Expectations

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Exercise 1. *Compute the expectation of a random normal* $\mathcal{N}(\mu, \sigma^2)$.

$$E(X) = \mu$$

Exercise 2. *Compute the expectation and the variance of a uniform distribution in the interval [0,2].*

$$E(X) = \frac{1}{2}$$

Exercise 3. *Let* X *be a random variable with mean* μ *and variance* σ^2*. Compute*

1. $E(2X - 3)$ *and* $V(2X - 3)$.
2. $V(5 - X)$.
3. $E((X - 2)(X + 1))$.

Done in class.

Exercise 4. *If* X *is a Poisson random variable of parameter* λ*, show that*

1. $E(X) = \lambda$.
2. $E(X(X - 1)) = \lambda^2$.

! *Compute* $E(X^2)$ *and* $E((X - E(X))^2)$

Exercise 5. *Compute the expectation of a continuous random variable* X *with distribution:*

1. *$f(x) = 6x(1-x)$ in $[0,1]$.*

$$E(X) = \frac{1}{2}$$

2. *$f(x) = \frac{3}{x^4}$ if $x > 1$.*

$$E(X) = \frac{3}{2}$$

3. *$f(x)$ if it's proportional to x^2 whenever $0 < x < 1$ and zero otherwise.*

$$E(X) = \frac{3}{4}$$

Exercise 6. *Compute the expectation of a discrete random variables such that $P(X = 1) = \frac{1}{4}$, $P(X = 2) = \frac{1}{2}$ and $P(X = 1000) = \frac{1}{4}$.*

$$E(X) = \frac{1}{4} + 1 + 250$$

Exercise 7. *Compute the expectation and the variance of a random variable with distribution $f(x) = \lambda^2 x e^{-\lambda x}$ for $x > 0$.*

Exercise 8. *! Let X be a random variable with uniform distribution in $[0,1]$. Compute $E(e^{5X})$. Can you find $E(1/X)$?*

Exercise 9. *Let X and Y be two independent random variables such that $E(X) = 2$ $E(X^2) = 6$, $E(Y^2) = 13$ and $E(Y = 3)$. Compute*

- *$E(X^2 - 3X + 2)$, $E((X+1)^2)$, $E((X - E(X))^2)$ and $E(X^2) - E(X)^2$.*
- *$E(X+Y)$, $E(2XY)$, $E((3X-Y)^2)$, $E(3X-Y)^2$ and $E(X \mid Y = 2)$.*