Probability exercises - Discrete variables

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Exercise 1. *Let X be a random variable with $f_X(x) = \frac{x}{10}$ and $\mathbb{D} = \{1, 2, 3, 4\}$.*

1. *Can you prove that $f_X(x)$ meets the two conditions of probability functions?*

   $f_X(x)$ is always positive. And $\sum_{x=1}^{4} \frac{x}{10} = 1$.

2. *Compute:*

   - $P(X > 1) = \frac{9}{10}$
   - $P(X \leq 1) = \frac{1}{10}$
   - $P(X = 1) = \frac{1}{10}$
   - $P(X \neq 1) = \frac{9}{10}$
   - $P(1.5 \leq X \leq 5) = \frac{9}{10}$
   - $P(X < 5) = 1$
   - $P(X = 6) = 0$
   - $P(X > 1 \mid X \leq 2) = \frac{2}{3}$
   - $P(X > 2 \mid X \leq 1) = 0$
   - $P(X \in \mathbb{N}) = 1$
   - *Can you guess what is $F_X(X)$?*

$$F_X(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{10} & 1 \leq x < 2 \\ \frac{3}{10} & 2 \leq x < 3 \\ \frac{6}{10} & 3 \leq x < 4 \\ 10 & 4 \leq x \end{cases}$$

Exercise 2. *Show what is the relation between $F_X(X)$ and $f_X(X)$. Knowing F_X can we obtain f_X? How? Knowing f_X can we obtain F_X? How? What are $F_X(X)$ and $f_X(X)$ if $P(X > x) = \frac{1}{2^x}$ for $x = 1, 2, 3, ...$?*

Done in class.

Exercise 3. *A certain hardware store sells a bag with 25 screws. From this 25, we know that 5 are defective. If we take 4 screws, let X be the random variable that counts the number of defective screws that we take. Find $f_X(x)$ if*

- *There is replacement when we take the screws.*

$$p = \frac{5}{25}, n = 4$$

$$X \sim Bi\left(4, \frac{1}{5}\right)$$

Exercise 4. *From a box, we take two balls. In this box there are 8 white balls, 4 black balls and 2 orange balls. If we win two euros for each black ball that we take and lose one for each ball of the other color, what is $f_X(x)$? X is the random variable that determines how much money can we win. Determine first the support of X.*

$$\mathbb{D} = \{-1, 1, 4\}$$

$$\begin{aligned}
P(X = -1) &= \frac{10 \cdot 9}{14 \cdot 13} \\
P(X = 1) &= 2 \cdot \frac{4 \cdot 10}{14 \cdot 13} \\
P(X = 4) &= \frac{4 \cdot 3}{14 \cdot 13}
\end{aligned}$$

$$f_X(X) = \begin{cases} P(X = x) & \text{if } x \in \mathbb{D} \\ 0 & \text{Otherwise} \end{cases}$$

Exercise 5. *What is the probability of throwing a coin and getting 4 heads. Use Bernouilli notation to show how you compute it. Solve it also doing Binomial notation.*

Let X be the number of heads when we throw 4 coins. Then,

$$X \sim Bi(n, p)$$

If we assume that the coin is a normal one, $p = \frac{1}{2}$. And,

$$P(X=4) = \begin{cases} \begin{pmatrix} n \\ 4 \end{pmatrix} \frac{1}{2^4}\left(1-\frac{1}{2}\right)^{n-4} & \text{if } n \geq 4 \\ 0 & \text{Otherwise} \end{cases}$$

Exercise 6. *The probability of a machine producing a defective piece is $\frac{1}{10}$, and producing one bad piece is independent of producing another. What is the probability of having one defective piece if the machine produces ten pieces? And of having two? What is the probability of having at least three defective pieces? And of having more than three?*

$X \sim Bi\left(10, \frac{1}{10}\right)$

Just substitute the following.

$$P(X=1),\ P(X=2),\ P(X \leq 3) = P(X=1) + P(X=2) + P(X=3), P(X>3) = 1 - P(X \leq 3)$$

Exercise 7. *Let*

$$X \sim Po(1)$$

Compute $P(X=1)$, $P(X=2)$ and $P(X \leq 3)$.

Just substitute $f_X(x)$ for $x = 1, 2$ and $F_X(3)$.

Exercise 8. *The number of car accidents that happen in a certain highway each day are distributed as a Poisson with parameter $\lambda = 4$. What is the probability of not having any accident in one day?*

$P(X=0) = e^{-4}$