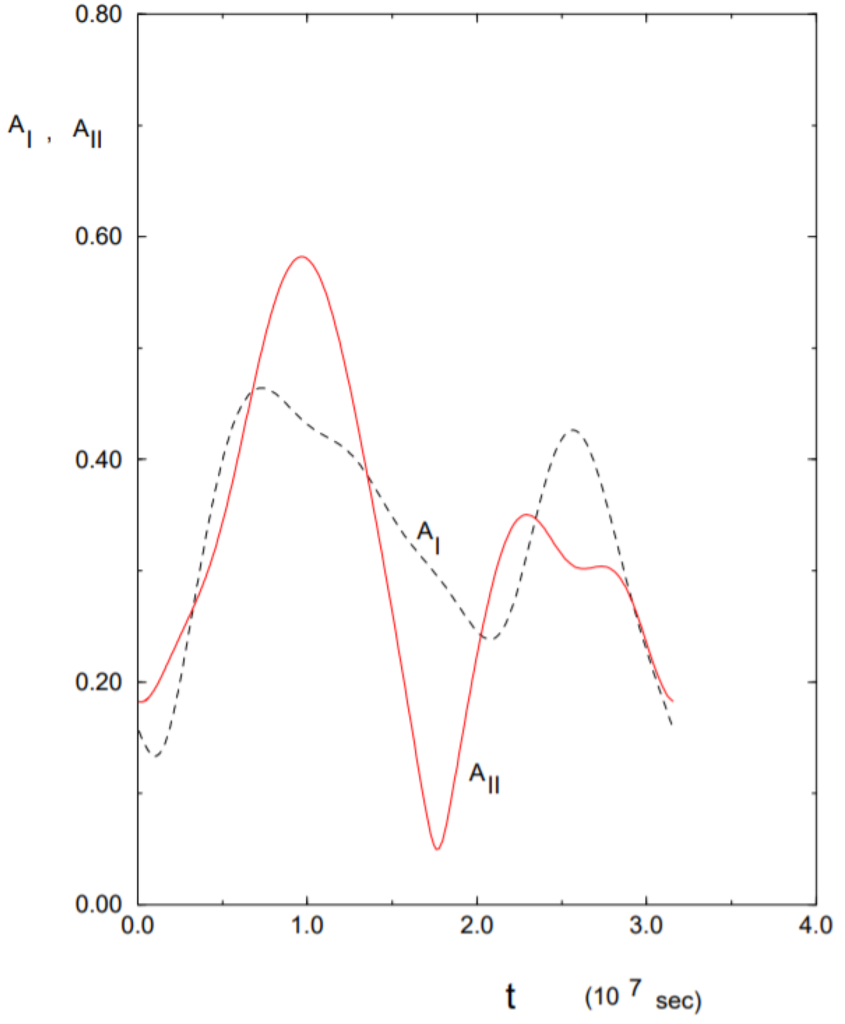
Detecting Exoplanets and recovering Parameters

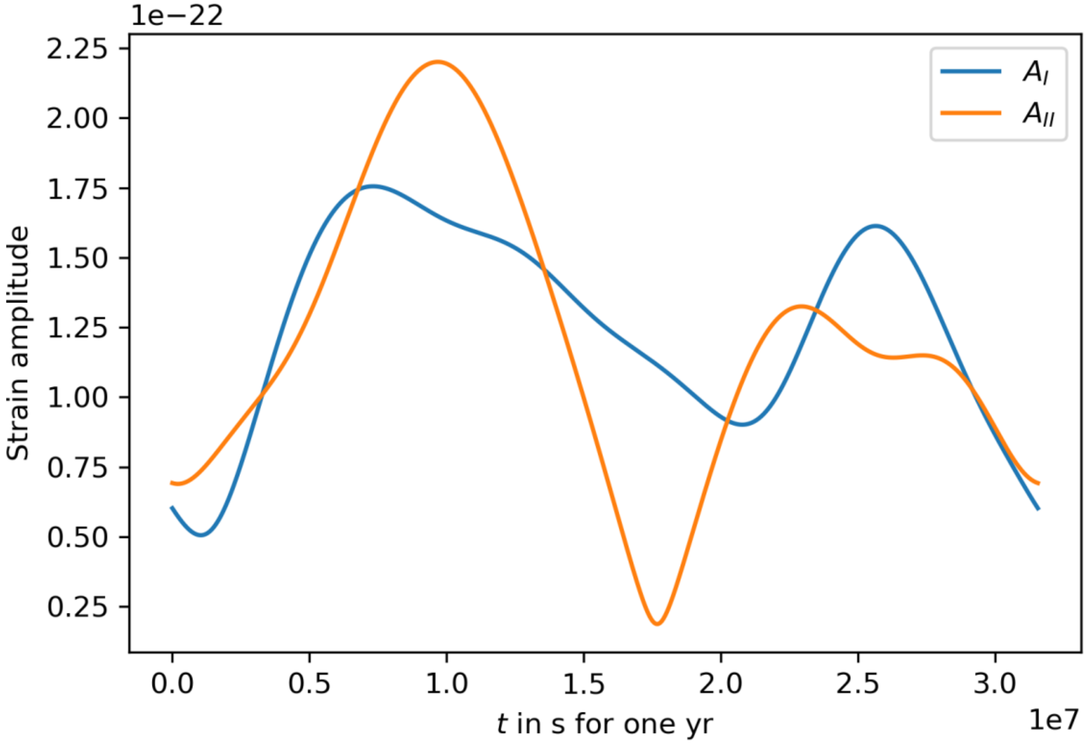
A short recap of my work in week 3

What did Tamanini and Danielski do?

1. Steal the important equations for the waveform of the strain h(t) from Cutler -> $h_{I,II}(t) = \frac{\sqrt{3}}{2} A_{I,II}(t) \cos\left[\Psi_{obs}(t) + \Phi_{I,II}^{(p)}(t) + \Phi_D(t)\right]$
2. Look at the Doppler-signal of a circumbinary exoplanet:
$$f_{obs}(t) = \left(1 + \frac{v_{\parallel}(t)}{c}\right) f_{GW} = \left(1 - \frac{K}{c}\cos\left(\frac{2\pi}{P}t + \varphi_0\right)\right) f_{GW}$$
3. Derive $\frac{\partial h_{I,II}}{\partial \lambda}(t)$ analyticallay to compute numerically the integral
$$\Gamma_{ij} = \frac{2}{S_n(f_0)} \sum_{\alpha=I,II} \int_0^{T_0} dt \frac{\partial h_\alpha}{\partial \lambda_i}(t) \frac{\partial h_\alpha}{\partial \lambda_j}(t)$$
4. The inverse of the Fisher matrix is the covariance matrix $\Sigma_{ij} = (\Gamma^{-1})_{ij}$ with $\sigma_i^2 = \Sigma_{ii}$ -> After numerial inversion we are done ☺

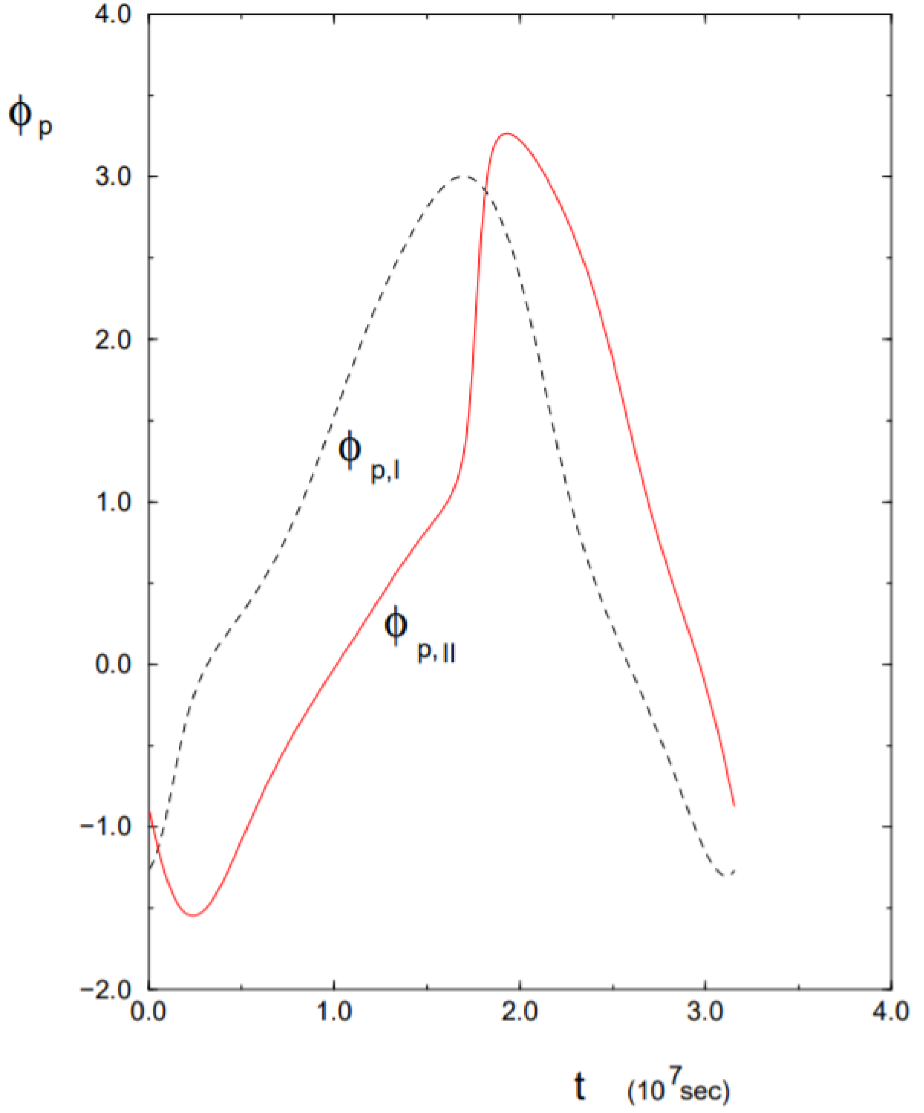
1. Steal from Cutler: Amplitude

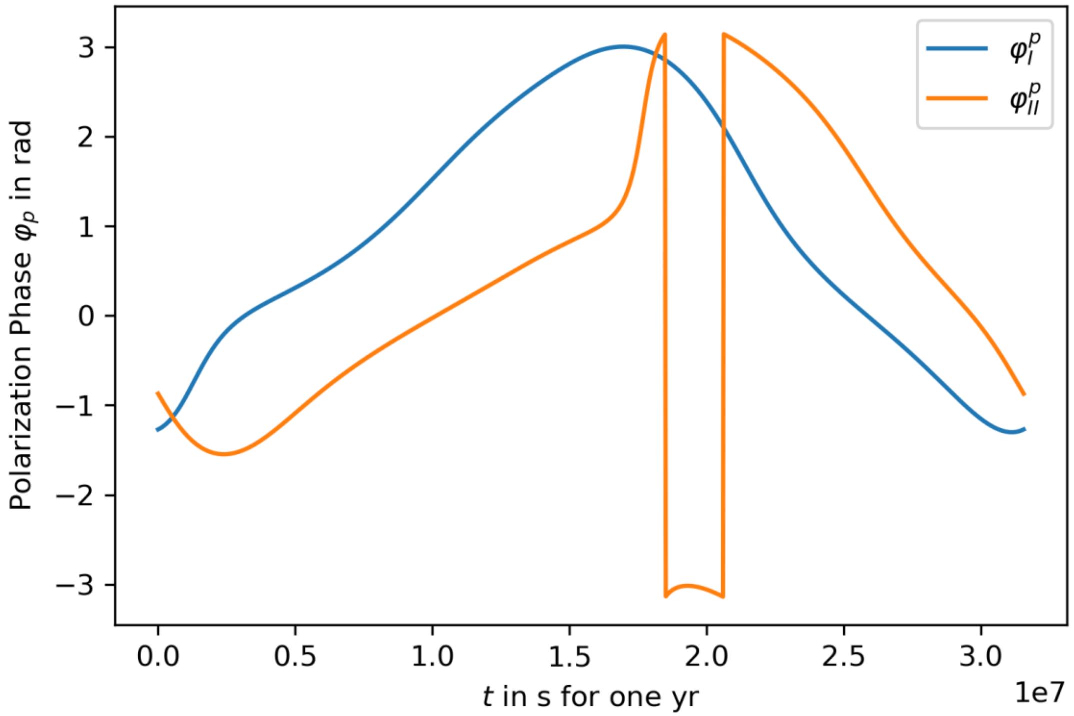




The amplitude and pattern function of course depends strongly on the eclptic coordinates of the source – thankfully Tamanini and Danielski used the same example coordinates to place their binary as Cutler

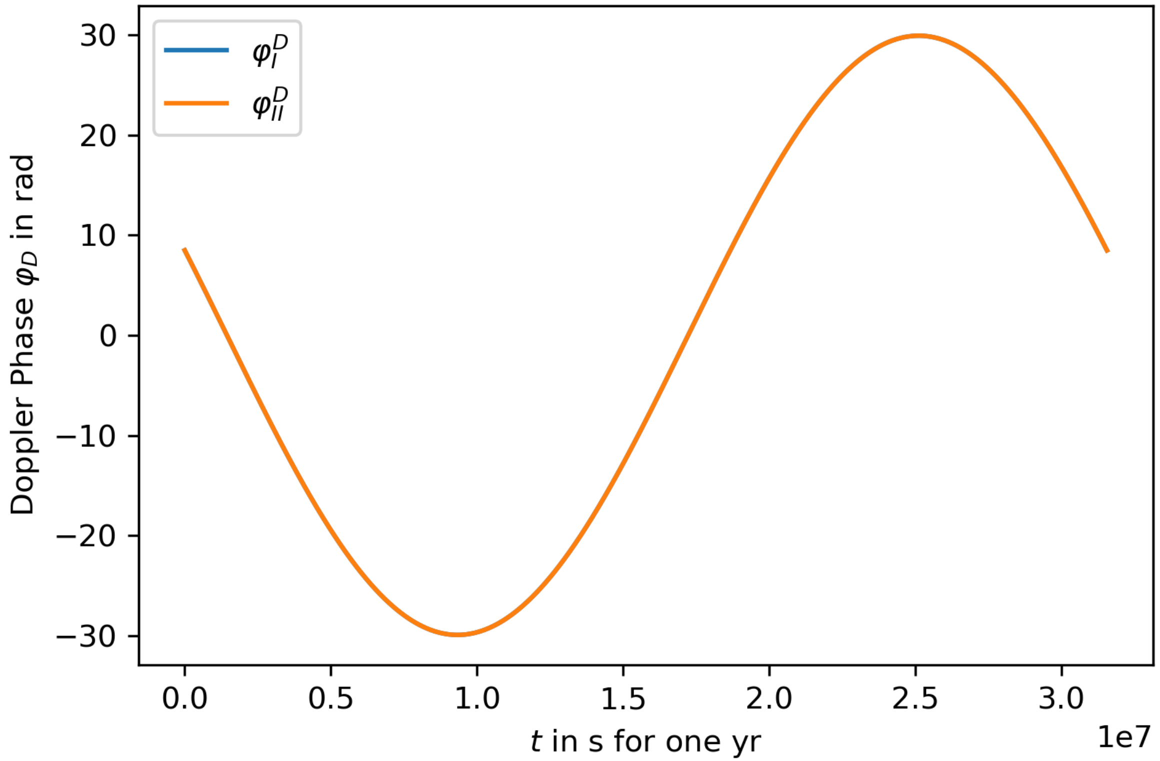
1. Steal from Cutler: Polarization Phase





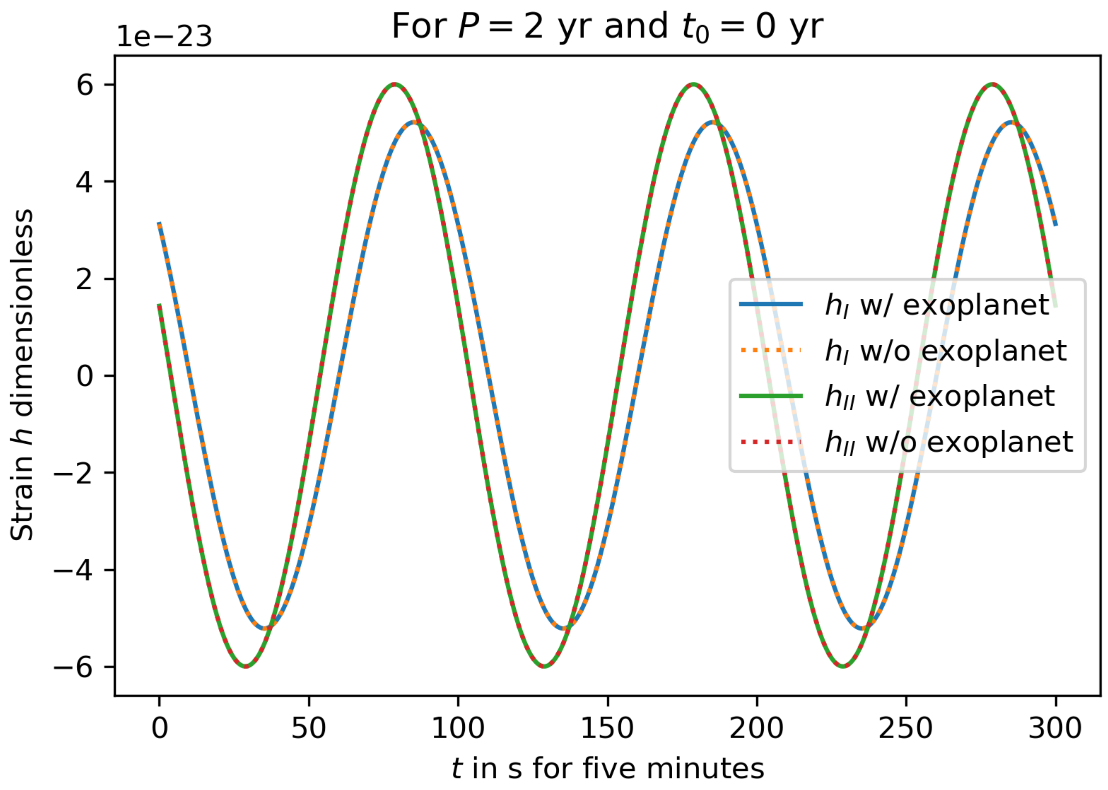
Note the incontinuity due to 2π modulus

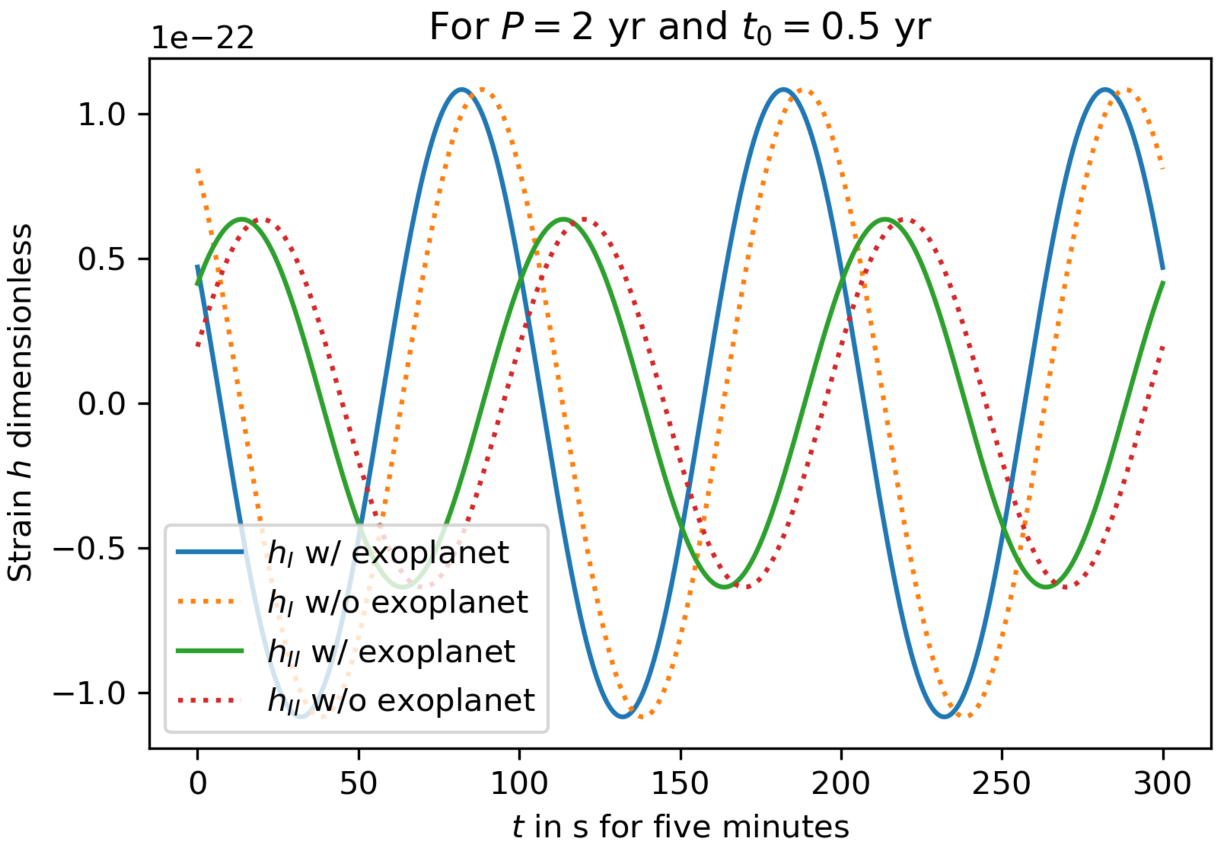
1. Steal from Cutler: Doppler Phase



The amplitude depends on the GW frequency as it should, and it should be the same for the two arms (same barycenter)

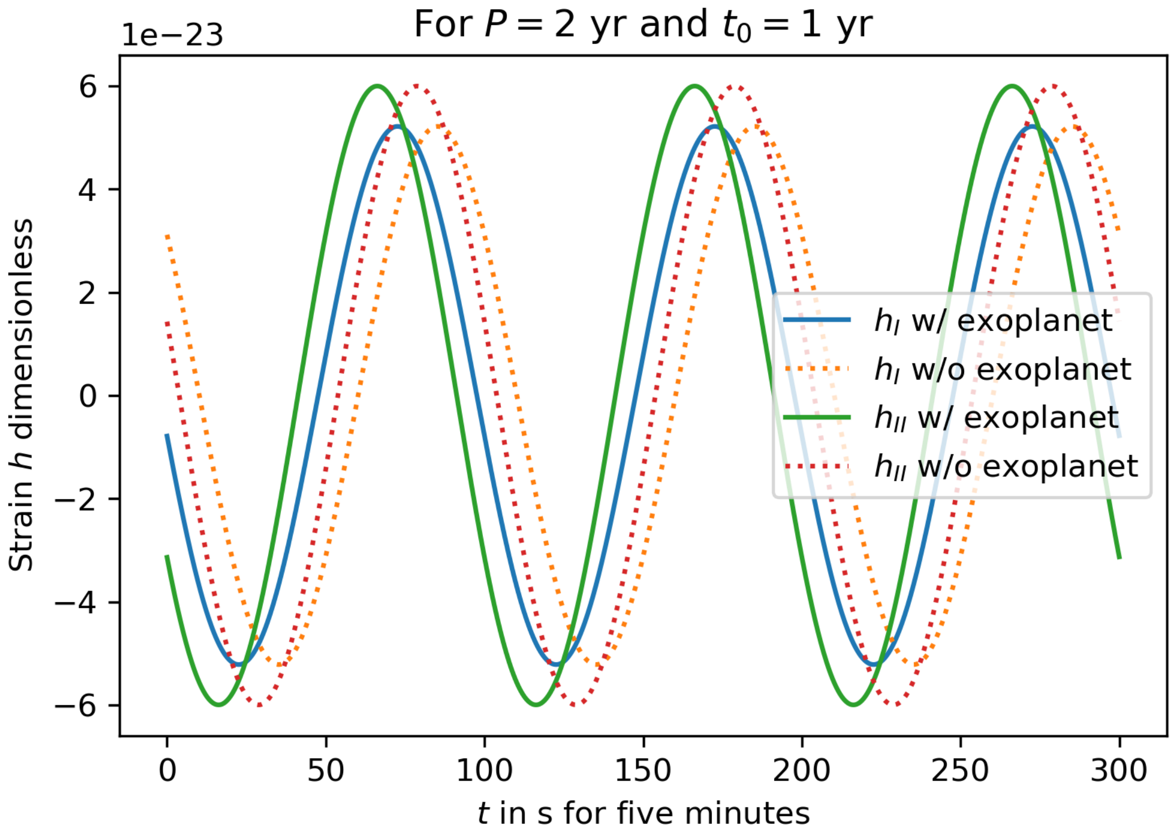
2. Look at the Doppler signal of CBPs

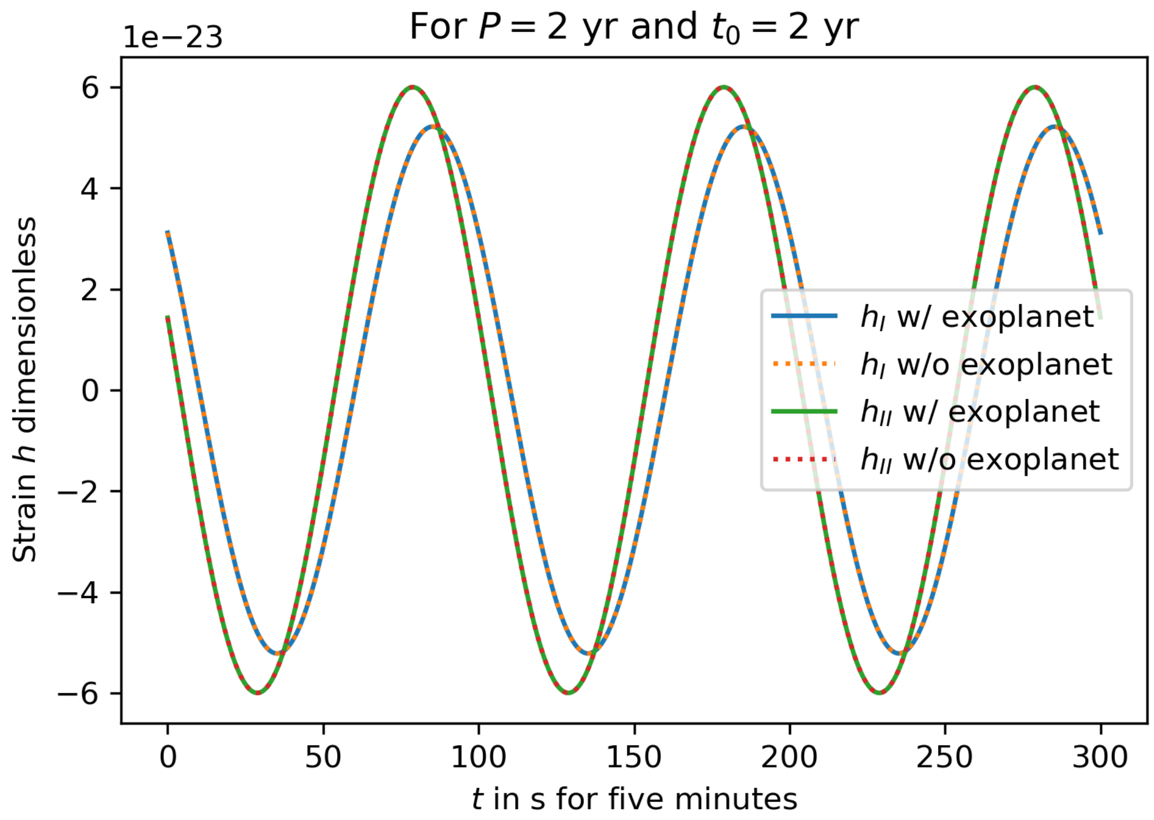




One can see the desyncing due to the presence of an exoplanet quite clearly, here a shift in the waveform due to an explanet of 5 M_J over half a year… (different start time of the detector t_0)

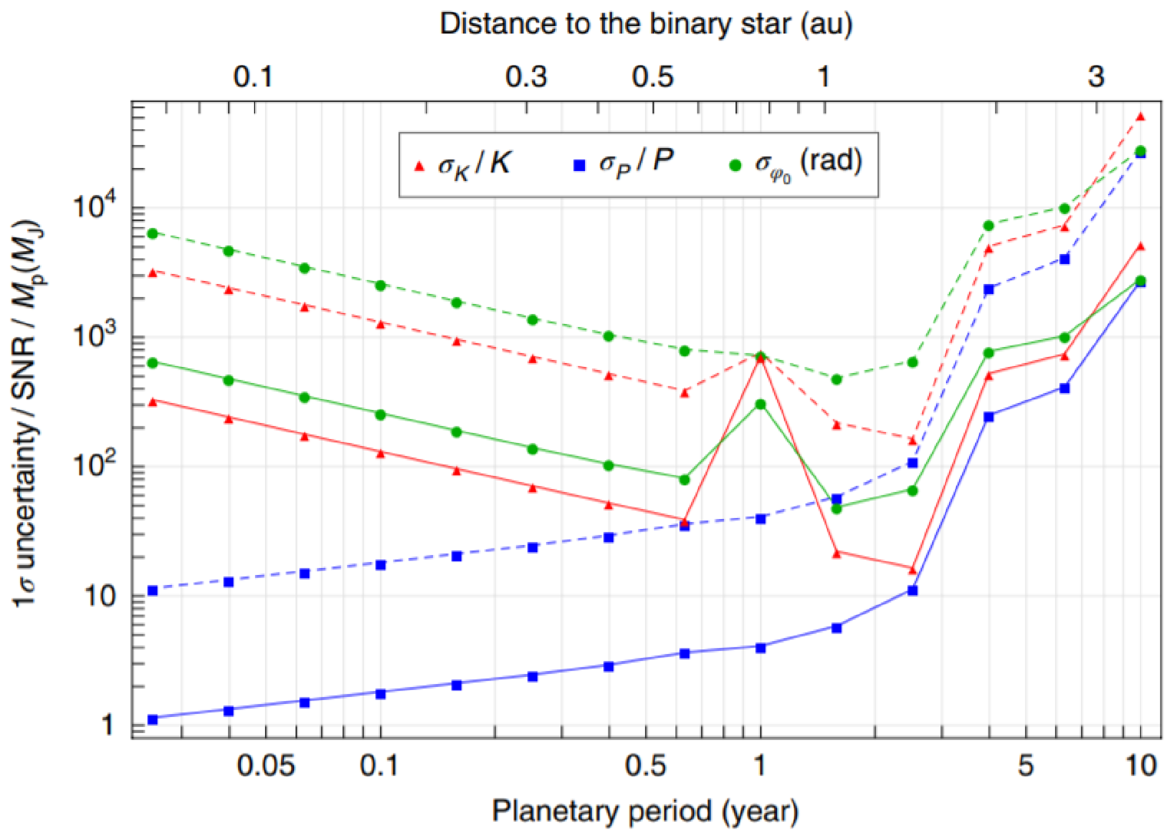
2. Look at the Doppler signal of CBPs

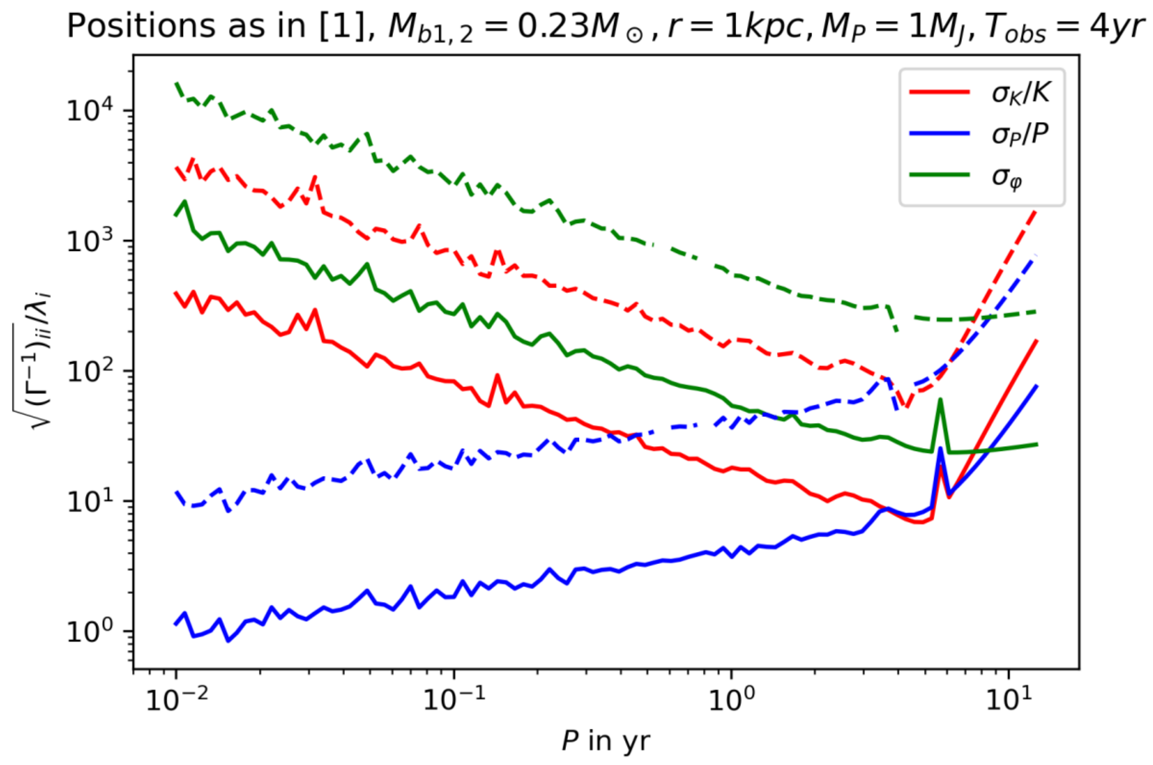




…but after two years we will sync up again (as we should)

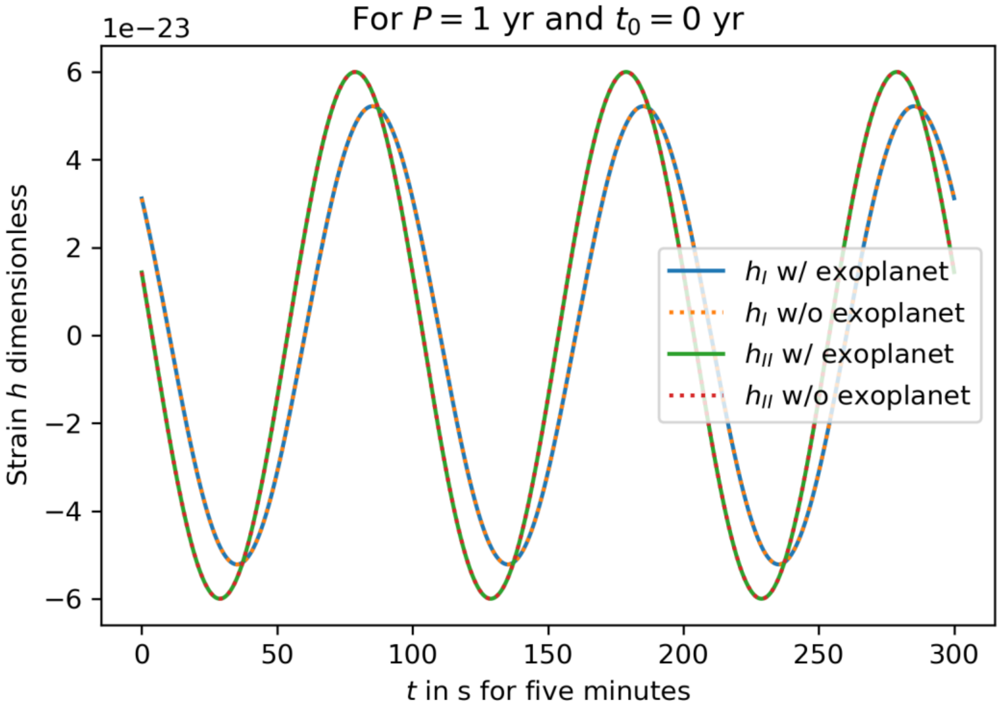
3. Compute the uncertainties

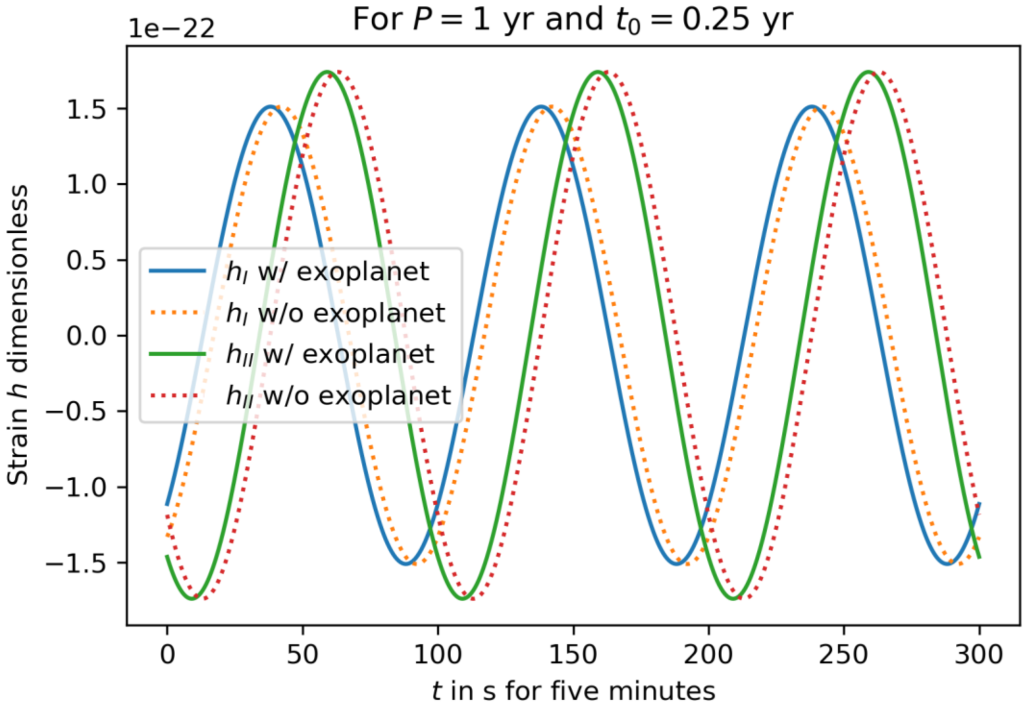




It‘s quite similiar, but: It‘s not the same! Where is the degeneracy at 1 yr?
Note #1: Wigliness probably due to floating point errors in integration + matrix inversion
Note #2: Different amplitude probably due to me using the wrong strain sensitivity, WIP

4. Why don‘t I see the degeneracy at P=1 yr?





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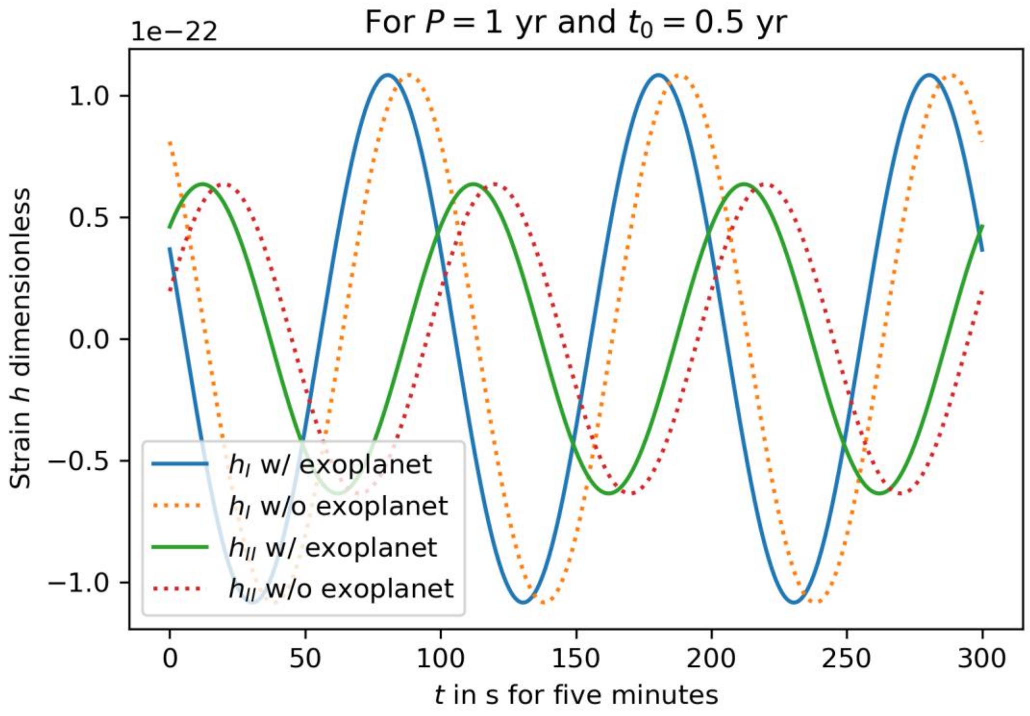

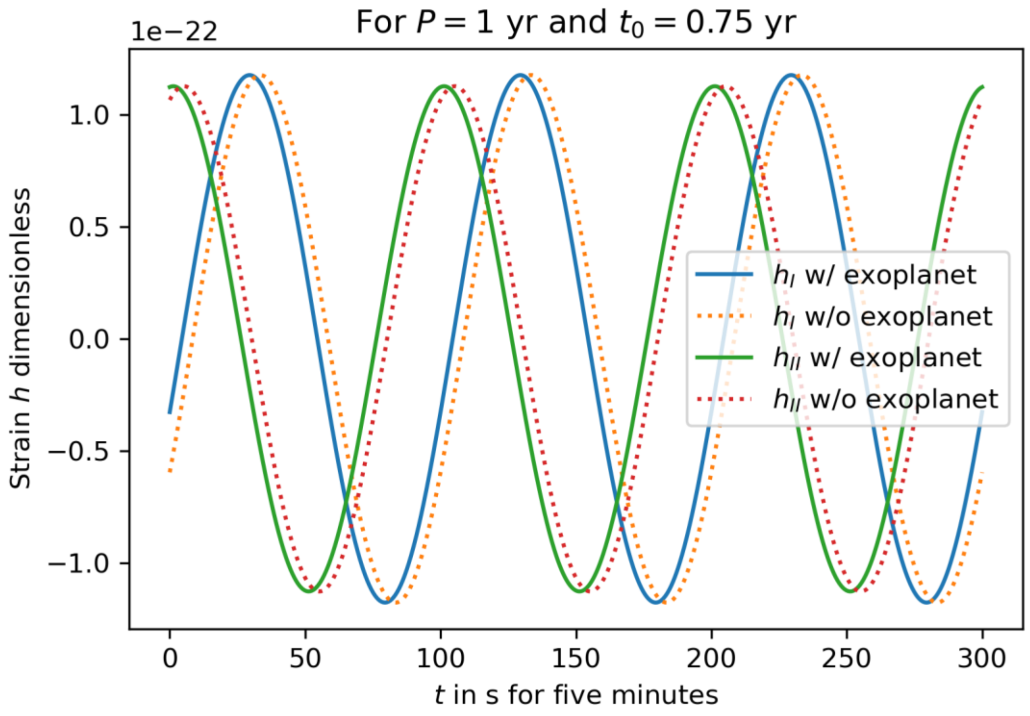

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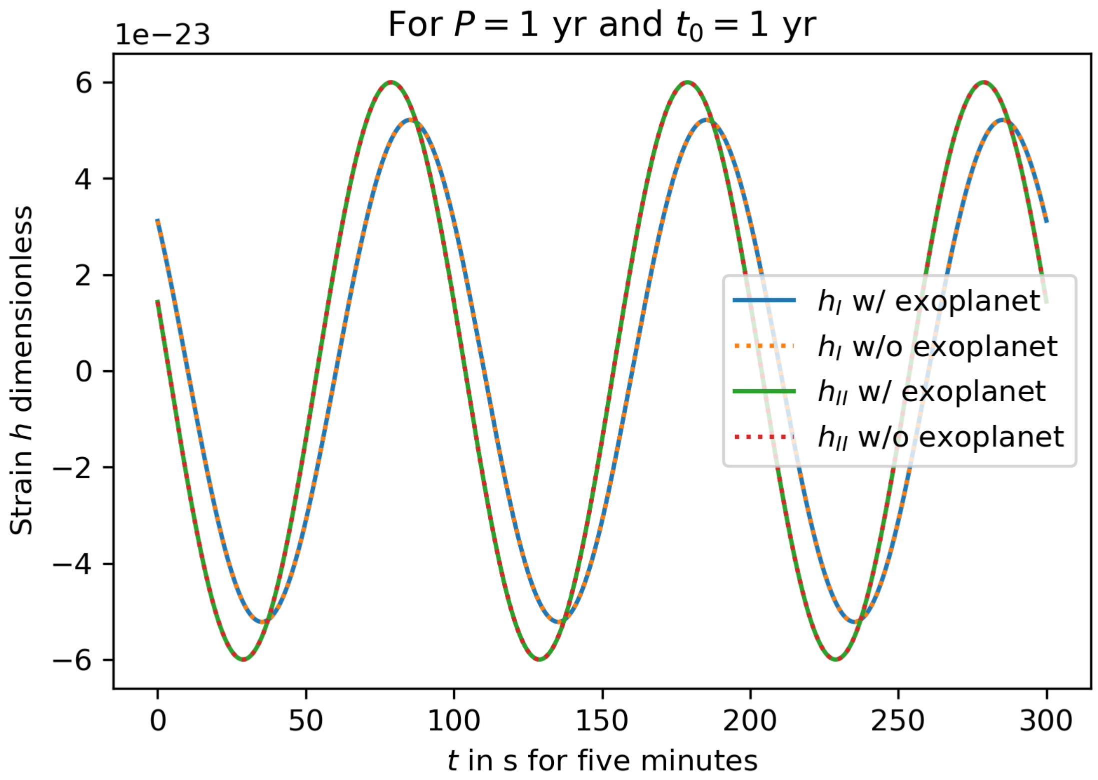

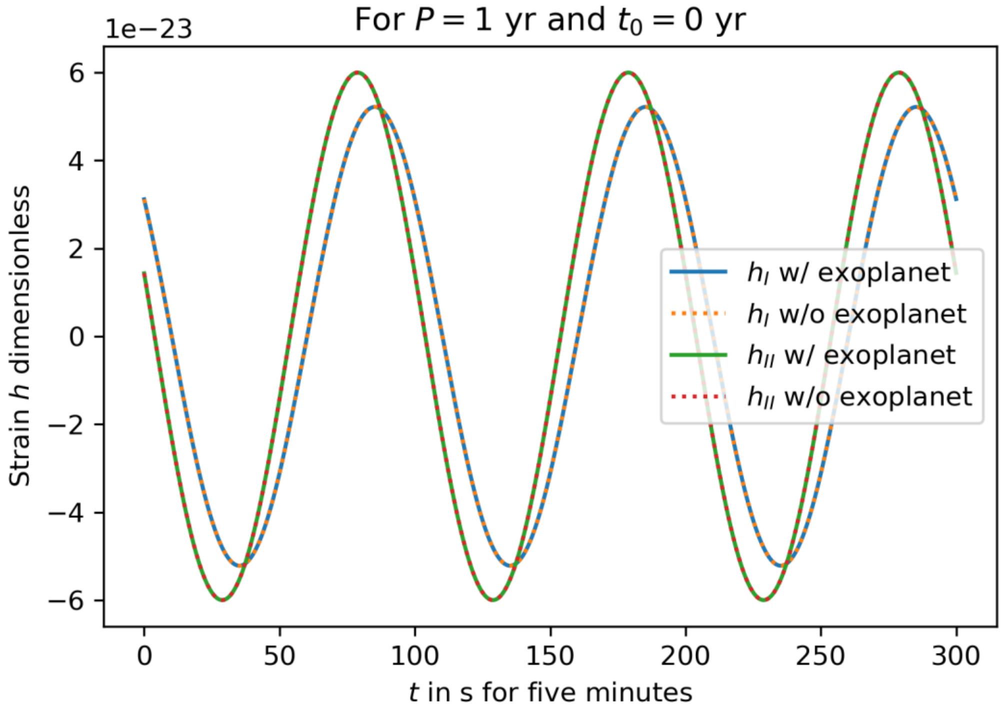

I don‘t see exactly, why there should be one: The amplitude of the frequency shift is still given by K and should in principle be read out by a search algorithm independent of the P=1 yr thing
-> I‘m still looking for bugs in my code though

5. New inputs

- The waveform is indeed quite important: To repeat the same approach with IceCube, all I‘d need is a parametric line of sight between the detector and the spacecrafts for each detection period
  -> Then we could also think about combining measurements
- The uncertainties of the parameters scale $\sigma \propto$ SNR and actually the fisher matrix estimation of the uncertainties will be the same as for example a log-likelihood parameter estimation of the real data only for SNR>>1 (see Maggiore eq. 7.73 in Chapter 7, we have to ignore a term O(SNR)) which could become quite problematic for IceGiant
- Could we maybe meet and talk next week for lunch or coffee?