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Research Article

Propagating Degrees of Freedom in f(R) **Gravity**

Yun Soo Myung

Institute of Basic Science and Department of Computer Simulation, Inje University, Gimhae 621-749, Republic of Korea

Correspondence should be addressed to Yun Soo Myung; ysmyung@inje.ac.kr

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We have computed the number of polarization modes of gravitational waves propagating in the Minkowski background in f(R) gravity. These are three of two from transverse-traceless tensor modes and one from a massive trace mode, which confirms the results found in the literature. There is no massless breathing mode and the massive trace mode corresponds to the Ricci scalar. A newly defined metric tensor in f(R) gravity satisfies the transverse-traceless (TT) condition as well as the TT wave equation.

1. Introduction

f(R) gravity theory is considered as a representative theory of modified gravities. f(R) gravity [1–4] has much attentions as a strong candidate for explaining the current accelerating universe [5, 6]. When choosing the Hu-Sawicki model [7], the theory could give rise to the late time cosmic acceleration without violating the gravity tests in the solar system and without affecting high redshift physics. Very recently, the observational constraints on this model were reported from weal lensing peak abundances [8]. Particularly, f(R) = R + $R^2/(6M^2)$ gravity [9–11] has shown a strong evidence for inflation to support recent Planck data [12]. An important feature of this model indicates that the inflationary dynamics were driven by the purely gravitational interaction R^2 and the scale of inflation is linked to the mass parameter M^2 . This theory could thus provide a unified picture of both inflation in the early universe and the accelerated expansion at later times. In addition, black hole [13-15] and traversal wormhole solutions [16, 17] have been found within f(R) gravity in recent years. The recent detection of gravitational waves by the LIGO Collaboration [18] is surely a milestone in gravitational waves research and opens new perspectives in the study of Einstein gravity (general relativity) and astrophysics. Hence, it is meaningful to explore gravitational waves in the modified theory of gravity, especially in f(R) gravity. The observation of the polarization modes of gravitational waves

will be a crucial tool to obtain valuable information about the black holes and the physics of the early universe.

It is well-known that the Einstein gravity with two polarization degrees of freedom (DOF) is distinguished from the metric f(R) gravity with three DOF [19]. Importantly, it is worth noting that the Einstein equation derived from f(R) gravity contains fourth-order derivative terms. A simple way to avoid a difficulty dealing with the fourth-order equation is to transform f(R) gravity into a scalar-tensor theory which is surely a second-order theory.

Very recently, it was reported that a polynomial f(R) model could provide two additional scalars of a massive longitudinal mode (perturbed Ricci scalar: $R^{(1)}$) and a massless transverse mode (breathing mode: \hat{h}_b), in addition to the two TT tensor modes $(\hat{h}^+, \hat{h}^\times)$ [20]. A breathing mode seems to be overlooked in the literature because of the assumption that the application of the Lorentz gauge implies the TT wave equation. Also, it was insisted that four DOF found in [20] is consistent with the result obtained from the Newman-Penrose (NP) formalism. However, the presence of a breathing mode contradicts the well-known fact in the literature that the f(R) gravity involves three DOF of a massive longitudinal mode and two spin-2 modes. Hereafter, we wish to call this the issue of DOF in f(R) theories.

In this work, we wish to point out that f(R) gravity still involves three DOF by investigating the fourth-order

equation composed of a second-order tensor and a fourth-order scalar.

It seems that there is no breathing mode because the

perturbed Ricci scalar $R^{(1)}$ is related closely to the trace "h" of perturbed metric tensor. Hence, the allocation of the Ricci scalar as a newly scalar represents the trace of metric tensor. Also, we wish to remind the reader that the Ricci scalar equation is not an independent equation and is not separated from the perturbed Einstein equation because it comes out just from taking the trace of the latter equation. This implies that the Ricci scalar is an emergent mode from h. Furthermore, it is instructive to note that, in the TT gauge, there is a close connection between the metric perturbation and the linearized Riemann tensor, implying that δR_{itjt} = $-\ddot{h}_{ii}^{TT}/2$ [21]. This gauge is very convenient because it fixes all local gauge freedoms. But it might be unclear that there exists a close relation between the metric perturbation and the NP formalism unless one chooses the TT gauge. One could not naively choose the TT gauge in the perturbed f(R)gravity because of $h \neq 0$, whereas the Lorentz gauge is easily implemented to eliminate the gauge DOF. However, one might choose the TT gauge to obtain a massless spin-2 in the perturbed f(R) gravity when one introduces a newly metric perturbation $h_{\mu\nu}$.

The organization of our work is as follows. In Section 2, we briefly describe f(R) gravity and its scalar-tensor theory and derive two sets of perturbed equations around the Minkowski background in Section 3. Section 4 is focused on obtaining the number of propagating DOF when one chooses the Lorentz gauge. Finally, we will discuss our result which shows that there is no breathing mode in Section 5.

2. f(R) Gravity and Its Scalar-Tensor Theory

Instead of a polynomial model of $f(R) = R + \alpha R^2 + \beta R^3 + \cdots$ [20, 22], we start with a specific f(R) gravity (Starobinsky model [9])

$$S_f = \frac{M_P^2}{2} \int d^4x \sqrt{-g} f(R), \quad f(R) = R + \frac{R^2}{6M^2},$$
 (1)

where the R^2 -term was originally motivated by one-loop correction to Einstein gravity. Here the mass parameter M^2 is chosen to be a positive value, which is consistent with the stability condition of f''(0) > 0 [2]. This model of f(R) gravity is enough to find the propagating DOF around the Minkowski background. The Einstein equation takes the form

$$R_{\mu\nu}f'(R) - \frac{1}{2}g_{\mu\nu}f(R) + (g_{\mu\nu}\nabla^2 - \nabla_{\mu}\nabla_{\nu})f'(R) = 0, \quad (2)$$

where the prime (') denotes the differentiation with respect to its argument.

On the other hand, one might represent (1) as a scalartensor theory by introducing an auxiliary field ψ [10]:

$$S_A = \int d^4x \sqrt{-g^I} \left[\frac{M_P^2}{2} R + \frac{M_P}{M} R \psi - 3 \psi^2 \right],$$
 (3)

where the superscript J means the Jordan frame. Varying S_A with respect to ψ provides

$$\psi = \frac{M_P}{6M}R\tag{4}$$

which means that the Ricci scalar is treated as an independent scalar degree of freedom. Plugging (4) into S_A again leads to the original f(R) gravity (1) exactly.

Making use of the conformal transformation and redefining the scalar field $(\psi \rightarrow \phi)$

$$g^{I}_{\mu\nu} \longrightarrow \frac{1}{1 + 2\psi/MM_{\scriptscriptstyle P}} g^{E}_{\mu\nu} \longrightarrow e^{-\sqrt{2/3}(\phi/M_{\scriptscriptstyle P})} g^{E}_{\mu\nu},$$
 (5)

one arrives at the Starobinsky model in the Einstein frame [11]

$$S_{S} = \int d^{4}x \sqrt{-g^{E}} \left[\frac{M_{P}^{2}}{2} R - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V_{S}(\phi) \right]$$
 (6)

with the Starobinsky potential

$$V_S(\phi) = \frac{3M_P^2 M^2}{4} \left[1 - e^{-\sqrt{2/3}(\phi/M_P)} \right]^2. \tag{7}$$

At this stage, we note that the conformal transformation (5) is a purely classified transformation of coordinates and results in one frame are classically equivalent to the ones obtained in other frame. Hence, it is plausible that the number of DOF in the scalar-tensor theory (6) is three because of two TT tensor modes and one scalar mode. From (6), the Einstein and scalar equations are derived as

$$G_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu}^{\phi},$$

$$T_{\mu\nu}^{\phi} = \partial_{\mu}\phi \partial_{\nu}\phi - \frac{1}{2} g_{\mu\nu} \left[(\partial \phi)^2 + V_S \right],$$

$$\nabla^2 \phi - V_S' = 0,$$

$$V_S' = \sqrt{\frac{3}{2}} M_P M^2 e^{-\sqrt{2/3}(\phi/M_P)} \left[1 - e^{-\sqrt{2/3}(\phi/M_P)} \right].$$
(8)

The above describes a process of $[R^2 \to R\psi - 3\psi^2 \to -(\partial\phi)^2 - V]$ briefly.

3. Two Sets of Perturbed Equations

We introduce the metric perturbation around the Minkowski background to find out the propagating DOF:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}.\tag{9}$$

The Taylor expansions around R=0 are employed to define the linearized Ricci scalar $\delta R(h)$ as [23]

$$f(R) = f(0) + f'(0) \delta R(h) + \cdots,$$

$$f'(R) = f'(0) + f''(0) \delta R(h) + \cdots,$$
(10)

with f(0) = 0, f'(0) = 1, and $f''(0) = 1/3M^2$. We note that $\delta R(h)$ will be used here instead of $R^{(1)}$ in [20]. The perturbed (linearized) equation to (2) is given by the fourth-order coupled equation

$$\delta R_{\mu\nu}(h) + \frac{1}{3M^2} \left[\eta_{\mu\nu} \left(-\frac{3M^2}{2} + \Box \right) - \partial_{\mu} \partial_{\nu} \right] \delta R(h)$$

$$= 0, \quad \Box = \partial^2,$$
(11)

where the linearized Ricci tensor and scalar are given by

$$\delta R_{\mu\nu}(h) = \frac{1}{2} \left[\partial^{\rho} \partial_{\mu} h_{\nu\rho} + \partial^{\rho} \partial_{\nu} h_{\mu\rho} - \Box h_{\mu\nu} - \partial_{\mu} \partial_{\nu} h \right],$$

$$\delta R(h) = \partial^{\rho} \partial^{\sigma} h_{\rho\sigma} - \Box h.$$
(12)

When using (12), the linearized equation (11) becomes a second (fourth)-order differential equation with respect to $h_{\mu\nu}(h)$. Obviously, it is not a tractable equation. Furthermore, its trace equation leads to the linearized Ricci scalar equation

$$\left(\Box - M^2\right) \delta R = 0. \tag{13}$$

Introducing the linearized Einstein tensor $\delta G_{\mu\nu} = \delta R_{\mu\nu} - \eta_{\mu\nu} \delta R/2$, (11) takes a compact form:

$$\delta G_{\mu\nu}(h) + \frac{1}{3M^2} \left[\eta_{\mu\nu} \Box - \partial_{\mu} \partial_{\nu} \right] \delta R(h) = 0. \tag{14}$$

We note that the Bianchi identity is satisfied when acting ∂^{μ} on (14).

On the other hand, two linearized equations of (8) together with $\phi = 0 + \varphi$ take the simple forms with $\delta T^{\phi}_{\mu\nu} = 0$ and $\delta R = 0$:

$$\delta R_{\mu\nu}(h) = 0, \tag{15}$$

$$\left(\Box - M^2\right)\varphi = 0. \tag{16}$$

We note that (13) and (16) are the same when replacing δR by φ , but the fourth-order coupled equation (11) is quite different from the linearized Einstein equation (15). This indicates that (11) can be reduced to two decoupled second-order equations (15) and (16) if one employs the conformal transformation and redefinition of scalar appropriately after choosing (3), leading to a canonical scalar action with the Starobinsky potential in the Einstein frame. In the scalar-tensor theory approach, one assigns the perturbed Ricci scalar to an independent scalar φ . Instead, it does not have the trace of metric perturbation h.

4. Propagating DOF with the Lorentz Gauge

In order to take into account the propagating DOF in f(R) gravity, it is convenient to separate the metric tensor $h_{\mu\nu}$ into the traceless part $h_{\mu\nu}^T$ and the trace part h as

$$h_{\mu\nu} = h_{\mu\nu}^{T} + \frac{h}{4} \eta_{\mu\nu} \tag{17}$$

with $h^{T\mu}_{\mu} \equiv h^T = 0$. This splitting is meaningful because the issue of DOF in f(R) theories is related to the presence of h.

First of all, let us choose the Lorentz (harmonic) gauge to eliminate the gauge DOF:

$$\partial_{\mu}h^{\mu\nu} = \frac{1}{2}\partial^{\nu}h \longrightarrow$$

$$\partial_{\mu}h^{T\mu\nu} = \frac{1}{4}\partial^{\nu}h.$$
(18)

Here, we note that the transverse condition of $\partial_{\mu}h^{T\mu\nu}=0$ cannot be achieved in f(R) gravity because of $h\neq 0$. Then, the linearized Ricci tensor and scalar are given by

$$\delta R(h)_{\mu\nu} \equiv \delta R_{\mu\nu}^{T} + \frac{\eta_{\mu\nu}}{4} \delta R = -\frac{1}{2} \left[\Box h_{\mu\nu}^{T} + \frac{\eta_{\mu\nu}}{4} \Box h \right],$$

$$\delta R(h) = -\frac{1}{2} \Box h,$$
(19)

where the last equation indicates that the linearized Ricci scalar exists iff $h \neq 0$ under the Lorentz gauge. This implies that δR cannot be defined without h. That is, if h = 0, $\delta R = 0$.

Now, the fourth-order equation (11) leads to

$$\Box h_{\mu\nu} + \frac{1}{3M^2} \left[\eta_{\mu\nu} \left(-\frac{3M^2}{2} + \Box \right) - \partial_{\mu} \partial_{\nu} \right] \Box h = 0. \tag{20}$$

The other form of (20) takes the form

$$\Box h_{\mu\nu}^T + \frac{1}{3M^2} \left[\eta_{\mu\nu} \left(-\frac{3M^2}{4} + \Box \right) - \partial_{\mu} \partial_{\nu} \right] \Box h = 0. \tag{21}$$

If $\Box h \neq 0$, its trace equation takes the form

$$\left(\Box - M^2\right)\Box h = 0\tag{22}$$

which is actually the same equation as in (13). Here, (22) implies

$$\left(\Box - M^2\right)h = 0\tag{23}$$

because h could represent a massive (scalar) graviton mode in f(R) gravity. The other case of $\Box h = 0$ is not allowed since if $\Box h = 0$, one could not derive the trace equation (22) itself. Importantly, this issue should be carefully treated because the massless mode satisfying $\Box h = 0$ may correspond to the breathing mode, which is the main subject of this work. In general, it seems that the solution of (22) is given by the sum of the massive mode and massless mode which are independent of each other. However, the massless mode which is a solution to $\Box h = 0$ does not exist in f(R) gravity. We stress that *h* plays the role of a propagating massive mode instead of δR . Here is the additional reason to understand why the massless mode (breathing mode) cannot survive in f(R) gravity. If one requires $\Box h = 0$ [$\delta R = 0$, via (19)], (21) reduces to $\Box h_{\mu\nu}^T = 0$, which is just the tensor equation in Einstein gravity when choosing the Lorentz gauge. It is worth noting that the last fourth-order term of (21) indicates a feature of the perturbed f(R) gravity clearly. If one chooses $\Box h = 0$, this term disappears. Therefore, we clarify that the massless scalar mode does not exist.

Acting ∂^{μ} on (21) leads to (18), which implies that the Lorentz-gauge condition is satisfied in the perturbed equation level. Plugging (22) into (20), we have

$$\Box h_{\mu\nu} - \frac{1}{3M^2} \Box \left[\frac{M^2}{2} \eta_{\mu\nu} + \partial_{\mu} \partial_{\nu} \right] h = 0. \tag{24}$$

Substituting (22) into (21) implies a fourth-order equation

$$\Box h_{\mu\nu}^{T} + \frac{1}{3M^{2}} \Box \left[\frac{M^{2}}{4} \eta_{\mu\nu} - \partial_{\mu} \partial_{\nu} \right] h = 0$$
 (25)

which shows clearly that the traceless metric perturbation $h_{\mu\nu}^T$ is closely coupled to the trace of metric perturbation h. It is clear that the trace mode h cannot be decoupled from the traceless tensor mode $h_{\mu\nu}^T$. This is the origin of difficulty met when taking into account DOF which had arisen from the f(R) gravity. Interestingly, (25) can be transformed to the Ricci tensor-Ricci scalar equation

$$\delta R_{\mu\nu}^T + \frac{1}{3M^2} \left[\frac{M^2}{4} \eta_{\mu\nu} - \partial_{\mu} \partial_{\nu} \right] \delta R = 0, \tag{26}$$

which indicates that the traceless Ricci tensor is coupled to the Ricci scalar.

At this stage, we observe that (25) may become a massless propagating tensor equation for $\tilde{h}_{\mu\nu}$ as

$$\widetilde{h}_{\mu\nu} \equiv h_{\mu\nu}^T + \frac{1}{3} \left(\frac{\eta_{\mu\nu}}{4} - \frac{\partial_{\mu}\partial_{\nu}}{M^2} \right) h, \quad \Box \widetilde{h}_{\mu\nu} = 0, \tag{27}$$

which suggests a way of defining a massless spin-2 in f(R) gravity. That is, plugging $h_{\mu\nu}^T$ in (27) into (25) leads to $\Box \tilde{h}_{\mu\nu} = 0$. We find from (17) that $h_{\mu\nu}^T$ differs from $h_{\mu\nu}$ by h. In particular, the splitting of h in $\tilde{h}_{\mu\nu}$ is nontrivial, which reflects a feature of f(R) gravity. This is compared to that of $h_{\mu\nu}$ in Einstein gravity. For this purpose, we may express $\tilde{h}_{\mu\nu}$ as

$$\widetilde{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{3} \left(\frac{\eta_{\mu\nu}}{2} + \frac{\partial_{\mu}\partial_{\nu}}{M^2} \right) h \tag{28}$$

$$= \overline{h}_{\mu\nu} - \frac{1}{3} \left(\eta_{\mu\nu} - \frac{\partial_{\mu}\partial_{\nu}}{M^2} \right) \overline{h}, \tag{29}$$

where $\overline{h}_{\mu\nu}=h_{\mu\nu}-\eta_{\mu\nu}h/2$ is the trace-reversed metric perturbation $(\overline{h}=-h)$ [21]. Here, the Lorentz gauge is given by $\partial^{\mu}\overline{h}_{\mu\nu}=0$. We may rewrite (20) in terms of $\overline{h}_{\mu\nu}$:

$$\Box \overline{h}_{\mu\nu} - \frac{1}{3M^2} \left[\eta_{\mu\nu} \Box - \partial_{\mu} \partial_{\nu} \right] \Box \overline{h} = 0, \tag{30}$$

which is surely the same equation found in [19]. Substituting $\overline{h}_{\mu\nu}$ defined in (29) into that in (30) arrives at $\Box \widetilde{h}_{\mu\nu} = 0$ which is the same equation as in (27).

Using the trace equation (23) and the Lorentz gauge (18), we may impose the TT condition for $\tilde{h}_{\mu\nu}$:

$$\partial_{\mu}\tilde{h}^{\mu\nu} = 0,$$
 (31)
$$\tilde{h}^{\mu}_{\ \mu} = 0,$$

which indicates that $\tilde{h}_{\mu\nu}$ is a newly tensor mode defined in f(R) gravity. This may imply that f(R) gravity accommodates three DOF of two from $\tilde{h}_{\mu\nu}$ and one from h. We note that if $h=0, \tilde{h}_{\mu\nu}$ reduces to $h^T_{\mu\nu}$ and to $h^{\rm TT}_{\mu\nu}$ finally, leading to Einstein gravity.

Considering a gravitational wave that propagates in z direction, (27) together with (31) exhibits the fact that gravitational waves have two polarization components. Explicitly, (31) implies

$$\widetilde{h}_{tt} = \widetilde{h}_{ti} = 0,$$

$$\widetilde{h}_{i}^{i} = 0,$$

$$\partial^{i}\widetilde{h}_{ii} = 0,$$
(32)

where the last two expressions correspond to the TT gauge. $\tilde{h}_{ij}^{\text{TT}} = \tilde{h}_{ij}^{\text{TT}}(t-z)$ is a valid solution to the TT wave equation $\Box \tilde{h}_{ij}^{\text{TT}} = 0$. The TT gauge condition of $\partial_z \tilde{h}_{zj}^{\text{TT}} = 0$ implies $\tilde{h}_{zj}^{\text{TT}}(t-z)$ is constant and, however, this component should be zero to satisfy a condition of the asymptotic flatness: $\tilde{h}_{\mu\nu} \to 0$ as $z \to \infty$. The remaining nonzero components of $\tilde{h}_{ij}^{\text{TT}}$ are given by $\tilde{h}_{xx}^{\text{TT}}, \tilde{h}_{xy}^{\text{TT}}, \tilde{h}_{yx}^{\text{TT}}$, and $\tilde{h}_{yy}^{\text{TT}}$. Requiring the symmetry and traceless condition leads to the two independent components

$$\widetilde{h}_{xx}^{\mathrm{TT}} = -\widetilde{h}_{yy}^{\mathrm{TT}} \equiv \widetilde{h}^{+} (t - z),$$

$$\widetilde{h}_{xy}^{\mathrm{TT}} = \widetilde{h}_{yx}^{\mathrm{TT}} \equiv \widetilde{h}^{\times} (t - z).$$
(33)

Now, considering (23), one finds the trace solution [24]

$$h = h^0 e^{ik_\mu x^\mu} \longrightarrow h \left(t - v_G z \right), \tag{34}$$

where $v_G = k/\omega = \sqrt{\omega^2 - M^2}/\omega < 1$ ($\omega^2 = M^2 + k^2$) is the group velocity of a massive scalar graviton.

Finally, we obtain $h_{\mu\nu}$ as the solution to (24) with (28):

$$h_{\mu\nu}(t,z) = \tilde{h}^{+}(t-z)e_{\mu\nu}^{(+)} + \tilde{h}^{\times}(t-z)e_{\mu\nu}^{(\times)} + \frac{1}{3}\left(\frac{\eta_{\mu\nu}}{2} - \frac{k_{\mu}k_{\nu}}{M^{2}}\right)h(t-\nu_{G}z).$$
(35)

The other solution $\overline{h}_{\mu\nu}$ as the solution to (30) with (29) takes the form

$$\overline{h}_{\mu\nu}(t,z) = \widetilde{h}^{+}(t-z)e_{\mu\nu}^{(+)} + \widetilde{h}^{\times}(t-z)e_{\mu\nu}^{(\times)} + \frac{1}{3}\left(\eta_{\mu\nu} + \frac{k_{\mu}k_{\nu}}{M^{2}}\right)h(t-\nu_{G}z),$$
(36)

which is the same solution found in [19]. This encodes that three DOF of $(\tilde{h}^+, \tilde{h}^\times, h)$ are found from f(R) gravity.

In order to compare (35) and (36) with the Ricci scalar-like solution [20, 24, 25], we write down its solution by replacing $\delta R = -\Box h/2$ with h_f

$$h_{\mu\nu}^{R}(t,z) = A^{+}(t-z)e_{\mu\nu}^{(+)} + A^{\times}(t-z)e_{\mu\nu}^{(\times)} + h_{f}(t-\nu_{G}z)\eta_{\mu\nu}.$$
(37)

Even though (A^+, A^{\times}, h_f) are similar to $(\widetilde{h}^+, \widetilde{h}^{\times}, h)$, the last term of solution $h_{\mu\nu}^R$ (37) differs from that of $h_{\mu\nu}$ (35).

On the other side of the scalar-tensor theory, (15) together with $\delta R = 0$ reduces to

$$\delta R_{\mu\nu}^{T} = 0 \longrightarrow$$

$$\Box h_{\mu\nu}^{T} = 0.$$
(38)

Also, as was shown in (16), the scalar mode φ is decoupled completely from $h_{\mu\nu}^T$. Requiring the transverse condition of $\partial_\mu h^{T\mu\nu}=0$ (Lorentz condition with h=0) leads to two DOF of (h^+,h^\times) , which describe the general relativity. Hence, it is obvious that the scalar-tensor theory has three DOF (one scalar DOF + two tensor DOF).

5. Discussions

First of all, we note that three DOF of $(\widetilde{h}^+, \widetilde{h}^\times, h)$ (35) are found from analyzing the perturbed f(R) gravity. Here, we did not introduce the Ricci scalar mode $(\delta R = R^{(1)})$ separately because it is closely related to the trace of metric tensor h. We have solved the fourth-order coupled equation (24) together with the trace equation (23) directly.

We have found that there is no breathing mode in f(R) gravity. The four DOF including breathing mode have been obtained in [20] by assuming that the traceless condition of h=0 cannot be imposed on the perturbed f(R) gravity, after counting the Ricci scalar mode. The authors in [20] have discovered the breathing mode (\hat{h}_b) from the condition of $h\neq 0$ when the background space-time is not Minkowski. The approach used in [20] was based on the observation that $R^{(1)}$ is considered as a different mode from h initially [25]. This might lead to overcounting of DOF. However, noting an expression of $\delta R = -\Box h/2$ (19) in the Lorentz gauge implies that δR is closely connected to h.

One might attempt to argue from (22) that the massless mode satisfying $\Box h=0$ may correspond to the breathing mode. In general, it seems that the solution to the fourth-order equation (22) is given by the sum of the massive mode and massless mode which are independent of each other. However, the massless mode which is a solution to $\Box h=0$ ($h\neq 0$) does not exist in f(R) gravity since $\Box h=0$ [via (19)] means $\delta R=0$ in the Lorentz gauge.

Consequently, we have clarified the issue of DOF in f(R) theories. The number of polarization modes of gravitational waves in f(R) gravity is still "three" in Minkowski spacetime, which is consistent with the results in the literature

(especially for [19]). Also, we would like to mention that the DOF counting of f(R) theories should be independent of propagating space-time.

Competing Interests

The author declares no competing interests.

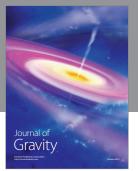
Acknowledgments

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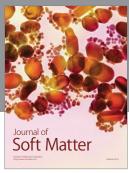
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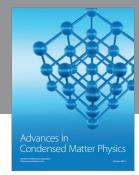
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