UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN

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FACULTAD DE CIENCIAS FÍSICO MATEMÁTICAS

FACULTAD DE CIENCIAS FISÍCO MATEMÁTICAS

Maestría en Ciencia de Datos

Materia

Métodos Estadísticos Multivariados

Profesora

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Tarea 1

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Matrícula

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$A = \begin{bmatrix} 2 & 0 & 7 & 8 \\ 6 & 4 & 6 & -2 \\ 0 & 7 & 9 & 2 \\ 6 & 5 & 4 & -8 \end{bmatrix}_{4\times4}$ $B = \begin{bmatrix} 2 & 0 \\ -5 & 3 \\ 6 & 9 \\ 0 & -2 \end{bmatrix}_{4\times2}$ $C = \begin{bmatrix} 3 & -3 & 6 & 9 \\ 4 & 0 & -2 & 0 \end{bmatrix}$

Calcular:

a) $B'A$ b) $2A+5B$ c) $3BC+4A$ d) $5C'-6B$ e) BA

1.- a) $B'A$

$B' = \begin{bmatrix} 2 & -5 & 6 & 0 \\ 0 & 3 & 9 & -2 \end{bmatrix}_{2\times4}$ $\therefore B'A = \begin{bmatrix} 2 & -5 & 6 & 0 \\ 0 & 3 & 9 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 7 & 8 \\ 6 & 4 & 6 & -2 \\ 0 & 7 & 9 & 2 \\ 6 & 5 & 4 & -8 \end{bmatrix} =$

$2\times4 \quad 4\times4$

2×4

$\overset{4}{2(2)}$ $\overset{-30}{-5(6)}$ $\cancel{6(0)}$ $\cancel{0(6)}$ | $\cancel{2(0)}$ $\overset{-20}{-5(4)}$ $\overset{42}{6(7)}$ $\cancel{0(5)}$

$\cancel{0(2)}$ $\overset{18}{3(6)}$ $\cancel{9(0)}$ $\overset{-12}{-2(6)}$ | $\cancel{0(0)}$ $\overset{12}{3(4)}$ $\overset{63}{9(7)}$ $\overset{-10}{-2(5)}$

$\overset{14}{2(7)}$ $\overset{-30}{-5(6)}$ $\overset{54}{6(9)}$ $\overset{68}{\cancel{0(4)}}$ | $\overset{16}{2(8)}$ $\overset{10}{(-5)(-2)}$ $\overset{12}{6(2)}$ $\cancel{0(-8)}$

$\cancel{0(7)}$ $\overset{18}{3(6)}$ $\overset{81}{9(9)}$ $\overset{-8}{-2(4)}$ | $\cancel{0(8)}$ $\overset{-6}{3(-2)}$ $\overset{18}{9(2)}$ $\overset{+16}{-2(-8)}$

$= \begin{bmatrix} -26 & 22 & 38 & 38 \\ 6 & 65 & 91 & 28 \end{bmatrix} = B'A$

1.- b) 2A + 5B

$$2\begin{bmatrix} 2 & 0 & 7 & 8 \\ 6 & 4 & 6 & -2 \\ 0 & 7 & 9 & 2 \\ 6 & 5 & 4 & -8 \end{bmatrix} + 5\begin{bmatrix} 2 & 0 \\ -5 & 3 \\ 6 & 9 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 14 & 16 \\ 12 & 8 & 12 & -4 \\ 0 & 14 & 18 & 4 \\ 12 & 10 & 8 & -16 \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ -25 & 15 \\ 30 & 45 \\ 0 & -10 \end{bmatrix}$$

como no tienen la misma dimensión, entonces se quedan expresadas así

1.- c) 3 B C + 4A

$$3\begin{bmatrix} 2 & 0 \\ -5 & 3 \\ 6 & 9 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 3 & -3 & 6 & 9 \\ 4 & 0 & -2 & 0 \end{bmatrix} + 4\begin{bmatrix} 2 & 0 & 7 & 8 \\ 6 & 4 & 6 & -2 \\ 0 & 7 & 9 & 2 \\ 6 & 5 & 4 & -8 \end{bmatrix} =$$

(4x2)(2x4)

(4x4) + (4x4) ✓

$$\begin{bmatrix} 6 & 0 \\ -15 & 9 \\ 18 & 27 \\ 0 & -6 \end{bmatrix}\begin{bmatrix} 3 & -3 & 6 & 9 \\ 4 & 0 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 28 & 32 \\ 24 & 16 & 24 & -8 \\ 0 & 28 & 36 & 8 \\ 24 & 20 & 16 & -32 \end{bmatrix} =$$

3 B C =

$$\begin{bmatrix} 6(3)+0(4) & 6(-3)+0(0) & 6(6)+0(-2) & 6(9)+0(0) \\ -15(3)+9(4) & -15(-3)+9(0) & -15(6)+9(-2) & -15(9)+9(0) \\ 18(3)+27(4) & 18(-3)+27(0) & 18(6)+27(-2) & 18(9)+27(0) \\ 0(3)+-6(4) & 0(-3)+-6(0) & 0(6)-6(-2) & 0(9)+-6(0) \end{bmatrix}$$

$$\begin{bmatrix} 18 & -18 & 36 & 54 \\ -9 & +45 & -108 & -135 \\ 162 & -54 & 54 & 162 \\ -24 & 0 & +12 & 0 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 28 & 32 \\ 24 & 16 & 24 & -8 \\ 0 & 28 & 36 & 8 \\ 24 & 20 & 16 & -32 \end{bmatrix} = \begin{bmatrix} 26 & -18 & 64 & 86 \\ 15 & 61 & -84 & -143 \\ 162 & -26 & 90 & 170 \\ 0 & 20 & 28 & -32 \end{bmatrix} = 3BC+4A$$

(4x4) + (4x4)

1-d) $5C' - 6B$

$(2\times4) - (2\times4)$ → si se puede restar

$$5\begin{bmatrix} 3 & 4 \\ -3 & 0 \\ 6 & -2 \\ 9 & 0 \end{bmatrix} - 6\begin{bmatrix} 2 & 0 \\ -5 & 3 \\ 6 & 9 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 20 \\ -15 & 0 \\ 30 & -10 \\ 45 & 0 \end{bmatrix} - \begin{bmatrix} 12 & 0 \\ -30 & 18 \\ 36 & 54 \\ 0 & -12 \end{bmatrix} = \begin{bmatrix} 3 & 20 \\ 15 & -18 \\ -6 & -64 \\ 45 & +12 \end{bmatrix} = 5C' - 6B$$

1-e) BA

$$\begin{bmatrix} 2 & 0 \\ -5 & 3 \\ 6 & 9 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 7 & 8 \\ 6 & 4 & 6 & -2 \\ 0 & 7 & 9 & 2 \\ 6 & 5 & 4 & -8 \end{bmatrix}$$

$(4\times2)\ (4\times4)$

→ la fila de B no es de la misma longitud que la columna B, por tanto no es viable la multiplicación

2

a) $6x + 2y = 5$
$3x + y = 8$

en notación $Ax + C = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

b) $4x - 5y + 10z = 0$
$6x + 2y - 5z = 12$
$5x + 3y + 0 = 9$

en notación $Ax + C = \begin{bmatrix} 4 & -5 & 10 \\ 6 & 2 & -5 \\ 5 & 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 9 \end{bmatrix}$

c) $2x + 5y - 8z + 4w = 6$
$6x + 0 + 9z - 10w = 8$
$4x + 3y + 0 - 9w = 15$
$7x + 0 - 8z + 0 = 30$

en notación $AX + C = \begin{bmatrix} 2 & 5 & -8 & 4 \\ 6 & 0 & 9 & -10 \\ 4 & 3 & 0 & -9 \\ 7 & 0 & -8 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 15 \\ 30 \end{bmatrix}$

③

a) Si $A = \begin{bmatrix} 2 & 5 \\ 4 & 8 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} -2 & 1.25 \\ 1 & -0.5 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 4 & 8 & 0 & 1 \end{array}\right] \left(\tfrac{1}{2}\right) = \left[\begin{array}{cc|cc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ 4 & 8 & 0 & 1 \end{array}\right] 4①-② = \left[\begin{array}{cc|cc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ 0 & 2 & 2 & -1 \end{array}\right] ②\left(\tfrac{1}{2}\right)$$

$$= \left[\begin{array}{cc|cc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 1 & -\frac{1}{2} \end{array}\right] ① - \tfrac{5}{2}② = \left[\begin{array}{cc|cc} 1 & 0 & -2 & \frac{5}{4} \\ 0 & 1 & 1 & -\frac{1}{2} \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1.25 \\ 0 & 1 & 1 & -0.5 \end{array}\right]$$

$$\begin{array}{cccc} 1 & \frac{5}{2} & \frac{1}{2} & 0 \\ 0 & \frac{-5}{2} & -\frac{5}{2} & +\frac{5}{4} \\ \hline 1 & 0 & -\frac{4}{2} & \frac{5}{4} \end{array}$$

b) Si $B = \begin{bmatrix} 2 & 4 & 0 \\ -3 & 5 & 2 \\ 8 & 0 & -2 \end{bmatrix} \Rightarrow B^{-1} = \begin{bmatrix} -0.5 & 0.4 & 0.4 \\ 0.5 & -0.2 & -0.2 \\ -2 & 1.6 & 1.1 \end{bmatrix}$

$$\begin{array}{c} ① \\ ② \\ ③ \end{array} \left[\begin{array}{ccc|ccc} 2 & 4 & 0 & 1 & 0 & 0 \\ -3 & 5 & 2 & 0 & 1 & 0 \\ 8 & 0 & -2 & 0 & 0 & 1 \end{array}\right] \left(\tfrac{1}{2}\right) = \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{1}{2} & 0 & 0 \\ -3 & 5 & 2 & 0 & 1 & 0 \\ 8 & 0 & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ ②+3① \\ ③-8① \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 11 & 2 & \frac{3}{2} & 1 & 0 \\ 0 & -16 & -2 & -4 & 0 & 1 \end{array}\right] \left(\tfrac{1}{11}\right) = \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{2}{11} & \frac{3}{22} & \frac{1}{11} & 0 \\ 0 & -16 & -2 & -4 & 0 & 1 \end{array}\right] \begin{array}{l} ①-2② \\ \\ ③+16② \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{4}{11} & \frac{5}{22} & -\frac{2}{11} & 0 \\ 0 & 1 & \frac{2}{11} & \frac{3}{22} & \frac{1}{11} & 0 \\ 0 & 0 & \frac{10}{11} & -\frac{20}{11} & \frac{16}{11} & 1 \end{array}\right] \left(\tfrac{11}{10}\right) = \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{4}{11} & \frac{5}{22} & -\frac{2}{11} & 0 \\ 0 & 1 & \frac{2}{11} & \frac{3}{22} & \frac{1}{11} & 0 \\ 0 & 0 & 1 & -2 & \frac{8}{5} & \frac{11}{10} \end{array}\right] \begin{array}{l} ①+\frac{4}{11}③ \\ ②-\frac{2}{11}③ \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & \frac{2}{5} & \frac{2}{5} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{5} & -\frac{1}{5} \\ 0 & 0 & 1 & -2 & \frac{8}{5} & \frac{11}{10} \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -0.5 & 0.4 & 0.4 \\ 0 & 1 & 0 & 0.5 & -0.2 & -0.2 \\ 0 & 0 & 1 & -2 & 1.6 & 1.1 \end{array}\right]$$

```
import numpy as np
```

```
A = [[0,0,0,-5],
     [1,1,1,0],
     [0,2,0,1],
     [6,0,1,1]]

a = np.matrix(A)
a
```

```
matrix([[ 0,  0,  0, -5],
        [ 1,  1,  1,  0],
        [ 0,  2,  0,  1],
        [ 6,  0,  1,  1]])
```

```
np.linalg.inv(a)
```

```
matrix([[ 0.06, -0.2 ,  0.1 ,  0.2 ],
        [ 0.1 ,  0.  ,  0.5 ,  0.  ],
        [-0.16,  1.2 , -0.6 , -0.2 ],
        [-0.2 , -0.  , -0.  , -0.  ]])
```

4. Sea A una matriz tal que $A = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$ con $a_{ii} \neq 0$ $\forall_i$

Verificar que:

$$A^{-1} = \begin{bmatrix} 1/a_{11} & 0 & 0 & 0 \\ 0 & \frac{1}{a_{22}} & 0 & 0 \\ 0 & 0 & \frac{1}{a_{33}} & 0 \\ 0 & 0 & 0 & \frac{1}{a_{44}} \end{bmatrix}$$

$$A^{-1} = \left[\begin{array}{cccc|cccc} a_{11} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \left(\frac{1}{a_{11}}\right) \\ \left(\frac{1}{a_{22}}\right) \\ \left(\frac{1}{a_{33}}\right) \\ \left(\frac{1}{a_{44}}\right) \end{matrix} = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{a_{11}} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{a_{22}} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{a_{33}} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{a_{44}} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 & 0 & 0 \\ 0 & \frac{1}{a_{22}} & 0 & 0 \\ 0 & 0 & \frac{1}{a_{33}} & \\ 0 & 0 & 0 & \frac{1}{a_{44}} \end{bmatrix}$$

⑤ Verificar que la matriz $C = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & 3 \end{bmatrix}$ es idempotente $(C^2 = CC = C)$

$$\begin{array}{cc} 3\times3 & 3\times3 \end{array}$$

$$CC = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} \overset{4}{2(2)}\ \overset{+2}{-2(-1)}\ \overset{-4}{-4(1)} & \overset{-4}{2(-2)}\ \overset{-6}{-2(3)}\ \overset{8}{-4(-2)} & \overset{-8}{2(4)}\ \overset{-8}{-2(4)}\ \overset{12}{-4(-3)} \\ \overset{-2}{-1(2)}\ \overset{-3}{3(-1)}\ \overset{4}{4(1)} & \overset{2}{-1(-2)}\ \overset{9}{3(3)}\ \overset{-8}{4(-2)} & \overset{4}{-1(4)}\ \overset{12}{3(4)}\ \overset{-12}{4(-3)} \\ \overset{2}{1(2)}\ \overset{2}{-2(-1)}\ \overset{-3}{-3(1)} & \overset{-2}{1(-2)}\ \overset{-6}{-2(3)}\ \overset{+6}{-3(-2)} & \overset{-4}{1(4)}\ \overset{-8}{-2(4)}\ \overset{9}{-3(-3)} \end{bmatrix} =$$

$$\underline{\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}} = \underline{C^2 = CC = C}$$

⑥ Ejemplo de $A^2 = 0$ = Matriz nilpotente

$$A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \quad \text{si } A^2 = AA = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 2(2)\ -4(1) & 2(-4)\ -4(-2) \\ 1(2)\ -2(1) & 1(-4)\ -2(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

7 Matriz singular cuyo determinante $=0$

si es singular

$$A=\begin{bmatrix} x & x+1 \\ 2 & x+3 \end{bmatrix}$$

$\det(A) = (x)(x+3) - 2(x+1) =$

$x^2+3x-2x-2 = x^2+x-2 = (x+2)(x-1)$

$\therefore$ si $x=-2$ ó $x=1 \Rightarrow$ la matriz será singular

ejemplo si $x=1$

$$A=\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = 4-4=0$$

ejemplo si $x=-2$

$$A=\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} = -2-(-2) = -2+2=0$$

Para que sea no singular, entonces $\boxed{x \neq 1 \text{ y } x \neq -2}$

8 obtener la norma de los sig. vectores

a) $v=[+2,-5]' \in \mathbb{R}^2$

b) $w=[1,-2,8]' \in \mathbb{R}^3$

c) $c=[2,5,-3,1,-1]' \in \mathbb{R}^5$

a) $|v| = \sqrt{4+25} = \sqrt{29} \approx 5.3851$

b) $|w| = \sqrt{1+4+64} = \sqrt{69} \approx 8.3066$

c) $|c| = \sqrt{4+25+9+1+1} = \sqrt{40} = 2\sqrt{10} \approx 6.3245$

9) Obtener los valores y vectores característicos de:

a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

b) $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

Tenemos que $(A - \lambda I) = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & -2 \\ 3 & -\lambda \end{bmatrix}$

$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & -2 \\ 3 & -\lambda \end{vmatrix} = (1-\lambda)(-\lambda) - \overset{-6}{(-2)(3)} = -\lambda + \lambda^2 + 6$

$= \lambda^2 - \lambda + 6 = (\lambda - 3)(\lambda + 2) \therefore \boxed{\lambda = 3 \text{ y } \lambda = -2}$

$p(\lambda) = \lambda^2 - \lambda + 6$ donde $\lambda = 3$ y $\lambda = -2$

b)

$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & 0 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 5-\lambda \end{bmatrix}$

$\det(A) = \begin{vmatrix} 2-\lambda & 0 & 0 & 2-\lambda & 0 \\ 0 & 3-\lambda & 0 & 0 & 3-\lambda \\ 0 & 0 & 5-\lambda & 0 & 0 \end{vmatrix}$

$[(2-\lambda)(3-\lambda)(5-\lambda) + 0 + 0] - [0 + 0 + 0] = (2-\lambda)(3-\lambda)(5-\lambda)$

$= (6 - 2\lambda - 3\lambda + \lambda^2)(5 - \lambda)$

$= (\lambda^2 - 5\lambda + 6)(5 - \lambda)$

$= 5\lambda^2 - 25\lambda + 30 - \lambda^3 + 5\lambda^2 - 6\lambda$

$= -\lambda^3 + 10\lambda^2 - 31\lambda + 30 = p(\lambda)$

$\therefore$

$p(\lambda) = (2-\lambda)(3-\lambda)(5-\lambda) = -\lambda^3 + 10\lambda^2 - 31\lambda + 30$

y con $\lambda \cdot \ni \cdot\cdot$ $\lambda = 2$, $\lambda = 3$ y $\lambda = 5$