Exercise 9

Matthias Gollwitzer, Jan Schalkamp

July 10, 2013

1 EXERCISE 1

$$\begin{aligned}
\binom{n}{(n-k)} &= \frac{n!}{(n-(n-k))!(n-k)!} \\
&= \frac{n!}{(n-n+k)!(n-k)!} \\
&= \frac{n!}{k!(n-k)!} \\
&= \frac{n!}{(n-k)!k!} \\
&= \binom{n}{k}
\end{aligned}$$

$\square$

2 EXERCISE 2

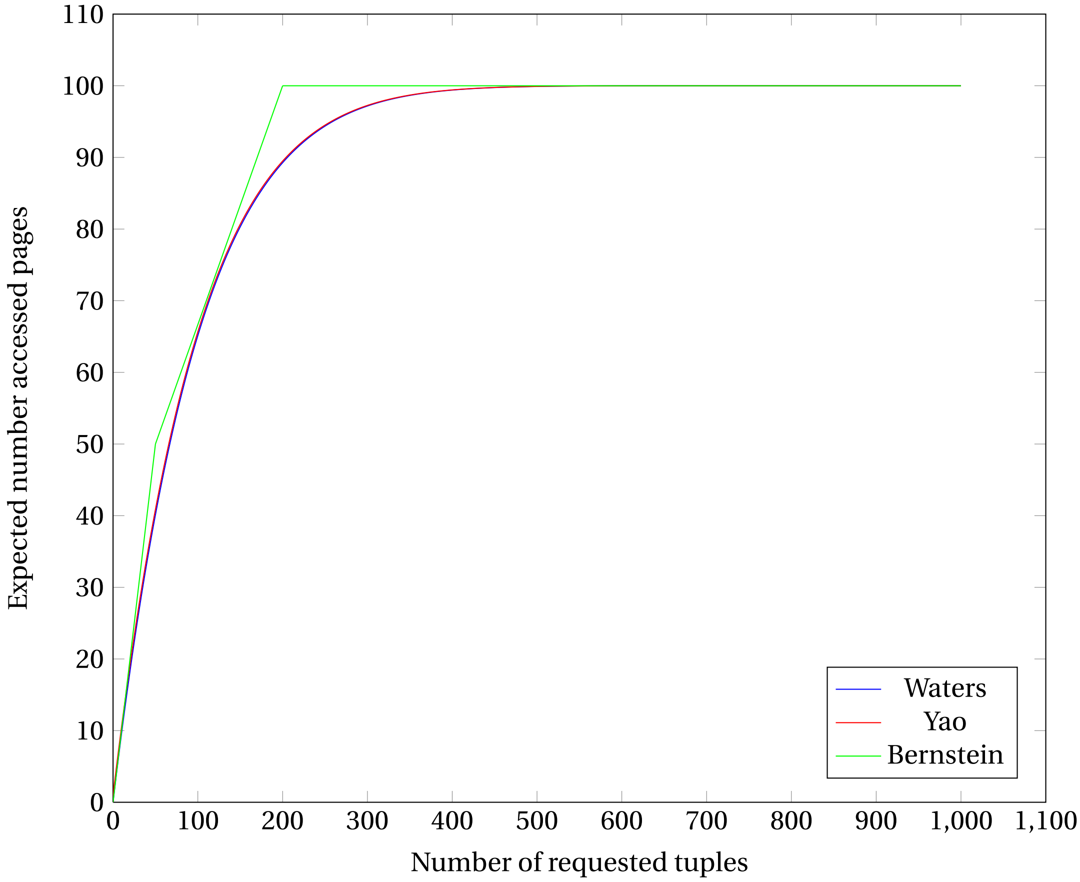
$$\begin{aligned}
|R| &= N = 6 \\
||R|| &= m = 3 \\
\text{tupels/page} &= n = 2 \\
|\text{requested tuples}| &= k = 2
\end{aligned}$$

$$\begin{aligned}
P_1 &= \binom{N}{k} = \binom{6}{2} = 15 \\
P_2 &= 3 \\
p &= \frac{3}{15} \\
\text{accesses}_{\text{avg}} &= 1 * p + 2 * (1 - p) = \frac{3}{15} + \frac{24}{15} = 1.8
\end{aligned}$$

With P_1 and P_2 being the number of ways to pick two tuples and to pick both tuples from a single bucket. Hence, p determines the possibility of picking a bucket with both requested tuples inside and $(1 - p)$ being the possibility of having to look in two different buckets (note, that there aren't any more options, as we are only looking for two tuples). Therefore, the average number of page accesses is $\text{accesses}_{\text{avg}} = 1.8$. It basically is *number of accessed pages∗ possibility to access this number of pages.* To show that it yields the correct solution, we evaluate Yao's formula using the same parameters as above:

$$\begin{aligned}
& \bar{y}_n^{N,m}(k) && = && m * y_n^{N,m}(k) \\
\implies & \bar{y}_2^{6,3}(2) && = && 3 * y_2^{6,3}(2) = \\
= & 3 * \left(1 - \frac{\binom{(6-2)}{2}}{\binom{6}{2}}\right) && = && 3 * \left(1 - \frac{\binom{4}{2}}{\binom{6}{2}}\right) = \\
= & 3 * 0.8 && = && 1.8
\end{aligned}$$

3 Exercise 3



Expected number accessed pages
Number of requested tuples
0
10
20
30
40
50
60
70
80
90
100
110
0
100
200
300
400
500
600
700
800
900
1,000
1,100
Waters
Yao
Bernstein