences:

$$h_{I,II}(t) = \frac{\sqrt{3}}{2} A_{I,II}(t) \cos\left[\Psi_{obs}(t) + \Phi_{I,II}^{(p)}(t) + \Phi_D(t)\right]$$

modulated through LISA's orbit around the sun



1. Short Recap: Fisher information approach

1. Steal the important equations for the waveform of the strain h(t) from Cutler:

$$h_{I,II}(t) = \frac{\sqrt{3}}{2} A_{I,II}(t) \cos\left[\Psi_{obs}(t) + \Phi^{(p)}_{I,II}(t) + \Phi_D(t)\right]$$

2. Look at the Doppler-signal of a circumbinary exoplanet:

$$f_{obs}(t) = \left(1 + \frac{v_{\parallel}(t)}{c}\right) f_{GW} = \left(1 - \frac{K}{c}\cos\left(\frac{2\pi}{P}t + \varphi_0\right)\right) f_{GW} \;\rightarrow\; \Psi_{obs}(t) = 2\pi \int_0^t f_{obs}(t')dt'$$

3. Derive $\frac{\partial h_{I,II}}{\partial \lambda}(t)$ for the 10 par.s of interest: $\ln(A)\,, f_0, f_1, \theta_S, \varphi_S, \theta_L, \varphi_L, K, P, \varphi_0$

4. Compute numerically the integral

$$\Gamma_{ij} = \frac{2}{S_n(f_0)} \sum_{\alpha=I,II} \int_0^{T_0} dt \, \frac{\partial h_\alpha}{\partial \lambda_i}(t) \frac{\partial h_\alpha}{\partial \lambda_j}(t) \;\Rightarrow\; \sigma_i^2 = \Sigma_{ii} = \left(\Gamma^{-1}\right)_{ii}$$

1. Short recap: IceGiant mission

$$\frac{\Delta \upsilon}{\nu_0} = \mathrm{y}(t) = \frac{\mu-1}{2}\Psi(t) - \mu\Psi\left(t - \frac{\mu+1}{2}T_2\right) + \frac{\mu+1}{2}\Psi(t - T_2)$$

[Armstrong 2006], doppler shift of carrier frequency due to GW

With $\Psi(t) = \frac{\hat{n}\cdot\boldsymbol{h}(t)\cdot\hat{n}}{1-\mu^2} = \frac{h_{\hat{n}\hat{n}}(t)}{1-\mu^2} = \frac{A}{1-\mu^2}\cos\left(\int_0^t \omega\, dt' + \phi_p\right)$

And through some calculations:

$$A = \sqrt{A_+^2 F_+^2 + A_\times^2 F_\times^2},\ \phi_p = \arctan\left(\frac{-A_\times F_\times}{A_+ F_+}\right)$$

$$\mu = \hat{k}\cdot\hat{n} = (\boldsymbol{R}\hat{z})\cdot\hat{x} = \cos\varphi\sin\theta$$

$$F_+(\hat{n},\psi = 0) = \cos^2\theta\cos^2\varphi - \sin^2\varphi,$$

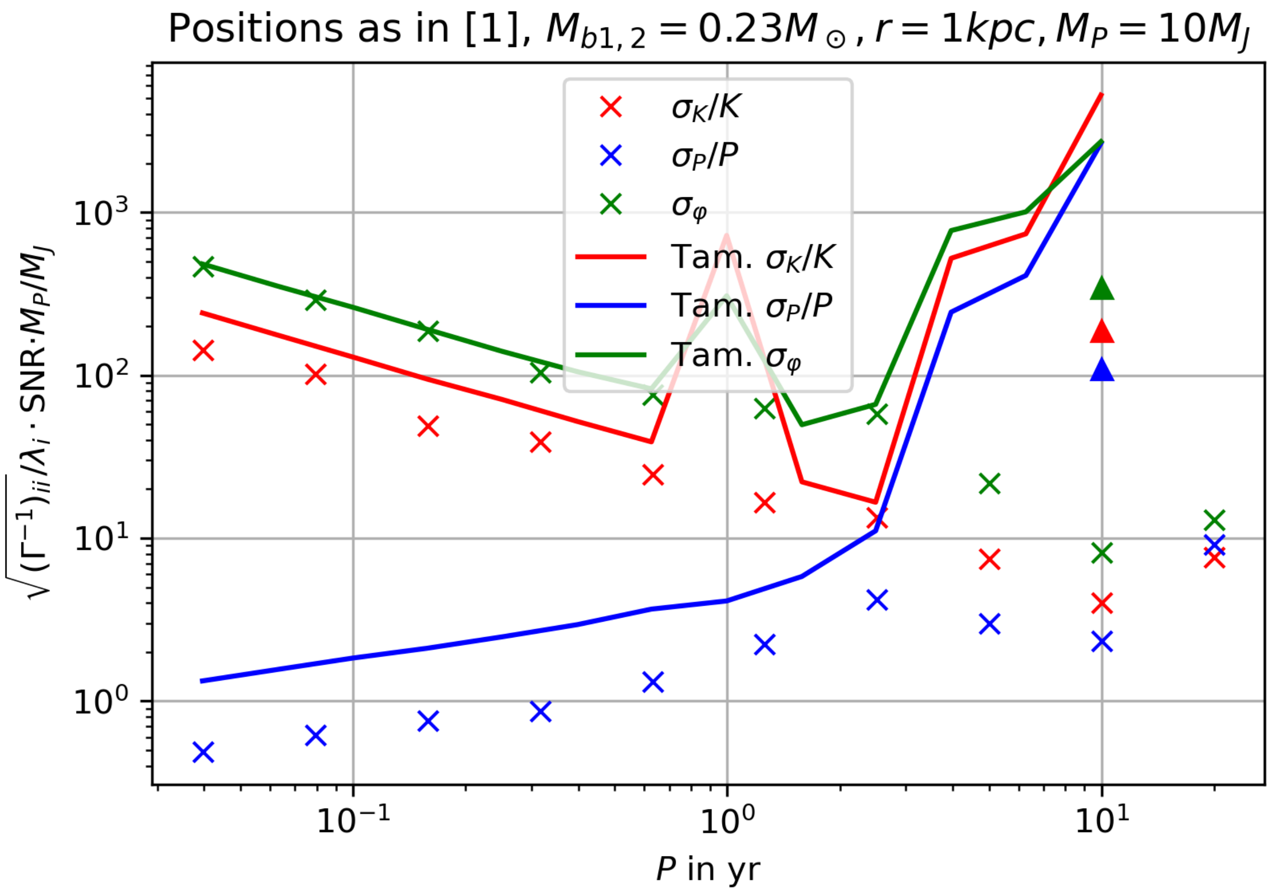
$$F_\times(\hat{n},\psi = 0) = 2\cos\theta\sin\varphi\cos\varphi$$

2. IceGiant bugfixes

Found a stupid mistake in my handling of derivatives and corrected it – now we do the proper analysis for ice giant mission

Crosses: $\mathrm{Var}(K, P, \varphi_0)$ – fixed position, orbital axis etc.

Triangles: $\mathrm{Var}(all\ parameters)$ – can't be trusted for IceGiant alone, it is insensitive to the polarization!



2. The big mistake

If the measurement is insensitive to some parameter (IceGiant or only LISA‘s $h_I(t)$ on polarization), the Fisher matrix will be **singular** -> no variance prediction possible

We use numerical integration for the calculation of the Fisher matrix and the set of the singular matrices $\mathrm{GL}_n(\mathbb{R})^C$ is a Lebesgue zero set -> we instead get a wrong variance prediction (e.g. occasionally negative variances)

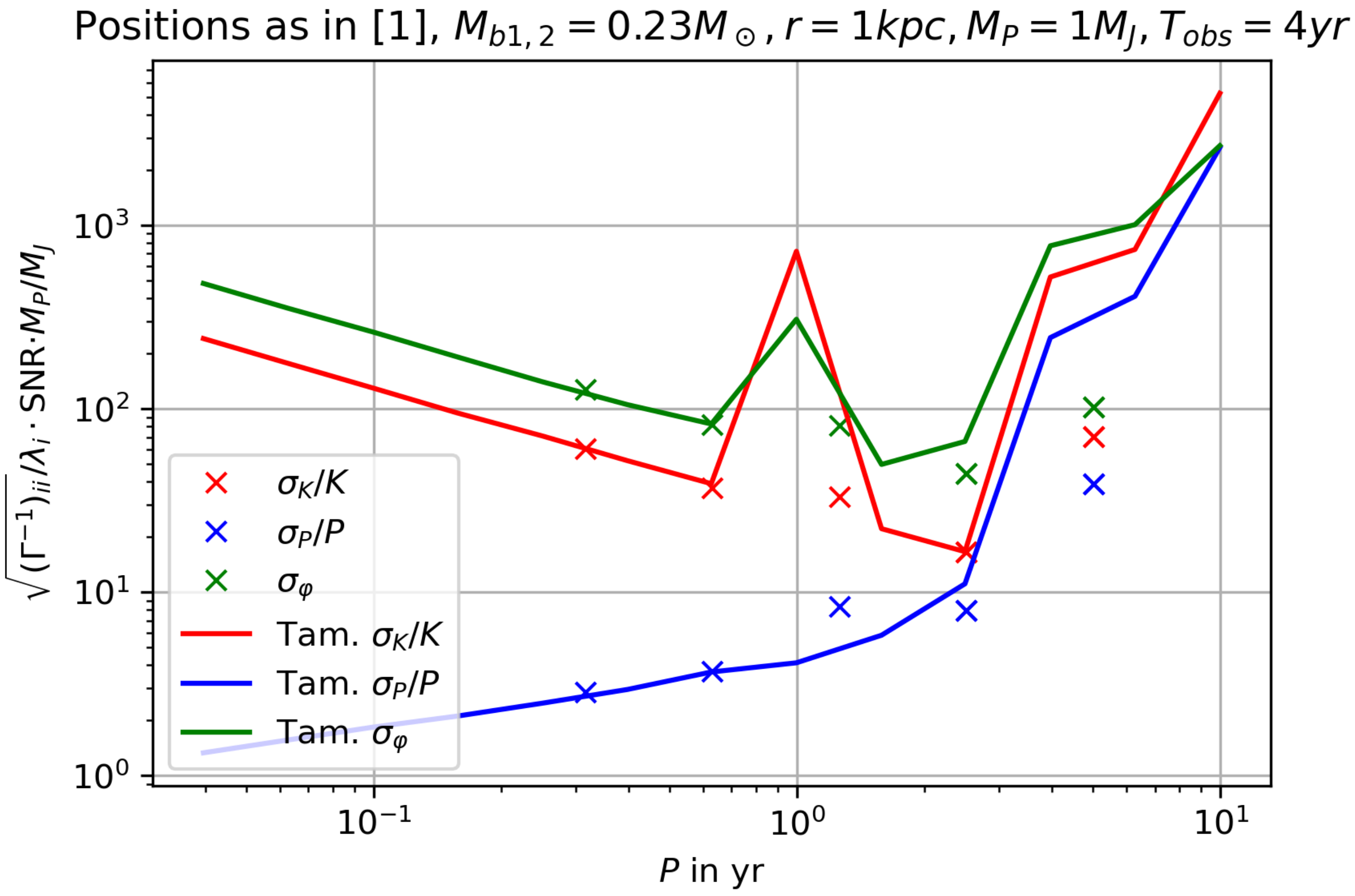
Only a combined measurement IceGiant + LISA will be sensitive to polarization, position, etc. $\rightarrow\ \Gamma_{IG} + \Gamma_{LISA,I} + \Gamma_{LISA,II} \in \mathrm{GL}_n(\mathbb{R})$

3. LISA improvements

Same error also with LISA:

I wanted to halve the computational time and only looked at $h_I(t)$ and doubled the $(S/N)^2, \Gamma_{ij}$...

Now I do the whole calculation with $h_I(t)$ & $h_{II}(t)$, limiting factor: numerical integration

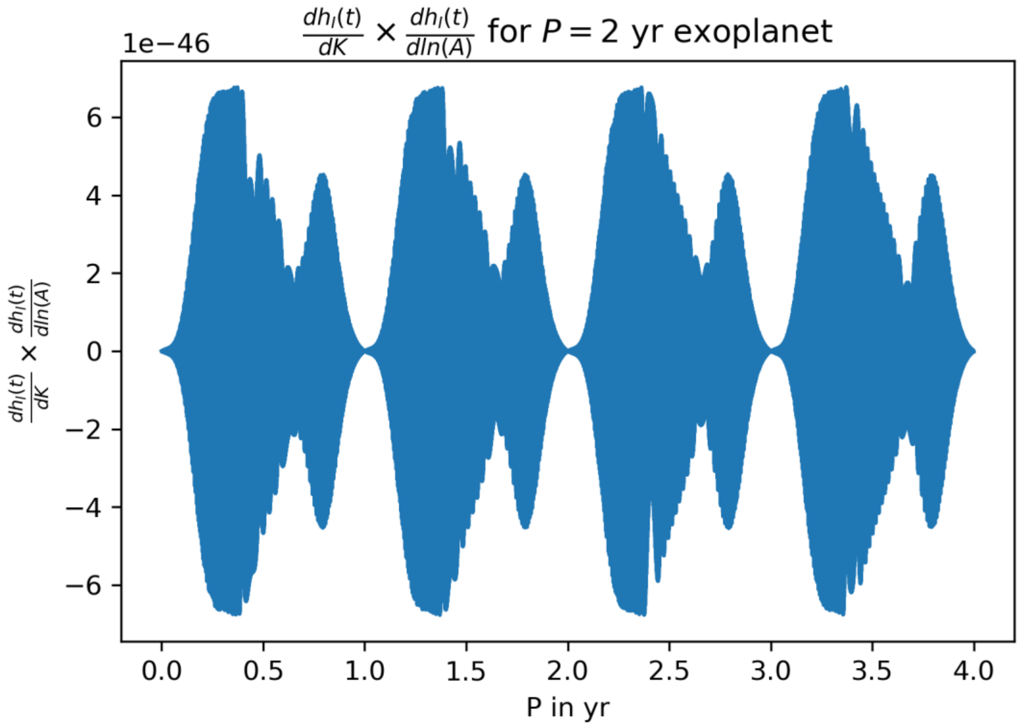


3. LISA improvements: Integration

Played around with vegas for MC integration
-> didn‘t work

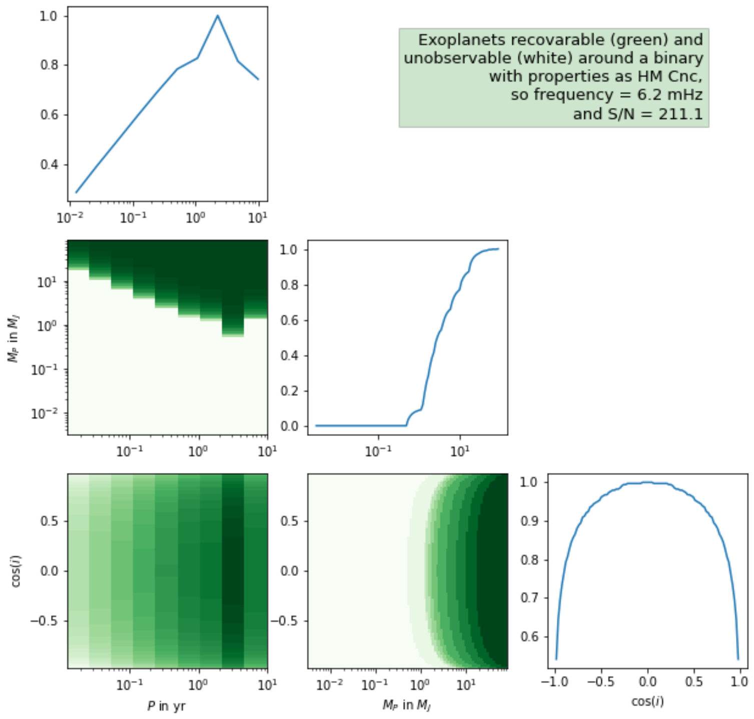
Scipy.quad still return integrals with relative errors of up to 200% for some integrals of the form $\int_0^{T_0} dt \frac{\partial h_\alpha}{\partial \lambda_i}(t) \frac{\partial h_\alpha}{\partial \lambda_j}(t)$ if I limit the algorithm s.t. a single Fisher matrix takes "only" 20 h on my machine

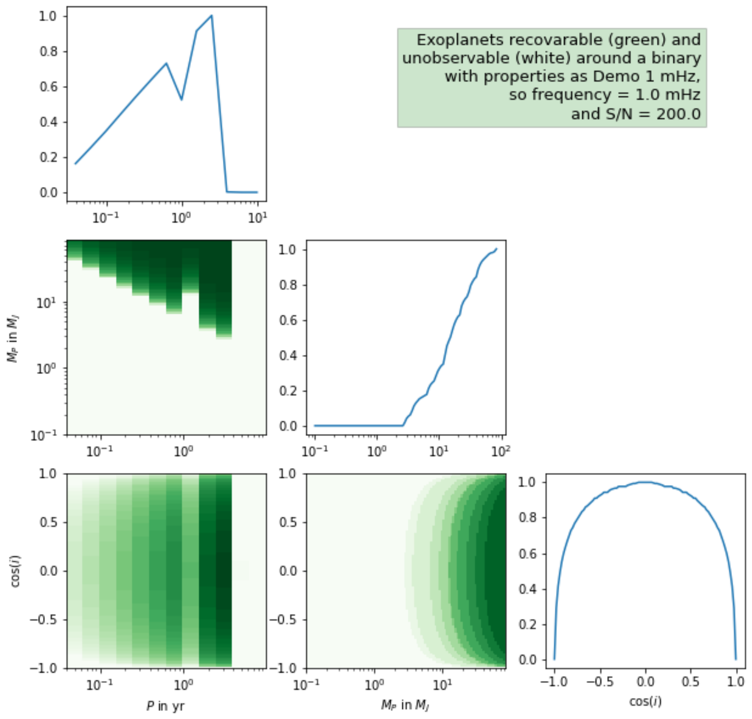
Using Euler would be great, but I struggle with Euler’s batch system and virtual python environments without any guidance…

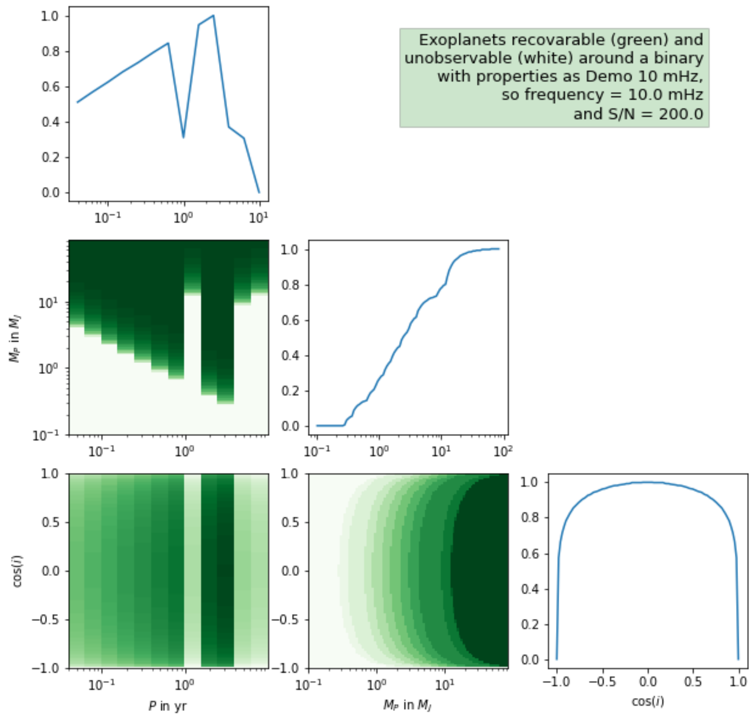


4. Parameter space exoplanets

With Lorenz and Deniz: For which exoplanet parameters are we able to observe them?
-> Impossible to answer without priors on parameter space



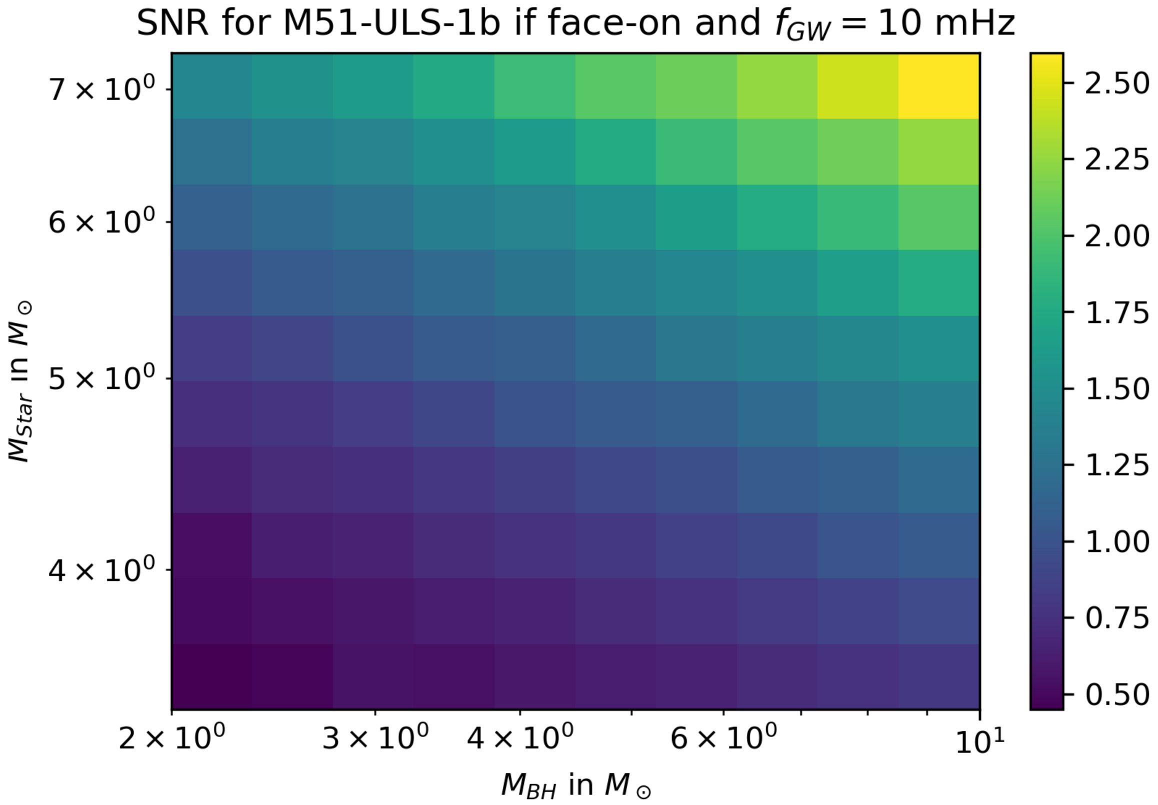




4. X-Ray binaries

We also took a small look at exoplanets around X-Ray binaries, but don‘t know how to constrain the orbital frequencies of such systems

Idea: Minimal orbit around BH s.t. the Roche lobe isn‘t traversed by the star, i.e. the star isn‘t tidally disrupted



5. Combining measurements

By defining the weight $w = \frac{S_n^{(L)}(f_0)}{S_n^{(IG)}(f_0)} < 1$ we find

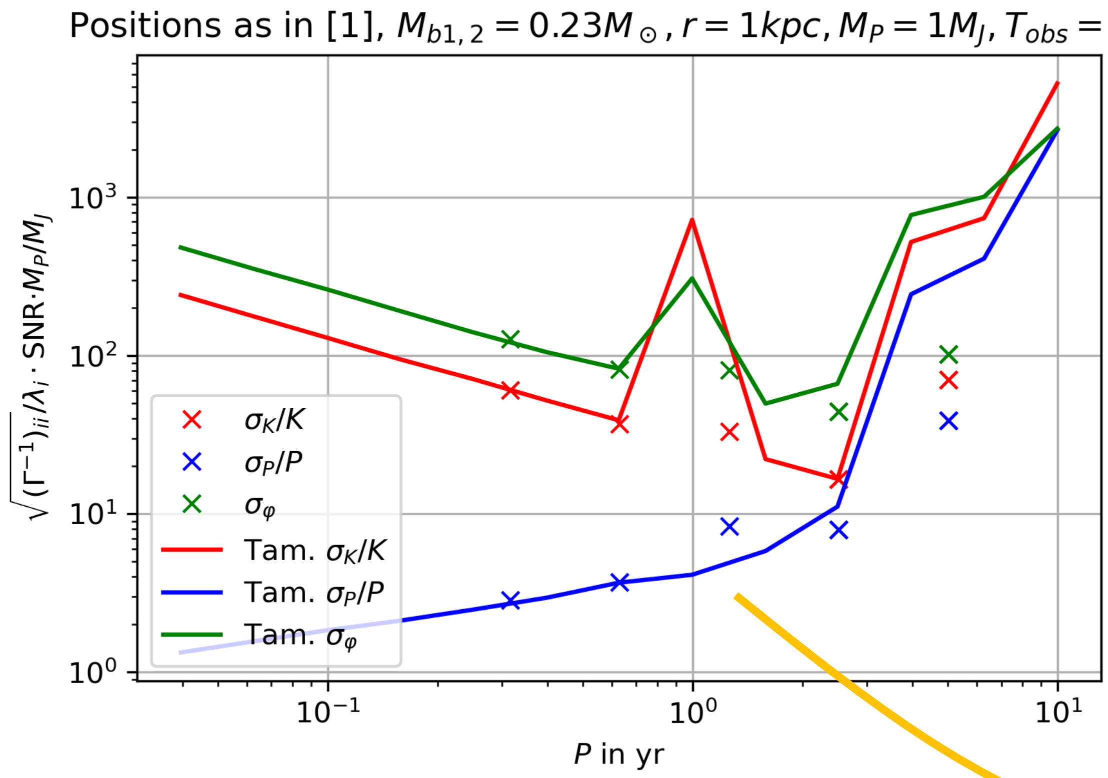
$SNR^2 = SNR_{(L)}^2 + SNR_{(IG)}^2 = SNR_{(L)}^2 \left(1 + w\frac{\int y^2 dt}{\int h^2 dt}\right)$ and in total

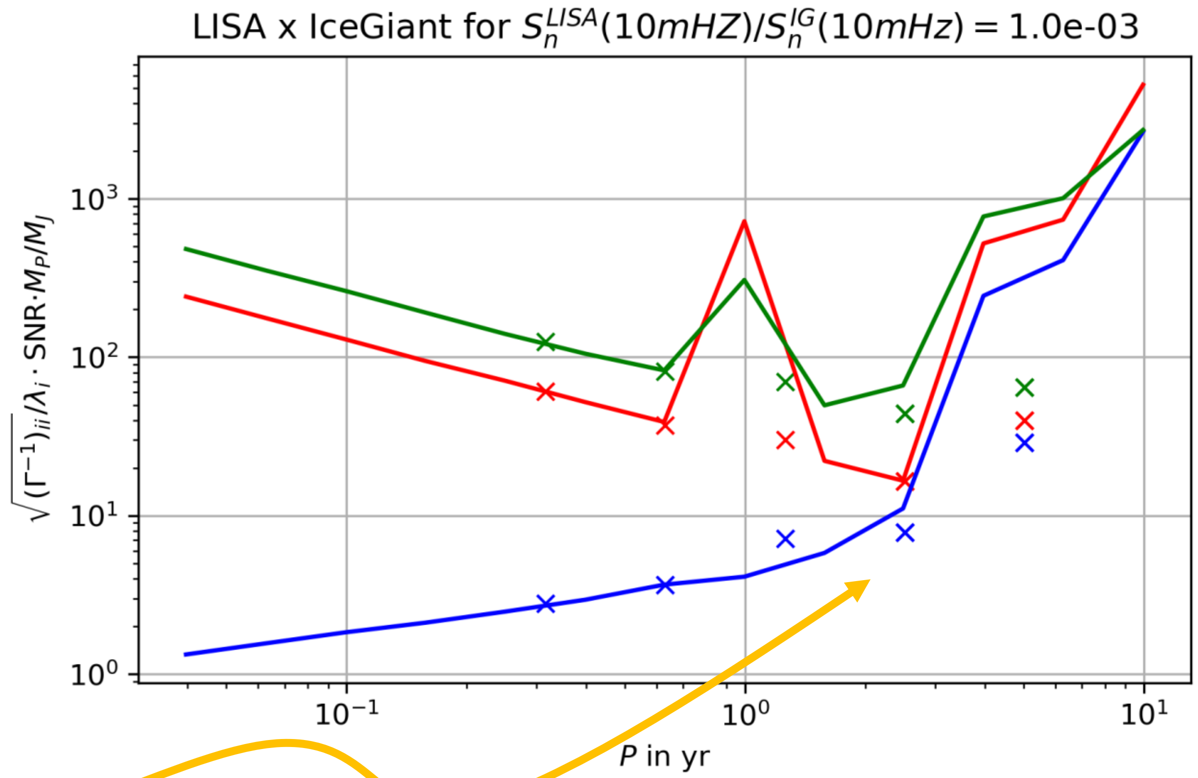
$$\frac{\Gamma_{ij}^{(tot)}}{SNR^2} = \frac{\Gamma_{ij}^{(L)}}{SNR_{(L)}^2}\left(1 + w\frac{\int y^2 dt}{\int h^2 dt}\right)^{-1} + w\frac{\int \frac{\partial y}{\partial \lambda_i}\frac{\partial y}{\partial \lambda_j}dt}{\int h^2 dt + \int y^2 dt}$$

Note that now w is a free parameter, dependant on the strain sensitivity of an icegiant mission in comparison to LISA's strain sensitivity at a given frequency, still holds -> can additionally look at improvements in all other parameters

5. Combining measurements

Very slight decrease in uncertainties -> waiting for higher accuracy integration





+ Ice Giant mission

References

[1] Tamanini, N., Danielski, C. (2019). The gravitational-wave detection of exoplanets orbiting white dwarf binaries using LISA. *Nat Astron 3*, 858–866. https://doi.org/10.1038/s41550-019-0807-y

[2] Danielski, C., Korol, V., Tamanini, N., & Rossi, E.M. (2019). Circumbinary exoplanets and brown dwarfs with the Laser Interferometer Space Antenna. *Astronomy and Astrophysics, 632.*

[3] Cutler, C. (1998). Angular resolution of the LISA gravitational wave detector. *Physical Review D, 57*, 7089-7102.

[4] Soyuer, D., Zwick, L., D'Orazio, D., Saha, P. (2021). Searching for gravitational waves via Doppler tracking by future missions to Uranus and Neptune. *MNRAS: Letters*, 503, 1, L73-79. https://doi.org/10.1093/mnrasl/slab025

[5] Maggiore, M. (2008). *Gravitational Waves Volume 1: Theory and Experiments*. Oxford University Press

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