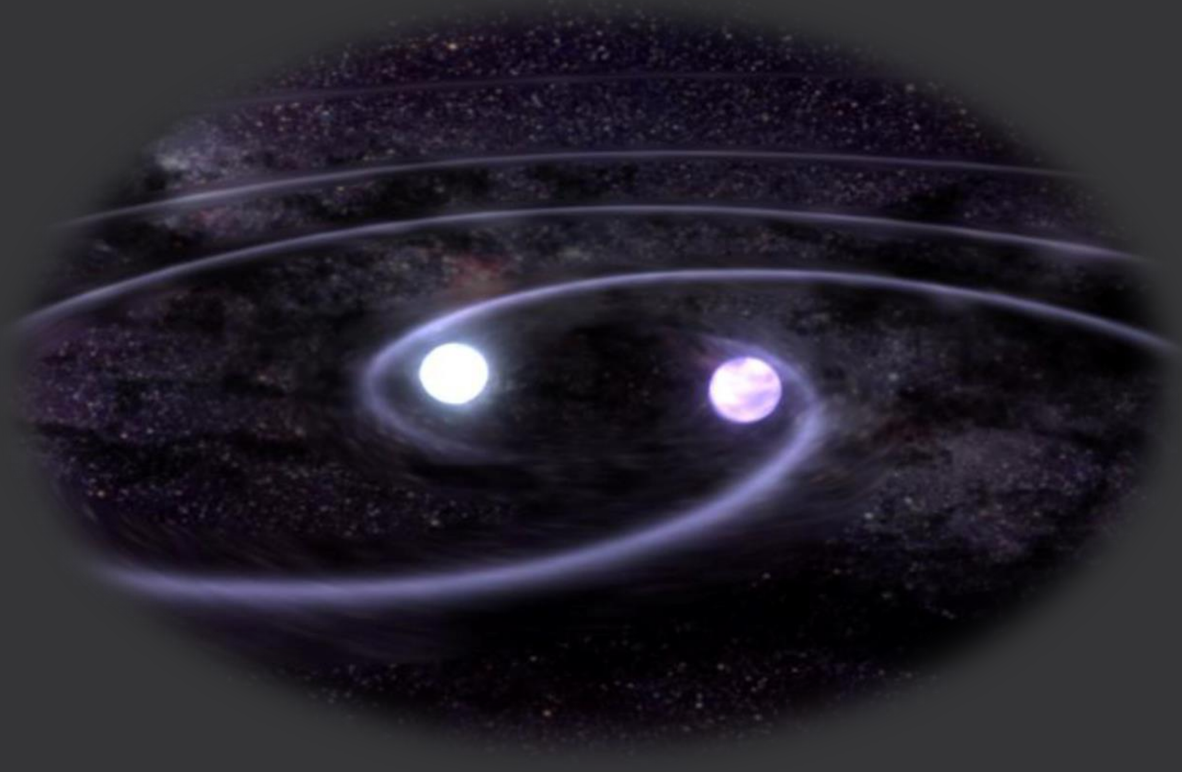
Detecting Exoplanets via Gravitational Waves



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General Idea

Double White Dwarf System (DWD)

If orbit ~circular with orbital period T, then GW emission with frequency

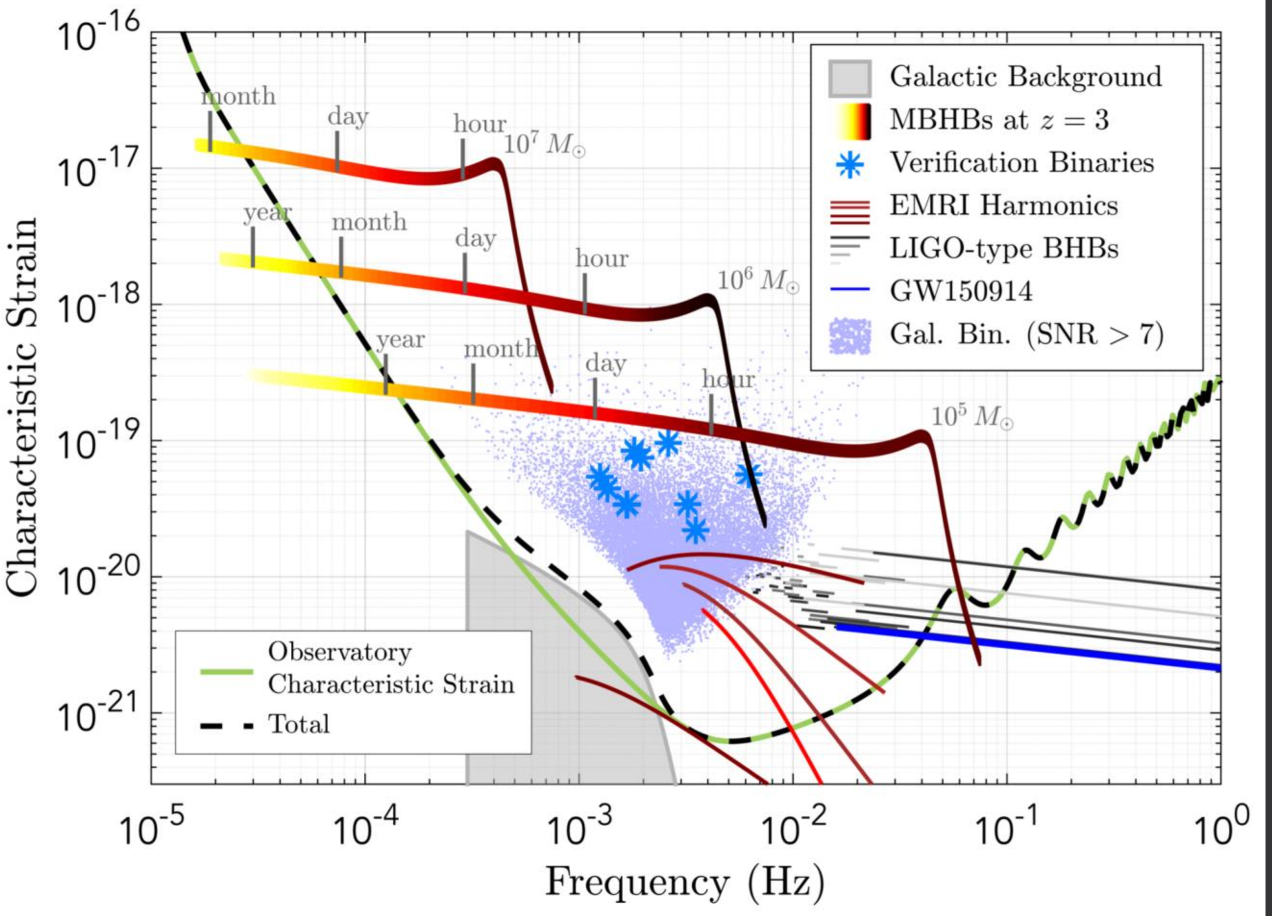
$$f_{GW} = 2 f_{DWD} = \frac{2}{T}$$

So for LISA sensitive in

$$0.1\ \text{mHz} < f_{GW} < 1\ \text{Hz} \Rightarrow 5\ h > T > 2\ s$$

In Newtonian approximation:

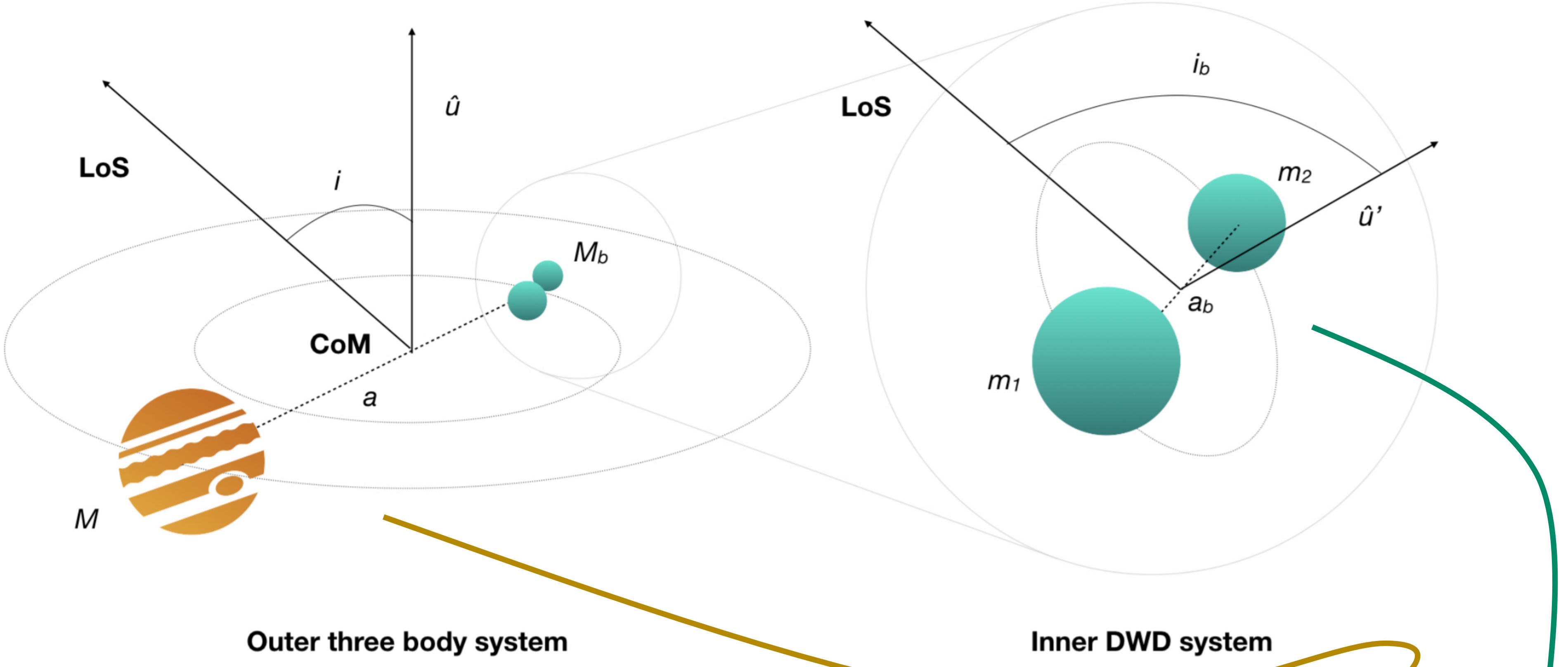
$$\omega^2 = \frac{G(m_1 + m_2)}{r^3}$$



Note: For elliptical orbitals with $e > 0.2$ instead overtone spectrum:

$$\omega_n = n\omega_0 \text{ with } \omega_0^2 = \frac{G(m_1 + m_2)}{a^3}$$

General Idea



Induced Doppler velocity by exoplanet/brown dwarf: $f_{obs}(t) = \left(1 + \frac{v_{\parallel}(t)}{c}\right) f_{GW}(t)$

Where newtonian calculation gives $v_{\parallel}(t) = -K \cos \frac{2\pi}{P} t$ with $K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M}{(M_b+M)^{2/3}} \sin i$

From emission to detection

Via phase of GW: $\cos \omega t \rightarrow \cos \int_{t_0}^{t} \omega(t')dt'$, so $\varphi(t) \rightarrow \Psi(t)$:

$$\Psi_{obs}(t) = 2\pi \int f_{obs}(t')dt' + \Psi_0$$

Which in the amplitude-and-phase form results in a measured wave, see [3]:

$$h_{I,II}(t) = \frac{\sqrt{3}}{2} A_{I,II}(t) \cos\left[\Psi_{obs}(t) + \Phi_{I,II}^{(p)}(t) + \Phi_D(t)\right]$$

Quantity we want to measure for information on planetary orbital period and mass (lower bound)

Doppler phase induced by rotation around the sun

Different polarization basis especially for LISA setup (equilateral triangle)

Polarization phase induced by rotation of detector wrt. Source (spin 2 graviton)

From emission to detection

From this we get via the one-sided spectral density noise $S_n(f_0)$ and thus the assumption of stationary and gaussian noise:

$$S/N^2 = \frac{2}{S_n(f_0)} \sum_{\alpha=I,II} \int_0^{T_0} dt\, h_\alpha(t) h_\alpha(t)$$

Parameter estimation from signal (via Fisher Information, see [3])! For signal-to-noise > 7 and relative uncertainties less than 30% on P, K
→ detection

Estimation:
3 to 83 (14 to 2218) detections of CBPs (BDs),
Observed over 4 (8) year LISA mission [2]
Dependance on probability for such objects

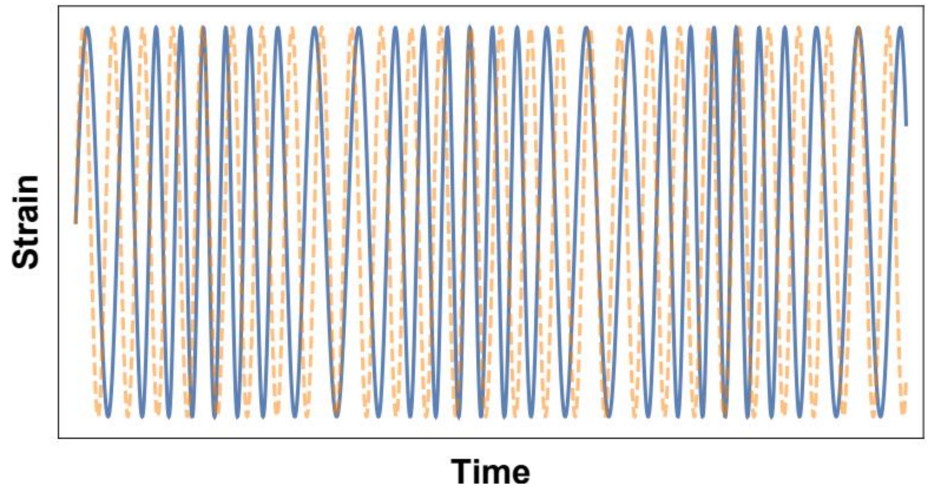


Fig. 3: Qualitative example of a DWD waveform with (blue) and without (orange dashed) the presence of a third body. The Doppler modulation is extremely exaggerated for visualisation purposes.

Why is this interesting?

1) ~93% of stars end as WDs: ~~Yet no exoplanets around WDs found!~~ Only one exoplanet around WD found (Vanderburg, Rappaport, 2020)
Question: Are WDs just to small and faint for detection of exoplanets or is it simply that planets can‘t survive the red giant phase?

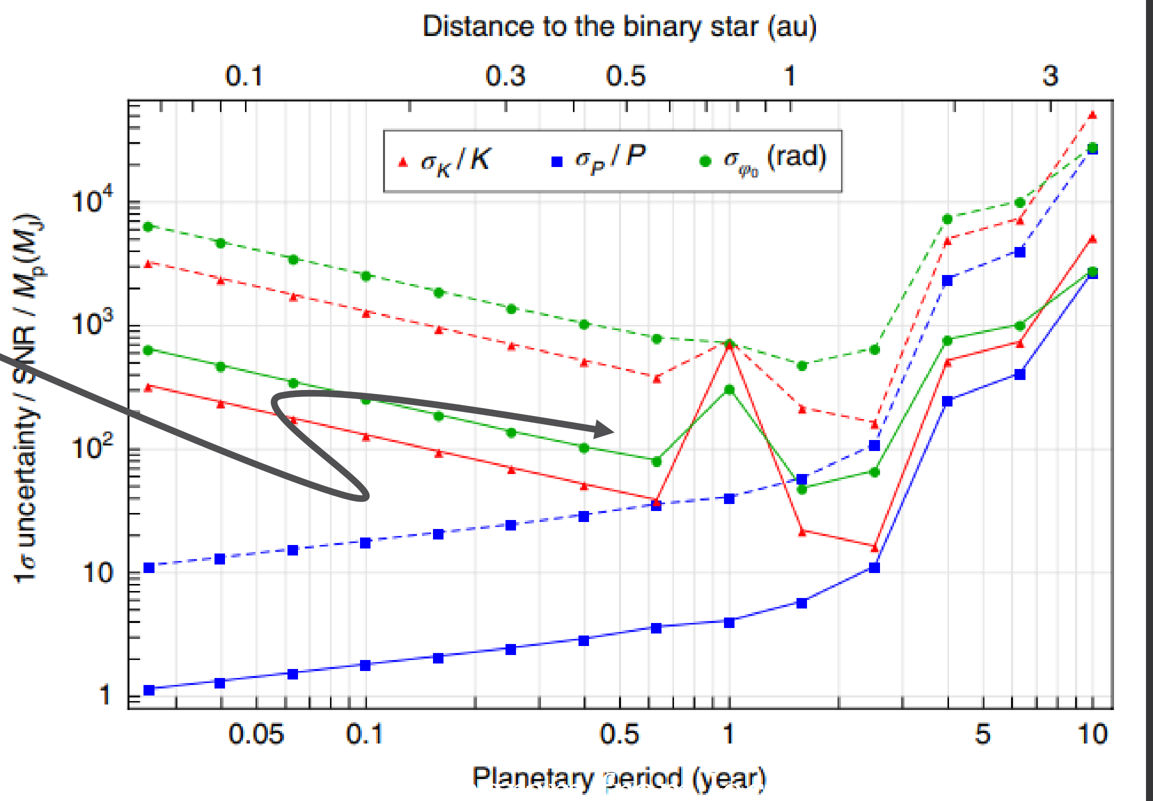
2) Roughly 25’000 DWDs assumed to be resolved by LISA → exoplanet detection probable

3) Bounds on planetary evolution and migration in extreme conditions → 2^{nd} or 3^{rd} generation of planets from accretion disk after giant phase of star?

4) ~~Potentially first detection of extragalactic exoplanet~~ (Di Stefano,Berndtsson, Urquhart *et al.* 2021, published last monday)

General assumptions for detection/parameter estimation

- Resolution of individual DWD systems (from confusion noise) → Need long mission lifetime T_0: Discrete Fourier Transform sorts monochromatic signals into frequency bins $\Delta f = {}^{1}/_{T_0} \ll \frac{v_\parallel}{c} f_{GW}$
- Less uncertainty for planet orbital period comparable to life-time T_0 and distinct from LISA‘s orbital period 1 yr
- Newtonian validity: ${}^{v}/_{c}\,, {}^{Rs}/_{d} \ll 1$
- Only one exoplanet dominating the radial motion

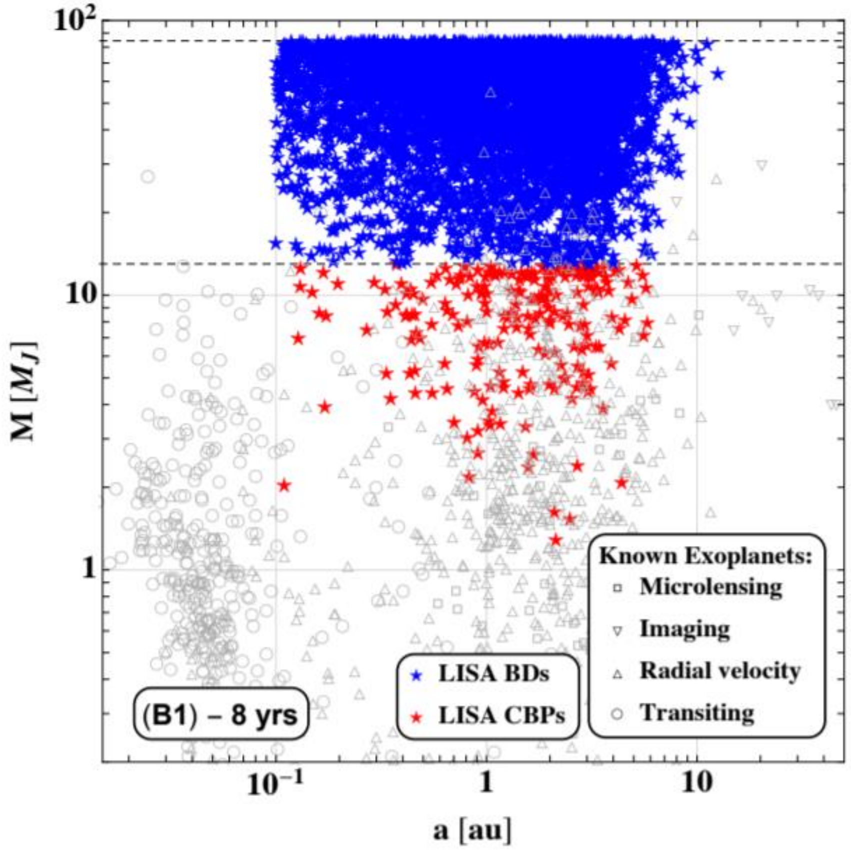


General assumptions for detection/parameter estimation

- Bias towards: high M, $f_{GW}\sim 10$ mHz and face on binaries (highest GW power output) in Local Group

- Stability criterion for three-body system:
  $P \gtrsim 4.5\, P_b$

- Far away from inspiral phase:
  $$\tau_0 \approx 10\ \mathrm{Gyr} \left(\frac{T_0}{1\ \mathrm{hr}}\right)^{8/3} \left(\frac{M_\odot}{M_{tot}}\right)^{2/3} \left(\frac{M_\odot}{\mu}\right) \gg 3300\ \mathrm{yr}\ [2]$$
  for detectable DWDs



Mock data [2] of detected exoplanets for optimistic case

References

[1] Tamanini, N., Danielski, C. (2019). The gravitational-wave detection of exoplanets orbiting white dwarf binaries using LISA. *Nat Astron 3*, 858–866. https://doi.org/10.1038/s41550-019-0807-y

[2] Danielski, C., Korol, V., Tamanini, N., & Rossi, E.M. (2019). Circumbinary exoplanets and brown dwarfs with the Laser Interferometer Space Antenna. *Astronomy and Astrophysics, 632.*

[3] Cutler, C. (1998). Angular resolution of the LISA gravitational wave detector. *Physical Review D, 57*, 7089-7102.

[4] Maggiore, M. (2008). *Gravitational Waves Volume 1: Theory and Experiments*. Oxford University Press