Machine Learning & Data Mining — Caltech CS/CNS/EE 155
Set 4 — February 7th, 2023
Manuel Rodriguez

Policies

- Due 9 PM, February 7th, via Gradescope.
- You are free to collaborate on all of the problems, subject to the collaboration policy stated in the syllabus.
- You should submit all code used in the homework. We ask that you use Python 3.6+ and PyTorch version 1.4.0 for your code, and that you comment your code such that the TAs can follow along and run it without any issues.
- This set requires the installation of PyTorch. There will be a recitation and office hour dedicated to helping you install these packages if you have problems.

Submission Instructions

- Submit your report as a single .pdf file to Gradescope.
- In the report, **include any images generated by your code** along with your answers to the questions.
- Submit your code by **sharing a link in your report** to your Google Colab notebook for each problem (see naming instructions below). Make sure to set sharing permissions to at least ”Anyone with the link can view”. **Links that can not be run by TAs will not be counted as turned in.** Check your links in an incognito window before submitting to be sure.

TA Office Hours

- **Julio Arroyo**
  - Sunday, 2/5: 3:00 pm - 4:00 pm
  - Monday, 2/6: 8:00 pm - 9:00 pm
- **Sreemanti Dey**
  - Friday, 2/3: 7:00 pm - 8:00 pm
  - Monday, 2/6: 7:00 pm - 8:00 pm

1 Deep Learning Principles [35 Points]

Relevant materials: lectures on deep learning

For problems A and B, we'll be utilizing the Tensorflow Playground to visualize/fit a neural network.

Problem A [5 points]: Backpropagation and Weight Initialization Part 1

Fit the neural network at this link for about 250 iterations, and then do the same for the neural network at this link. Both networks have the same architecture and use ReLU activations. The only difference between the two is how the layer weights were initialized – you can examine the layer weights by hovering over the edges between neurons.

Give a mathematical justification, based on what you know about the backpropagation algorithm and the ReLU function, for the difference in the performance of the two networks.

Solution A.: *The difference between the two demos is that the second demo does not seem to learn. We will analyze why the second demo does not learn.*
Using the notation from lecture, for any given Loss Function $\mathbb{L}$*, the contribution of all the weights* $\mathbf{W}^l$*, the weights in layer* l*, is given by*

$$\nabla L_{\mathbf{W}^l} = \frac{\partial \mathbb{L}}{\partial \mathbf{W}^l} = \frac{\partial \mathbb{L}}{\partial \mathbf{x}^l} \cdot \frac{\partial \mathbf{x}^l}{\partial \mathbf{s}^l} \cdot \frac{\partial \mathbf{s}^l}{\partial \mathbf{W}^l}$$

With $\frac{\partial \mathbf{s}^l}{\partial \mathbf{W}^l} = \mathbf{x}^{l-1}$*, and* $\mathbf{x}^m$ *is the output layer at layer* m*. In the second demo, all these* $\mathbf{x}^m$ *are zero and thus it makes the* $\nabla L_{\mathbf{W}^l} = 0$ *for all* $\mathbf{W}^l$ *except for those in the first layer since* $\mathbf{x}^0$ *are not necessarily zero, but even in this case, we know that the signal* s^l *is zero and thus* $\partial \mathbf{x}^l \partial \mathbf{s}^l = ReLU'(\mathbf{s}^l) = ReLU'(0)$ *which is undefined or zero* $\implies$ *the loss* $\mathbb{L}$ *does not change according to the backpropagation algorithm.*

Problem B [5 points]: Backpropagation and Weight Initialization Part 2

Reset the two demos from part i (there is a reset button to the left of the "Run" button), change the activation functions of the neurons to sigmoid instead of ReLU, and train each of them for 4000 iterations.

Explain the differences in the models learned, and the speed at which they were learned, from those of part i in terms of the backpropagation algorithm and the sigmoid function.

Solution B.: *From the two demos, the first one, the one with proper weights, is the only one that really changed (the second is still not learning at all). The first demo with Sigmoid at 4000 epochs ends up having a higher test error than that same demo with ReLU after just 250 epochs (though they are both low test errors). This can be explained by Sigmoid's asymptotic behavior near its edges which results in SGD taking very small steps since the gradient in that area is near zero. In contrast to this, ReLU has a constant gradient which allows it to more quickly adjust when learning.*

Problem C: [10 Points]

When training any model using SGD, it's important to shuffle your data to avoid correlated samples. To illustrate one reason why this is particularly important for ReLU networks (i.e. it has ReLU activation functions between its hidden layers, but its output layer is still softmax/tanh/linear), consider a dataset of 1000 points, 500 of which have positive (+1) labels, and 500 of which have negative (-1) labels. What happens if we train a fully-connected network with ReLU activations using SGD, looping through all the negative examples before any of the positive examples? Do not assume that the derivative of ReLU at 0 is implemented as 0. *Hint: this is called the "dying ReLU" problem, although it is possible with other activation functions.*

Solution C: *If we feed ReLU all the negative examples first, then our backpropagation will train our model to have negative weights everywhere. This is to cancel out all the negative values with negative weights so that the input to ReLU is positive. Once we start incorporating positive examples after training on the 500 negative values, we have a situation similar to the second demo where all the weights are zero. Since all the weights are negative and our input values are positive, the input to ReLU will be negative and thus the output will be zero, this makes it so that our backpropagation does not change the weights since our gradient of ReLU will be zero.*

Problem D: Approximating Functions Part 1 **[7 Points]**

Draw or describe a fully-connected network with ReLU units that implements the OR function on two 0/1-valued inputs, x_1 and x_2. Your networks should contain the minimum number of hidden units possible. The OR function OR(x_1, x_2) is defined as:

$$\begin{aligned}\text{OR}(1,0) &\geq 1\\ \text{OR}(0,1) &\geq 1\\ \text{OR}(1,1) &\geq 1\\ \text{OR}(0,0) &= 0\end{aligned}$$

Your network need only produce the correct output when $x_1 \in \{0,1\}$ and $x_2 \in \{0,1\}$ (as described in the examples above).

Solution D.:
Contains two layers: input layer and an output layer. Model would take the inputs x_1 and x_2, have weights $w_1 = 1$ and $w_2 = 1$ and use $ReLU$ as an activation function. If either of x_1 or x_2 are nonzero, then the output will be $ReLU(x_1w_1 + x_2w_2) = x_1w1 + x2w2 \geq 0$. If both x_1 and x_2 are zero though, then the output will be $ReLU(x_1w_1 + x_2w_2) = 0$.

Problem E: Approximating Functions Part 2 **[8 Points]**

What is the minimum number of fully-connected layers (with ReLU units) needed to implement an XOR of two 0/1-valued inputs x_1, x_2? Recall that the XOR function is defined as:

$$\text{XOR}(1,0) \geq 1$$
$$\text{XOR}(0,1) \geq 1$$
$$\text{XOR}(0,0) = \text{XOR}(1,1) = 0$$

For the purposes of this problem, we say that a network f computes the XOR function if $f(x_1, x_2) = \text{XOR}(x_1, x_2)$ when $x_1 \in \{0,1\}$ and $x_2 \in \{0,1\}$ (as described in the examples above).

Explain why a network with fewer layers than the number you specified cannot compute XOR.

Solution E.: *The minimum number of fully-connected layers would be 2, with 1 input layer, 1 hidden layer, and 1 output layer. This is because XOR is not a linearly separable classification problem and thus must be modeled by multiple Linear Functions combined.*

2 Inside a Neural Network [17 Points]

Relevant Materials: Lectures on Deep Learning

Although this is no longer the peak of the pandemic, coronavirus datasets are still very salient due to the number of people in the US that are still being affected by the disease. In this problem, you will investigate the workings of a simple neural network by designing linear neural nets to classify coronavirus cases.

Problem A: Installation [2 Points]

Before any modeling can begin, PyTorch must be installed. PyTorch is an automatic differentiation framework that is widely used in machine learning research. We will also need the **torchvision** package later on, which will make downloading the MNIST dataset much easier.

To install both packages, follow the steps on
`https://pytorch.org/get-started/locally/#start-locally`. Select the 'Stable' build and your system information. We highly recommend using Python 3.6+. CUDA is not required for this class, but it is necessary if you want to do GPU-accelerated deep learning in the future.

Once you have finished installing, write down the version numbers for both **torch** and **torchvision** that you have installed.

Solution A: *The PyTorch version is 1.13.1+cu116, and the torchvision version is 0.14.1+cu116.*

Problem B: The Data [5 Points]

Load and preprocess the tabular dataset, "COVID-19_Case_Surveillance_Public_Use_Data_Subset.csv." This is a small subset of the CDC's Covid-19 Case Surveillance data (`https://data.cdc.gov/Case-Surveillance/COVID-19-Case-Surveillance-Public-Use-Data/vbim-akqf`). Limit the number of input variables in your final dataset to be between 7 and 12, and use the "death_yn" column as the dependent variable. You might find various features in the pandas package useful here.

Explain your preprocessing decisions.

Solution B: *In my preprocessing decisions, I took out the `cdc_report_dt` field because it is redundant information it is recommended to use `cdc_case_earliest_dt` instead. Also, I decided to take out the `race_ethnicity_combined` field because in a real study, it would be unethical to use and thus whatever information is discovered here, in order to be similar to results in the real world, should not include this field. This leaves us with 9 input fields to use. We changed all the date-time fields to be integer values and normalized all columns (individually) using `sklearn's StandardScaler`.*

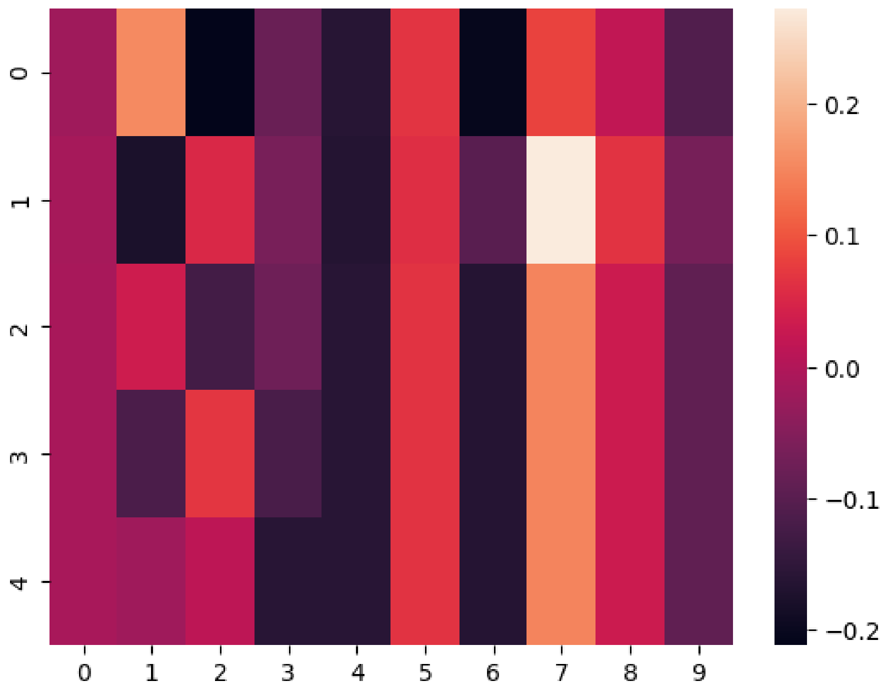
Problem C: Linear Neural Network [5 Points]

Now, use PyTorch's "Sequential" class to make a neural network with one linear layer of size 5 and a binary output layer (with softmax, since this is a classification task). Do not use any activation function in your linear layer. Choose an appropriate optimizer, learning rate, and loss function (either Adam or RMSProp will probably work best as an optimizer). Use these to train and test your model on your dataset–look at the problem 3 sample code to see how to train and test.

Finally, visualize the weight vectors in your model with a heatmap.

Make sure to show training losses and test accuracy clearly in your notebook.

Solution C: *The heatmap for the weights:*



My average Training Loss is around 0.00002 and my average Test Loss is around 0.0290. The Test Accuracy achieved is around 97.75%.

Problem D: 2-Layer Linear Neural Network [5 Points]

Finally, create and train a 2-layer linear neural network and assess its performance on your dataset. It is expected that the 1-layer and 2-layer models have very similar losses–why is this the case?

Solution D:
The 1-layer and 2-layer models are expected to have similar losses because applying two linear models on top of each other is mostly redundant unless some new information is introduced. In other words, all linear information

that could have been learned was already done in the first layer.

3 Depth vs Width on the MNIST Dataset [23 Points]

Relevant Materials: Lectures on Deep Learning

MNIST is a classic dataset in computer vision. It consists of images of handwritten digits (0 - 9) and the correct digit classification. In this problem you will implement a deep network using PyTorch to classify MNIST digits. Specifically, you will explore what it really means for a network to be ”deep”, and how depth vs. width impacts the classification accuracy of a model. You will be allowed at most N hidden units, and will be expected to design and implement a deep network that meets some performance baseline on the MNIST dataset.

Problem A: The Data [3 Points]

Load the MNIST dataset using torchvision; see the problem 3 sample code for how.

Image inputs in PyTorch are generally 3D tensors with the shape (no. of channels, height, width). Examine the input data. What are the height and width of the images? What do the values in each array index represent? How many images are in the training set? How many are in the testing set? You can use the **imshow** function in matplotlib if you'd like to see the actual pictures (see the sample code).

Solution A.: *The height and width of the images is 28 by 28 pixels. Each value in an array index represents the RGB colors of each pixel in the image represented by that index. There are 60,000 images in the training dataset, and there are 10,000 images in the test dataset.*

Model submission instructions:

For each problem 3C-3E and 4G there should be a separate notebook. In your notebook, include the code you used to train your model and make sure your results are visible.

Problem B: Modeling Part 1 [8 Points]

Using PyTorch's ”Sequential” model class, build a deep network to classify the handwritten digits. You may **only** use the following layers:

- **Linear:** A fully-connected layer
- **ReLU (activation):** Sets negative inputs to 0
- **Softmax (activation):** Rescales input so that it can be interpreted as a (discrete) probability distribution.
- **Dropout:** Takes some probability and at every iteration sets weights to zero at random with that probability (effectively regularization)

A sample network with 20 hidden units is in the sample code file. (Note: activations, Dropout, and your last Linear layer do not count toward your hidden unit count, because the final layer is "observed" and not *hidden*.)

Use categorical cross entropy as your loss function. There are also a number of optimizers you can use (an optimizer is just a fancier version of SGD), and feel free to play around with them, but RMSprop and Adam are the most popular and will probably work best. You also should find the batch size and number of epochs that give you the best results (default is batch size = 32, epochs=10).

Look at the sample code to see how to train your model. PyTorch should make it very easy to tinker with your network architecture.

Your task. Using at most 100 hidden units, build a network using only the allowed layers that achieves test accuracy of at least 0.975.

Imporant note on stochasticity: For problems 3C-3E and 4G, you might notice that your model's accuracy fluctuates every time you train it. This is caused by weight initialization, shuffled mini-batching for SGD, dropout probabilities, etc. You may want to consider controlling the effects of randomness by manually setting the seed. In any case, when we say "achieve test accuracy of at least x", we mean that your model should achieve the stated accuracy more than half the times you train it.

Hint: for best results on this problem and the two following problems, normalize the input vectors by dividing the values by 255 (as the pixel values range from 0 to 255).

Solution B:

Problem C: Modeling Part 2 [6 Points]

Repeat problem C, except that now you may use 200 hidden units and must build a model with at least 2 hidden layers that achieves test accuracy of at least 0.98.

Solution C:

Problem D: Modeling Part 3 [6 Points]

Repeat problem C, except that now you may use 1000 hidden units and must build a model with at least 3 hidden layers that achieves test accuracy of at least 0.983.

Solution D:

4 Convolutional Neural Networks [40 Points]

Relevant Materials: Lecture on CNNs

Problem A: Zero Padding **[5 Points]**

Consider a convolutional network in which we perform a convolution over each 8×8 patch of a 20×20 input image. It is common to zero-pad input images to allow for convolutions past the edges of the images. An example of zero-padding is shown below:

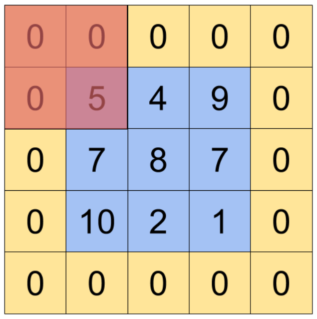


Figure: A convolution being applied to a 2×2 patch (the red square) of a 3×3 image that has been zero-padded to allow convolutions past the edges of the image.

What is one benefit and one drawback to this zero-padding scheme (in contrast to an approach in which we only perform convolutions over patches entirely contained within an image)?

Solution A: *A benefit of using the zero-padding is that it creates a more accurate model for analysis. This is because as the CNN goes through its filter layers, and especially if they are large filters, the dimensions of the input image will shrink and could potentially become so small that certain features become unlearnable. The padding makes it so that the description of the image that the CNN has is more descriptive.*
A drawback of using the zero-padding is that it increases the amount of computation that you can do since now there isa larger image with more information being supplied to the CNN to process.

5 x 5 Convolutions

Consider a single convolutional layer, where your input is a 32×32 pixel, RGB image. In other words, the input is a $32 \times 32 \times 3$ tensor. Your convolution has:

- Size: $5 \times 5 \times 3$
- Filters: 8
- Stride: 1
- No zero-padding

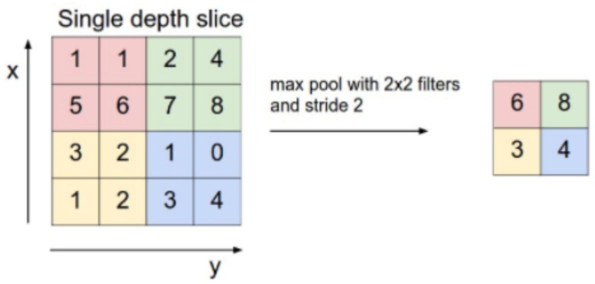
Problem B [2 points]: What is the number of parameters (weights) in this layer, including a bias term? What is the shape of the output tensor?

Solution B.: *The number of convolutions* $= (32 - (5-1))^2 = (28)^2 = 784$, *this is equivalent to the number of parameters (weights) in this layer that need to be determined but adding the bias term we get* 785. *The shape of the output tensor is then* $28 \times 28 \times 8$.

Max/Average Pooling

Pooling is a downsampling technique for reducing the dimensionality of a layer's output. Pooling iterates across patches of an image similarly to a convolution, but pooling and convolutional layers compute their outputs differently: given a pooling layer B with preceding layer A, the output of B is some function (such as the max or average functions) applied to patches of A's output.

Below is an example of max-pooling on a 2-D input space with a 2×2 filter (the max function is applied to 2×2 patches of the input) and a stride of 2 (so that the sampled patches do not overlap):



Average pooling is similar except that you would take the average of each patch as its output instead of the maximum.

Consider the following 4 matrices:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Problem C [3 points]:

1. Apply 2×2 average pooling with a stride of 2 to each of the above images.
2. Apply 2×2 max pooling with a stride of 2 to each of the above images.

Solution C.:

1. *Average pooling - Here are the results for each of the 4 matrices:*

$$\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}, \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ 1 & \frac{1}{2} \end{bmatrix}$$

2. *Max pooling - Since all blocks in each of the 4 matrices has a '1', all the matrices would reduce to:*

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Problem D [4 points]:

Consider a scenario in which we wish to classify a dataset of images of various animals, taken at various angles/locations and containing small amounts of noise (e.g. some pixels may be missing). Why might pooling be advantageous given these distortions in our dataset?

Solution D.: *Pooling would be advantageous to minimizing the effects of these distortions because pooling can give you an estimate of the average pattern or behavior in a given section of the image. By taking this average, we can get a sense of the overall idea of an image without getting distracted by fitting to noise by trying to learn from each pixel individually.*

Hyperparameter Tuning

The performance of neural networks depends to a large extent on the choice of hyperparameters. However, theory does not give us a systematic way of how to pick them. In practice, there are at least three different strategies to tune a network.

1. **"Babysitting":** When tuning a model, you can manually choose a set of hyperparameters, monitor in- and out-of-sample performance, adjust according to intuition, and train again.

2. **Grid search:** For each hyperparameter (learning rate, dropout probability, etc), you define a *range* of values to search, and an *interval* at which to sample points. For example, you could try learning rates lr= $\{10^{-1}, 10^{-2}, \ldots, 10^{-5}\}$ and dropout probabilities $p = \{0, 0.1, \ldots, 0.5\}$, and then train your model for each (lr, p) pair, and keep the setting that yields best results.
3. **Random search:** You can again define a search range, but instead of testing points at pre-determined intervals, you could sample points at random and train your model with those hyperparameters. You could take it one step further and implement a ”coarse-to-fine” approach: i.e. define a large search space, sample and test a few hyperparameters, and then repeat the process inside a finer search space around those hyperparameters that yielded best results in the first pass.

Problem E [5 points]: Pick one of the three hyperparameter tuning strategies, and explain at least one strength and one limitation of using that strategy to tune your model.

Solution E.: *Choosing* ***Random search:*** *to study, we can see that one strength that it has is that it is much less susceptible to human bias than the other two methods. With Random Search, we are allowing a lot of freedom to choose a wide range of hyperparameters which increases the likelihood that our ideal hyperparameters are somewhere in this space. However, one limitation to this approach is that is will likely take the longest to find a decent, let-alone 'strong,' hyperparameter. This is because with so much variety in choice, it is unlikely that you will randomly stumble across the optimal hyperparameter and once you decide to refine your search to a narrower region, you begin to incorporate human bias like in the other methods.*

PyTorch implementation

Problem F [20 points]:

Using PyTorch “Sequential” model class as you did in 2C, build a deep *convolutional* network to classify the handwritten digits in MNIST. You are now allowed to use the following layers (but **only** the following):

- **Linear:** A fully-connected layer
  - In convolutional networks, Linear (also called dense) layers are typically used to knit together higher-level feature representations.
  - Particularly useful to map the 2D features resulting from the last convolutional layer to categories for classification (like the 1000 categories of ImageNet or the 10 categories of MNIST).
  - Inefficient use of parameters and often overkill: for A input activations and B output activations, number of parameters needed scales as $O(AB)$.
- **Conv2d:** A 2-dimensional convolutional layer
  - The bread and butter of convolutional networks, conv layers impose a translational-invariance prior on a fully-connected network. By sliding filters across the image to form another image, conv layers perform “coarse-graining” of the image.

  - Networking several convolutional layers in succession helps the convolutional network knit together more abstract representations of the input. As you go higher in a convolutional network, activations represent pixels, then edges, colors, and finally objects.
  - More efficient use of parameters. For N filters of $K \times K$ size on an input of size $L \times L$, the number of parameters needed scales as $O(NK^2)$. When N, K are small, this can often beat the $O(L^4)$ scaling of a Linear layer applied to the L^2 pixels in the image.

- **MaxPool2d:** A 2-dimensional max-pooling layer
  - Another way of performing "coarse-graining" of images, max-pool layers are another way of ignoring finer-grained details by only considering maximum activations over small patches of the input.
  - Drastically reduces the input size. Useful for reducing the number of parameters in your model.
  - Typically used immediately following a series of convolutional-activation layers.
- **BatchNorm2d:** Performs batch normalization (Ioffe and Szegedy, 2014). Normalizes the activations of previous layer to standard normal (mean 0, standard deviation 1).
  - Accelerates convergence and improves performance of model, especially when saturating non-linearities (sigmoid) are used.
  - Makes model less sensitive to higher learning rates and initialization, and also acts as a form of regularization.
  - Typically used immediately before nonlinearity (Activation) layers.
- **Dropout:** Takes some probability and at every iteration sets weights to zero at random with that probability
  - An effective form of regularization. During training, randomly selecting activations to shut off forces network to build in redundancies in the feature representation, so it does not rely on any single activation to perform classification.
- **ReLU (activation):** Sets negative inputs to 0
- **Softmax (activation):** Rescales input so that it can be interpreted as a (discrete) probability distribution.
- **Flatten:** Flattens any tensor into a single vector (required in order to pass a 2D tensor output from a convolutional layer as input into Linear layers)

Your tasks. Build a network with only the allowed layers that achieves **test accuracy of at least 0.985**. You are required to use categorical cross entropy as your loss function and to train for 10 epochs with a batch size of 32. Note: your model must have fewer than 1 million parameters, as measured by the method given in the sample code. Everything else can change: optimizer (RMSProp, Adam, ???), initial learning rates, dropout probabilities, layerwise regularizer strengths, etc. You are not required to use all of the layers, but *you must have at least one dropout layer and one batch normalization layer in your final model*. Try to figure out the best possible architecture and hyperparameters given these building blocks!

In order to design your model, you should train your model for 1 epoch (batch size 32) and look at the final **test accuracy** after training. This should take no more than 10 minutes, and should give you an immediate sense for how fast your network converges and how good it is.

Set the probabilities of your dropout layers to 10 equally-spaced values $p \in [0, 1]$, train for 1 epoch, and report the final model accuracies for each.

You can perform all of your hyperparameter validation in this way: vary your parameters and train for an epoch. After you're satisfied with the model design, you should train your model for the full 10 epochs.

In your submission. Turn in the code of your model, the test accuracy for the 10 dropout probabilities $p \in [0, 1]$, and the final test accuracy when your model is trained for 10 epochs. We should have everything needed to reproduce your results.

Discuss what you found to be the most effective strategies in designing a convolutional network. Which regularization method was most effective (dropout, layerwise regularization, batch norm)?

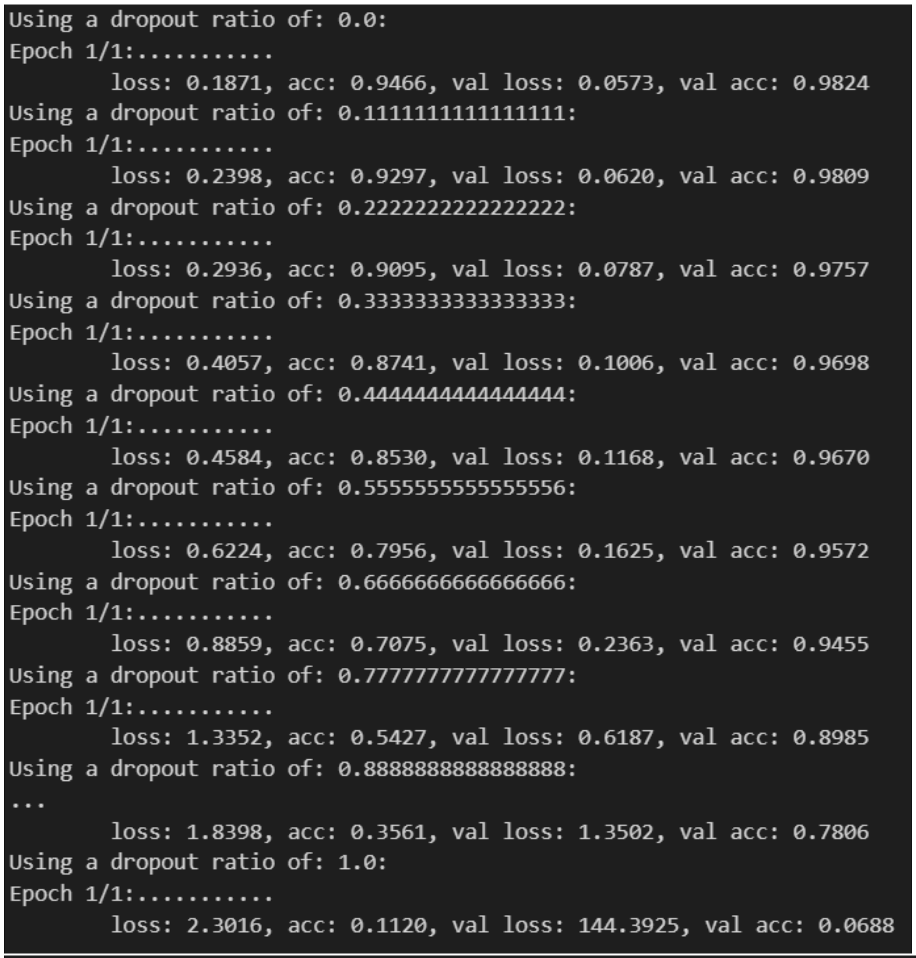
Do you foresee any problem with this way of validating our hyperparameters? If so, why?

Hints:

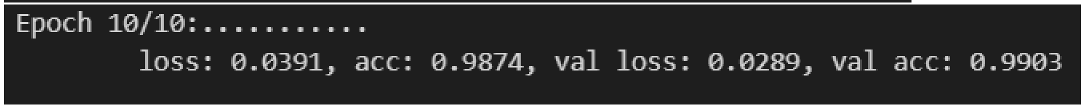
- You are provided with a sample network that achieves a high accuracy. Starting with this network, modify some of the regularization parameters (layerwise regularization strength, dropout probabilities) to see if you can maximize the test accuracy. You can also add layers or modify layers (e.g. changing the convolutional kernel sizes, number of filters, stride, dilation, etc.) so long as the total number of parameters remains under the cap of 1 million.
- You may want to read up on successful convolutional architectures, and emulate some of their design principles. Please cite any idea you use that is not your own.
- To better understand the function of each layer, check the PyTorch documentation.
- Linear layers take in single vector inputs (ex: *(784,)*) but Conv2D layers take in tensor inputs (ex: *(28, 28, 1)*): width, height, and channels. Using the transformation `transforms.ToTensor()` when loading the dataset will reshape the training/test X to a 4-dimensional tensor (ex: *(num_examples, width, height, channels)*) and normalize values. For the MNIST dataset, *channels=1*. Typical color images have 3 color channels, 1 for each color in RGB.
- If your model is running slowly on your CPU, try making each layer smaller and stacking more layers so you can leverage deeper representations.
- Other useful CNN design principles:
  - CNNs perform well with many stacked convolutional layers, which develop increasingly large-scale representations of the input image.

- Dropout ensures that the learned representations are robust to some amount of noise.
- Batch norm is done after a convolutional or dense layer and immediately prior to an activation/nonlinearity layer.
- Max-pooling is typically done after a series of convolutions, in order to gradually reduce the size of the representation.
- Finally, the learned representation is passed into a dense layer (or two), and then filtered down to the final softmax layer.

Solution F:
I found switching to Adam as an optimizer and using batch norm to be the most effective strategies in increasing the accuracy. Dropout was also good to adjust to try to correct the effects of layer sizes but it was much more difficult to come up with good values for them. Also, Dropout worked best at the start of the model (in the early layers) which makes sense because later into the model, we have feature detection and dropping a weight is much more unlikely to help the model rather than make it forget some important feature.
`https://colab.research.google.com/drive/1qO5-9S98jp6PNXnVc65kZAZiCojroRxH`



```
Using a dropout ratio of: 0.0:
Epoch 1/1:..........
        loss: 0.1871, acc: 0.9466, val loss: 0.0573, val acc: 0.9824
Using a dropout ratio of: 0.1111111111111111:
Epoch 1/1:..........
        loss: 0.2398, acc: 0.9297, val loss: 0.0620, val acc: 0.9809
Using a dropout ratio of: 0.2222222222222222:
Epoch 1/1:..........
        loss: 0.2936, acc: 0.9095, val loss: 0.0787, val acc: 0.9757
Using a dropout ratio of: 0.3333333333333333:
Epoch 1/1:..........
        loss: 0.4057, acc: 0.8741, val loss: 0.1006, val acc: 0.9698
Using a dropout ratio of: 0.4444444444444444:
Epoch 1/1:..........
        loss: 0.4584, acc: 0.8530, val loss: 0.1168, val acc: 0.9670
Using a dropout ratio of: 0.5555555555555556:
Epoch 1/1:..........
        loss: 0.6224, acc: 0.7956, val loss: 0.1625, val acc: 0.9572
Using a dropout ratio of: 0.6666666666666666:
Epoch 1/1:..........
        loss: 0.8859, acc: 0.7075, val loss: 0.2363, val acc: 0.9455
Using a dropout ratio of: 0.7777777777777777:
Epoch 1/1:..........
        loss: 1.3352, acc: 0.5427, val loss: 0.6187, val acc: 0.8985
Using a dropout ratio of: 0.8888888888888888:
...
        loss: 1.8398, acc: 0.3561, val loss: 1.3502, val acc: 0.7806
Using a dropout ratio of: 1.0:
Epoch 1/1:..........
        loss: 2.3016, acc: 0.1120, val loss: 144.3925, val acc: 0.0688
```



```
Epoch 10/10:..........
        loss: 0.0391, acc: 0.9874, val loss: 0.0289, val acc: 0.9903
```