Middle School Challenge ’17

Mathematical Puzzle Programs

Version 2016/11/04

Middle School Challenge '17

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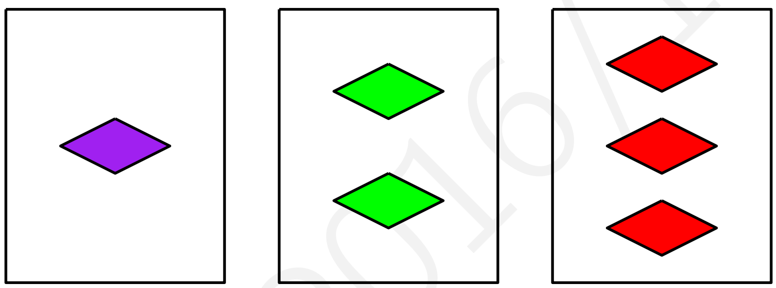
Ready, SET, Go!

Opening Puzzle

You'll be faced with quite a few puzzling challenges today, so let's start with a quick warm-up! Each team is given a table for their home base for this game.

Your team will begin by collecting game cards from our staff. To collect a card, you'll need to answer the arithmetic question tossed up by the card's holder. If you're the first person to respond with the correct answer, you can take their card back to your team's table. Once you've delivered your card, you may then attempt to earn more cards.



As your team collects many game cards, you will eventually have a three-card SET. Not every combination of three cards forms a SET, however... in a SET, each of the following features is either *all the same* or *all different*:

- **Color:** red, green, or purple.
- **Shape:** ovals, squiggles, or diamonds.
- **Number:** one, two, or three.
- **Shading:** empty, solid, or striped.

The collection above is a SET because it has all different colors, all the same shapes, all different numbers, and all the same shadings. After you've found a SET, all of the cards at your table are returned to our staff, and you'll start over again.

Once your team has found 5 SETs, you can move on to the first round of puzzles. Good luck!

Middle School Challenge '17

Ready, SET, Go!

Random Arithmetic

- $30 + 57 = 87$
- $25 - 15 = 10$
- $11 \times 23 = 253$
- $62 + 100 = 162$
- $46 + 95 = 141$
- $81 - 70 = 11$
- $234 \div 18 = 13$
- $21 + 174 = 195$
- $120 \div 15 = 8$
- $25 \times 9 = 225$
- $10 + 34 = 44$
- $22 - 11 = 11$
- $23 \times 8 = 184$
- $552 \div 24 = 23$
- $166 - 137 = 29$
- $62 - 52 = 10$
- $176 - 32 = 144$
- $18 \times 24 = 432$
- $62 - 13 = 49$
- $144 \div 9 = 16$
- $391 \div 23 = 17$
- $92 + 10 = 102$
- $15 \times 15 = 225$
- $135 + 58 = 193$
- $15 \times 7 = 105$
- $56 - 20 = 36$
- $27 + 165 = 192$
- $84 \div 12 = 7$
- $50 - 17 = 33$
- $54 - 14 = 40$
- $73 - 30 = 43$
- $40 + 21 = 61$
- $189 - 126 = 63$
- $209 \div 19 = 11$
- $24 \div 6 = 4$
- $11 \times 6 = 66$
- $177 - 85 = 92$
- $14 + 13 = 27$
- $128 - 25 = 103$
- $59 + 22 = 81$
- $91 + 26 = 117$
- $27 - 16 = 11$
- $6 \times 4 = 24$
- $32 + 125 = 157$
- $240 \div 10 = 24$
- $7 \times 4 = 28$
- $6 \div 2 = 3$
- $21 \times 22 = 462$
- $9 \times 14 = 126$
- $105 - 86 = 19$
- $40 + 41 = 81$
- $104 \div 8 = 13$
- $294 \div 21 = 14$
- $108 \div 6 = 18$
- $110 \div 10 = 11$
- $153 - 91 = 62$
- $220 \div 22 = 10$
- $151 + 39 = 190$
- $46 + 18 = 64$
- $193 - 164 = 29$
- $46 - 22 = 24$
- $21 \times 19 = 399$
- $54 + 98 = 152$
- $10 \times 21 = 210$
- $90 + 45 = 135$
- $124 + 66 = 190$
- $24 - 13 = 11$
- $62 - 23 = 39$
- $9 \times 20 = 180$
- $19 \times 25 = 475$
- $315 \div 15 = 21$
- $164 - 51 = 113$
- $94 - 63 = 31$
- $98 + 29 = 127$
- $10 + 32 = 42$
- $25 + 23 = 48$
- $380 \div 20 = 19$
- $43 - 15 = 28$
- $5 \times 8 = 40$
- $68 + 84 = 152$
- $29 - 18 = 11$
- $396 \div 22 = 18$
- $110 - 61 = 49$
- $73 - 47 = 26$
- $16 + 12 = 28$
- $59 - 25 = 34$
- $120 \div 6 = 20$
- $25 + 139 = 164$
- $25 - 11 = 14$
- $270 \div 15 = 18$
- $12 \times 18 = 216$
- $168 - 101 = 67$
- $12 \times 6 = 72$
- $6 \times 11 = 66$
- $2 \times 14 = 28$
- $16 + 12 = 28$

- $18 \times 19 = 342$
- $18 \times 3 = 54$
- $23 + 107 = 130$
- $17 \times 11 = 187$
- $11 \times 16 = 176$
- $87 + 29 = 116$
- $11 \times 20 = 220$
- $35 + 11 = 46$
- $4 \times 13 = 52$
- $84 \div 12 = 7$
- $17 \times 9 = 153$
- $20 \div 2 = 10$
- $61 - 30 = 31$
- $5 \times 3 = 15$
- $48 - 33 = 15$
- $8 \times 11 = 88$
- $117 - 48 = 69$
- $199 - 29 = 170$
- $9 \times 6 = 54$
- $48 - 26 = 22$
- $58 + 98 = 156$
- $96 \div 4 = 24$
- $149 - 104 = 45$
- $56 + 106 = 162$
- $113 - 71 = 42$
- $23 \times 15 = 345$
- $74 - 30 = 44$
- $18 \times 10 = 180$
- $176 \div 8 = 22$
- $110 - 68 = 42$
- $29 + 117 = 146$
- $66 \div 22 = 3$
- $18 \div 2 = 9$
- $7 \times 2 = 14$
- $50 \div 25 = 2$
- $147 - 23 = 124$
- $17 + 174 = 191$
- $28 \div 4 = 7$
- $144 - 73 = 71$
- $36 - 25 = 11$
- $142 - 16 = 126$
- $19 \times 7 = 133$
- $67 - 42 = 25$
- $163 - 143 = 20$
- $19 + 35 = 54$
- $42 + 50 = 92$
- $82 - 41 = 41$
- $16 \times 14 = 224$
- $4 \times 10 = 40$
- $70 + 26 = 96$
- $178 - 61 = 117$
- $14 \times 19 = 266$
- $111 - 55 = 56$
- $100 - 18 = 82$
- $17 \times 6 = 102$
- $20 + 151 = 171$
- $33 + 12 = 45$
- $81 \div 9 = 9$
- $63 \div 7 = 9$
- $69 - 13 = 56$
- $85 \div 17 = 5$
- $552 \div 24 = 23$
- $14 \times 20 = 280$
- $12 \div 3 = 4$
- $114 \div 6 = 19$
- $30 + 69 = 99$
- $256 \div 16 = 16$
- $13 \times 25 = 325$
- $45 + 110 = 155$
- $10 + 57 = 67$
- $25 \times 20 = 500$
- $14 + 119 = 133$
- $17 + 27 = 44$
- $10 \times 10 = 100$
- $10 + 32 = 42$
- $79 + 14 = 93$
- $12 + 22 = 34$
- $139 - 37 = 102$
- $38 + 44 = 82$
- $73 - 21 = 52$
- $92 + 106 = 198$
- $10 \times 5 = 50$
- $22 \times 10 = 220$
- $18 + 18 = 36$
- $118 + 25 = 143$
- $76 + 49 = 125$
- $6 \times 8 = 48$
- $17 \times 16 = 272$
- $11 \times 20 = 220$
- $24 \div 3 = 8$
- $195 - 73 = 122$
- $193 - 156 = 37$
- $5 \times 5 = 25$
- $73 - 11 = 62$
- $2 \times 18 = 36$
- $71 - 14 = 57$
- $17 + 50 = 67$
- $75 + 44 = 119$
- $67 - 41 = 26$
- $69 - 22 = 47$
- $20 \div 4 = 5$
- $86 - 11 = 75$
- $91 - 43 = 48$
- $50 + 53 = 103$

Middle School Challenge '17

Ready, SET, Go!

Staff Instructions

To run this opening event, each team should have one staff member assigned to their table, to judge when the team thinks they have found a SET. For each SET, the judge should collect all cards that were on the table when the SET was found (not just the three forming a SET), and return these cards to a nearby caller. The judge is responsible for keeping score for their own team, and for guiding the team to their headquarters once they have scored a total of five SETs.

Several staff members (approximately one for every 2-3 teams) should serve as callers. The callers hold many SET cards, and call out arithmetic problems from the Random Arithmetic pages (or other randomly generated arithmetic problems; order need not be followed). When the caller hears the first correct response from a player, they give a SET card to that player (unless the player already has a SET card in their possession, in which case they are disqualified for that question and a new question should be called). They should replenish their cards from each other or from teams who have recently scored a SET.

This game requires at least one copy of the SET card game for every five teams.

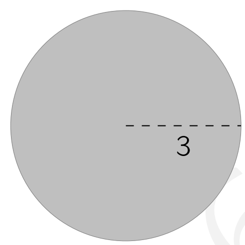
Middle School Challenge ’17

Which Witch?

Main Puzzle 1

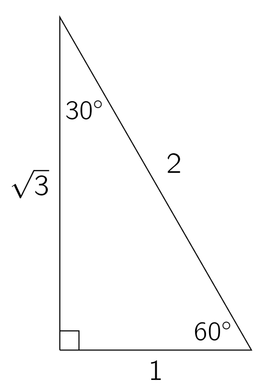
Grendel the Witch is doing what witches do: brewing potions!

In order to work efficiently, Grendel wants to use three cauldrons at the same time. Viewed from the top-down, each cauldron is perfectly circular, with a radius of 3 feet.



In order to stir each position successfully, Grendel must be equally close to all three cauldrons used. **How can Grendel position the cauldrons so that she’s as close as possible to all three? How far would she be from the center of each cauldron in this configuration?** She’s a frail old thing, so don’t be afraid to squeeze her into a small space if necessary...

By the way, some of Grendel’s *mathemagics* may help here. She has a feeling that the following triangle is important to this particular puzzle. It is drawn below such that the shortest side has length 1, but you may need to scale things a bit...

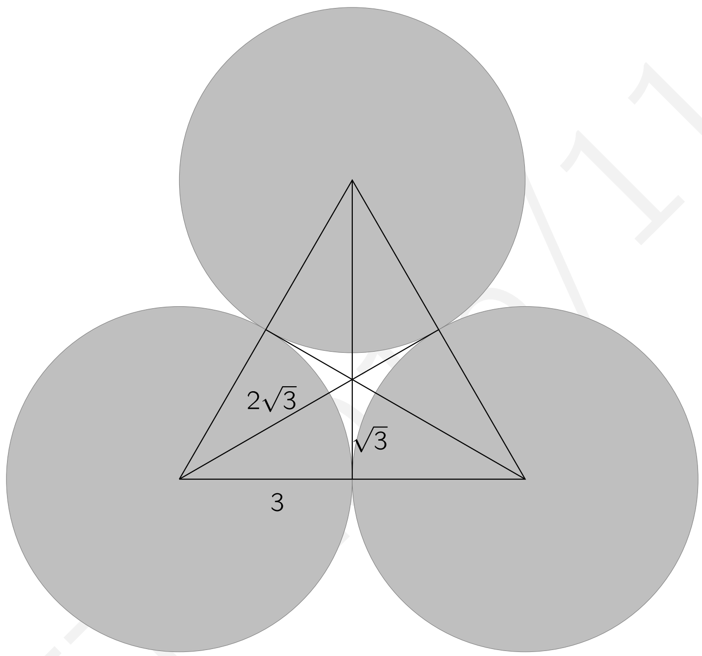


Middle School Challenge '17

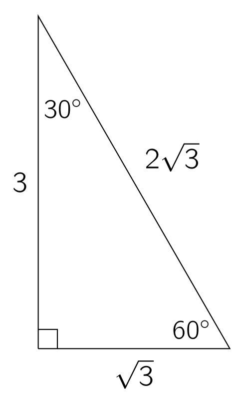
Which Witch?

Solution - Main Puzzle 1

The solution, $2\sqrt{3}$ feet, is shown by the following diagram.



Each small triangle is 30-60-90 as they were dissected from an equilateral 60-60-60 triangle. The appropriate side lengths are obtained by scaling the triangle given in the puzzle by a factor of $\sqrt{3}$.

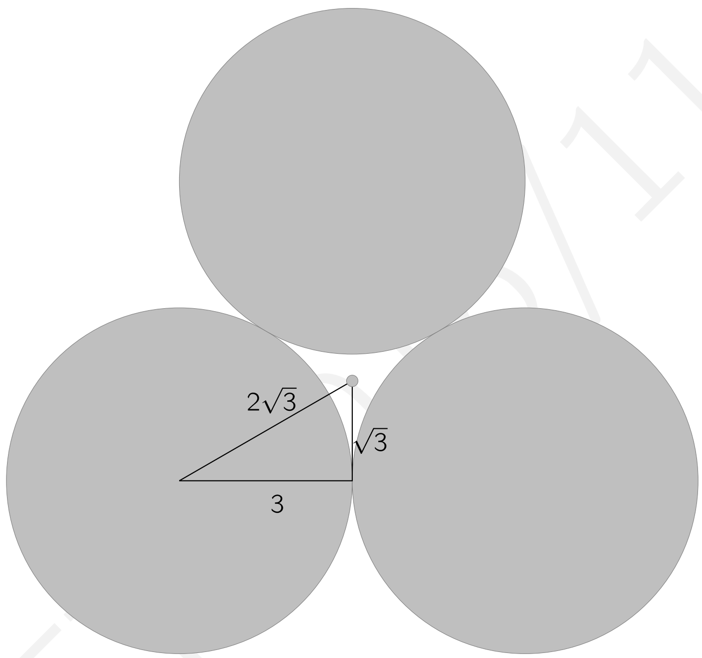


Middle School Challenge '17

Which Witch?

Bonus Puzzle 1

Now Grendel has bewitched a broom to do the hard work of stirring her caudlrons for her, as shown below.



Magicked brooms are rather finicky however, and unlike Grendel won't work in too tight a space. So, **can you figure out the area of the small space between the three cauldrons**? If it helps, the area of a circle is πr^2 where r is the radius of the circle, and the area of a triangle is $\frac{1}{2}bh$ where b is the length of its base and h is the height of the triangle measured perpendicularly to the base.

Mark your answer below.

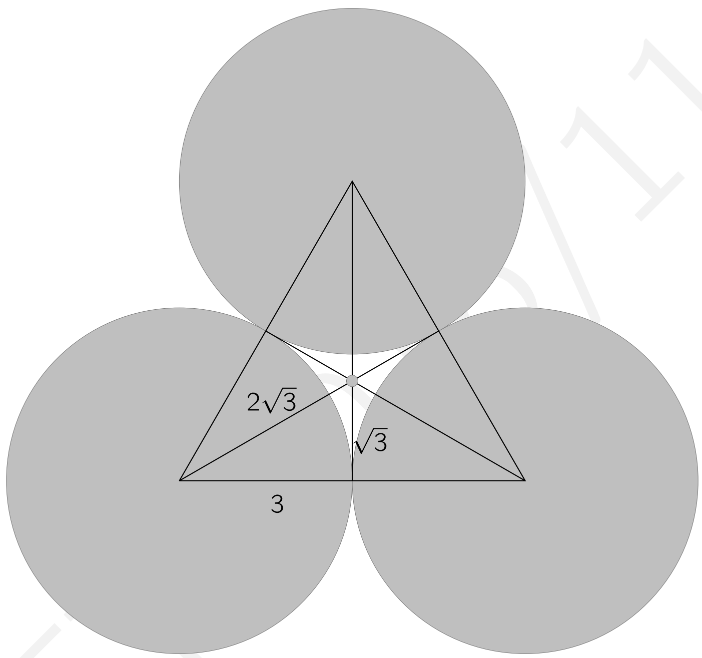
- ○ $2\pi\sqrt{3}$
- ○ $9\sqrt{3} - \frac{9}{2}\pi$
- ○ $4\sqrt{6} + 3\pi$
- ○ $6\pi\sqrt{6}$
- ○ $\pi\sqrt{3} + \frac{9\sqrt{3}}{4}$

Middle School Challenge '17

Which Witch?

Solution - Bonus Puzzle 1

The solution may be obtained by counting the triangles and circle wedges from the solution to the Main Puzzle:



Each small triangle has area $A_T = \frac{1}{2}(3)(\sqrt{3}) = \frac{3\sqrt{3}}{2}$, so the total area in the large triangle is $6A_T = 9\sqrt{3}$.

Meanwhile, each gray circle has area $A_C = \pi(3)^2 = 9\pi$.

Since the large triangle contains six circle wedges, each of which (witch?) is $\frac{1}{12}$ a circle, the area containing the broom is $A = 6A_T - \frac{1}{2}A_C = 9\sqrt{3} - \frac{9}{2}\pi$.

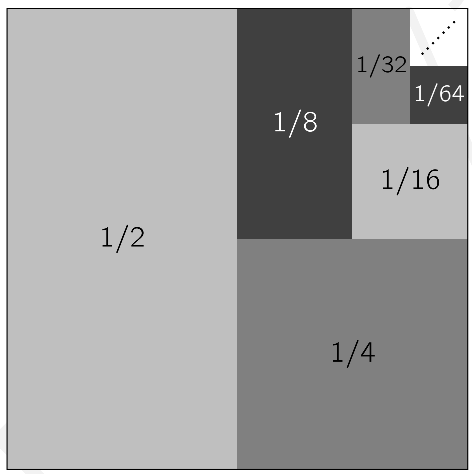
Middle School Challenge ’17

Fear the Hungry Dead

Main Puzzle 2

The insatiable Zom Bea eats anything in her path. Last year at Count Calcula’s party she ate ALL of the Count’s famous meat pie! She started by eating half of the pie... then she ate half of the remaining pie! Actually, she continued to have more and more servings (and blocking anyone else from enjoying even one bite), each time eating half of the remaining pie.

Here’s a picture to give you the idea. (Hey, what kind of monster bakes a square pie, though?)



Since there was no pie left for anyone else, the Count has given Zom some stipulations for the amount she is allowed to eat this year. (Otherwise, guess who isn’t getting invited to any future parties!) The Count has made it clear: “Zom’s first serving must be no larger than $\frac{1}{4}$ the entire pie, and any future serving cannot be larger than $\frac{1}{4}$ the most recent serving!”

Zom is terrified that this delicious meat pie will be her last, so maybe you can help her out. **What fraction of the pie is she allowed to eat? Make sure you draw a picture which explains why.** Bear in mind, she feels like she could eat servings of the delectible pie *forever*.

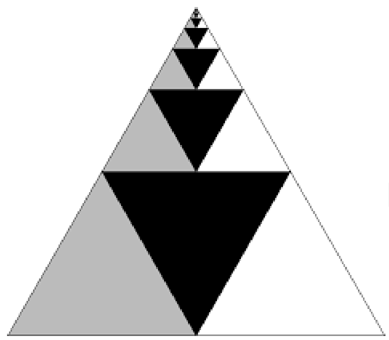
Oh, and Zom doesn’t know what shape the pie will be this year, but does it matter?

Middle School Challenge '17

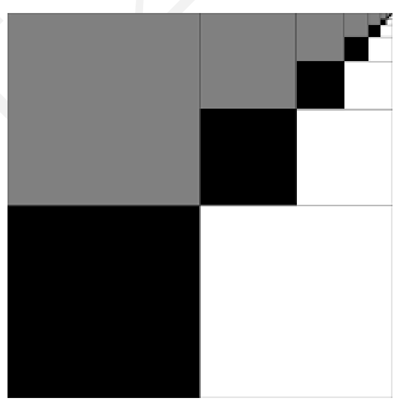
Fear the Hungry Dead

Solution - Main Puzzle 2

The answer is 1/3. Here's a handy image which explains why: each gray triangle represents 1/4 of the previous serving, and each row contains 1/3 gray triangles.



This image also works (as do many others); simliar to the above triangle, each gray square represents 1/4 of the previous serving, and each L-shape contains 1/3 gray squares.



Middle School Challenge ’17

Fear the Hungry Dead

Bonus Puzzle 2

After quickly gobbling up her third of the pie, Zom Bea realized that Count Calcula hadn’t counted on her eating all her servings at once!

“Very well!” said the Count. “I see that you will never be satisfied. So, I will continue to bake you my meat pies. In 2017, I will bake you 1/17 of a pie. In 2018, I will bake you 1/18 of a pie. And so on! Are you happy now?”

Zom’s not sure, exactly. Since zombies and vampires never die, **how many total meat pies can Zom count on enjoying over the years to come?**

Mark your answer below.

- ○ Less than 1
- ○ At least 1, but less than 2
- ○ At least 2, but less than 10
- ○ At least 10, but less than 100
- ○ At least 100

Middle School Challenge '17

Fear the Hungry Dead

Solution - Bonus Puzzle 2

The Count is offering up what mathematicians call a "harmonic series" of meat pies:

$$\sum_{i=17}^{\infty} \frac{1}{i} = \frac{1}{17} + \frac{1}{18} + \frac{1}{19} + \dots$$

Actually, unlike in the Main Puzzle, this infinte sum is just that: infinite! Here's a nice way to see that this sum cannot be finite.

$$\begin{aligned} \textstyle\sum_{i=17}^{\infty} \frac{1}{i} &= (\tfrac{1}{17} + \dots + \tfrac{1}{32}) + (\tfrac{1}{33} + \dots + \tfrac{1}{64}) + \dots \\ &\geq (\tfrac{1}{32} + \dots + \tfrac{1}{32}) + (\tfrac{1}{64} + \dots + \tfrac{1}{64}) + \dots \\ &= \tfrac{16}{32} + \tfrac{32}{64} + \dots \\ &= \tfrac{1}{2} + \tfrac{1}{2} + \dots \\ &= \infty \end{aligned}$$

So, it might take a while, but Zom has at least 100 pies ahead of her to enjoy.

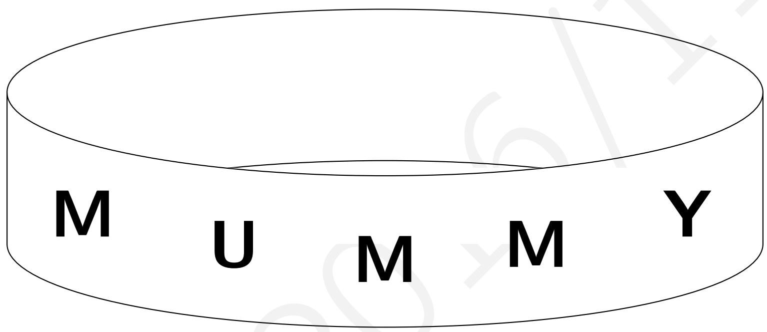
Middle School Challenge ’17

Mummy Masquerade

Main Puzzle 3

Marvin Mummy has decided that this is finally the year he doesn’t go as himself for Halloween. And he’s figured out the costume that will finally make him the winner of Count Calcula’s annual costume contest. “I’ll dress up as *Mummy Man*, the undead superhero with the power to wrap up crime!”

His lack of imagination aside, Marvin has at least decided to put a little creativity into the design of his superbelt (the source of Mummy Man’s powers, duh!).



“I want curves connecting each letter to each of the other letters, but I do NOT want those curves to cross! They can go around or even behind on the back of the belt, but I don’t want the curve crossing another curve!”

Tape a strip of paper to make your own version of Mummy Man’s belt, writing the letters M U M M Y anywhere you like. **How can you draw ten non-crossing curves which connect each pair of letters?** Remember, these curves are allowed to go around and behind the belt; actually, they’ll have to!

Middle School Challenge ’17

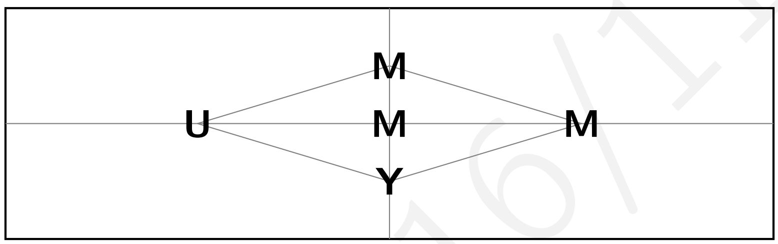
Mummy Masquerade

Solution - Main Puzzle 3

This problem is equivalent to asking one to embed a complete graph K_5 with 5 vertices onto the torus.

A solution can be drawn on a two-dimensional rectangle where points on opposite sides are identifed (taped) together.

There are multiple solutions to this problem; here’s one as an example.



Middle School Challenge '17

Mummy Masquerade

Bonus Puzzle 3

Thanks to your help, Marvin's got the coolest super belt in town!

"Awesome! But... could I do any better? I'd love a similar belt with the letters M A R V I N. Oh oh! Could I fit M O N S T E R? I gots to know!"

What do you think? **What's the maximum number of letters which can be connected by non-crossing lines on Marvin's belt?**

Mark your answer below.

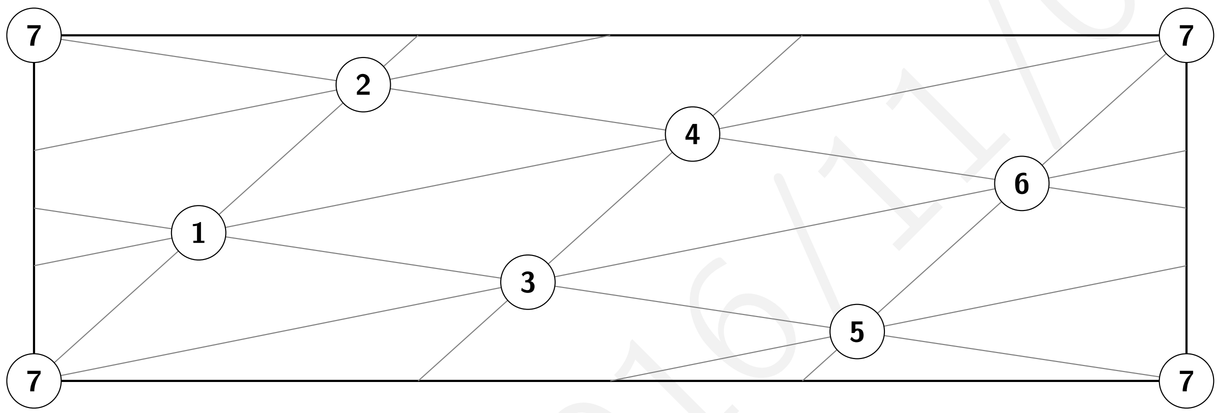
- ○ 5
- ○ 6
- ○ 7
- ○ 8
- ○ 9 or more

Middle School Challenge '17

Mummy Masquerade

Solution - Bonus Puzzle 3

The answer is 7 (although, the proof that 8 is impossible is non-trivial). Here's one such embedding:

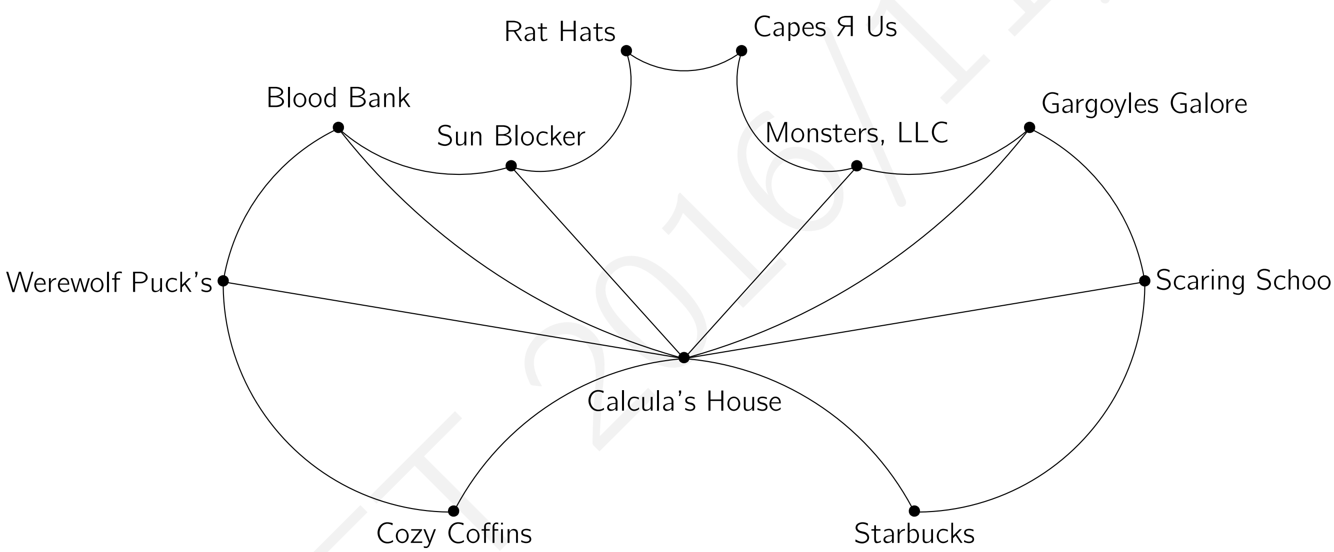


Middle School Challenge '17

Count on It

Main Puzzle 4

Count Calcula is planning construction for a new town called "Batty Borough" outside Transylvania. As everyone knows, the cost to construct a building is **100 dragon teeth** per road leading into that building. "Ack! Vat a pain in the neck! I vish someone vould help me by *biting*, er, writing out the cost of my building plan!"



What would be the total cost of building Batty Borough?

Actually, the Count's ulimate goal is to renovate all of Transylvania itself, a monsterous metropolis with **57 buildings** and **73 roads**. He calculates that renovating each building would only cost **50 dragon teeth** per road leading into it, but he cannot seem to track down a roadmap similar to his plan for Batty Borough. All he knows is that each road begins and ends at a building. **Does the Count have enough information to calculuate the cost of renovating Transylvania? If so, what would the renovation cost?**

Middle School Challenge ’17

Count on It

Solution - Main Puzzle 4

This problem is equivalent to the finding degree-sum formula in graph theory.

Rather than counting how many roads go into each building, and calculating each buliding’s cost, it’s much easier to simply count the total number of roads. Then, since each road leads into two buildings, we may multiply this number by two to calculate the total number of roads entering all buildings.

In particular, Batty Bourough has 17 roads, and therefore 34 road-ends. Since each building costs 100 teeth for each road end leading to it, the total cost to renovate Batty Bourough is 3400 teeth.

As for Transylvania, we can ignore the total number of buildings, and use the same process: 73 roads multiplied by 2 equals 146 road-ends, making the total cost to renovate Transylvania $146 \times 50 = 7300$ teeth.

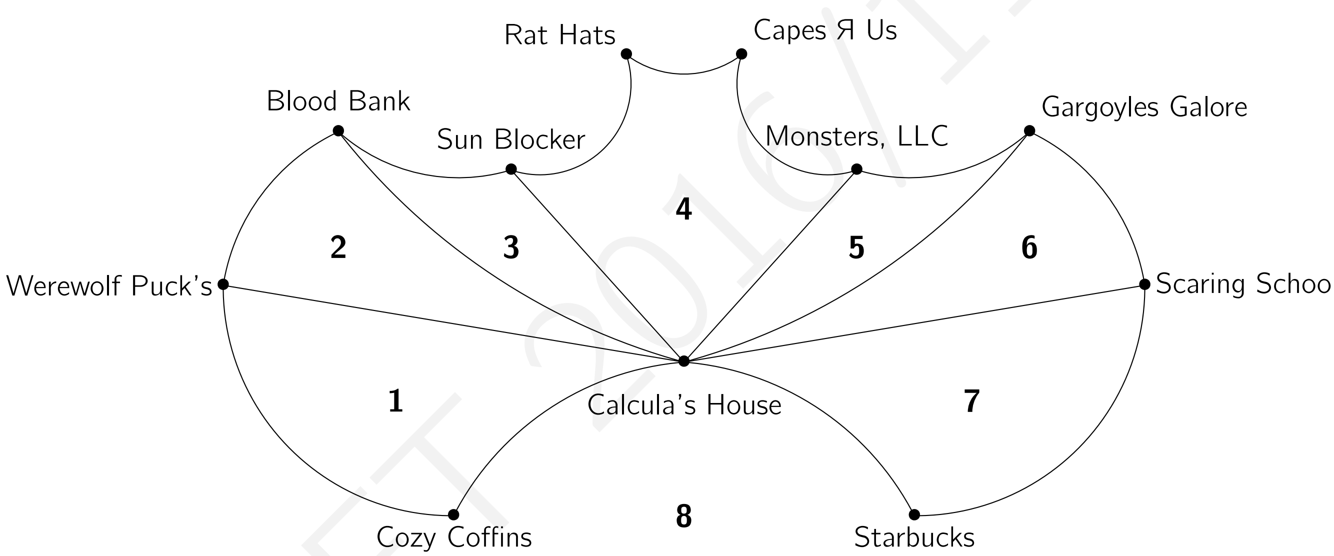
Middle School Challenge ’17

Count on It

Bonus Puzzle 4

Count Calcula now has to face his scariest task yet: meeting with a certain Ms. Knope of the Transylvania Parks and Recreation office.

According to city code, there must be exactly one park in each area of the city which may be walked to without entering a building or crossing a road. For example, if Batty Borough had the same code, the Count would need to build 8 parks:



Remember, Transylvania has **57 buildings** and **73 roads**, and most importantly, no bridges (so roads cannot cross one another). Believe it or not, it’s possible to predict for Ms. Knope how many parks should be in Transylvania. Can you figure out the pattern? (Here’s a hint: try drawing some small towns and adding up the numbers of buildings, roads, and parks... do you see a pattern?)

How many parks should there be in Transylvania?

Mark your answer below.

- ○ 11
- ○ 18
- ○ 27
- ○ 39
- ○ 56

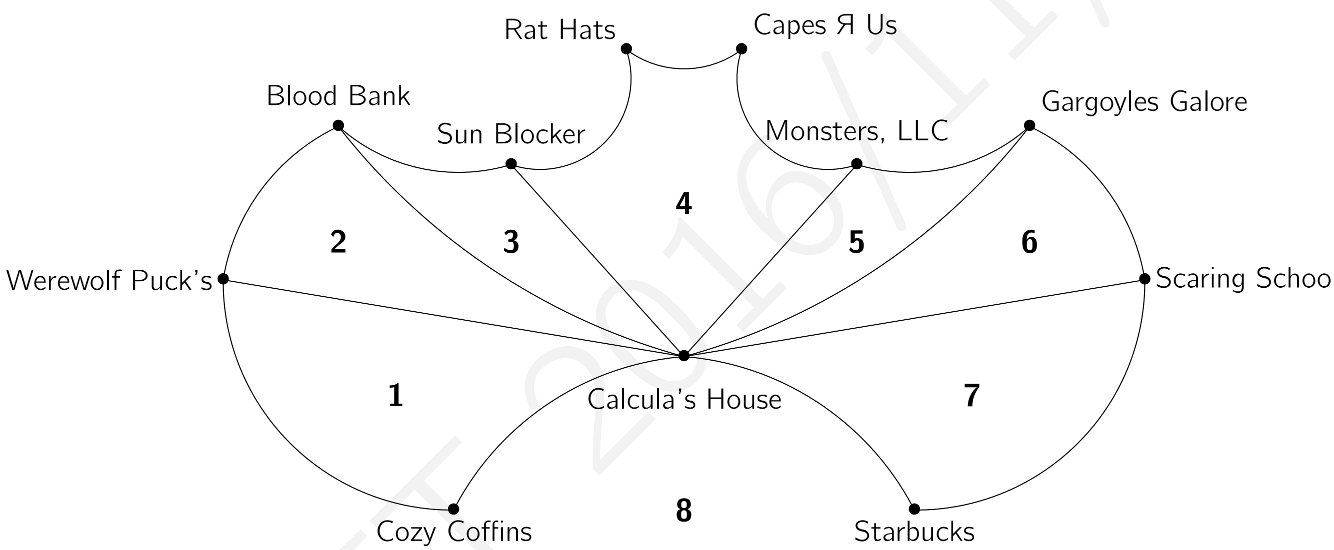
Middle School Challenge '17

Count on It

Solution - Bonus Puzzle 4

The pattern is called **Euler's formula**: for any such map where roads cannot cross, $B - R + P = 2$, where B is the number of buildings, R is the number of roads, and P is the number of parks.

You can see this in Batty Borough, where $11 - 17 + 8 = 2$.



So, in Transylvania, $57 - 73 + P = 2$ solves to $P = 18$.

Middle School Challenge '17

Ghostly Charm

Main Puzzle 5

Goolia the Ghost was floating around the Wayward Tavern yesterday when she saw three friends, Casper, Myrtle, and Blinky, seated around a table with the following cards:

She then witnessed the following:

- Casper told his friends: "*One of these cards is my favorite. I'll tell Casper its suit, and then I'll tell Blinky its rank.*"
- Casper secretly told Myrtle the suit: Bats, Black Cats, Pumpkins, or Candy Corn.
- Casper secretly told Blinky the rank: A, 2, 3, 4, 5, or 6.
- Myrtle mentioned offhand: "*Ooooohhh! I don't know what card Casper picked, but I know that Blinky doesn't know either!*"

Goolia immediately thought to herself, "*Well, that rules out a few of them, anyway!*" **Which of the cards was Goolia able to rule out, given what she saw and heard from the three card-playing ghosts?**

Middle School Challenge ’17

Ghostly Charm

Solution - Main Puzzle 5

The only way Blinky already knew the chosen card is if it was the only card of its rank on the table. The only two such cards are the 3 of Candy Corn and the 6 of Bats.

However, since Myrtle knows that Blinky cannot know the chosen card, she must know that the 3 of Candy Corn and the 6 of Bats were not chosen. Since all she knows is the suit, she must have been told by Capser that the suit is either Pumpkins or Black Cats.

Therefore, Goolia can rule out all the Candy Corn and Bats cards.

Middle School Challenge ’17

Ghostly Charm

Bonus Puzzle 5

Actually, the conversation Goolia the Ghost witnessed amongst Casper, Myrtle, and Blinky went on a bit further.

- Casper told his friends: “*One of these cards is my favorite. I’ll tell Casper its suit, and then I’ll tell Blinky its rank.*”
- Casper secretly told Myrtle the suit: Bats, Black Cats, Pumpkins, or Candy Corn.
- Casper secretly told Blinky the rank: Ace, 2, 3, 4, 5, or 6.
- Myrtle shrieked: “*Ooooohhh! I don’t know what card Casper picked, but I know that Blinky doesn’t know either!*”
- Blinky moaned: “*Ahhhhh! You are right, but now that you’ve said that, I just figured it out!*”
- Myrtle wailed: “*Aiiieee! And now I know as well!*”

Which card was chosen by Casper to be his favorite?

Mark your answer below.

○ The 2 of Pumpkins

○ The 4 of Pumpkins

○ The Ace of Black Cats

○ The 2 of Black Cats

○ The 5 of Black Cats

Middle School Challenge '17

Ghostly Charm

Solution - Bonus Puzzle 5

Myrtle's message told not just Goolia, but also Blinky that the suit must be one of these choices:

- The 2 of Pumpkins
- The 4 of Pumpkins
- The Ace of Black Cats
- The 2 of Black Cats
- The 5 of Black Cats

Since this was enough information for Blinky to figure out the card, its rank cannot be repeated, ruling out the cards of rank 2:

- The 4 of Pumpkins
- The Ace of Black Cats
- The 5 of Black Cats

Since Blinky told Myrtle that he knew the card was enough information for her to deduce the card, that rules out the repeated Black Cat suit. Thus only the 4 of Pumpkins is left as a possibility.

Middle School Challenge '17

Basic Experimentation

Main Puzzle 6

Dr. Frankenstein, between his usual experiments, is often found working on creating new ways to represent numbers.

"*Hmm... according to my tests, most numbers are expressed using a so-called base-10 or decimal system. Yes, yes! Look Igor! The number* 3047 *is simply a sum of powers of ten!! Hahahaha!*"

$$\begin{array}{rcccccccc} 3047 & = & 3000 & + & 000 & + & 40 & + & 7 \\ & = & 3 \times 10^3 & + & 0 \times 10^2 & + & 4 \times 10^1 & + & 7 \times 10^0 \end{array}$$

Apparently, this has given him an idea for a new experiment. "*Hmmm, yesss! What if I... do I dare?! What if I let each digit represent a number between* 0 *and* 15*?*" His new creation, which the doctor has named both "base-16" and "hexadecimal", represents the numbers ten through fifteen as the digits A through F:

$$\begin{array}{rcccccccc} 1C0F_{hex} & = & 1_{hex} \times 16^3 & + & C_{hex} \times 16^2 & + & 0_{hex} \times 16^1 & + & F_{hex} \times 16^0 \\ & = & 1 \times 16^3 & + & 12 \times 16^2 & + & 0 \times 16^1 & + & 15 \times 16^0 \\ & = & 1 \times 4096 & + & 12 \times 256 & + & 0 \times 16 & + & 15 \\ & = & 4096 & + & 3072 & + & 0 & + & 15 \\ & = & 7183 & & & & & & \end{array}$$

Word around the village has spread quickly about the doctor's numerical experimentation. This was facilitated by the accidental escape of another of his creations, a monster known affectionately by the doctor as $BAD_{hex} + FACE_{hex}$.

"*Ah, I'd love to recapture my creation, but it seems it will only respond to its five-digit hexadecimal serial code. I seem to recall that this code is related to his name* $BAD_{hex} + FACE_{hex}$*.*"

Since Dr. Frankenstein's memory is not what it used to be, perhaps you can help him out. **What is the monster's hexadecimal serial number, obtained by calculating** $BAD_{hex} + FACE_{hex}$**?**

Middle School Challenge '17

Basic Experimentation

Solution - Main Puzzle 6

The solution is 1067B. Rather than converting back-and-forth between base-10, it's best to use simple factoring

$$
\begin{array}{rcccccccccc}
BAD_{hex} & = & & & 0 \times 16^3 & + & 11 \times 16^2 & + & 10 \times 16^1 & + & 13 \times 16^0 \\
+FACE_{hex} & = & & & 15 \times 16^3 & + & 10 \times 16^2 & + & 12 \times 16^1 & + & 14 \times 16^0 \\
\hline
 & & & & 15 \times 16^3 & + & 21 \times 16^2 & + & 22 \times 16^1 & + & 27 \times 16^0 \\
 & = & & & 15 \times 16^3 & + & 16 \times 16^2 + 5 \times 16^2 & + & 16 \times 16^1 + 6 \times 16^1 & + & 16 \times 16^0 + 11 \times 16^0 \\
 & = & & & 16 \times 16^3 & + & 6 \times 16^2 & + & 7 \times 16^1 & + & 11 \times 16^0 \\
 & = & 1 \times 16^4 & + & 0 \times 16^3 & + & 6 \times 16^2 & + & 7 \times 16^1 & + & 11 \times 16^0 \\
 & = & 1 & & 0 & & 6 & & 7 & + & B_{hex}
\end{array}
$$

This can be made even cleaner by using the traditional long addition algorithm with hexadecimal digits directly, noting:

$$D_{hex} + E_{hex} = 1B_{hex}$$

$$1_{hex} + A_{hex} + C_{hex} = 17_{hex}$$

$$1_{hex} + B_{hex} + A_{hex} = 16_{hex}$$

$$1_{hex} + F_{hex} = 10_{hex}$$

$$
\begin{array}{ccccc}
 & 1 & 1 & 1 & \\
 & & B & A & D \\
+ & F & A & C & E \\
\hline
1 & 0 & 6 & 7 & B
\end{array}
$$

Middle School Challenge '17

Basic Experimentation

Bonus Puzzle 6

With $BAD_{hex} + FACE_{hex}$ back in its cell, Dr. Frankenstein has a bit of free time on his hands. "*Oh! My grandniece's fifteenth birthday is coming up! I should give her a pet. I have several experim-, er, adorable creatures that she might like, but I'd love to get her a pet appropriate for her age.*"

As terrifying as a gift pet from Dr. Frankenstein would probably be, you should help him choose the best one. **Of the following experiement serial numbers, which is divisible by fifteen?** Of course, that's 15 in base-10 and F_{hex} in hexadecimal.

Mark your answer below.

- ○ $A30D_{hex}$
- ○ $5AD1_{hex}$
- ○ $7BB0_{hex}$
- ○ 6105_{hex}
- ○ $CA44_{hex}$

Middle School Challenge '17

Basic Experimentation

Solution - Bonus Puzzle 6

$5AD1_{hex}$ is the solution. Again, rather than computing each number out to base-10 and dividing by fifteen, there's a useful trick which may be used instead:

$$\begin{array}{rcccccccc} 5AD1 & = & 5_{hex} \times 1000_{hex} & + & A_{hex} \times 100_{hex} & + & D_{hex} \times 10_{hex} & + & 1_{hex} \times 1_{hex} \\ & = & 5_{hex} \times FFF_{hex} + 5_{hex} & + & A \times FF_{hex} + A_{hex} & + & D_{hex} \times F_{hex} + D_{hex} & + & 1_{hex} \end{array}$$

Since $FFF_{hex}, FF_{hex}, F_{hex}$ are easily seen to be divisible by $15 = F_{hex}$, you need only check that $5_{hex} + A_{hex} + D_{hex} + 1_{hex} = 5 + 10 + 14 + 1 = 30$ is divisble by $15 = F_{hex}$.

Middle School Challenge '17

Balancing Act

Just For Fun Puzzle

We found a bunch of odd balance scales in MaPP national headquarters, tucked away in a dusty storage closet, and each labeled with a letter. Here's what we can tell.

- These are balance scales, so the total weights of the kettlebells on each side of a scale are exactly the same.
- The kettlebells A, B, C, D are the same for each picture, with A being the lightest to D being the heaviest.
- The ? kettlebell is always a unique weight between 1 and 40 from picture to picture. Perhaps it describes a way to rearrange the letters on the scales?

Can you figure out the hidden message hidden within our balance scales?

T

U

E

O

E

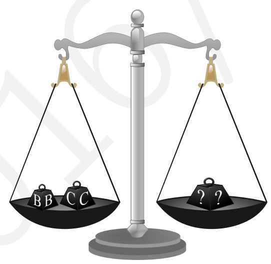
M

L

R

H

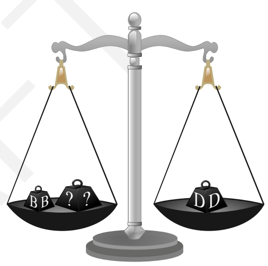
S



T

I

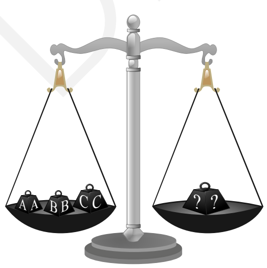
A



O

B

T



I

N

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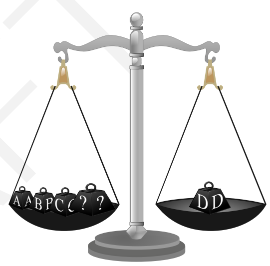
S

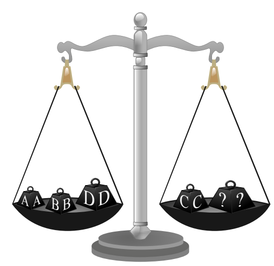
A

P

S

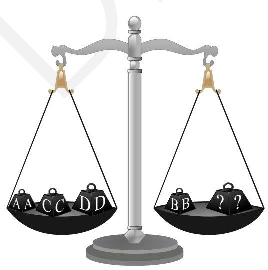


C



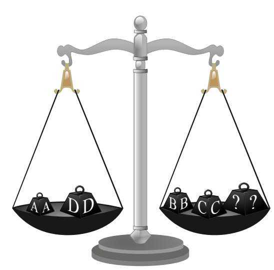
R

G

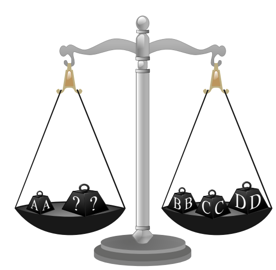


R

E



I



E

Middle School Challenge '17

Balancing Act

Solution

The kettlebells weigh $A = 1$, $B = 3$, $C = 9$, and $D = 27$ units each. By rearranging the letters according to the ? kettlebell weights, the message "TRUE MATHEMATICS IS THE ART OF SOLVING PROBLEMS" is revealed.