Introduction to Algorithms	**Problem Set 6 (2 problems)**
CS 4820 Spring 2023	**Due: March 23, 2023 (Thursday) 11:59 pm**

Hand in your solutions electronically on Gradescope (except for coding problems, which should be submitted as indicated in the question). Your submission should be typeset (except for hand-drawn figures). Collaboration is encouraged while solving the problems, but:

1. list the NetID's of those in your group;
2. you may discuss ideas and approaches, but you should not write a detailed argument or code outline together;
3. notes of your discussions should be limited to drawings and a few keywords; you must **write up the solutions and write code on your own**.

Remember that when a problem asks you to design an algorithm, you must also prove the algorithm's correctness and analyze its running time, i.e., the running time must be bounded by a polynomial function of the input size.

(1) (10 points) Recall that in homework 5 Question 1(c), you are given a flow network $G = (V, E)$, and each vertex $v \in V$ is assigned a score r_v. The goal was to find a minimum capacity cut (A, B) such that the total score in A, $\sum_{v \in A} r_u$, is maximized. In this coding question, we ask you to implement a program which solves HW5 Q1(c). For this coding version, you only need to output the maximum total score $\sum_{v \in A} r_u$ (instead of the cut (A, B)).
The input format (passed via stdin, terminated by a newline) is as follows:

- The first line contains two positive integers n, m (separated by a space), which represents the number of nodes and edges in G respectively. The nodes are labeled by numbers $1, \ldots, n$. In addition, 1 is the source, and n is the sink.
- The second line contains n integers $r_1, r_2, \ldots, r_n$, where each r_i is the score assigned to node i.
- The following m lines are the m edges in G. Each of the m lines contains three integers u_j, v_j, c_j (separated by spaces), which represents an edge from u_j to v_j with capacity c_j.

Your program should output, to stdout, one line:

- $\max_A \left(\sum_{u \in A} r_u\right)$ where the maximum is taken over all the minimum capacity cut $(A, V - A)$.
- Your output should be terminated by a newline.

The input is guaranteed to satisfy the following constraints:

- $3 \leq n \leq 2000$
- $m \leq 10000$
- No edge enters the source 1, and no edge leaves the sink n
- $1 \leq c_i \leq 1000$
- The maximum flow value of G is at most 20000
- $-10 \leq r_i \leq 10$ for every $i \in [n]$
- In at least 20% of the test cases, $r_i = -1$ for every $i \in [n]$.

Sample Input:
4 3
-1 -1 -1 -1
1 2 2
2 3 1
3 4 1

Sample Output:
-2

The sample input above corresponds to a path $1 \to 2 \to 3 \to 4$, where the edge $1 \to 2$ has capacity 2 and the edges $2 \to 3$ and $3 \to 4$ have capacity 1. There are two possible minimum cuts in this example, $(\{1,2\},\{3,4\})$ and $(\{1,2,3\},\{4\})$ (both of which have cut capacity 1). The first cut has score -2, and the second cut has score -3. Therefore your output should be the larger score, -2.

(2) (1+9=10 points) Let ϕ be a 3CNF formula on n variables. An $\neq$-assignment $z \in \{0,1\}^n$ to the variables of ϕ is such that each clause contains two literals that evaluate to unequal truth values (under the assignment z).

1. Show that the negation of any $\neq$-assignment to ϕ is also an $\neq$-assignment.
2. Let $\neq$SAT be the collection of 3CNF formulas that have an $\neq$-assignment. Prove that $\neq$SAT is NP-complete.