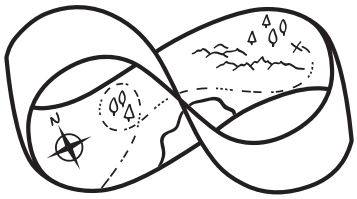


MaPP

MaPP Challenge '22:
Terry Kettler and the Mathemagical Mystery

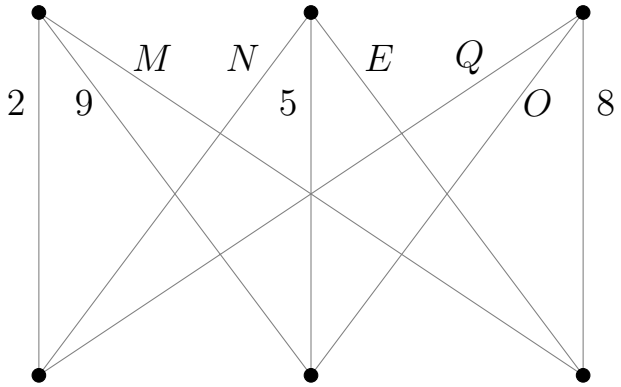
It finally happened! Although most people get their invitation when they turn eleven, I, Terry Kettler, finally received my letter from Professor Ellen Rudin's school for magicians. Unlike some other schools of magic that you may have read about or visited in a theme park, Rudin Academy is very real, and very exclusive.

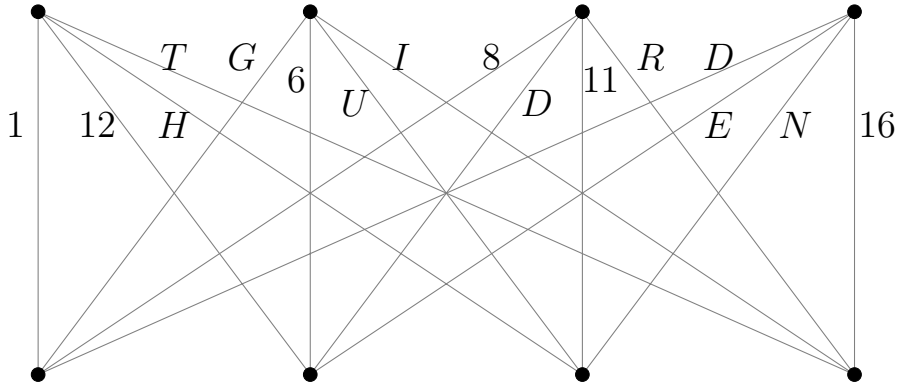
You see, the letter came with a "Welcome Packet", but it's actually a collection of puzzles that I'll need to solve in order to be accepted. Of course, by "me" I really mean "we" - I hear you're pretty great at solving puzzles, and I may have put off my application until the last minute...

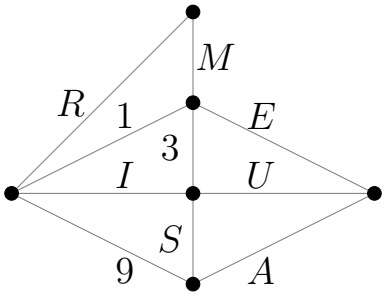
I've made you a copy of this welcome packet. It won't make much sense by itself, but I'll send you messages using the ClueKeeper app that will help your team solve the enigmata hidden within these pages. You'll be finding several "magic codes" and "magic words"; submit them using your ClueKeeper app so I can include them in my application. If you can solve enough of these puzzles, they'll have to let me join their school, right?

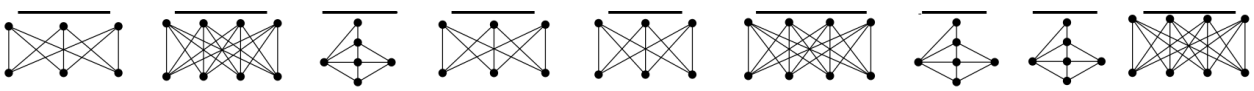
PDF last updated: July 17, 2022

Our Mentoring Program





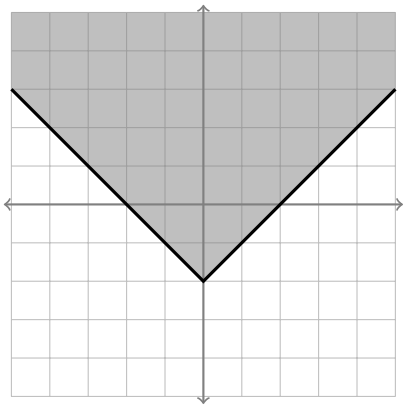




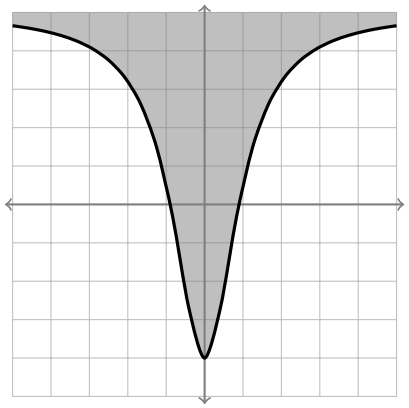
Our Tuition Schedule

			A	3		7			M
2	7	4					9	1	
	1	8			6			2	O
			2		E	G	P	5	4
T		7							9
1			R		U	I			5
	6		4				5	D	
L				8	S		2		
4	9	0	7	H	C		B		N
5	8	1				0			6

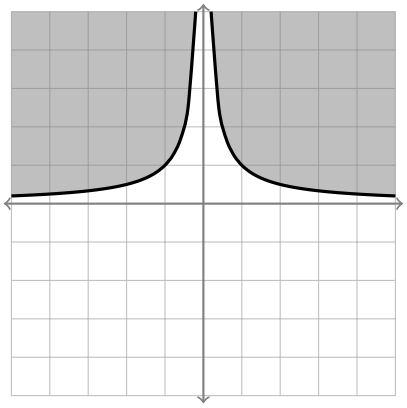
Our Curriculum



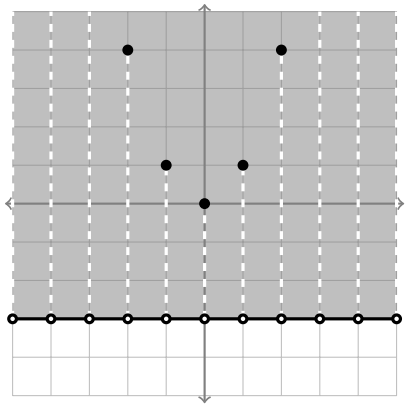
P_C_H_M



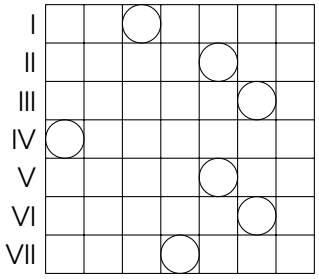
T_M_R_S



H_O_E_N



M_V_T_O



Our Placement Exam

$$8 \div 4 \times 3 + 1 \times 9 - 7$$

$$7 \times 4 \div 9 - 9 - 6 \div 3$$

$$3 + 5 \div 4 - 2 + 5 - 2$$

$$9 - 4 \times 3 - 3 + 4 \div 2$$

$$9 \div 3 + 2 \times 1 + 5 - 3$$

$$5 \div 3 + 7 - 6 - 5 - 4$$

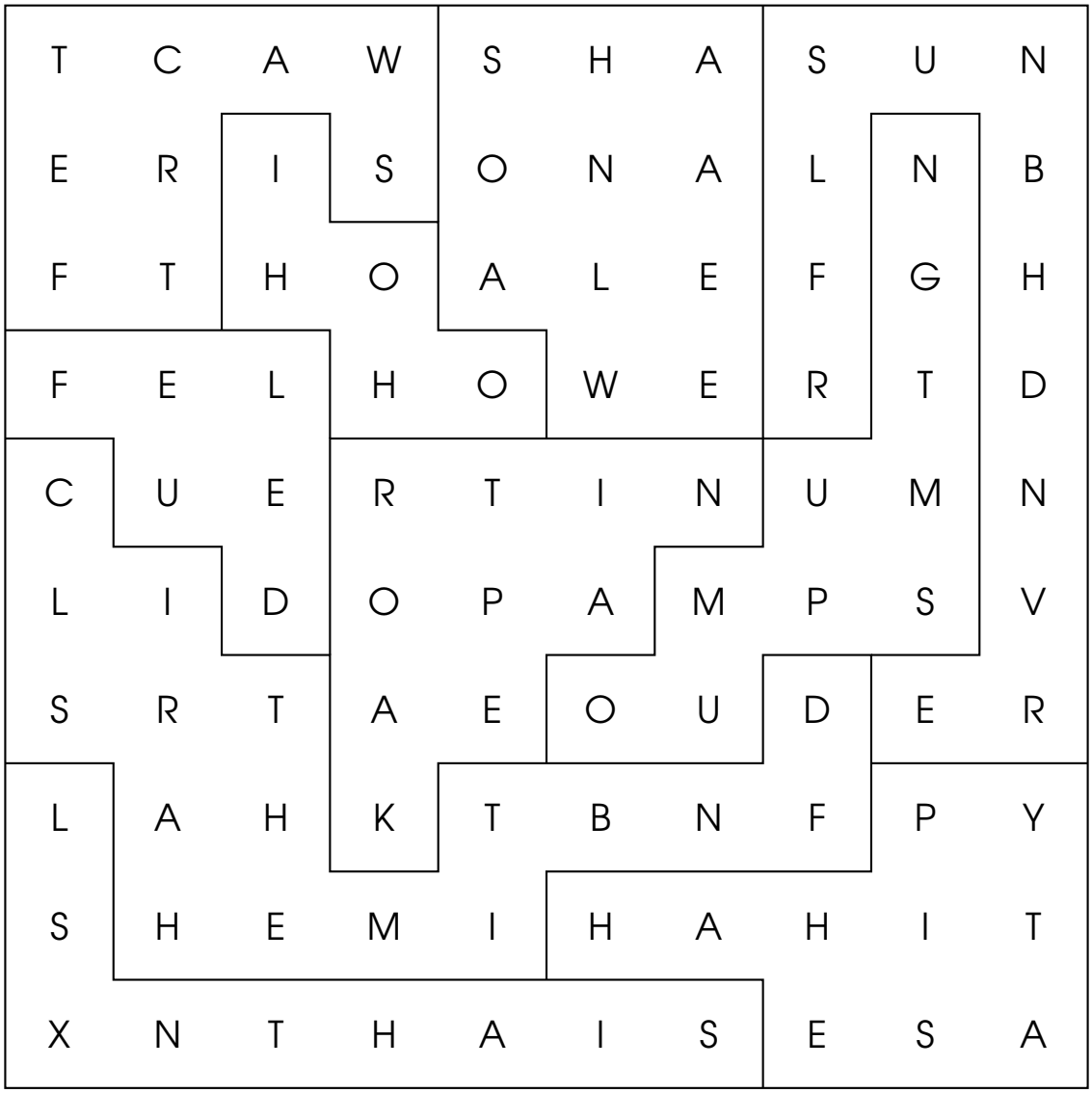
$$5 + 3 \div 2 \times 6 + 2 \div 2$$

$$2 + 1 - 9 - 2 + 3 \times 4$$

$$9 - 4 \times 2 + 8 \div 4 - 2$$

1 3 8 8 8 13 14 14 15

Our Night Sky



Our Calendar

☾	♂	☿	♃	♀	♄	☉	⧈
1	2	3	4	5	6	7	
8	9	10	11	12	13	14	
15	16	17	18	19	20	21	
22	23	24	25	26	27	28	(29)

FUSION: 443 days with the 261st day on a Magday
HERESY: 414 days with the 355th day on a Magday
LIVING: 430 days with the 223rd day on a Magday
MEMORY: 410 days with the 306th day on a Magday
MYTHOS: 425 days with the 114th day on a Magday
WISHES: 406 days with the 155th day on a Magday

Our Seal

	B	E	B	EB	BB	E	BC	BC	C	N
B B B										
B B B										
B B B B										
B B C B										
B B B B										
B B B B										
B B B B										
B B C										
B B										
B										

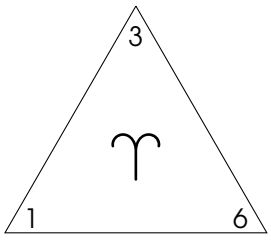
	CC	DBB	CCBB	BYB	BB	B	BYB	CCBB	DBB	CC
R										
C C										
C C										
C C										
C C										
C C										
B B B B										
D D										
N										
W										

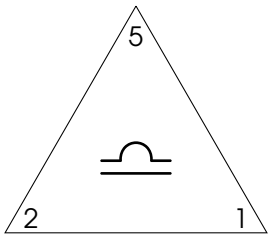
	YC	AY	YCC	RB	BYB	BYB	BR	CCY	YA	CY
E										
Y Y										
A C										
C C										
A Y										
Y A										
C C										
C A										
Y Y										
E										

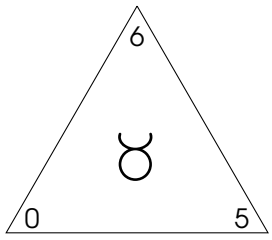
0 is MP1's fourth. 1 is CP1's third. 2 is MP3's first. 3 is CP3's second. 4 is CP2's first.

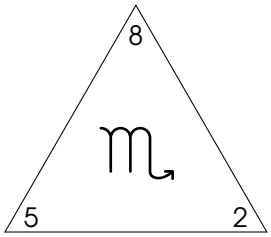
5 is MP2's second. 6 is MP1's last. 7 is MP3's fifth. 8 is CP3's first. There is no 9.

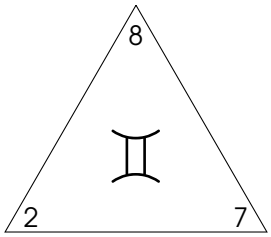
Our Twelve Signs

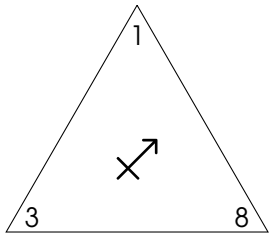


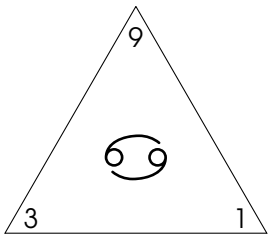


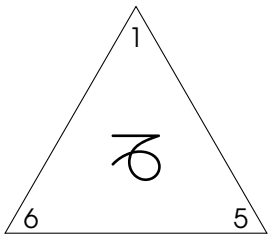


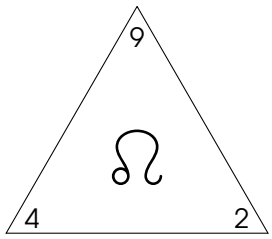


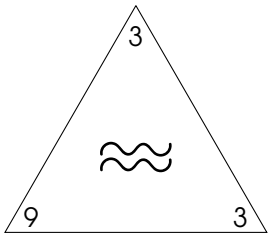


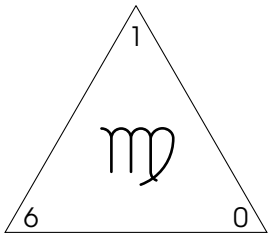


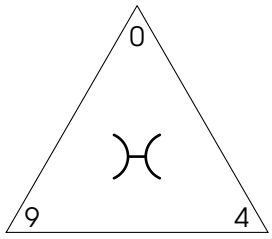












Code Reference

Letter	Decimal	Binary	Morse	Braille
A	1	00001	.-	
B	2	00010	-...	
C	3	00011	-.-.	
D	4	00100	-..	
E	5	00101	.	
F	6	00110	..-.	
G	7	00111	--.	
H	8	01000	....	
I	9	01001	..	
J	10	01010	.---	
K	11	01011	-.-	
L	12	01100	.-..	
M	13	01101	--	

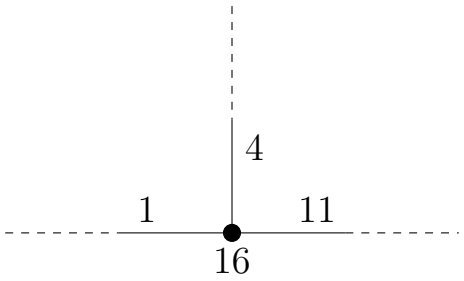
Letter	Decimal	Binary	Morse	Braille
N	14	01110	-.	
O	15	01111	---	
P	16	10000	.--.	
Q	17	10001	--.-	
R	18	10010	.-.	
S	19	10011	...	
T	20	10100	-	
U	21	10101	..-	
V	22	10110	...-	
W	23	10111	.--	
X	24	11000	-..-	
Y	25	11001	-.--	
Z	26	11010	--..	

Clues

Main Puzzle 1

To succeed at an institution like Rudin Academy, you must not only forge strong friendships, but you must also study under the appropriate mentors. But of course, it is not so easy that they would just tell you which mentors you are paired with. To find the appropriate mentors, everything must be in absolute balance. Below are pictures of several potential mentoring programs and hidden in these programs is a "magic word" to be deciphered.

The dots in the pictures represent the people (both students and mentors). Lines are connected between pairs of dots if they have a mentor/mentee relationship. Many of the lines are labeled with numbers already. Each person has a value equal to the sum of the labels of each connection (line) that person has. For instance, the person shown below has the value `16' because the lines incident to this person have labels 1,4, and 11.



In an ideal mentoring program, every person has the **same** value. (However, two people in different mentoring programs may or may not have the same value.)

Also, we are restricted with what numbers we can use to label the lines! In the first picture we can only use the integers $\{1, 2, \ldots, 9\}$ and each integer is only used once. In the second picture we can only use the integers $\{1, 2, \ldots, 16\}$ and each integer is only used once. For the third picture, we can use unique integers from the set $\{1, 2, \ldots, 12\}$ but now we notice that not all of these integers may be used!

Main Puzzle 2

Attending a private school of magic ain't cheap, and I'm pretty sure they make the tuition schedule complicated on purpose. In fact, I think the accountant in charge of billing is one of those evil "Death Drinkers" you may have heard about.

It seems what I'm supposed to do is fill out the 10×10 grid using the digits 0 through 9. It's kind of like a Sudoku: each digit should appear exactly once in every row and column, and also exactly once in every 2×5 subrectangle marked by bold borders the background shading. I've completed a 6×6 example using 2×3 subrectangles below to give you the idea.

What's especially evil is that I don't think the grid is uniquely solvable: there's gonna be some cells that you just don't have enough information to figure out. But that's okay: if you use the letters in the corners of cells that you are able to determine, you'll be able to spell out one of Rudin Academy's magic words.

2	5	1	3	0	4
4	3	0	5	2	1
0	1	2	4	5	3
3	4	5	2	1	0
1	2	4	0	3	5
5	0	3	1	4	2

Main Puzzle 3

The class I'm most excited to take is "Defense Against the Scary Spells". The curriculum found in my welcome packet includes four of the basic shield spells. Their names are basically nonsense, and it looks like the packet is missing a few letters as well. That said, if you can match each shield boundary with the graph equation that matches it below, you'll be able to figure them out.

What I've yet to learn is how to choose a shield to use against each attack. Attacks are infinitely-many points in the plane with vertical coordinates that increase to ∞. If the shield covers all except a finite number of the attack points, it's successful, but if the shield fails to cover infinitely-many of the attack points' it's defeated.

There's got to be a way to use the shield names to spell out one of Rudin Academy's magic words.

$$\texttt{\_A\_U\_U\_} \qquad y = |x| - 2$$

$$\texttt{\_I\_A\_U\_} \qquad y = \frac{5x^2 - 4}{x^2 + 1}$$

$$\texttt{\_A\_E\_R\_} \qquad y = \begin{cases} x^2 & \text{if } x \text{ is an integer} \\ -3 & \text{otherwise} \end{cases}$$

$$\texttt{\_O\_F\_I\_} \qquad y = \frac{1}{|x|} \quad (x \neq 0)$$

I Shield defeated by $\{(\frac{1}{2} - n, n - \frac{9}{2}) : n \in \mathbb{Z}^+\} = \{(-\frac{1}{2}, -\frac{7}{2}), (-\frac{3}{2}, -\frac{5}{2}), (-\frac{5}{2}, -\frac{3}{2}), \dots\}$

II Shield defeated by $\{(\frac{n^2+1}{n}, \sqrt{n}) : n \in \mathbb{Z}^+\}$

III Shield defeated by $\{(0, n^2 - 10) : n \in \mathbb{Z}^+\}$

IV Shield defeated by $\{(n - 3, 2n) : n \in \mathbb{Z}^+\}$

V Shield defeated by $\{(\frac{4}{n^2}, \frac{n}{2}) : n \in \mathbb{Z}^+\}$

VI Shield defeated by $\{(-1)^n n, n + 5) : n \in \mathbb{Z}^+\}$

VII Cannot be defeated by any attack with vertical coordinates increasing toward ∞.

Cryptic Puzzle 1

At Rudin Academy, Arithmancy is taught by a team of three professors: one who wields a wand in their left hand, one who wields a wand in their right hand, and one who wields one wand in each hand at the same time. While arithmancy problems look like the arithmetic you are used to in non-magical school, each magician computes their solutions in a unique way.

I'm told that each of these professors wrote three exercises for the school's placement exam. I expect that a magic word can be revealed by matching the nine provided solutions from the answer bank to each exercise, once you make a slight adjustment based on the nature of each exercise's author.

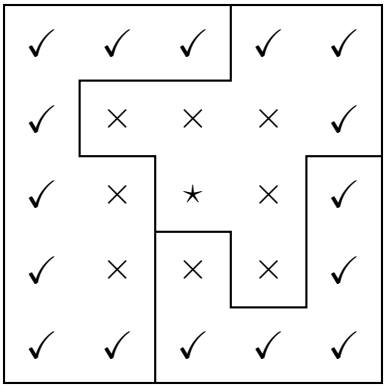
Cryptic Puzzle 2

Rudin Academy is well-known for its Astronomy course, which uses patterns in the night sky to provide cryptic predictions about the days to come.

"Our Night Sky" in the welcome packet is an example of such a divination. Hidden among the one hundred letters in this square are twenty "stars" that spell out a message. The location of these stars may be divined by considering the following four rules:

- Exactly two stars appear in each horizontal row.
- Exactly two stars appear in each vertical column.
- Exactly two stars appear in each shape within the square.
- No star ever appears directly next to another star (vertically, horizontally, or diagonally), as illustrated below.

Not only does this message answer the question "When did the magician know he had discovered something special?", but it also contains one of the academy's magic words.



Cryptic Puzzle 3

Like yourselves, magicians observe a 365 day year, except that every fourth year (a "leap year") must include a 366th day. Likewise, their weeks generally include seven days, Monday through Sunday. But the magical calendar uses 13 months rather than the usual 12: Jan, Feb, Mar, Apr, May, Jun, Sol, Jul, Aug, Sep, Oct, Nov, and Dec. This allows each month to include 28 days, a sensible pattern which guarantees that the 1st of each month is always a Monday, and the 28th is always a Sunday.

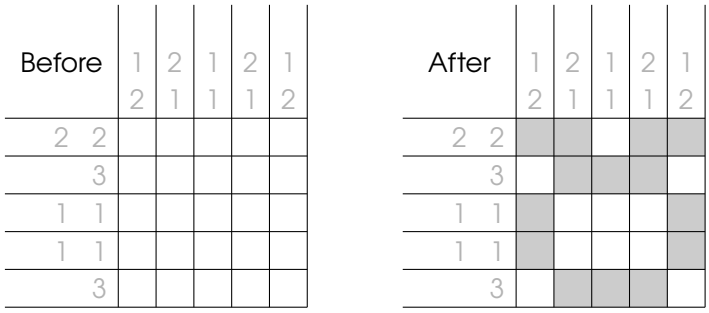
Ah you say, but that only adds up to 364 days a year? Very good! So to account for this, a 29th day, called Magday, occurs on December 29 every year, and on June 29 every leap year. As the name might suggest, magical powers are particularly powerful on Magday.

Knowing this will be important to help me plan my schedule. I have to complete six special courses throughout my time at the Academy. Each special course lasts a specific number of days, and is designed so that a specific day of the course must be a Magday. Additionally, each such course must begin on the day immediately following the end of the previous course: for example, if a course ends on Sol 28, the next course must begin on Jul 1. Also, these courses require a lot of work, so no skipping weekends or any other day.

I get the feeling that there's really only one way I can make this work, and if you can help me figure it out, you'll find yet another magic word.

Metapuzzle

Thanks for all your help revealing the school's magic words for me, but it's about time to uncover the most magical word of all. My guess is that the regular magic words need to be combined to create a spell that will reveal the school's Seal. The only other clues I have for you are the images below...



$$\gamma, \varepsilon, \beta, \delta - 30, \alpha + 3, \zeta - 5$$

Bonus Puzzle

Hey, it looks like you're pretty good at this Rune stuff, huh? For a few bonus points, maybe you can help me use all twelve unique Signs to design four Runes, such that as many of the following two-digit numbers are used for δ, ε, and ζ as possible:

- 02
- 03
- 12
- 13
- 26
- 30
- 34
- 41
- 59
- 79
- 81
- 83

Bring your ClueKeeper app to the location designated by your game's organizers to score your solution. If you do, you'll earn $N-4$ points, where N counts how many of the above twelve numbers are included in your four Runes (making this puzzle worth a maximum of 8 points).

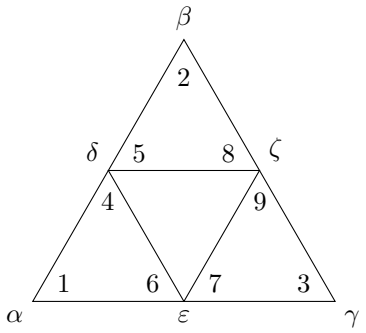
Unlocking puzzles (instructions/illustration omitted in ClueKeeper for Cryptic Puzzle unlocks)

To unlock this puzzle, we'll first need to determine its magic code, a three-digit number. A magic code is obtained by combining three of the school's twelve Signs from the welcome packet into the configuration illustrated below. Then the code is given by the Greek letters $\alpha\beta\gamma$; for the example below, the magic code would be 123.

In the example below, we also have $\delta = 45$, $\varepsilon = 67$, and $\zeta = 89$. For this puzzle's magic code, you'll need to find the only combination of Signs that satisfies the following requirements:

- δ VARIES
- ε VARIES
- ζ VARIES

Once you've figured out the magic code $\alpha\beta\gamma$, submit it in the ClueKeeper app to unlock this puzzle. (If your game is using ClueKeeper's GPS feature, you'll need to be at the location corresponding to this code given on your Campus Map to submit it.)



Unlocking puzzle clues
Main Puzzle 1

- δ: pentagonal number
- ε: digits sum to 10
- ζ: digits sum to 4

Main Puzzle 2

- δ: abundant, and digits sum to 15
- ε: digits sum to 3
- ζ: prime larger than 10

Main Puzzle 3

- δ: twin prime
- ε: least whole number
- ζ: two less than a multiple of 11

Cryptic Puzzle 1

- δ: twin prime
- ε: prime less than 10
- ζ: prime but not a twin prime

Cryptic Puzzle 2

- δ: even
- ε: digits sum to 14
- ζ: multiple of δ

Cryptic Puzzle 3

- δ: used as δ, ε or ζ for Cryptic Puzzle 1
- ε: sum of six consecutive primes
- ζ: pentagonal number

Hidden Puzzle
Have you noticed the "fantastic" connection between all the magic words?

Solutions

- Main Puzzle 1 [RUNE]: 353
- Main Puzzle 1: QUINOTAUR
- Cryptic Puzzle 1 [RUNE]: 611
- Cryptic Puzzle 1: HOBGOBLIN
- Main Puzzle 2 [RUNE]: 416
- Main Puzzle 2: MANTICHORE
- Cryptic Puzzle 2 [RUNE]: 664
- Cryptic Puzzle 2: DRAKAINA
- Main Puzzle 3 [RUNE]: 615
- Main Puzzle 3: CHIMERA
- Cryptic Puzzle 3 [RUNE]: 231
- Cryptic Puzzle 3: WYVERN
- Metapuzzle: AMAZED
- Hidden Puzzle: FANTASTIC BEASTS