



Middle School Challenge '17

Mathematical Puzzle Programs



Middle School Challenge '17

| | | |
|----------|-------------------------------------|-----------|
| 1 | Which Witch? | 1 |
| 1.1 | Main Puzzle 1 | 1 |
| 1.2 | Solution - Main Puzzle 1 | 2 |
| 1.3 | Bonus Puzzle 1 | 4 |
| 1.4 | Solution - Bonus Puzzle 1 | 5 |
| 2 | Fear the Hungry Dead | 6 |
| 2.1 | Main Puzzle 2 | 6 |
| 2.2 | Solution - Main Puzzle 2 | 7 |
| 2.3 | Bonus Puzzle 2 | 8 |
| 2.4 | Solution - Bonus Puzzle 2 | 9 |
| 3 | Count on It | 10 |
| 3.1 | Main Puzzle 1 | 10 |
| 3.2 | Solution - Main Puzzle 1 | 11 |
| 3.3 | Bonus Puzzle 1 | 12 |
| 3.4 | Solution - Bonus Puzzle 1 | 13 |



Middle School Challenge '17

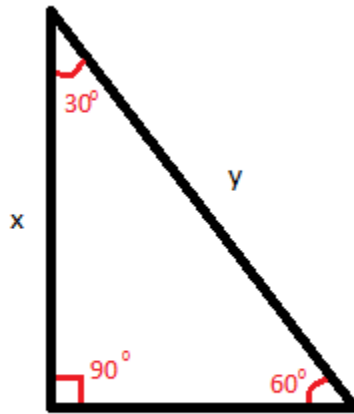
Which Witch?

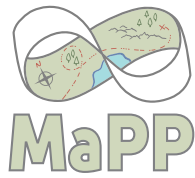
Main Puzzle 1

Grendel the Witch is cooking up a storm! She is brewing potions in several cauldrons.

Grendel needs to make at least three potions at a time, so she needs to use at least 3 cauldrons at once. If the cauldrons have a radius of 3 feet and Grendel must be equally close to all cauldrons used, what is the shortest distance Grendel can be from the center of a cauldron?

There is a special triangle which might help you answer this puzzle: the 30-60-90 triangle. Let x be the side opposite of the 60° angle. Let y be the side opposite the 90° angle. It must be that $y = \frac{2}{\sqrt{3}}x$.



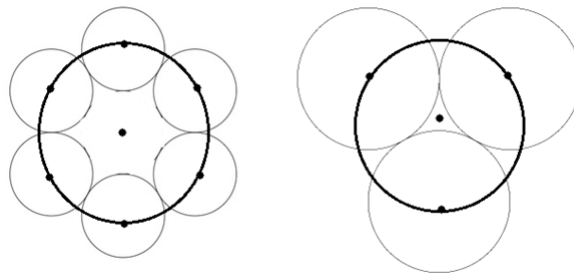


Middle School Challenge '17

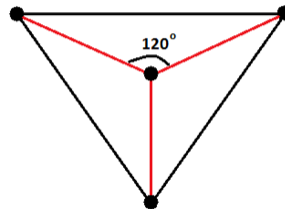
Which Witch?

Solution - Main Puzzle 1

Since Grendel is equally close to the center of every cauldron used, these centers lay on a big circle with Grendel as the center. The distance to Grendel is minimized by making no gaps between any two cauldrons. As we use fewer and fewer cauldrons the radius of this “big circle” is minimized. This means using 3 cauldrons minimizes the distance from Grendel to a cauldron.

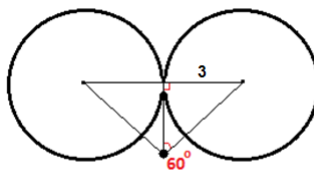


To determine how close Grendel is in the circle to the right we note the triangle formed by connecting the three circle centers and note Grendel is equally close to them.



The black lines are equal length and the red lines are equal length so we have 3 equivalent isosceles triangles within an equilateral triangle. So the three angles at the center all equal to one another and sum up to 360° . This means each of these angles is 120° .

Now let's look at just the “top two” circle centers, as well as Grendel.



By drawing upward from Grendel we will cut the segment between the two circle centers in half. However, what we've really done is form a 30, 60, 90 triangle whose side opposite to the 60° angle is 3. The hypotenuse

of this triangle happens to be the distance we are trying to find so all we must do now is use the formula we were given at the start: $y = \frac{2}{\sqrt{3}}x$. (Where y is the hypotenuse and x is the side opposite to the 60° angle)

Plugging in 3 for x yields $y = \frac{6}{\sqrt{3}} = 2\sqrt{3}$.

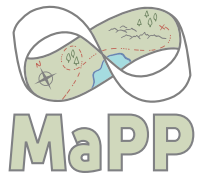


Middle School Challenge '17

Which Witch?

Bonus Puzzle 1

Grendel decided she's working too hard so she got her brooms to do the work for her. Grendel set up several brooms and cauldrons in her basement. Every broom is equally close to 3 cauldrons and is $\frac{2}{\sqrt{3}}$ feet away from them. What is the maximum number of brooms a single cauldron can be $\frac{2}{\sqrt{3}}$ feet away from? What shape do all the brooms surrounding one cauldron form?



Middle School Challenge '17

Which Witch?

Solution - Bonus Puzzle 1

6, a hexagon.

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Middle School Challenge '17

Fear the Hungry Dead

Main Puzzle 2

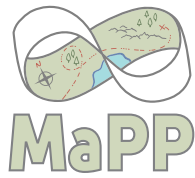
The insatiable Carli Ivor eats anything in her path. Last year at Count Calcula's party she ate ALL of the Counts famous meat pie! She started by eating half of the pie. Then she ate half of the remaining pie. She continued to have more servings, each time eating half of the remaining pie. A picture of what she ate last year is given below.

This year, the Count gave Carli some stipulations to the amount she could eat. If she eats more than her allotment she will not be invited to anymore parties! Carli is terrified she will not be able to eat anymore of the delicious meat pie, so she sent along a request.

..*grow!*...*moaning*.."Ahem, sorry, something was caught in my throat. It must have been who, I mean what, I ate earlier. Count Calcula gave me some odd rules to eat by and I need help understanding them! At first, I am allowed to eat one fourth of the entire meat pie! The Count then said if I was still hungry, I could eat a fourth of the portion that I just ate! Then if I was STILL hungry I could eat a fourth of the most recent portion! He said I could continue in this manner until I was full.

Well, since I will not get full this poses a problem. Just tell me the most I am allowed to eat in order to satisfy the Counts restrictions!

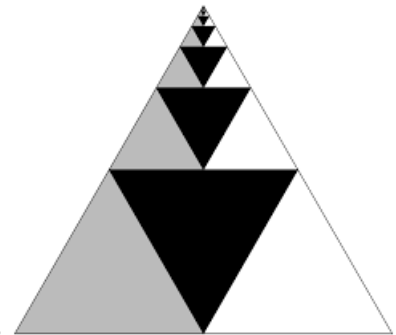
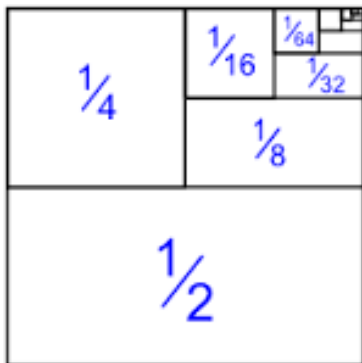
Carli Ivor has a puzzle piece if you can answer her question.



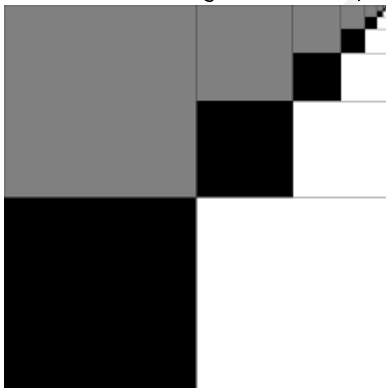
Middle School Challenge '17

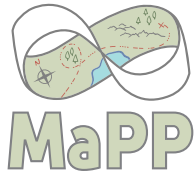
Fear the Hungry Dead

Solution - Main Puzzle 2



The following are two separate pictures to show that the answer is $\frac{1}{3}$





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Fear the Hungry Dead

Bonus Puzzle 2

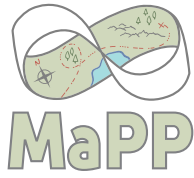
Carli was pleased with your previous answer, but Count Calcula is devious, and may yet stop Carli Ivor from eating his pies.

Carli Ivor's calculations are as follows $s_1 = \frac{1}{5} = .2$

$$s_2 = \frac{1}{5} + \frac{1}{5^2} = \frac{1}{5} + \frac{1}{25} = .2 + .04 = .24$$

$$s_3 = \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} = .2 + .04 + .008 = .248$$

She did a few more calculations and realized that the answer was becoming close to .25 and she concluded that if she continued forever, that one fourth was the most she would be able to eat (which is correct!).



Middle School Challenge '17

Fear the Hungry Dead

Solution - Bonus Puzzle 2

This is a well known series called a geometric series. If $|r| < 1$, then $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$. In particular, if $a = \frac{1}{x}$ and $r = \frac{1}{x}$, then $\sum_{n=1}^{\infty} ar^{n-1} = \sum_{n=1}^{\infty} (\frac{1}{x})(\frac{1}{x})^{n-1} = \frac{\frac{1}{x}}{1-\frac{1}{x}} = \frac{1}{x-1}$.

We expect students to plug in the first few terms of several different series, and take an educated guess for the solution. Try $x=6, 7$, and 8 !



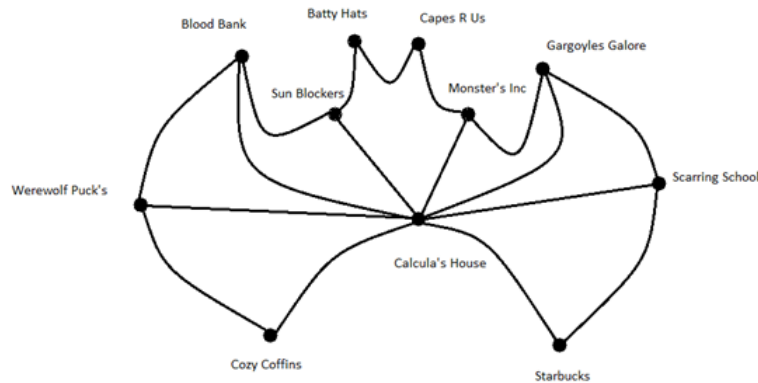
Middle School Challenge '17

Count on It

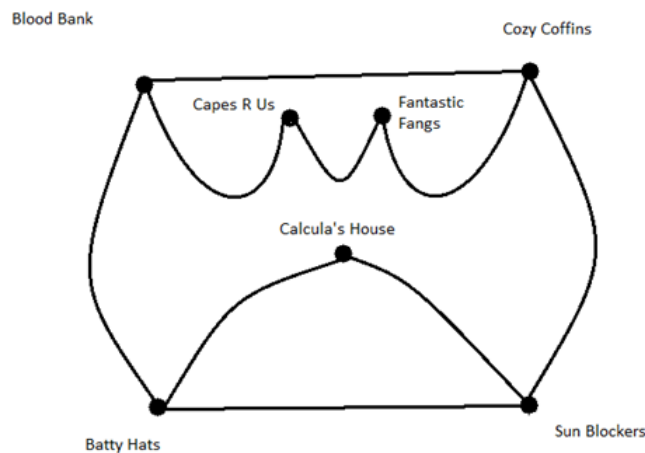
Main Puzzle 1

Count Calcula is making plans to build a new part of Transylvania. The cost to construct a building is 100 dragon teeth per road leading to that building. He wants you to determine the cost of the three plans below.

Plan A)



Plan B)



Plan C) Igor misplaced the blueprints to Plan C but says there are 75 roads.



Middle School Challenge '17

Count on It

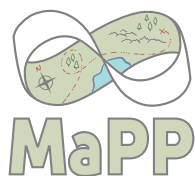
Solution - Main Puzzle 1

This problem is essentially the degree-sum formula for graph theory. The degree of a building is the number of roads which lead to it. Adding up the degrees of every building will actually count the number of road twice. Why? Every road leads to exactly two buildings, meaning every road contributes exactly 2 to the sum of degrees.

With this in mind we see Plan A has 17 roads, so the sum of degrees of every building is 34 meaning it will cost 3400 dragon teeth to construct Plan A.

Plan B has 9 roads, so 18 is the sum of degrees meaning it will cost 1800 dragon teeth.

Plan C has 75 roads, so 150 is the degree-sum meaning it will cost 15,000 dragon teeth.



Middle School Challenge '17

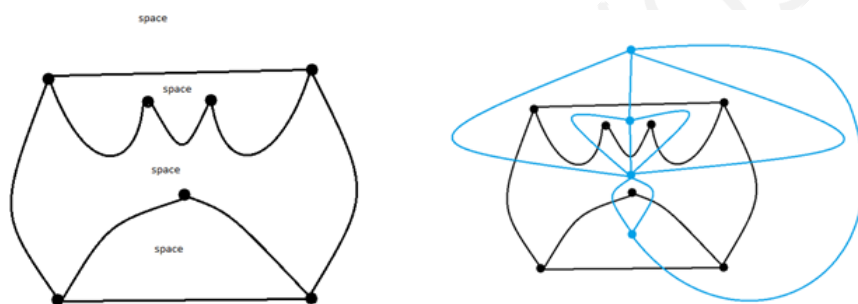
Count on It

Bonus Puzzle 1

A ghost city will be built around the new addition to Transylvania and follows a few rules which make haunting easier. The rules are as follows:

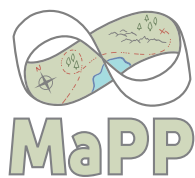
- 1) Every space in Transylvania must have exactly one ghost building.
- 2) A ghost road is made if it joins two ghost buildings by crossing over a road in Transylvania.

Below is the ghost city of Transylvania Plan B:



If Transylvania Plan D has 33 roads what is the cost of constructing the ghost city?

If the length of a space is the number of roads the space touches what is the sum of lengths of spaces in Plan D?



Middle School Challenge '17

Count on It

Solution - Bonus Puzzle 1

Cost: 6600 dragon teeth. Length-sum: 66

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