QUANTITATIVE METHODS

Rates and Returns

Interest Rates

- **Interest rate** = Nominal risk-free rate
  + Default risk premium
  + Liquidity risk premium
  + Maturity risk premium
- **Nominal risk-free rate** = Real risk-free rate
  + expected inflation rate

Rates of Return

- **Arithmetic or mean return**: $\overline{R}_i = \frac{1}{T}\sum_{t=1}^{T} R_{it}$
- **Geometric mean of return**:

  $\overline{R}_G = \sqrt[N]{(1+R_1)(1+R_2)...(1+R_n)} - 1$
- **Trimmed mean**: remove a small defined percentage of the largest and smallest values and calculate the mean by averaging the remaining observations.
- **Winsorized mean**: replace extreme values at both ends with the values of their nearest observations and calculate the mean by averaging the remaining observations.
- **Leverage return**= $R_L = R_P + \frac{V_B}{V_E}(R_P - R_D)$

Money Weighted and Time Weighted Return:

- **Money Weighted Rate of Return :** same as IRR calculation
- **Time Weighted Rate of Return :** measures compound growth and past-period return for N years (n periods)
- **HPR** =holding period yield $= \frac{P_1 - P_0 + CF_1}{P_0} = \frac{FV-PV}{PV}$
- **TWRR**= $\sqrt[N]{(1+r_1)(1+r_2)\ldots(1+r_n)} - 1$
- Time-weighted return **is not influenced by cash flow**, but money-weighted return will be affected by cash flow.

The Time Value of Money

EAR

- **EAR = Effective annual rate** $= (1+r_{period})^m - 1$

  $1+EAR = \left(1+\frac{r}{m}\right)^m$
- **If continuous compounding, then EAR = $e^{annual\ int} - 1$**

The future/present value of cash flow

- $FV = PV(1+r)^n$
- $PV = \frac{FV}{(1+r)^n}$

Annuities

- **Annuities:** is a finite set of level sequential cash flows.
  a) equal intervals
  b) equal amount of cash flows
  c) same direction
- **Ordinary annuity:** cash flow at end-of-time period.
- **Annuity due:** cash flow at beginning-of-time period.
- **Perpetuities:** annuities with infinite lives.
- Future value of annuity due = FV of an ordinary annuity×(1+r)
- Present value of annuity due = PV of an ordinary annuity×(1+r)
- Present Value of Perpetuity = PMT / r

Statistical Measures of Asset Returns

Measures of Central tendency and Location

- Population mean: $\mu_x = \frac{\sum_{i=1}^{N} X_i}{N}$
- Sample mean: $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$
- Weighted mean: $\overline{X} = \sum_{i=1}^{n}(w_i X_i)$

where w_i are weights that sum to 1.

- Arithmetic mean : $\bar{X}=\frac{\sum_{i=1}^{N} X_i}{n}$
- Geometric mean: $G=\sqrt[N]{X_1X_2X_3...X_N}=(\prod_{i=1}^{N} X_i)^{1/N}$
- Harmonic mean: $\bar{X}_H=\frac{N}{\sum_{i=1}^{N}(1/X_i)}$
- Arithmetic mean × Harmonic mean = Geometric mean^2
- Quantiles: $L_y=\left[(n+1)\frac{y}{100}\right]$
- Interquartile Range(IQR): $IQR=Q_3-Q_1$

Measures of Absolute Dispersion

- The range: Range=Maximum value − Minimum value
- MAD=mean absolute deviation= $\frac{\sum_{i=1}^{N}|X_i-\mu|}{N}$
- Population variance : $\sigma^2=\frac{\sum_{i=1}^{N}(X_i-\mu)^2}{N}$
- Population standard deviation :

$$\sigma=\sqrt{\frac{\sum_{i=1}^{N}(X_i-\mu)^2}{N}}=\sqrt{\sigma^2}$$

- Sample variance: $s^2=\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}$
- Sample standard deviation: $s=\sqrt{\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}}$
- Semivariance:

$$\text{Semivariance}=\frac{\sum_{\text{for all } X_i\le\bar{X}}(X_i-\bar{X})^2}{n-1}$$

- Target semivariance:

$$\text{Target Semivariance}=\frac{\sum_{\text{for all } X_i\le B}(X_i-B)^2}{n-1}$$

Measures of Relative Dispersion

- **Coefficient of Variation(CV):** express how much dispersion exists relative to mean of a distribution.

$$CV=\frac{s_X}{\bar{X}}$$

- **Sharpe ratio:** measures excess return per unit of risk.

$$\text{Sharpe ratio}=\frac{E(R_P)-R_f}{\sigma_P}$$

- **Roy's safety-first criterion**

$$SFR=\frac{E(R_P)-R_L}{\sigma_P}$$

Measures of Shape of a Distribution

- $$\text{Sample skewness}\approx\left(\frac{1}{n}\right)\frac{\sum_{i=1}^{n}(X_i-\bar{X})^3}{s^3}$$

Positive skewness: Mode < Median < Mean

Negative skewness: Mode > Median > Mean

- $$\text{Sample kurtosis}\approx\frac{1}{n}\frac{\sum_{i=1}^{n}(X_i-\bar{X})^4}{s^4}$$
- Excess kurtosis = sample kurtosis – 3

Covariance & Correlation

- Covariance: $\text{Cov}(X,Y)=E[(X-E(X))(Y-E(Y))]$
- Correlation coefficient: $\rho_{X,Y}=\frac{\text{Cov}(X,Y)}{\sigma_X\sigma_Y}$

Correlation coefficient only indicates linear relationship.

- ρ=0: an absence of any linear (straight-line) relationship; ρ=-1: a perfect inverse linear relationship; ρ==+1: a perfect linear relationship.
- A **scatter plots** is a graph that shows the relationship between the observations for two data series in two dimensions.

Probability Trees and Conditional Expectations

Expected Value & Variance

- Expected value:

$$E(X)=\sum_{i=1}^{n} X_i P(X_i)=X_1\times P(X_1)+\cdots+X_n\times P(X_n)$$

- Variance:

$$\sigma^2(X)=E\left[X-E(X)\right]^2$$
$$=\sum_{i=1}^{n} P(X_i)\left[x_i\text{-}E(X)\right]^2$$
$$=P(X_1)\left[x_1\text{-}E(X)\right]^2+P(X_2)\left[x_2\text{-}E(X)\right]^2$$
$$+\cdots+P(X_n)\left[x_n\text{-}E(X)\right]^2$$

Probability Rules

- Multiplication rule of probability

$$P(AB)=P(A)P(B\mid A)=P(B)P(A\mid B)$$

- Addition rule of probability:

$$P(A+B)=P(A)+P(B)-P(AB)$$

- Total Probability Rule:

$$P(R)=P(R\mid S_1)P(S_1)+\ldots+P(R\mid S_N)P(S_N)P(R)$$

$S_1, S_2,\ldots, S_N$ are mutually exclusive and exhaustive Events

- Conditional expected value:

$$E(X\mid S)=P(X_1\mid S)X_1+\cdots+P(X_n\mid S)X_n$$

- Conditional variance:

$$\sigma^2(X\mid S)=P(X_1\mid S)[X_1\text{-}E(X\mid S)]^2+\cdots$$
$$+P(X_n\mid S)[X_n\text{-}E(X\mid S)]^2$$

Bayes' formula:

Updated probability
= Probability of new information for a given event / Unconditional probability of new information × Prior probability of event

$$P(A\mid B)=\frac{P(B\mid A)}{P(B)}\times P(A)$$

Odds

- Odds for an event =P(E)/(1-P(E))
- Odds against an event =(1-P(E))/P(E)

Other Calculation Rules

- Labeling: $=\dfrac{n!}{n_1!\times n_2!\times\ldots\times n_k!}$
- Combination (binominal labelling):

$${}_nC_r=C_n^r=\frac{n!}{r!(n-r)!}$$

- Permutation:

$${}_nP_r=P_n^r=\frac{n!}{(n-r)!}$$

Common Probability Distributions

Binomial Distribution

- Bernoulli random variable :
  a) P(Y=1) = p P(Y=0) = 1-p
  b) E(X) = p Variance= p(1-P)
- Binomial Probability : $p(x)=C_n^x p^x(1-p)^{n-x}$
  a) E(X) = np
  b) Variance= np(1-p)

Continuous Uniform Distribution:

$$F(x)=P(X\le x)=\begin{cases}0, x<a\\ \dfrac{x-a}{b-a}, a\le x\le b\\ 1, x>b\end{cases}$$

Normal Distribution:

- Completely described by mean and variance (μ, σ^2)
- It is symmetric with skewness measure of 0, i.e., mean = mode = median
- Kurtosis = 3
- Linear combinations of normal random variables are normally distributed.

Standardized Normal Distribution $z=\dfrac{X-\mu}{\sigma}$

- F(-z)=1-F(z)
- P(Z>z) = 1 –F(z)

Confidence Intervals

50% of observations fall within $\pm\frac{2}{3}\sigma$

68% of observations fall within $\pm1\sigma$

90% of observations fall within $\pm1.65\sigma$

95% of observations fall within $\pm1.96\sigma$

99% of observations fall within $\pm2.58\sigma$

Lognormal Distribution

For asset price: **ln(X) ~ N** (μ,σ^2)

X~lognormal

The Chi-Square (χ^2) Distribution

The distribution of the sum of the squares of k independent standard normally distributed random variables.

$$\sum Z_i^2 = Z_1^2 + Z_2^2 + \cdots + Z_k^2 \sim \chi^2_{(k)}$$

- Asymmetrical and developed for testing hypotheses of positive parameters;
- As the degrees of freedom increase, the shape of the density function becomes more similar to a bell curve.

The Student's t-distribution

- Symmetric;
- Less peaked than a normal distribution ("fatter tails")
- Converges to the standard normal distribution as degrees of freedom goes to infinity.

The F-Distribution:

If U1 and U2 are two independent Chi-Squared distributions with k1 and k2 degrees of freedom, respectively, then X follows an F- distribution with k1 and k2 denominator degrees of freedom :

$$X = \frac{U_1 / k_1}{U_2 / k_2} \sim F(k_1, k_2)$$

- Asymmetrical distributions bounded from below by 0 ;
- As both the numerator ($k1$) and the denominator ($k2$) degrees of freedom increase, the shape of the density function will also become more bell curve–like

Sampling & Estimation

Sampling Distribution:

Probability distribution of all possible sample statistics computed from a set of equal-size samples randomly drawn from the same population. The *sampling distribution of the mean* is the distribution of estimates the mean.

Central Limit Theorem:

When selecting simple random samples of size n from a population with a mean μ and a finite variance σ^2, the sampling distribution of the sample mean approaches a normal probability distribution with mean μ and a variance equal to σ^2/n as the sample size becomes large (n≥30). $\bar{X}\sim N(\mu, \frac{\sigma^2}{n})$

Standard Error of Sample Mean:

Standard error of the sample mean is the standard deviation of distribution of the sample mean.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$ (known population variance)

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$ (unknown population variance)

Estimation

- **The desirable properties of an estimator: unbiasedness, efficiency, consistency**
- **Point estimation:** sample statistic is used to estimate the population parameter
- **Confidence interval estimation:**

Confidence interval: gives range of values the mean value will be between, with a given probability (say 90% or 95%). With known variance, formula for a confidence interval is:

Point estimate +/- (reliability factor × standard error)

$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ (with known variance)

$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ (with unknown variance)

Hypothesis Testing

Null and Alternative Hypotheses

Null hypothesis (H_0):

- the hypothesis that is actually tested; the basis for selection of the test statistics.

Alternative hypothesis (H_a):

- concluded if there is sufficient evidence to reject the null hypothesis.

Test Statistic

$$Test\ Statistic = \frac{Sample\ statistics - Hypothesized\ value}{Stanard\ error\ of\ the\ sample\ statistic}$$

Difference Between One- and Two-Tailed Tests:

One-tailed test: tests whether value is greater than or less than a given number.

$H_0 : \mu \le \mu_0 \quad H_a : \mu > \mu_0$
$or, H_0 : \mu \ge \mu_0 \quad H_a : \mu < \mu_0$

Two-tailed test: tests whether value is equal to a given number.

$H_0 : \mu = \mu_0 \quad H_a : \mu \ne \mu_0$

Type I and Type II Errors:

- **Type I error:** rejection of null hypothesis when it is actually true.
- **Type II error**: failure to reject null hypothesis when it is actually false.

Decision	If	
	H_0 **is true**	H_0 **is false**
Reject H_0	Type I error *Significant level* α is the probability of type I error	Correct *Power of test* is defined as 1-P (Type II error)
Fail to reject H_0	Correct	Type II error

Types of Hypothesis Tests:

Test of Mean against Point Value

- Normally distributed population, known population variance

  $H_0: \mu=\mu_0$

  $$Z=\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$$

- Normally distributed population, unknown population variance

  $H_0: \mu=\mu_0$

  $$t=\frac{\bar{x}-\mu_0}{s/\sqrt{n}},\ df=(n-1)$$

Test of Difference in Means

- Independent populations, unknown population variances assumed equal

  $H_0: \mu_1-\mu_2=0$

  $$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{s_p^2/n_1+s_p^2/n_2}}=\frac{\bar{x}_1-\bar{x}_2}{\sqrt{s_p^2/n_1+s_p^2/n_2}}$$

  where $s_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$,

  a.k.a. a pooled estimator of the common variance.

  Degree of freedom: $n_1 + n_2 - 2$

- Independent populations, unknown population variances not assumed equal

  $H_0: \mu_1-\mu_2=0$

  $$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{s_1^2/n_1+s_2^2/n_2}}=\frac{\bar{x}_1-\bar{x}_2}{\sqrt{s_1^2/n_1+s_2^2/n_2}}$$

Paired Comparisons Test

- Samples not independent, for example, two returns of stocks in the market, the return of gas and that of oil. In special cases, the same population after and before an event, or comparison with the two different styles, which are affected by the same macro factors.

  $H_0: \mu_d=0$

  $$t=\bar{d}/s_{\bar{d}}$$

Degree of freedom: n-1

Variances Hypothesis Testing:

- Normally distributed population

  H_0: $\sigma^2=\sigma_0^2$

  $$\chi^2=\frac{(n-1)s^2}{\sigma_0^2}$$

  Degree of freedom: n-1

- Two independent normally distributed populations

  H_0: $\sigma_1^2=\sigma_2^2$

  $F=\frac{s_1^2}{s_2^2}$; (always make $S_1 \geq S_2$)

  Degree of freedom: n_1-1, n_2-1

Correlation Hypothesis Testing:

- Both of the variables are normally distributed.

  H_0: $\rho = 0$

  $$t=\frac{r-0}{\sqrt{\frac{1-r^2}{n-2}}}$$

  Degree of freedom: n-2

Chi-square test of independence

- Test of independence a relationship between the size and investment type, using a nonparametric test statistic that is chi- square distributed

  $$\chi^2=\sum_{i=1}^{m}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$$

$$E_{ij}=\frac{\text{Total Row i} \times \text{Total Column j}}{\text{Overall Total}}$$

Degrees of freedom: (r − 1)(c − 1)

Introduction to Linear Regression

Simple Linear Regression

$$Y_i=b_0+b_1X_i+\varepsilon_i \quad i=1,2\cdots,n$$

Y_j = dependent/ explained/ predicted variable

X_j = independent/ explanatory/ predicting variable

Coefficient estimation

- $\hat{b}_1=\frac{Cov(X,Y)}{Var(X)}$
- $\hat{b}_0=\bar{Y}-\hat{b}_1\bar{X}$

Elements of ANOVA Table

- **The sum of squared errors or residuals (SSE)**

  $$SSE=\sum_{i=1}^{n}(Y_i-Y_i)^2$$

- **The sum of squares regression (SSR)**

  $$SSR=\sum_{i=1}^{n}(Y_i-\bar{Y})^2$$

- **The sum of squares total (SST)**

  $$SST=\sum_{i=1}^{n}(Y_i-\bar{Y})^2$$

Coefficient of determination (R^2)

- $R^2=\frac{SSR}{SST}=1-\frac{SSE}{SST}$
- R^2 of 0.9 means the independent variable explains approximately 90% of the variation in the dependent variable.

Standard error of estimate (SEE)

- $SEE=\sqrt{\frac{SSE}{n-k-1}}=\sqrt{MSE}$
- SEE is low if the regression is very strong and high if the relationship is weak.
- The SEE is the standard deviation of the error terms.

F-statistic

$$F=\frac{MSR}{MSE}=\frac{SSR/k}{SSE/(n-k-1)}$$

ECONOMICS

Topics in demand and supply analysis

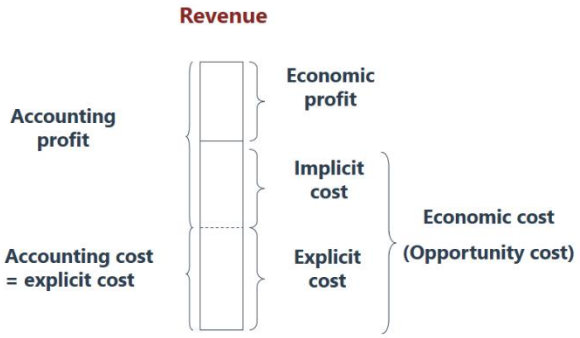
Accounting Profit and Economic Profit and Normal

Profit

Accounting profit = total revenue – total accounting (explicit) cost

Economic profit= accounting profit - implicit opportunity costs = total revenue – total economic costs

Normal profit is the accounting profit that makes economic profit zero.



Total, Average, and Marginal Revenue

- Under perfect competition MR=P
- Under imperfect competition
  Total revenue (TR) is maximized when MR=0.
  The relationship between MR, P, and price elasticity of demand: **MR=P[1-1/E_p]**

Cost

Total cost = total fixed cost + total variable cost

Marginal cost = change in total cost / change in output

Average fixed cost = total fixed cost / output

Average variable cost = total variable cost / output

Average cost = total cost / output = AFC+AVC

Profit Maximization

Profit maximization occurs when **marginal revenue (MR) equals marginal cost (MC); (MR=MC)**

Breakeven and Shutdown

Breakeven: total revenue = total cost.

Operate in short run if TR <TC but TR > TVC.

Shut down in short run if TR < TVC.

Profit maximization: MR=MC

Firms and Market Structures

Perfect competition

Perfect competition: Many firms with no pricing power; very low or no barriers to entry; homogeneous product.

Short-term equilibrium:

- P= MR = MC
- If P < AVC, a firm would shut down temporarily.

Long-term equilibrium:

- P = MR = MC = ATC
- All firms make zero economic profits.

Monopoly

Monopoly: Only one firm; single firm with significant pricing power; high barriers to entry; advertising used to compete with substitute products.

To **maximize profit**, monopolists will expand output until marginal revenue (MR) equals marginal cost (MC). (MR=MC)

The relationship between MR and price elasticity, Ep, is: MR=P[1-1/E_p]

Oligopoly

Oligopoly: More than one, usually less than five firms. Few firms that may have significant pricing power; high barriers to entry; products may be homogeneous or differentiated.

Profit Maximizing Condition: MR = MC

Monopolistic competition

Monopolistic competition: Many firms; some pricing power; low barriers to entry; differentiated products; large advertising expense.

Profit Maximizing Condition:

MR = MC. Also, Marginal Revenue (MR) < Price

In all market structures, profit is maximized at the output quantity for which marginal revenue=marginal cost.

Price Discrimination:

Firms charge different prices in markets with different demand elasticities to increase revenue and profits.

Understanding Business Cycles

Theories of the Business Cycle

The Neoclassical school: Expansions and contractions represent efficient operation of the economy in response to external real shocks; Governments should not to intervene in the economy with discretionary fiscal

and monetary policy.

Keynesian economist: Change in expectation shifts aggregate demand and expansion policy is necessary; the government should intervene to keep capital and labor employed by deliberately running a larger fiscal deficit.

Monetarists: Stead and predictable increase in money supply

Business cycle Phases:

Business cycle: Recovery, Expansion, Slowdown Contraction

Economic Indicators

Leading: Turning points occur ahead of peaks and troughs (stock prices, initial unemployment claims, and manufacturing new orders)

Coincident: Turning points coincide with peaks and troughs (Industrial production index, personal income, manufacturing sales)

Lagging: Turning points follow peaks and troughs (average duration of unemployment, inventory/sales ratio, prime rate)

Monetary and Fiscal Policy

Policy Multipliers

Money multiplier

- $\text{money multiplier} = \frac{1}{\text{reserve requirement}}$

Fiscal multiplier

- $\text{fiscal multiplier} = \frac{1}{1-\text{MPC}(1-t)}$

Where MPC=Marginal propensity to consume, t=tax rate.

Expansionary and Contractionary Policy

Monetary policy is expansionary when the policy rate is less than the neutral interest rate (real trend rate of economic growth+ inflation target) and contractionary when the policy rate is greater than the neutral interest rate.

Fiscal policy is expansionary when a budget deficit is increasing or surplus is decreasing and contractionary when a budget deficit is decreasing or surplus is increasing.

International trade and capital flows

Regional Trading Agreements

Free trade area: Removes barriers to goods and services trade among members.

Customs union: Members also adopt common trade policies with non-members.

Common market: Members also remove barriers to labor and capital movements among members.

Economic union: Members also establish common institutions and economic policy.

Monetary union: Members also adopt a common currency.

Currency Exchange Rates

Real Exchange Rate (domestic/foreign)

$$\text{Real Exchange Rate} = \text{nominal FX rate} \times \frac{\text{base currency CPI}}{\text{price currency CPI}}$$

Interest Rate Parity (IRP)

F (forward), S (spot) X/Y, r_X and r_Y is the nominal risk-free rate in X and Y

$$\frac{F}{S} = \frac{1+r_X}{1+r_Y}$$

$$\frac{F-S}{S} = \frac{1+r_X}{1+r_Y} - 1 = \frac{r_X - r_Y}{1+r_Y} \approx r_X - r_Y$$

Exchange Rate Regimes

Formal dollarization: country adopts foreign currency.

Monetary union: members adopt common currency.

Fixed peg: ±1% margin versus foreign currency or basket of currencies.

Target zone: Wider margin than fixed peg.

Crawling peg: Pegged exchange rate adjusted periodically.

Crawling bands: Width of margin increases over time.

Managed floating: Monetary authority acts to influence exchange rate but does not set a target.

Independently floating: Exchange rate is market-determined.

FINANCIAL STATEMENT ANALYSIS

Income Statement

Unusual or infrequent Items

- Gains/losses from disposal of a business segment.
- Gains/losses from sale of assets or investments in subsidiaries.
- Provisions for environmental remediation
- Impairments, restructuring costs, write-offs, and write-downs.
- Integration expenses associated with businesses recently acquired.

Discontinued Operations

- A business-asset, operations, investing, financing activities-must be physically/operationally distinct from reset of firm.
- When deciding to be disposed, income/losses are reported net of tax after net income from continuing operations.

Item in the income statement account

- $\text{Gross profit margin} = \frac{\text{Gross profit}}{\text{Revenue}}$
- $\text{Operating profit margin} = \frac{\text{Operating profit}}{\text{Revenue}}$
- $\text{Net profit margin} = \frac{\text{Net income}}{\text{Revenue}}$

Basic and Diluted EPS

- **Basic EPS =**

$$\frac{\text{net income} - \text{preferred dividends}}{\text{weighted average number of common shares outstanding}}$$

- **Diluted EPS =**

$$\frac{[\text{NI-Pref.D}] + \text{Conv.pref.D} + \text{Conv. INT} \times (1-t)}{\begin{pmatrix}\text{wtd}\\ \text{avg}\\ \text{sh's}\end{pmatrix} + \begin{pmatrix}\text{shares from}\\ \text{conversion}\\ \text{of conv. pfd. sh's}\end{pmatrix} + \begin{pmatrix}\text{sh's from}\\ \text{conversion}\\ \text{of conv. debt}\end{pmatrix} + \begin{pmatrix}\text{shares}\\ \text{issuable from}\\ \text{stock options}\end{pmatrix}}$$

Treasury stock method (options or warrants)

Number of shares that would be issued upon exercise
– number of shares that would have been purchased with the proceeds
= shares issuable from stock options

Balance Sheet

Accounting equation:

- Assets = liabilities + stockholder's equity
- Assets = liabilities + contributed capital + beginning retained earnings + revenue-expenses – dividend declared + accumulated OCI

Marketable Security Classification

		Unrealized G/L	Realized G/L
HTM	AMORT	Not report	I/S
AFS	Fair value	OCI	I/S
Trading	Fair value	I/S	I/S

Cash Flow Statement

Basic Format:

- Net change in cash = CFO + CFI + CFF
- Ending cash = Beginning cash + Net change in cash

CFO (direct method)

- Cash collected from customers = net sales - ΔA/R + Δunearned revenue
- Cash paid to suppliers = -COGS -Δinventories + ΔA/P + Dep included in COGS
- Cash interest =-interest expenses + ΔI/P+ amortization of bond discount - premium
- Cash taxes = -income tax expenses + ΔT/P + ΔDTL - ΔDTA

CFO (indirect method)

- Start with net income
- - gain + losses (resulting from CFF or CFI)
- + non-cash charges
- - non-cash revenue
- - increase in non-cash current asset
- + increase in current liability

CFI:

Cash used in purchase

- $BV_{end} = BV_{Begin}$ + Purchase – Disposal NBV – Depreciation

Proceeds received from sale of fixed assets

- Gain or loss = proceeds received – disposal NBV
- $CFI=-(BV_1+\text{depreciation}-BV_0)+\text{gain-loss}$

CFF:

- -Dividend paid= -dividend declared+ Δdividend payable
- Dividend declared=NI-ΔR/E

Free cash flow to the firm:

- FCFF is the cash flow available to the company's suppliers of debt and equity capital.
- FCFF=NI+NCC+[Int×(1-tax rate)]-FCInv-WCInv
- FCFF=CFO +[Int×(1-tax rate)]-FCInv

Free cash flow to equity:

- FCFE is the cash flow available to the company's common stockholders.
- FCFE=CFO – FCInv + Net borrowing

Cash flow ratios:

- Cash flow-to-revenue=CFO/net revenue
- Cash return-on-equity=CFO/average total equity
- Cash-to-income=CFO/operating income
- Cash flow per share=CFO - preferred dividends/weighted average number of common shares
- Debt coverage=CFO/total debt
- Interest coverage = (CFO + INT paid + taxes paid)/interest paid
- Reinvestment= CFO/cash paid for long-term debt repayment
- Dividend payment = CFO/dividends paid
- Investing and financing = CFO/cash outflows from investing and financing activities

Analysis of Financial Statement

Common-size statement:

- $\frac{\text{Balance sheet account}}{\text{Total assets}}$
- $\frac{\text{Income statement account}}{\text{Revenues}}$
- $\frac{\text{Cash flow statement account}}{\text{Revenues}}$

$\frac{\text{Cash outflow}}{\text{Total cash outflows}}$ $\frac{\text{Cash inflow}}{\text{Total cash inflows}}$

Profitability Ratios:

- $\text{gross profit margin} = \frac{\text{gross profit}}{\text{revenue}}$
- $\text{Operating profit margin} = \frac{\text{operating profit}}{\text{revenue}} = \frac{\text{EBIT}}{\text{net sales}}$
- $\text{Net profit margin} = \frac{\text{net income}}{\text{revenue}}$
- $\text{ROA} = \frac{\text{net income+Int}\times(1\text{-t})}{\text{average total assets}} = \frac{\text{Net Income}}{\text{average total assets}}$
- $\text{ROE} = \frac{\text{net income}}{\text{average total equity}}$

Liquidity ratios:

- $\text{Current ratio} = \frac{\text{current assets}}{\text{current liabilities}}$
- $\text{Quick ratio} = \frac{\text{cash+marketable securities+receivables}}{\text{current liabilities}}$
- $\text{Cash ratio} = \frac{\text{cash+marketable securities}}{\text{current liabilities}}$
- $\text{Defensive interval} = \frac{\text{cash+maketable securities+receivables}}{\text{daily cash expenditures}}$

Activity Ratios

- $\text{Total asset turnover} = \frac{\text{net revenue}}{\text{average total assets}}$
- $\text{Fixed asset turnover} = \frac{\text{net revenue}}{\text{average net fixed assets}}$
- $\text{Working Capital turnover} = \frac{\text{net revenue}}{\text{average Working Capital}}$
- Working capital =current assets - current liabilities
- $\text{Receivables turnover} = \frac{\text{annual sales}}{\text{average receivables}}$
- $\text{Days of sales outstanding} = \frac{365}{\text{receivables turnover}}$
- $\text{Inventory turnover} = \frac{\text{COGS}}{\text{average inventory}}$
- $\text{Days of inventory on hand} = \frac{365}{\text{inventory turnover}}$
- $\text{Payables turnover ratio} = \frac{\text{COGS}}{\text{average trade payables}}$
- $\text{Number of days of payables} = \frac{365}{\text{payables turnover ratio}}$
- Cash conversion cycle = days of inventory on hand + days of sales outstanding – number of days of payables

- Operating cycle = collection period + inventory period

Solvency Ratios

- Debt-to-equity ratio = D / E
- Debt-to-capital= D / (D + E)
- Debt-to-assets = D / A
- Financial leverage = A / E
- Debt-to-EBITDA= D/EBITDA
- $\text{Interest coverage} = \frac{\text{EBIT}}{\text{interest}}$
- $\text{Fixed charge coverage} = \frac{\text{EBIT} + \text{lease payment}}{\text{interest} + \text{lease payment}}$

Sustainable growth rate:

- g = RR×ROE

RR = retention ratio = 1-payout ratio

Payout ratio = PR = Dividend/earnings

DuPont analysis:

- $\text{ROE} = \text{Net profit margin} \times \text{Asset turnover} \times \text{Equity multiplier} = \frac{\text{Net income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Assets}} \times \frac{\text{Assets}}{\text{Equity}}$
- $\text{ROE} = \text{tax burden} \times \text{interest burden} \times \text{EBIT margin} \times \text{asset turnover} \times \text{leverage} = \left(\frac{\text{net income}}{\text{EBT}}\right) \times \left(\frac{\text{EBT}}{\text{EBIT}}\right) \times \left(\frac{\text{EBIT}}{\text{revenue}}\right) \times \left(\frac{\text{revenue}}{\text{average total assets}}\right) \left(\frac{\text{average total assets}}{\text{average equity}}\right)$

Inventories:

COGS

- COGS = Opening Inventory + Purchases – Ending Inventory
- Ending inventory = beginning inventory + purchase - COGS

Inventory Costing Methods:

- LIFO (only permitted under US GAAP)
- FIFO
- Weighted average
- Specific identification method

LIFO reserve:

- $INV_F = INV_L$ + LIFO reserve
- $COGS_F = COGS_L$ -Δ LIFO reserve
- $NI_F = NI_L$ + Δ LIFO reserve × (1-t)
- In periods of rising prices and stable or increasing inventory quantities:

	LIFO results in	FIFO results in
COGS	Higher	Lower
Grow profit	Lower	Higher
Inventory Balances	Lower	Higher

Long-Lived Assets

Capitalizing and expensing:

	Capitalizing	Expensing
Income variability	Lower	Higher
Profitability—first year	Higher	Lower
Profitability—last year	Lower	Higher
Total cash flow	Same	Same
CFO	Higher	Lower
CFI	Lower	Higher
Leverage ratios (D/E)	Lower	Higher

Depreciation:

- $\text{Straight line} = \frac{\text{cost-residual value}}{\text{useful life}}$
- DDB depreciation = $\frac{2}{\text{useful life}} \times \text{Book Value at beginning of period}$
- $\text{Units of production} = \frac{\text{cost - salvage value}}{\text{useful life in units}} \times \text{output units}$

Depreciation effect on ratios

	Straight line	Accelerated
Depreciation expense	Lower	Higher
Net income	Higher	Lower
Assets	Higher	Lower
Equity	Higher	Lower
ROA	Higher	Lower
ROE	Higher	Lower
Turnover ratios	Lower	Higher
Cash flow	Same	Same

Intangibles:

US GAAP:

- R&D costs expensed under GAAP
- Post-feasibility software costs can be capitalized and depreciated under straight line basis.

IFRS:

- Research cost are expensed
- Development costs can be capitalized when specific criteria are met.

Impairment Effects:

Impairment Effects	
Cash flow	No effect
Assets	Decrease
Deferred taxes	Decrease
Stockholder's equity	Decrease
Current net income, ROA,ROE	Decrease
Future net income, ROA,ROE	Increase
Depreciation expense	Decrease
Future asset turnover ratios	Increase
Debt/equity ratio	Increase

Revaluation of Long-Lived Assets

- IFRS: revaluation gain recognized in net income only to the extent it reverses previously recognized impairment loss; further gains recognized in equity as revaluation surplus.
- U.S. GAAP: revaluation is not permitted.

Analysis of Income Taxes

- Income tax expense = Tax payable +ΔDTL-ΔDTA

Effective tax rate:

- Reported effective tax rate = $\frac{\text{income tax expense(from the f/s)}}{\text{pretax income(from the f/s)}}$

Quick memorizing tip for defer tax:

- DTL Creation situations: Depreciation
- DTA creation situations: R&D; Account receivables; customer advance; warranty liability;

Deferred taxes

- It arises due to **temporary differences**.
- Deferred tax liabilities are created when taxable income<pretax income.
  - ✓ Treat DTL as equity if not expected to reverse during the process of analysis.
  - ✓ If the reversal or non-reversal is uncertain, ignored DTL
- Deferred tax assets are created when taxable income>pretax income.
  - ✓ Must recognize valuation allowance if more likely than not that DTA will not be realized.

Long-Term Liabilities

Bond

- Premium bond: coupon rate > market rate at issurance.
- Discount bond: coupon rate < market rate at issurance.
- Interest expense =$BV_0 \times r_m$
- Interest expense = $\begin{pmatrix}\text{market rate}\\ \text{at issue}\end{pmatrix} \times \begin{pmatrix}\text{balance sheet value of}\\ \text{liability at beginning of period}\end{pmatrix}$
- Coupon (payment) = coupon rate of bond issued * face value (par value)
- Primium/Disounc amortization = |Coupon (payment) – interest expense|

Accounting treatment for lessee

	B/S	I/S	CFS
All IFRS lease and finance lease under US GAAP	Recognize "right-of-use" (ROU) asset and lease liability	Report depreciation expense on ROU asset	Reduction of lease liability -----CFF
		Report interest expense on lease liability	Interest payment----CFO or CFF (IFRS)
Operating lease (US GAAP)	Recognize "right-of-use" (ROU) asset and lease liability	Reporting single lease expense (a straight line allocation of lease cost)	Entire cash payment is CFO

Accounting treatment for lessors

	B/S	I/S	CFS
Operating lease under IFRS and US GAAP	Retain asset on B/S	Report lease income Report depreciation expense on leased asset	Lease payments received -----CFO
Finance lease (IFRS) Sales-type lease (US GAAP)	Remove leased asset from B/S Recognize lease asset (lease receivable and residual)	Report interest revenue on lease receivable. If applicable, report revenue, COGS, and selling profit.	Interest portion of lease received-----CFO or CFI (IFRS) Receipt of lease principal----- CFI
Direct financing lease (US GAAP)	Remove leased asset from B/S Recognize lease receivable	Report interest revenue on lease receivable	Interest portion of lease payment received----CFO Receipt of lease principal-----CFI

Pensions

- Defined contribution: employer contribution expensed in period incurred.
- Defined benefit: overfunded plan recognized as asset, underfunded plan recognized as liability. Balance sheet value equals funded status of plan under U.S.GAAP but may be not under IFRS.

CORPORATE ISSUERS

Capital Allocation

Net Present Value (NPV)

- $NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t}$
- Project NPV = NPV (without options) – Option cost + Option value.

Internal Rate of Return

- $\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$

Return on Invested Capital

- ROIC

$$= \frac{\text{After-tax operating profit}_t}{\text{Average invested capital}}$$

$$= \frac{(1 - \text{Tax rate}) \times \text{Operating profit}_t}{\text{Average total LT liabilities and equity}_{t-1,t}}$$

Cost of Capital

Weighted Average Cost of Capital

- $WACC=(w_d)[k_d(1-t)]+(w_{ps})(k_{ps})+(w_{ce})(k_s)$

Working Capital & Liquidity

- **Primary sources of liquidity:** cash balances, short-term funding, cash flow management of collections and payment.
- **Secondary sources of liquidity:** liquidating assets, negotiating debt agreements, bankruptcy protection.

Cash Conversion Cycle

- Cash Conversion Cycle = Days of inventory on hand + Days sales outstanding - Days payable outstanding

- Effective Annual Rate of Supplier Financing = $\left(\left(1+\frac{\text{Discount\%}}{100\%-\text{Discount\%}}\right)^{\frac{\text{Days in Year}}{\text{Payment Period - Discount period}}}\right)-1$

Working Capital

- Total Working Capital =
  Current assets
  Minus: Current liabilities

- Net Working Capital =
  Current assets, excluding cash and marketable securities
  Minus: Current liabilities, excluding short-term and current debt

Measures of cash flows

- Cash flow from operations =
  Cash received from customers
  Plus: Interest and dividends received on financial investments
  Minus: Cash paid to employees and suppliers
  Minus: Taxes paid to governments
  Minus: Interest paid to lenders
- Free cash flow =
  Cash flows from operations
  Minus: Investments in long-term assets

Liquidity ratios

- Current ratio= $\frac{\text{current assets}}{\text{current liabilities}}$
- Quick ratio= $\frac{\text{cash+marketable securities+receivables}}{\text{current liabilities}}$
- Cash ratio= $\frac{\text{cash+marketable securities}}{\text{current liabilities}}$

Capital structure

MM I without Taxes

- $V_L = V_U$

MM II without Taxes

- $r_e = r_0 + (r_0 - r_d)\frac{D}{E}$

MM I with Taxes

- $V_L = V_U + tD$

MM II with Taxes

- $r_e = r_0 + (r_0 - r_d)(1\text{-}t)\frac{D}{E}$

Static Trade-off Theory

- $V_L = V_U + tD$ - PV(Costs of financial distress)

EQUITY

Market Organization and Structure

Well-Functioning Security Markets

Operational efficiency (lowest possible transaction costs).

Informational efficiency (asset prices reflect new information quickly and rationally).

Allocational efficiency.

Complete market.

Market Structures

Quote-driven markets: investors trade with dealers.

Order-driven markets: buyers and sellers matched by rules.

Brokered markets: brokers find counterparties.

Call Market: Trading takes place at specific times and uses the auction process to match trades.

Continuous Market: Trading takes place at all times.

Types of Orders:

Execution instructions: how to trade; e.g. market orders; limit orders.

Validity instructions: when to execute; e.g. stop orders, day orders, fill-or-kill orders.

Clearing instructions: how to clear and settle; for sell orders, specify short sale or sale of owned security.

Market order: Executed at current price

Limit order:

- Limit buy order: Maximum purchase price specified below the current price
- Limit sell order: Minimum selling price specified above the current price

Stop loss order:

- Stop loss sell order: A price specified below the current price to liquidate a long position
- Stop loss buy: A price specified above the current price to reverse a current short position

Margin

Margin Purchases

- Leverage factor = 1/margin percentage.

Margin Call Price

- Long margin=$P_0 \times \frac{\text{1-initial margin\%}}{\text{1-maintenance margin\%}}$

Security Market Indexes

Index return

Price return:

- Price return=$\frac{P_1-P_0}{P_0}$

Total return:

- Total return=$\frac{P_1-P_0+\text{Income}}{P_0}$

Method of index construction

Price Weighting:

- $\text{index value} = \frac{\text{sum of stock prices}}{\text{number of stocks in index}}$
- $r_p = \frac{\text{index value}_1 - \text{index value}_0}{\text{index value}_0}$
- Stock split adjust the value of the divisor

$$\frac{\text{sum of stock prices after stock split}}{\text{adjusted divisor}} = \frac{\text{sum of stock prices before stock split}}{\text{number of stocks in index}}$$

Market Value Weighting:

- $r_m = \frac{\sum(P_i \text{ of time T} \times \#\text{ of shares of i at T})}{\sum(P_i \text{ of time t} \times \#\text{ of shares of i at t})} - 1$

Equal weighting:

- change in average index value = $\frac{\sum X_i}{n} - 1$

  (arithmetic mean)

  where: X = 1 + HPY
- change in average index value =

  $\sqrt[n]{X_1 \times X_2 \times \ldots \times X_n} - 1$ (geometric mean)

  where: X = 1 + HPY

Biases:

- PWI is biased toward high priced stocks;
- VWI toward high market value stocks;
- EWI toward smaller stocks.

Market Efficiency

Forms of EMH:

Weak-form:

- Current stock prices fully reflect available security market info. Volume information/past price do not relate to future direction of security prices. Investor cannot achieve excess returns using technical analysis.

Semi-strong form:

- Security prices instantly adjust to new public information. Investor cannot achieve excess returns using fundamental analysis.

Strong form:

- Stock prices fully reflect all information from public and private sources.
- Assumes perfect markets in which all information is cost free and available to everyone at the same time. Even with inside information, investor cannot achieve excess returns.

Overview of Equity Securities

Types of Equity Investments

Common Shares:

- Callable common shares and putable common shares
- Cumulative voting and statutory voting

Preference Shares:

- Cumulative preference shares
- Participating preference shares
- Convertible preference shares

Non-domestic Equity Securities

Global registered shares(GRS):

- GRS are traded in different currencies on stock exchanges around the world.

Depository receipts(DRs):

- **Sponsored DR:** the firm is involved with the issue. A sponsored DR provides the investor voting rights.
- **Unsponsored DR:** the firm is not involved with the issue. The depository bank retains the voting rights.

Risk and Return of Equity Securities:

- Return on Equity:

$$ROE_t = \frac{NI_t}{\text{average BV}} = \frac{NI_t}{(BV_t + BV_{t-1})/2}$$

$$ROE_t = \frac{NI_t}{BV_{t-1}}$$

Industry and Competitive Analysis

Market share trends and major player

- $HHI = \sum_{n=1}^{\infty} S_n^2$

Porter's Five Forces

- Rivalry among existing competitors.
- Threat of new entrants.
- Threat of substitute products.
- Bargaining power of buyers.
- Bargaining power of suppliers.

PESTLE Frameworks

- Political influences
- Economic influences
- Social influences
- Technological influences
- Legal influences
- Environmental influences

Competitive Strategies

- Cost leadership
- Differentiation
- Focus

Compamy Analysis: Forecasting

Forecast objects

- Drivers of financial statement lines
- Individual financial statement lines
- Summary measures
- Ad hoc objects

Forecast approaches

- Historical results approach
- Historical base rates and convergence
- Management guidance approach
- Analyst's discretionary forecast approach

Equity Valuation:

Discount dividend model (DDM):

Preferred stock:

- Preferred stock value =

$$V_p = \frac{D_p}{(1+r_p)} + \frac{D_p}{(1+r_p)^2} + \cdots + \frac{D_p}{(1+r_p)^{\infty}} = \frac{D_p}{r_p}$$

Common stock:

- General DDM:

$$V_j = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \cdots + \frac{D_{\infty}}{(1+r)^{\infty}} = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t}$$

- Constant growth model (Gordon growth model):

$$V_0 = \frac{D_0(1+g_c)}{r_e - g_c} = \frac{D_1}{r_e - g_c}$$

$$D_0 = (1-RR) \times EPS$$

$$g_c = ROE \times RR$$

$$E(R_i) = R_f + \beta_i \left[E(R_m) - R_f \right]$$

Critical relationship between r and g:

- As difference between r and g widens, value of stock falls.
- As difference narrows, value of stock rises.
- Small changes in difference between r and g cause large changes in stock's value.

Critical assumptions of infinite period DDM:

- The company is assumed to be going concern.
- Stock pays dividends with a constant growth rate.
- r must be greater than g.

Required rate of return:

- CAPM: $E(R_i) = R_f + \beta_i \left[E(R_m) - R_f \right]$
- R_E=current bond yield + equity risk premium

Free Cash Flow to Equity:

- FCFE= Net Income + Non-Cash Charge – WCInv – FCInv + Net Borrowing
- FCFF = Net Income + Non-Cash Charge + Int (1-t) – FCInv – WCInv
- $V_0 = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1+r_e)^t}$
- $V_0 = \frac{FCFE_0(1+g)}{r-g}$

Price multiplier:

Trailing P/E: (b=retention ratio=RR)

$$P_0/E_0 = \frac{D_0(1+g)/E_0}{r-g} = \frac{(1-b)(1+g)}{r-g} = \frac{\text{price per share}}{\text{EPS previous 12 mo.}}$$

Leading P/E:

$$P_0/E_1 = \frac{D_1/E_1}{r-g} = \frac{(1-b)}{r-g} = \frac{\text{price per share}}{\text{forecast EPS next 12 mo.}}$$

$$P/B = \frac{\text{Price per share}}{\text{Book value per share}}$$

$$P/S = \frac{\text{Price per share}}{\text{Sale per share}}$$

$$P/CF = \frac{\text{Price per share}}{\text{Cash flow per share}}$$

EV/EBITDA

- EBITDA = Earnings before interest, taxes, depreciation and amortization
- EV= market value of common stock + market value of preferred equity + market value of debt– cash and short-term investments

FIXED INCOME

Bond Valuation

Bond Pricing

There are two equivalent ways to price a bond:

- Constant discount rate applied to all cash flows (YTM) to find PV. This is a bond's flat price (does not include accrued interest).
- Discount each cash flow using appropriate spot rate for each. This is a bonds no-arbitrage price.
- Bond price(Annual compounding) =

$$\frac{CPN_1}{(1+YTM)} + \frac{CPN_2}{(1+YTM)^2} + \ldots + \frac{CPN_N + Par}{(1+YTM)^N}$$

- Bond price(Semi-annual compounding) =

$$\frac{CPN_1}{(1+YTM/2)} + \frac{CPN_2}{(1+YTM/2)^2} + \ldots + \frac{CPN_{2N} + Par}{(1+YTM/2)^{2N}}$$

- Valuation Using Forward Rates

  Bond value=

$$\frac{CF_1}{(1+S_1)} + \frac{CF_2}{(1+S_1)(1+1y1y)} + \ldots \frac{CF_n}{(1+S_1)(1+1y1y)\ldots(1+(T-1)y1y)}$$

- Value of a zero-coupon bond (semi-annual paid)

$$Bond\ value = \frac{Par\ value}{(1+\frac{YTM}{2})^{2N}}$$

- The no-arbitrage price of a bond is calculated using spot rates=

$$\frac{CPN_1}{(1+S_1)} + \frac{CPN_2}{(1+S_2)^2} + \cdots\cdots + \frac{CPN_N + Par}{(1+S_N)^N}$$

Full price, clean price, and Accrued Interest

- Full price includes accrued interest. Government bonds use actual day counts; corporate bonds use 30/360 method.
- Clean(flat) Price: the agreed upon price of the bond.
- Full price=PV at last coupon date $\times(1+YTM)^{t/T}$

  Accrued interest=coupon payment $\times(t/T)$
- Full Price = Clean price + Accrued interest

Matrix Pricing

For illiquid bonds, use yields of bonds with same credit quality to estimate yield; adjust for maturity differences with linear interpolation.

Yield measure

- Bond Equivalent Yield= $\frac{365}{t} \times \frac{FV - PV}{PV}$
- Discount yield= $\frac{360/365}{t} \times \frac{FV - PV}{FV}$
- Add-on yield= $\frac{360/365}{t} \times \frac{FV - PV}{PV}$
- Effective yield= $(1+\frac{YTM}{m})^m - 1$
- Convert an annual percentage rate for m periods per year (APR_m), to an annual percentage rate for n per year (APR_n):

$$(1+\frac{APR_m}{m})^m = (1+\frac{APR_n}{n})^n$$

- Current yield(income or interest yield)=

$$\frac{\text{sum of coupon payment received over the year}}{\text{flat bond price}}$$

Relationship between Coupon Rate, Current Yield, YTM

- Par bond: coupon rate = current yield = YTM
- Discount bond: coupon rate < current yield < YTM

- Premium bond: coupon rate > current yield > YTM

Spot Rates & Forward Rates

To derive a bond's value using spot rates, discount the individual cash flows at appropriate rate for each flow's time horizon; sum PV of the cash flow to get bond's current value. This value is the arbitrage-free value.

- Spot rate from forward rate:

$$S_3 = \left[(1+S_1)(1+1y1y)(1+2y1y)\right]^{\frac{1}{3}} - 1$$

- Relationship Between Forward Rates and Spot Rates

$$(1+S_T)^T = (1+S_1)(1+1y1y)\ldots(1+(T-1)y1y)$$

- Forward rate from spot rate:

$$1y1y = \frac{(1+S_2)^2}{(1+S_1)} - 1$$

Repayment/Prepayment Provisions

- Bullet bonds. Lump sum at maturity pays entire principal.
- Amortizing securities. Periodic principal and interest payments.

$$A = \frac{r \times Principal}{1-(1+r)^{-N}}$$

- Sinking fund provision. Provide for bond retirement through predefined principal payments over life of the issue.
- Call provisions. Issuer has right (but not obligation) to retire all or part of issue prior to maturity. Issuer owns option to call the bonds away from investor.

Spread

- G-spread: the benchmark is government bond yield
- I-spread: the benchmark is swap rate
- Z-spread: the spread that must be added to each rate on the benchmark yield curve to make the present value of a bond equal to its price

Option Cost = Z-spread – OAS

- Callable bond: ZS > OAS
- Putable bond: ZS < OAS

Asset-Backed Securities:

MBS

- Residential MBS: home mortgages are collateral.
- Agency RMBS include only conforming loans; non-agency RMBS may include nonconforming loans and need credit enhancement.
- Prepayment risk: contraction risk from faster prepayments; extension risk from slower prepayments.
- CMOs: pass-through MBS are collateral. May have sequential-pay or PAC/support structure.
- Commercial MBS: non-recourse mortgages on commercial properties are collateral.

ABS

- Credit card ABS: credit card receivables are collateral.
- CDOs: Bonds, bank loans, MBS, ABS, or other CDOs are collateral.

Collateral and Credit Enhancement:

- Secured bonds are backed by specific collateral and senior to unsecured bonds.
- Unsecured bonds are general claims to issuers cash flows and assets.
- Internal credit enhancement: Excess spread, overcollateralization, waterfall structure.
- External credit enhancement: Surety bonds, letters of credit, bank guarantees.

Bond Risk and Return

Source of Return

- annualzed holding period return = $\left(\frac{\text{total return}}{\text{bond price}}\right)^{1/n} - 1$

Types of Risks

Interest rate risk: price fluctuations due to interest rate changes. Interest rates and bond prices move inversely.

- Longer maturity bonds. Higher interest rate risk (all else same).
- Smaller coupon bonds. Higher interest rate risk (all else same).
- If market interest rates are high, price volatility will be lower than if market interest rates are low.

Prepayment risk: a bond's principal will be paid early

when the interest rates are lower.

Reinvestment risk: bond coupon and amortization payments will be reinvested at lower rates.

Credit risk: that a bond may default on its promise to make coupon and principal payments.

- Default risk. Issuer might not make payments.
- Credit spread risk. Difference in bond's yield from yield on risk-free security. All else equal, the riskier the bond, the higher the spread.
- Downgrade risk. Bond may be reclassified as riskier security by a major rating agency.

Liquidity risk: that a bond cannot be sold at fair value quickly. Measured by bid ask spread = Ask – Bid.

Duration and Convexity

Macaulay duration

- is a weighted average of time (in years) until cash flow will be received. The weights are the proportions of the total bond value that each cash flow represents.
- Macaulay duration= $\sum_{t=1}^{n} t \times (PVCF_t / P_0)$
- Modified Duration

$$\text{Modified duration}=\frac{\text{Macaulay duration}}{1+\text{periodic market yield}}$$

$$\text{Approximate modified duration}=\frac{V_- - V_+}{2\times V_0 \times \Delta YTM}$$

Measures percent change in bond price for a one percent change in interest rates.

$$\text{duration}=-\frac{\text{percentage change in bond price}}{\text{yield change in percent}}$$

- Duration

  is the slope of a bond's price-yield function. It is steeper at low interest rates, flatter at high interest rates. So, duration (interest rate sensitivity) is high at low rates and low at higher rates. This holds for non-callable bonds.
- Money duration=

  annual modified duration × full price of bond
- Money duration per 100 units of par value=

  annual modified duration × full price of bond per 100 of par

Effective duration

- curve duration, measures the price sensitivity with respect to changes in the U.S. Treasury par curve.
- Effective duration = $\frac{V_- - V_+}{2V_0(\Delta y)}$

PVBP

- is the money change in full price of a bond when its YTM changes by one basis point(0.01%)
- PVBP= $\frac{V_- - V_+}{2}$
- Convexity is a measure of degree of curvature or convexity in the price/yield relationship.
- Convexity accounts for amount of error in estimated price (based on duration).
- A callable bond is likely to be called as yields fall, so no one will pay a price higher than the call price. The price won't rise significantly as yield falls and you'll see negative convexity at work as yields fall, prices rise at a decreasing rate. For a positively convex bond, as yields fall, prices rise at an increasing rate.

Convexity

$$\text{approximate convexity}=\frac{V_- + V_+ - 2V_0}{(\Delta YTM)^2 V_0}$$

$$\text{effective convexity}=\frac{V_- + V_+ - 2V_0}{(\Delta curve)^2 V_0}=\frac{\frac{V_- - V_0}{\Delta curve}-\frac{V_0 - V_+}{\Delta curve}}{\Delta curve}\times\frac{1}{V_0}$$

Portfolio Duration & Key Rate Duration

Portfolio duration

The Macaulay and modified durations for the portfolio are calculated as the weighted average of the statistics for the individual bonds. The shares of overall portfolio market value are the weights.

$$\text{Portfolio duration}=w_1D_1+w_2D_2+\cdots+w_ND_N$$

$$\text{Portfolio convexity}=w_1C_1+w_2C_2+\cdots+w_NC_N$$

Key Rate Duration:

partial duration, the sensitivity of the value of a bond or portfolio to changes in spot rate for a specific maturity, holding other spot rates constant.

Bond's value of a change in spread

Return impact of a change in spread

$$\%\Delta\text{bond value} = -\text{duration}(\Delta\text{spread}) + \frac{1}{2}\text{convexity}(\Delta\text{spread})^2$$

Features & Interest Rate Sensitivity:

Longer bonds more are sensitive

Higher coupon bonds are less sensitive

Callable bonds are less sensitive than non-callable bonds.

Credit Analysis

Credit analysis

Investment grade: Baa3/BBB- or above

Non-investment grade: Ba1/BB+ or below

"The Cs": capacity, capital, collateral, covenants, character, conditions, country, currency.

Default risk=probability of default

Loss severity=percent of value lost if borrower defaults

Expected loss=default risk*loss severity

Recovery rate=1- Loss severity given default

The Cs of Credit Analysis

Capacity refers to the ability of the borrower to make its debt payments on time.

Capital addresses other company resources available that reduce reliance on debt.

Collateral refers to the quality and value of the assets supporting the issuer's indebtedness.

Covenants are the terms and conditions of lending agreements that the issuer must comply with.

Character refers to the quality of management.

Conditions refers to the general economic, competitive, and business environment faced by all borrowers that may affect their ability to service or refinance debt.

Country involves the geopolitical environment as well as the legal and political system faced by all issuers in a jurisdiction that may affect debt payment.

Currency affects issuers whose cash flows are affected by exchange rate changes or who borrow in a currency outside of their jurisdiction, such as sovereign issuers with foreign currency debt.

DERIVATIVES

Instruments, Pricing & Valuation

Forward Contracts

A **cash settlement** forward contract does not require actual delivery of the underlying asset, but a cash payment to the party disadvantaged by the difference between market price of the asset and contract price at settlement date;

Early termination can be achieved by entering into a new forward contract with the opposite position, at the current expected future price. This will fix the amount of payment to be made/received at settlement date.

FRA

It is a forward contract on an interest rate, such as Shibor. Can be viewed as a forward contract to borrow/lend money at a certain rate at some future date.

Futures (Three methods for risk management)

Margin

- **Initial margin:** the first deposit.
- **Maintenance margin:** If the margin balance in the trader's account falls below the maintenance margin, the trader will get a margin call.
- **Variation margin:** Shortfall amount that needs to be deposited by the next trading day to bring the margin back up to initial margin for a futures contract, in the event of a margin call is called variation margin.

Mark to market: The margin requirement of a futures contract is low because at the end of every day there is a daily settlement process called marking to market.

Daily price limit: Price limits are exchanged-imposed limits on how much the contract price can change from the previous day's settlement price.

Futures vs. forwards

Forward contract:

One party agrees to buy, (the counterparty sells) a physical asset/security at specific price on specific future date. If asset's future price increases, the buyer (at the older, lower price) has a gain and seller has a loss.

Futures contract:

Standardized, exchange-traded forward contract. Different from forwards in that futures trade in the active secondary market, are regulated, backed by

clearinghouse, and require daily settlement of gains/losses.

Forwards	Futures
OTC(private contract)	Exchange-traded
Customized contracts	Standardized contracts
Default risk	Guaranteed by clearinghouse
Little or no regulation	Regulated
Settlement at maturity	Daily settlement (mark to market)
No margin deposit required	Margin required and adjusted

Options

Intrinsic Value:

Call Payoff = Max [0, Stock price –Exercise price]

Put Payoff = Max [0, Exercise price – Stock price]

Option value = intrinsic value (exercise value) + time value of Option

Options and Exercise Price

A call with a lower exercise price is worth more. A put with a higher exercise price is worth more.

Time to Expiration

Longer time is beneficial for calls. It is also beneficial for American puts. (Exceptions: European puts due to waiting disadvantage.)

American vs. European Options

American options let the owner exercise the option any time before or at expiration.

European options can be exercised only at expiration. Value of the American option will equal/exceed value of the European option.

Lower Bounds & Maximum Value:

Option	Minimum Value	Maximum Value
European/ American call	$\text{Max}[0, S_t - X/(1+r_f)^{(T-t)}]$	S_t
European put	$\text{Max}[0, X/(1+r_f)^{(T-t)} - S_t]$	$X/(1+r_f)^{(T-t)}$
American put	Max[0, X -S_t]	X

Factors that Affect Option Values

Increase in:	Calls	Puts
Asset price	Increase	Decrease
Exercise price	Decrease	Increase
Risk-free rate	Increase	Decrease
Volatility	Increase	Increase
Time to expiration	Increase	Increase*
Holding costs	Increase	Decrease
Holding benefits	Decrease	Increase

***** There is an exception to the general rule that European put option thetas are negative. The put value may increases as the option approaches maturity if the option is deep in-the-money and close to maturity.

Interest rate SWAPs

May be replicated by a series of off-market FRAs with present values at swap initiation that sum to zero.

Arbitrage, Replication, and Derivatives

Law of one price

Two assets with identical cash flows in the future, regardless of future events, should have the same price. If A and B have identical future payoffs, and A is priced lower than B, buy A and sell B.

Second type of arbitrage

Two assets with uncertain returns can be combined in a portfolio that will have a certain payoff. If a portfolio of A and B has a certain payoff, the portfolio should yield the risk-free rate.

Forward price

Pricing with cost and benefit:

$$\boldsymbol{FP} = \left(\boldsymbol{S}_0 - \boldsymbol{PVB}_0 + \boldsymbol{PVC}_0\right) \times \left(1 + \boldsymbol{R}_f\right)^T$$

$$\text{or } \boldsymbol{FP} = \boldsymbol{S}_0 \times \left(1 + \boldsymbol{R}_f\right)^T + FVC - FVB$$

T-Bill Forward

Price: $\boldsymbol{FP} = \boldsymbol{S}_0 \times \left(1 + \boldsymbol{R}_f\right)^T$

Value: $\boldsymbol{V}_{long} = \boldsymbol{S}_t - \dfrac{\boldsymbol{FP}}{\left(1 + \boldsymbol{R}_f\right)^{T-t}}$

$$V_{short} = -V_{long} = \frac{FP}{\left(1 + \mathrm{R}_f\right)^{T-t}} - S_t$$

Equity Forward

Price: $FP = (S_0 - PVD_0) \times (1 + R_f)^T$

$FP = S_0 e^{(r+c-i)\times T}$ (continuous compounding)

Value: $V_{long} = (S_t - PVD_t) - \frac{FP}{(1+R_f)^{T-t}}$

Bond Forward

Price: $\boldsymbol{FP = (S_0 - PVC_0)\times(1+R_f)^T}$

Value: $\boldsymbol{V_{long} = (S_t - PVC_t) - \frac{FP}{(1+R_f)^{T-t}}}$

Interest rate forward contracts

Price: $(1+IFR_{B-A})^{B-A} = \frac{(1+Z_A)^A}{(1+Z_B)^B}$

Foreign exchange forward Contracts

Price: $(1+IFR_{B-A})^{B-A} = \frac{(1+Z_A)^A}{(1+Z_B)^B}$

$FP_{f/d} = S_{f/d} \times \frac{1+r_f}{1+r_d}$

$FP_{f/d} = S_{f/d} \times e^{(r_f - r_d)\times T}$

Futures Pricing

Yield is expressed in percentage terms:

$f_{A,B-A} = 100 - (100 \times MRR_{A,B-A})$

Put-Call Parity

- $c + X/(1+R_f)^T = S + p$

Each security in the put-call parity relationship can be expressed as:

$-s = -c + p - X/(1+R_f)^T$

$p = c + X/(1+R_f)^T - S$

$c = p + S - X/(1+R_f)^T$

$-p = -c + S - X/(1+R_f)^T$

$-c = -p + X/(1+R_f)^T - S$

Put-call-forward parity

$\boldsymbol{C_0 + \frac{X}{(1+R_f)^T} = P_0 + \frac{FP}{(1+R_f)^T}}$

Option Put–call Parity Applications: Firm Value

- **Solvency:** If $V_T > D$, the firm is *solvent* and able to return capital to *both* its shareholders and debtholders.
  a) Debtholders receive D and are repaid in full.
  b) Shareholders receive the residual: $E_T = V_T - D$.
- **Insolvency**: If $V_T<D$, the firm is insolvent. In the event of insolvency, shareholders receive nothing and debtholders are owed more than the value of the firm's assets. Debtholders therefore receive V_T to settle their debt claim of D at time T.
  a) Debtholders have a priority claim on assets and receive $V_T<D$.
  b) Shareholders receive the residual, $E_T=0$.

Option Pricing-binomial Model

- Value of an option: $c_0 = [\pi_u C_1^u + \pi_d C_1^d] \times \frac{1}{(1+R_f)^T}$
- Risk-neutral probability (π) of an up move is: $\pi_u = \frac{1+R_f-R_d}{R_u-R_d}$
- Hedge ratio (h^*: *shares per option*)
  a) Call option: $h^* = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}$
  b) put option: $h^* = \frac{p_1^u - p_1^d}{S_1^u - S_1^d}$
- A risk-free portfolio:
  a) Call option:

$$V_0 = h^* S_0 + c_0 = PV(h^* S_1^u - c_1^u) = PV(h^* S_1^d - c_1^d)$$

  b) Put option:

$$V_0 = h^* S_0 + p_0 = PV(h^* S_1^u + p_1^u) = PV(h^* S_1^d + p_1^d)$$

ALTERNATIVE INVESTMENTS

Hedge Funds:

Event-driven strategies

- Seek to profit from short-term events that will affect individual companies. Merger arbitrage; distressed/restructuring; activist shareholders; special situations.

Relative value strategies

- Seek to profit from a pricing discrepancy between related securities. Fixed Income Convertible arbitrage; Asset Backed Fixed Income; General Fixed Income; Volatility; Multi-Strategy.

Macro hedge strategies

- "Top down" strategies based on global economic trends.

Equity hedge strategies

- Seek to profit from long or short positions in publicly traded equities and derivatives with equities as their underlying assets. Strategies include market neutral, fundamental growth, quantitative directional, short bias and sector specific.

Private Capital

Leveraged buyouts

Venture capital stages of development:

- Formative stage, later stage, mezzanine stage.

Growth capital:

- Stage between VC and Maturity.

Exit strategies:

- Trade sale; IPO; recapitalization; secondary sales; write off/liquidation;SPAC.

The stage of venture capital investing

- Formative stage:
  Angel investing: At the idea stage, funds are used to transform the idea into a business plan and to assess market potential. Often provided by individuals.
  Seed stage: support product development and/or marketing efforts. The first stage at which VC funds invest.
  Early stage: help companies move toward operation but before commercial production and sales have occurred.
- Later stage: after commercial production and sales have begun but before any IPO. Funds may be used for expansion.
- Mezzanine stage: prepare to go public. Represent the bridge between the expanding company and the IPO.

Private debt :

- Direct Lending
- Mezzanine Debt
- Venture Debt
- Distressed Debt
- Other Private Debt Strategies
  Collateralized loan obligations (CLOs)
  Unitranche debt
  Infrastructure debt
  Real estate debt
  Specialty loans

Natural Resouces

Investing in Commodities

Contango:

- futures price>spot price

Backwardation:

- futures price < spot price

Futures price ≈ spot price $(1+R_f)$ + storage costs - convenience yield

Timberland and Farmland

- Timber (trees) can be grown and easily "stored" by simply not harvesting.
- Farm products must be harvested when ripe, so there is little flexibility in the production process.

Infrastructure

Long-lived assets for public use, including transportation, utility, communications, social

- Brownfield: Existing infrastructure
- Greenfield: Infrastructure to be built

Performance Appraisal

- Multiple of invested capital (MOIC) = (Realized value of investment + Unrealized value of investment)/Total amount of invested capital
- Use of Borrowed Funds: rL = Leveraged portfolio return/Cash position = [r×(Vc+Vb)-(Vb×rb)]/ Vc

PORTFOLIO MANAGEMENT

Portfolio risk and return

A Single Asset

Expected return:

- $$E(R)=\sum_{i=1}^{n}P_iR_i=P_1R_1+P_2R_2+\cdots+P_nR_n$$

Variance of return:

- $$Var=\sigma^2=\frac{1}{T}\sum_{i=1}^{T}(R_t-\mu)^2$$

Sample variance of return:

- $$s^2=\frac{1}{T-1}\sum_{t=1}^{T}(R_t-\overline{R})^2$$

Standard deviation of return:

- $$\sigma=\sqrt{\frac{1}{T}\sum_{i=1}^{T}(R_t-\mu)^2}\ (population)$$
- $$s=\sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(R_t-\overline{R})^2}\ (sample)$$

Two-asset Portfolio

Covariance of return:

- $$Cov_{1,2}=\frac{\sum_{t=1}^{T}(R_{t,1}-\mu_1)(R_{t,2}-\mu_2)}{T}\ (population)$$
- $$Cov_{1,2}=\frac{\sum_{t=1}^{T}(R_{t,1}-\overline{R}_1)(R_{t,2}-\overline{R}_2)}{T-1}\ (sample)$$

Correlation of return:

- $$\rho_{1,2}=\frac{Cov_{1,2}}{\sigma_1\sigma_2}$$

Variance of return for the portfolio:

- $$\sigma_P^2=w_1^2\sigma_1^2+w_2^2\sigma_2^2+2w_1w_2\sigma_1\sigma_2\rho_{1,2},\ w_1+w_2=1$$

N-assets Portfolio:

Variance of n assets: (given same variances, weights and covariances for all n assets)

- $$\sigma_p^2=\frac{(\overline{\sigma})^2}{N}+\frac{(N-1)}{N}\overline{Cov}$$

Efficient Frontier

Markowitz efficient frontier is the set of portfolios that have highest return for a given level of risk and have lowest risk for a given level of return.

Efficient portfolios: A portfolio that offers the highest return for a given risk level, or has the lowest risk level for a given return.

Global minimum variance portfolio: The portfolio on the efficient frontier that has the lowest risks.

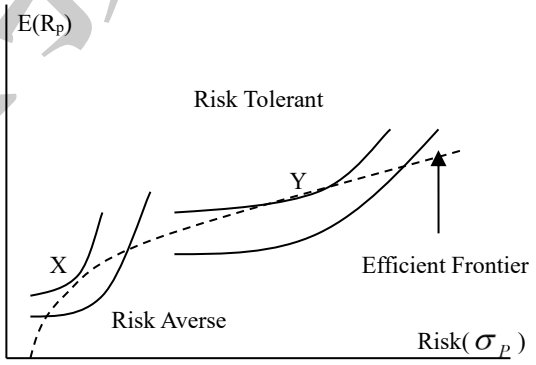
Indifference curve

$$U=E(r)-\frac{1}{2}A\sigma^2$$

A = a measure of risk aversion, which is measured as the marginal reward that an investor requires to accept additional risk.

Optimal portfolio

- Tangent of the highest indifference curve and the efficient frontier.
- Different investors may have different optimal portfolios.
- X is more risk-averse than Y.

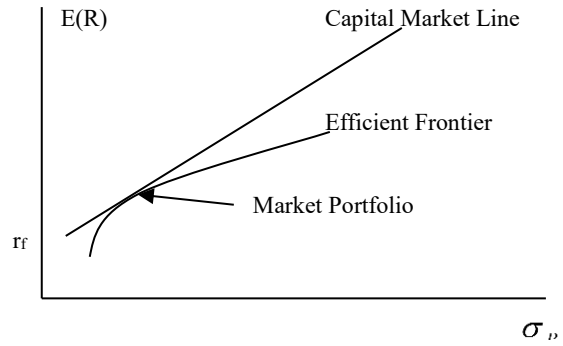


Capital Market Line (CML)

Capital market line (CML): When investors share identical expectations about the mean returns, variance of returns, and correlations of risky assets, the CAL for all investors is the same and is known as the capital market line (CML).

Market portfolio: tangent of CML and efficient frontier.

CML: $$E(R_P)=R_F+\frac{E(R_M)-R_F}{\sigma_M}\sigma_P$$



SML and CAPM

Systematic and unsystematic risks:

Unsystematic risks are diversified away; investors are compensated solely for systematic risk.

- Total Risk = Systematic Risk + Nonsystematic Risk

Security Characteristic Line:

Linear regression between the excess return on a security and the excess return of market.

$$R_{i,t} - R_f = \alpha_i + \beta_i(R_{M,t} - R_f) + \varepsilon_i$$

Beta:

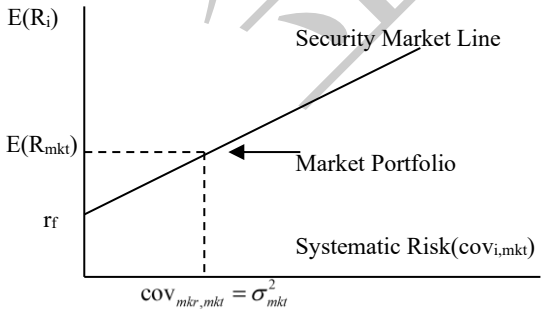
a measure of how sensitive an asset's return is to the market as a whole. A standardized measure of systematic risk.

$$\beta_i = \frac{Cov_{i,mkt}}{\sigma^2_{mkt}} = (\frac{\sigma_i}{\sigma_{mkt}}) \times \rho_{i,mkt}$$

Security market line (SML):

Graphical representation of **CAPM**

$$E(R_i) = r_f + \beta_i[E(R_M) - r_f]$$



How to judge if a stock is properly valued

- **Undervalued**

Estimated return > Required return from the SML

Investors should buy.

- **Overestimated**

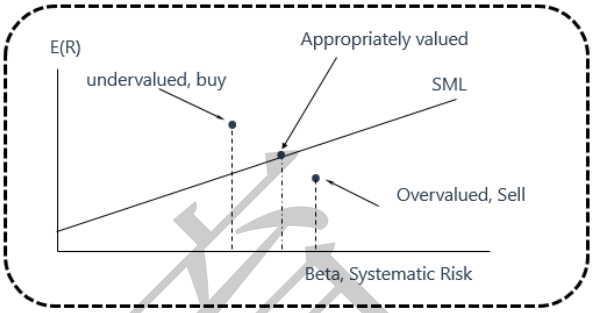
Estimated return < Required return from the SML

Investors should sell.

- **Properly valued**

Estimated return = Required return from the SML

Investors are indifferent between buying or selling.



Risk Adjusted Returns

Sharpe ratio and M-squared alpha measure excess return per unit of total risk.

Treynor measure and Jensen's alpha measure excess return per unit of systematic risk.

- $Sharpe\ ratio = \frac{R_P - R_f}{\sigma_P}$
- $M^2 alpha = [E(R_p) - R_f]\frac{\sigma_m}{\sigma_p} - [E(R_m) - R_f]$
  $= (SR_P - SR_m)\sigma_m$
- $Treynor\ measure = \frac{R_P - R_f}{\beta_P}$
- $\alpha_P = R_p - \{R_f + \beta_p[E(R_m) - R_f]\}$
  $= (R_P - R_f) - \beta_P(R_M - R_f)$

Portfolio Management: An Overview

Characteristics of different types of investors

Investor	Time horizon	Risk tolerance	Liquidity needs	Income needs
Individuals	Varies by individual	Varies by individual	Varies by individual	Varies by individual
DB plan	Long	High	Quite low	High for mature funds; Low for growing funds

Banks	Short	Quite low	High	Pay interest and operational expenses
Endowments and Foundations	Very long	High	Quite low	Meet spending commitments
Insurance	Long-life Short-P&C	Quite low	High	Low
Mutual funds	Varies by fund	Varies by fund	High	Varies by fund
Sovereign wealth funds	Varies by fund	Varies by fund	Varies by fund	Varies by fund

The Behavioral Biases of Individuals

Behavioral biases

- Cognitive errors
  - Belief perseverance
    - Conservatism bias
    - Confirmation bias
    - Representativeness bias
    - Illusion of control bias
    - Hindsight bias
  - Processing error
    - Anchoring & adjustment
    - Mental accounting bias
    - Framing bias
    - Availability bias
- Emotional biases
  - Loss aversion bias
  - Overconfidence bias
  - Self-control bias
  - Status quo bias
  - Endowment bias
  - Regret-aversion bias