stochastic_process

November 13, 2017

1 Experiment 1

```
In [102]: import numpy as np
          import pandas as pd
          from matplotlib import pyplot as plt
          import sympy as s
          from sympy import Symbol
          from sympy import integrate
```

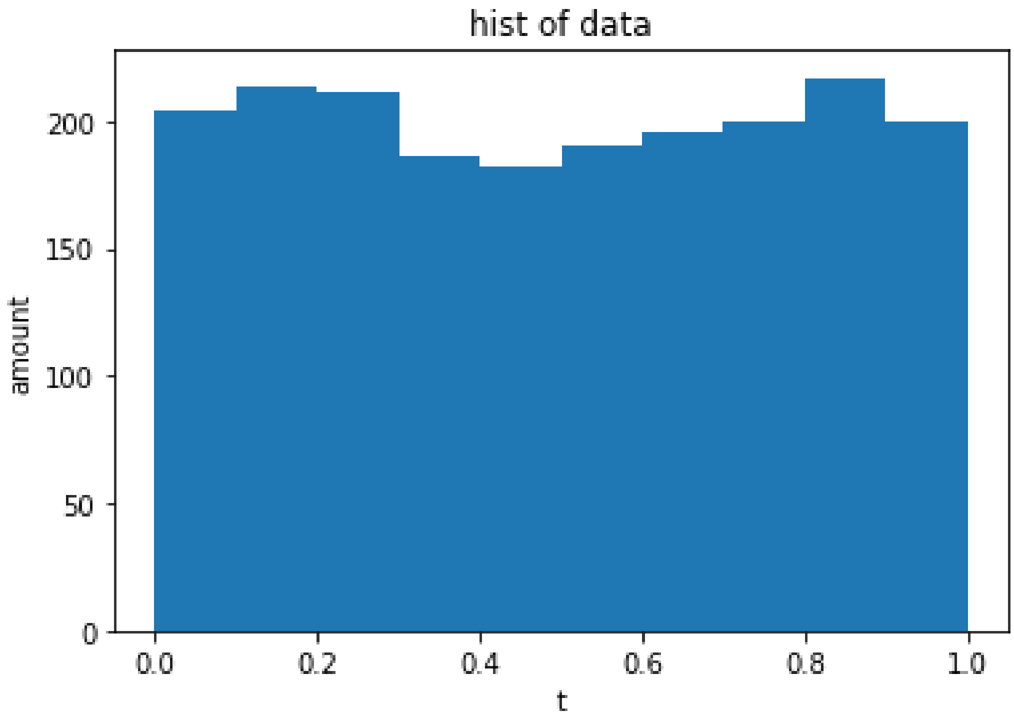
```
In [103]: def Bx_hat(X, i):
              N = np.shape(X)[0]
              EX = np.mean(X)
              abs_i = abs(i)
              val = 0
              for n in range(0, N-abs(i)):
                  val += (X[n+abs_i]-EX)*(X[n]-EX)
              val /= N
              return val
```

```
In [104]: np.random.seed(10)
          X = np.random.rand(2000)
          print(X[0:50])
```

```
[ 0.77132064  0.02075195  0.63364823  0.74880388  0.49850701  0.22479665
  0.19806286  0.76053071  0.16911084  0.08833981  0.68535982  0.95339335
  0.00394827  0.51219226  0.81262096  0.61252607  0.72175532  0.29187607
  0.91777412  0.71457578  0.54254437  0.14217005  0.37334076  0.67413362
  0.44183317  0.43401399  0.61776698  0.51313824  0.65039718  0.60103895
  0.8052232   0.52164715  0.90864888  0.31923609  0.09045935  0.30070006
  0.11398436  0.82868133  0.04689632  0.62628715  0.54758616  0.819287
  0.19894754  0.8568503   0.35165264  0.75464769  0.29596171  0.88393648
  0.32551164  0.1650159 ]
```

```
In [105]: plt.figure()
          amounts, _, _ = plt.hist(X)
          plt.title('hist of data')
```

```
plt.ylabel('amount')
plt.xlabel('t')
plt.show()
```




```
In [106]: data = np.zeros((3,10))
          data[0,:] = 200
          data[1,:] = amounts
          data[2,:] = np.absolute(amounts-data[0,:])/data[0,:]
          df = pd.DataFrame(data)
          df.index = ['theoretical value', 'practical value', 'error']
          df.columns = df.columns + 1
          df
```

Out[106]:

	1	2	3	4	5	6	7 \
theoretical value	200.00	200.00	200.000	200.00	200.00	200.00	200.00
practical value	204.00	214.00	211.000	186.00	182.00	190.00	196.00
error	0.02	0.07	0.055	0.07	0.09	0.05	0.02

	8	9	10
theoretical value	200.0	200.000	200.0
practical value	200.0	217.000	200.0
error	0.0	0.085	0.0

```
In [107]: print('theoretical mean: ', 0.5)
          print('practical mean: ', np.mean(X))
```

```
theoretical mean:  0.5
practical mean:  0.498595175369
```

```
In [108]: print('theoretical variance: ', 1/12)
          print('practical variance: ', np.var(X))
```

```
theoretical variance:  0.08333333333333333
practical variance:  0.0856778615785
```

```
In [111]: Bx_h = np.zeros((21, 1))
          x = np.arange(-10, 10+1, 1)
          for i, v in enumerate(x):
              Bx_h[i] = Bx_hat(X, v)

          plt.plot(x, np.zeros_like(x), '.-')
          plt.plot(x, Bx_h, '.-')
          plt.legend(['theoretical Bx', 'practical Bx'])
          plt.title('Bx')
          plt.show()
```

```
In [226]: pd.set_option('display.max_columns',100)
          pd.set_option('display.precision',5)
          data = np.zeros((2,np.shape(x)[0]))
          data[0,:] = [0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0]
          data[1,:] = np.squeeze(Bx_h)
          df = pd.DataFrame(data)
          df.index = ['theoretical Bx', 'practical Bx']
          df.columns = x
          df
```

```
Out[226]:                     -10  \
          theoretical Bx  0.00000
          practical Bx   -0.00128

                              -9   \
          theoretical Bx  0.00000
          practical Bx    0.03957

                              -8   \
          theoretical Bx  0.00000
          practical Bx    0.00206

                              -7   \
          theoretical Bx  0.00000
          practical Bx    0.01233

                              -6   \
```

```
theoretical Bx  0.00000
practical Bx   -0.01337

                    -5   \
theoretical Bx  0.00000
practical Bx   -0.00381

                    -4   \
theoretical Bx  0.00000
practical Bx    0.03668

                    -3   \
theoretical Bx  0.00000
practical Bx    0.00511

                    -2   \
theoretical Bx  0.00000
practical Bx   -0.03513

                    -1   \
theoretical Bx  0.00000
practical Bx   -0.00877

                     0   \
theoretical Bx  1.00000
practical Bx    1.01861

                     1   \
theoretical Bx  0.00000
practical Bx   -0.00877

                     2   \
theoretical Bx  0.00000
practical Bx   -0.03513

                     3   \
theoretical Bx  0.00000
practical Bx    0.00511

                     4   \
theoretical Bx  0.00000
practical Bx    0.03668

                     5   \
theoretical Bx  0.00000
practical Bx   -0.00381

                     6   \
```

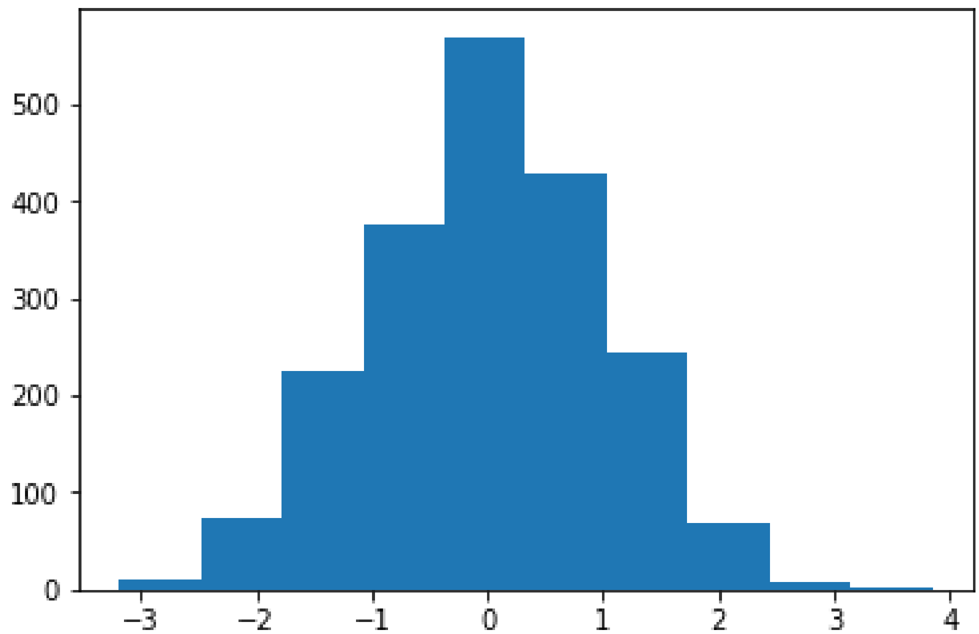
```
theoretical Bx  0.00000
practical Bx   -0.01337

                     7   \
theoretical Bx  0.00000
practical Bx    0.01233

                     8   \
theoretical Bx  0.00000
practical Bx    0.00206

                     9   \
theoretical Bx  0.00000
practical Bx    0.03957

                     10
theoretical Bx  0.00000
practical Bx   -0.00128
```

2 Experiment 2

In [227]:
```
np.random.seed(55)
X = np.random.rand(25000)
# Y = np.random.randn(2000)
Y = np.zeros(2000)
for i in range(0,2000):
    Y[i] = np.sum(X[12*i:12*i+12]-0.5)
print(Y[0:50])
```

```
[-0.78580149  0.91558336  0.26679742  1.03427115  0.01360813 -1.43302117
  0.02838285  0.93930686 -0.27297461  0.71313708 -0.67394771 -1.18851013
 -1.03358154 -0.97556141 -0.05937913 -1.0852331   0.61210618 -0.49002067
 -1.15550763  0.31011511  0.10845409  0.83946053 -0.14980455 -0.56878991
  0.94269235  0.1884211   0.34181886 -1.59142244  1.04672648 -0.11098559
 -1.61946593  0.3348359   1.38315576 -0.36999028 -0.33928221  0.01447092
 -1.02190267 -0.24710888 -2.32144219 -0.401806    1.70194589  0.49332755
  0.1851737  -0.8163923   0.07346427  1.42283872  1.07912717  0.62912514
  1.23192427 -0.6240719 ]
```

In [228]:
```
plt.figure()
amounts, _, _ = plt.hist(Y)
plt.show()
```




```
In [229]: def gauss(x, mu1, sigma1):
              return 1/(s.sqrt(2*s.pi)*sigma1)*s.exp(-(x-mu1)**2/(2*sigma1**2))

In [230]: x = Symbol('x')
          y = gauss(x, 0, 1)
          lis_ther = []
          for i in ([-s.oo, -3], [-3, -2], [-2, -1], [-1, 0], [0, 1], [1, 2], [2, 3], [3, s.oo])
              lis_ther.append(int(integrate(y, (x, i[0],i[1])).evalf()*2000))
          lis_pret = []
          lis_pret.append(np.sum(Y<-3))
          for i in ([-3, -2], [-2, -1], [-1, 0], [0, 1], [1, 2], [2, 3]):
              lis_pret.append(np.sum((Y>i[0])&(Y<i[1])))
          lis_pret.append(np.sum(Y>3))

          # pd.set_option('display.max_colwidth',100)
          arr_ther = np.array(lis_ther)
          arr_pret = np.array(lis_pret)
          data = np.zeros((3,arr_ther.shape[0]))
          data[0,:] = arr_ther
          data[1,:] = arr_pret
          data[2,:] = np.abs(arr_pret-arr_ther)/arr_ther * 100
          df = pd.DataFrame(data)
          df.index = ['theoretical amounts', 'practical amounts', 'error(%)']
          df.columns = ['(-oo,-3]','(-3,-2]','(-2,-1]','(-1,0]','(0,1]','(1,2]','(2,3]','(3,oo)'
          df
```

```
Out[230]:                       (-oo,-3]  \
          theoretical amounts       2.0
          practical amounts         2.0
          error(%)                  0.0

                                 (-3,-2]  \
          theoretical amounts  42.00000
          practical amounts    45.00000
          error(%)              7.14286

                                  (-2,-1]  \
          theoretical amounts  271.00000
          practical amounts    284.00000
          error(%)               4.79705

                                   (-1,0]  \
          theoretical amounts  682.00000
          practical amounts    668.00000
          error(%)               2.05279

                                    (0,1]  \
          theoretical amounts  682.00000
          practical amounts    667.00000
          error(%)               2.19941

                                    (1,2]  \
          theoretical amounts  271.00000
          practical amounts    287.00000
          error(%)               5.90406

                                   (2,3]  \
          theoretical amounts  42.00000
          practical amounts    45.00000
          error(%)              7.14286

                               (3,oo)
          theoretical amounts     2.0
          practical amounts       2.0
          error(%)                0.0
```

```
In [231]: data = np.zeros((2,10))
          data[0,:] = 200
          data[1,:] = amounts
          df = pd.DataFrame(data)
          df.index = ['theoretical value', 'practical value']
          df.columns = df.columns + 1
          df
```

```
Out[231]:                          1   \
```

```
theoretical value  200.0
practical value     10.0

                      2   \
theoretical value  200.0
practical value     73.0

                      3   \
theoretical value  200.0
practical value    225.0

                      4   \
theoretical value  200.0
practical value    375.0

                      5   \
theoretical value  200.0
practical value    569.0

                      6   \
theoretical value  200.0
practical value    428.0

                      7   \
theoretical value  200.0
practical value    243.0

                      8   \
theoretical value  200.0
practical value     67.0

                      9   \
theoretical value  200.0
practical value      9.0

                     10
theoretical value  200.0
practical value      1.0
```

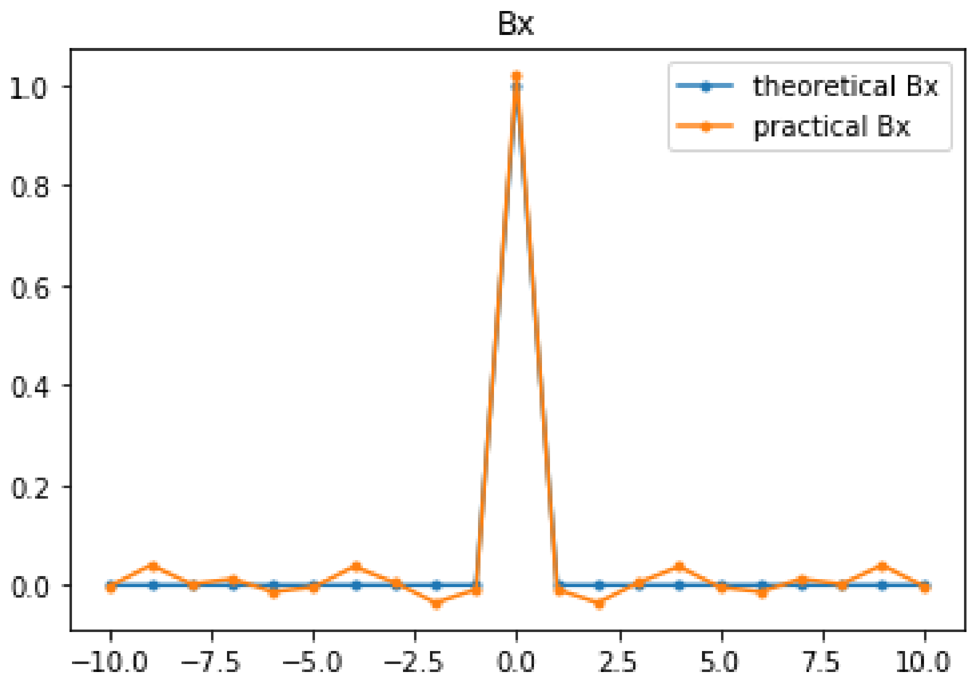
In [209]:
```
print('theoretical mean value: ', 0.0)
print('practical mean value: ', np.mean(Y))
```

```
theoretical mean value:  0.0
practical mean value:  0.00705865846988
```

In [210]:
```
print('theoretical variance: ', 1)
print('practical variance: ', np.var(Y))
```

```
theoretical variance:  1
practical variance:  1.01861058877
```

```
In [235]: Bx_h = np.zeros((21, 1))
          x = np.arange(-10, 10+1, 1)
          for i, v in enumerate(x):
              Bx_h[i] = Bx_hat(Y, v)
          plt.figure()
          plt.plot(x, [0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0], '.-')
          plt.plot(x, Bx_h, '.-')
          plt.legend(['theoretical Bx', 'practical Bx'])
          plt.title('Bx')
          plt.show()
```



3 Experiment 3

```
In [238]: cosi = np.random.randn(2000)
          X = np.zeros(1000)
          for i in range(1000):
              X[i] = cosi[i+1] + 4*cosi[i]

In [239]: print('X theoretical mean:', 0.0)
          print('X practical mean: ', np.mean(X))
```

```
X theoretical mean: 0.0
X practical mean:  -0.0380992637205
```

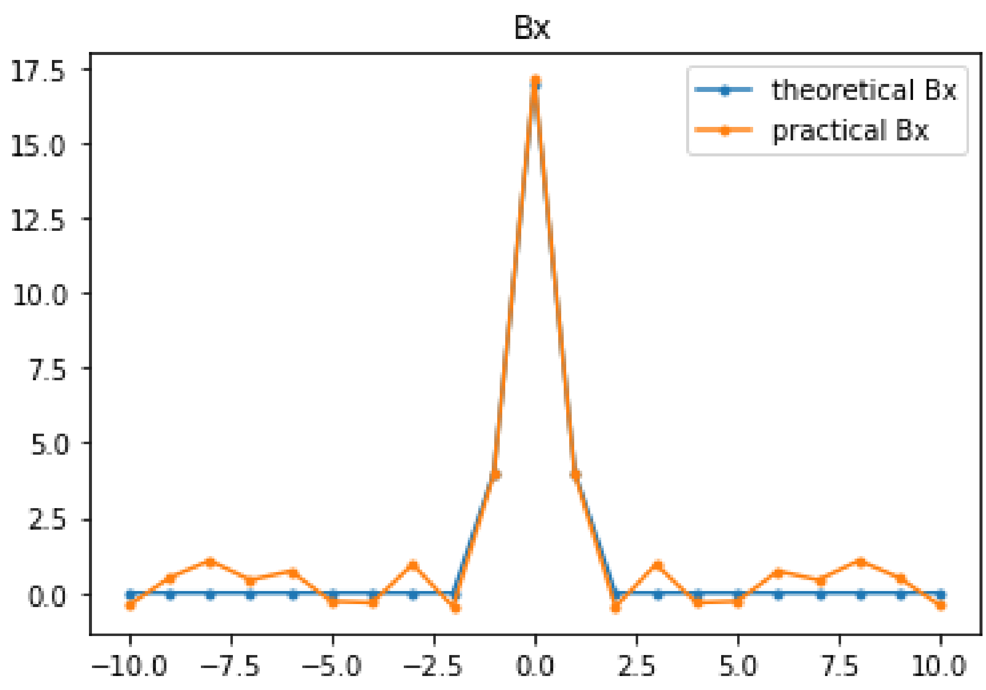
```
In [240]: print('X^2 theoretical mean: ', 17)
          print('X^2 practical mean: ', np.mean(X**2))
```

```
X^2 theoretical mean:  17
X^2 practical mean:  17.113515764
```

```
In [241]: print('X theoretical variance: ', 17)
          print('X practical variance: ', np.mean(X**2)-np.mean(X)**2)
```

```
X theoretical variance:  17
X practical variance:  17.1120642101
```

```
In [243]: Bx_h = np.zeros((21, 1))
          x = np.arange(-10, 10+1, 1)
          for i, v in enumerate(x):
              Bx_h[i] = Bx_hat(X, v)

          plt.figure()
          plt.plot(x, [0,0,0,0,0,0,0,0,0,4,17,4,0,0,0,0,0,0,0,0,0], '.-')
          plt.plot(x, Bx_h, '.-')
          plt.legend(['theoretical Bx', 'practical Bx'])
          plt.title('Bx')
          plt.show()
```



4 Expriment 4

```
In [246]: cosi = np.random.randn(2000)
          k = 10
          X = np.zeros(k)
          X[0] = np.random.randn()
          for i in range(1, k):
              X[i] = cosi[i] - 0.707*X[i-1]
```

```
In [247]: print('X theoretical mean: ', np.power(-0.707, k)*X[0])
          print('X practical mean: ', np.mean(X))
```

```
X theoretical mean:  -0.0164790067915
X practical mean:  0.00717533116642
```

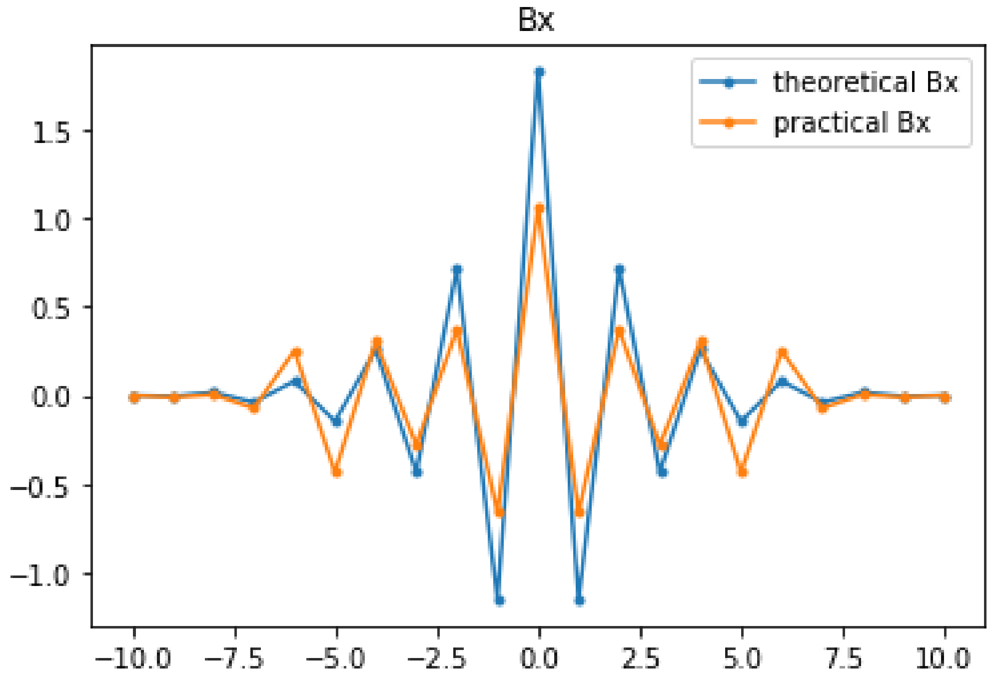
```
In [248]: print('X theoretical variance: ', np.power(-0.707, k)*X[0])
          print('X practical variance ', np.var(X))
```

```
X theoretical variance:  -0.0164790067915
X practical variance  1.06180793256
```

```
In [269]: def f_gamma_x(X, m, a):
              X_0 = X[0]
              N = np.shape(X)[0]
              EX = np.mean(X)
              abs_m = abs(m)
              val = 0
              for k in range(1, N-abs_m+1):
                  s = k + abs_m
                  val += np.power((-a),k+s)*X_0**2+(np.power(-a,np.abs(k-s))-np.power(-a,np.abs(
              val /= N
              return val
```

```
In [270]: Bx_h = np.zeros((21, 1))
          Bx_ex = np.zeros((21, 1))
          x = np.arange(-10, 10+1, 1)

          for i, v in enumerate(x):
              Bx_ex[i] = f_gamma_x(X, v, 0.707)

          for i, v in enumerate(x):
              Bx_h[i] = Bx_hat(X, v)
```

```
plt.figure()
plt.plot(x, Bx_ex, '.-')
plt.plot(x, Bx_h, '.-')
plt.legend(['theoretical Bx', 'practical Bx'])
plt.title('Bx')
plt.show()
```



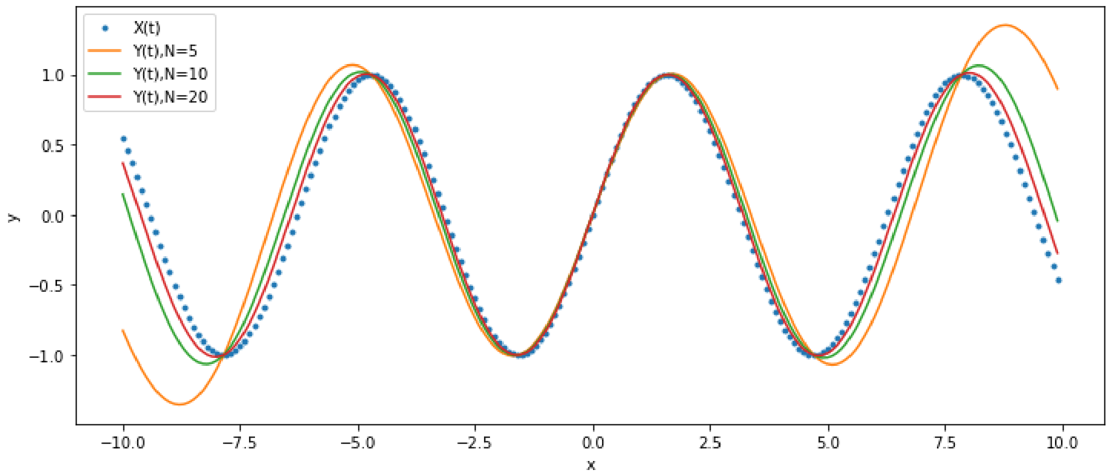
5 Experiment 5

```
In [26]: def func(n):
             X = np.arange(-n, n+1, 1)
             X = np.sin(X*np.pi/2)
             return X
```

```
In [27]: def X_t(t):
             val = np.sin(t)
             return val
         def Y_t(t, N):
             val = 0
             for n in range(-N, N+1):
                 val += X_t(n*np.pi/2) * np.sin(t-n*np.pi/2) / (t-n*np.pi/2)
             return val
```

```
In [28]: x = np.arange(-10,10,0.1)
         x_t = X_t(x)
```

```
plt.figure(figsize=[12,5])
plt.plot(x, x_t, '.')
for N in [5, 10, 20]:
    y_t = Y_t(x, N)
    plt.plot(x, y_t)
plt.legend(['X(t)', 'Y(t),N=5', 'Y(t),N=10', 'Y(t),N=20'])
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```



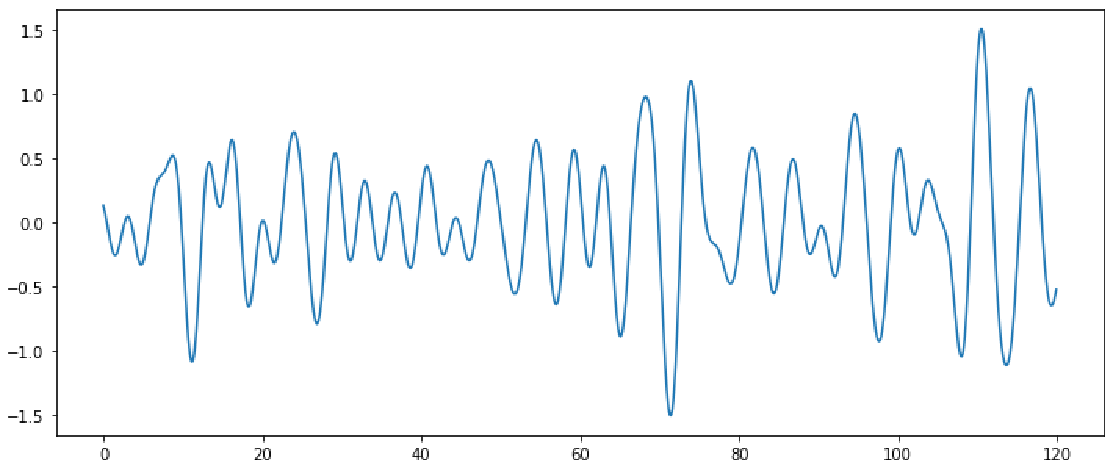
6 Experiment 6

7 Experiment 7

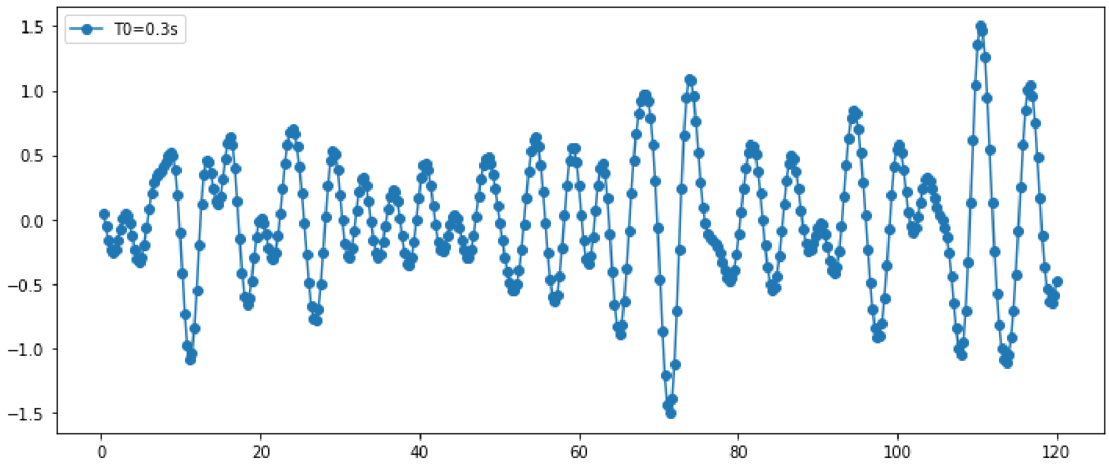
8 Experiment 8

In [461]:
```
A = 0.78
B = 3.11/4
N = 40
np.random.seed(12)
epsilon = np.random.rand(N+1)*2*np.pi-np.pi
w = 0.05*np.arange(0, 41)

def sqrt_calculate(w_n, w_n_1):
    if w_n==0:
        w_n=1e-8
    if w_n_1==0:
        w_n_1=1e-8
    val = np.exp(-B*np.power(w_n, -4))-np.exp(-B*np.power(w_n_1,-4))
```

```
    val = np.sqrt(2*A/(4*B)*val)
    return val

def cosi(t):
    val = 0
    for i in range(1, N+1):
        w_n = w[i]
        w_n_1 = w[i-1]
        val += sqrt_calculate(w_n, w_n_1)*np.cos(w_n*t+epsilon[i])
    return val

t = np.arange(0, 120, 0.1)
val = cosi(t)
plt.figure(figsize=[12,5])
plt.plot(t, val, '-')
plt.show()
```



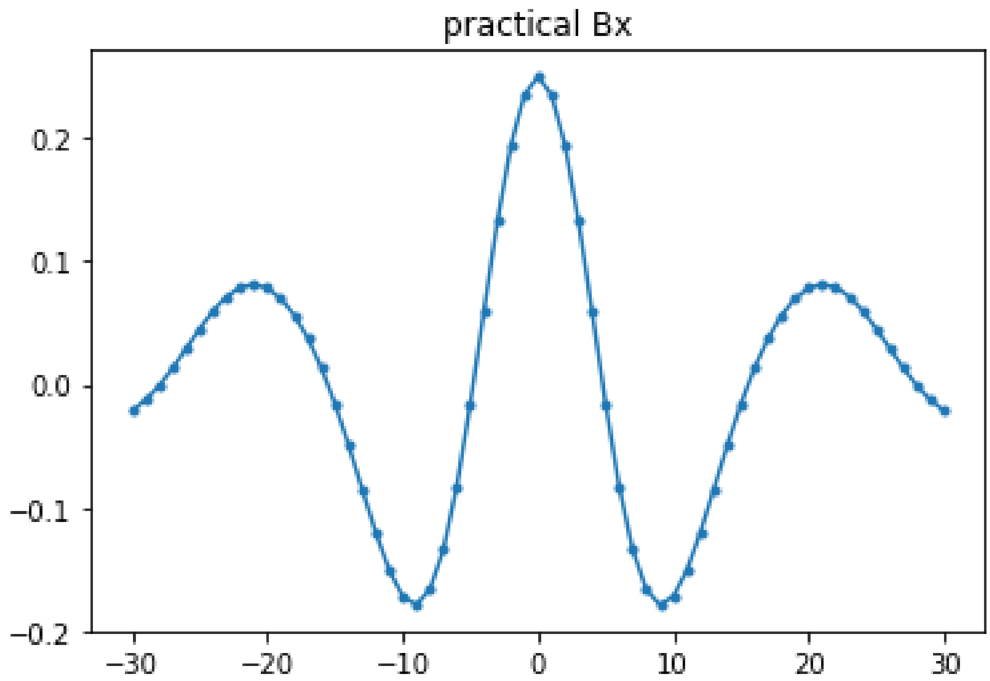

```
In [462]: T0 = 0.3
          nT0 = T0*np.arange(1,400+1,1)
          lis = cosi(nT0)
          plt.figure(figsize=[12,5])
          plt.plot(nT0, lis, '-o')
          plt.legend(['T0=0.3s'])
          plt.show()
```



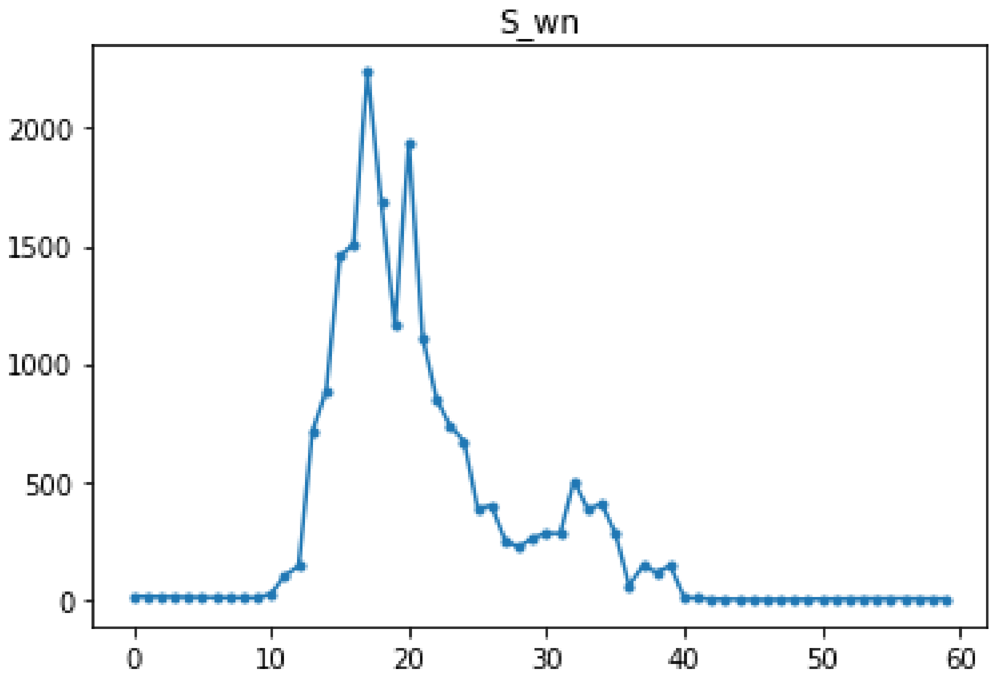
In [463]:
```
print('mean: ', np.mean(lis))
```

```
mean:  -0.00436546577416
```

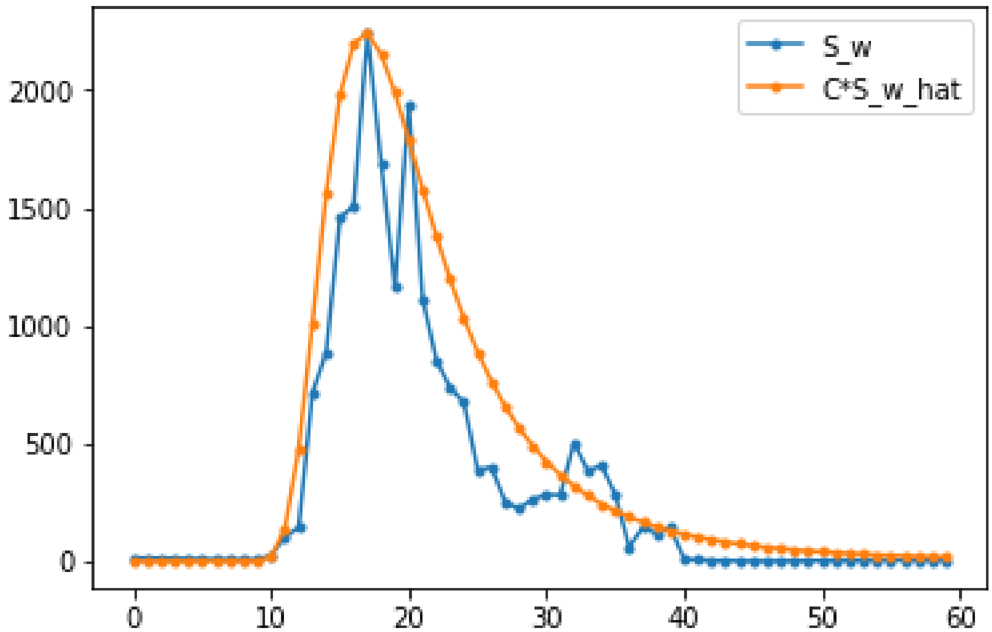
In [464]:
```
Bx_h = np.zeros((61, 1))
x = np.arange(-30, 30+1, 1)
for i, v in enumerate(x):
    Bx_h[i] = Bx_hat(lis, v)

plt.figure()
plt.plot(x, Bx_h, '.-')
plt.title('practical Bx')
plt.show()
```




```
In [465]: def S_cosi_hat(w):
              T0 = 0.3
              N = 400
              val = 0
              for m in range(1,N+1):
                  val += cosi(m*T0)*np.exp(np.complex(0,-w*m*T0))
              return val.imag**2+val.real**2

In [466]: S_cosi_w_n_hat = np.zeros((60,1))
          for i,v in enumerate(range(1, 61)):
              w_n = 0.05*v
              S_cosi_w_n_hat[i] = S_cosi_hat(w_n)
          plt.plot(tmp, '.-')
          plt.title('S_wn')
          plt.show()
```




```
In [467]: def S_cosi(w):
              return A/np.power(w,5)*np.exp(-B/np.power(w,4))
          S_cosi_w_n = np.zeros((60,1))
          for i,v in enumerate(range(1, 61)):
              w_n = 0.05*v
              S_cosi_w_n[i] = S_cosi(w_n)

          C = np.max(tmp)/np.max(S_cosi_w_n)

In [468]: plt.plot()
          plt.plot(tmp, '.-')
          plt.plot(S_cosi_w_n*C, '.-')
          plt.legend(['S_w', 'C*S_w_hat'])
          plt.show()
```


S_w
C*S_w_hat
2000
1500
1000
500
0
0
10
20
30
40
50
60