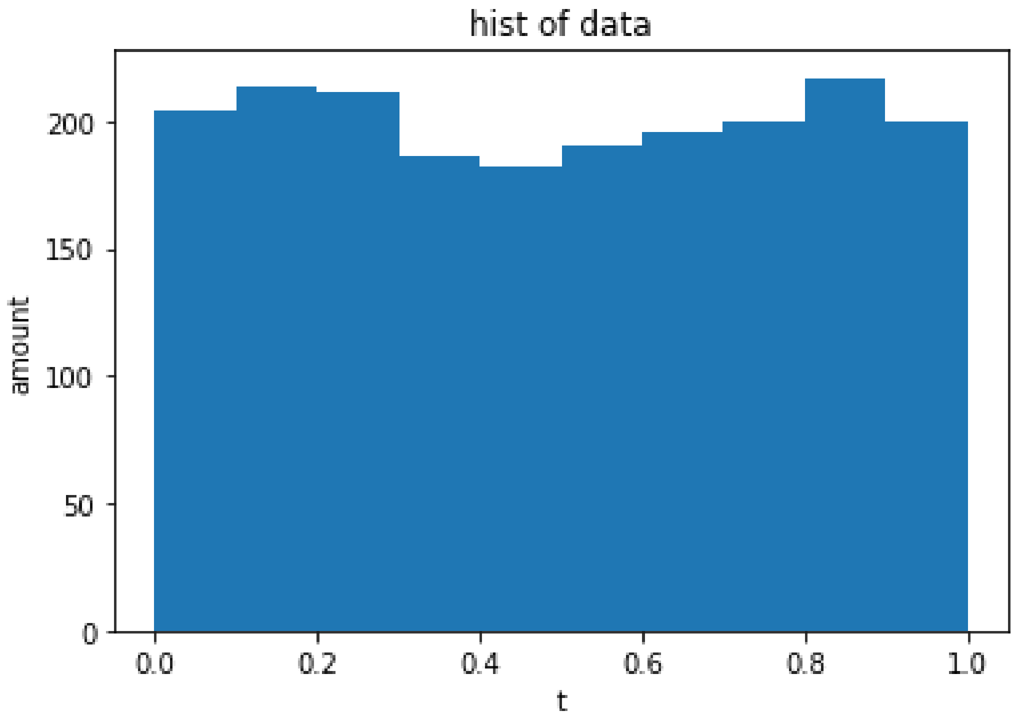
Stochastic Process Experiment

November 23, 2017

1 Experiment 1

```
In [3]: import numpy as np
        import pandas as pd
        from matplotlib import pyplot as plt
        import sympy as s
        from sympy import Symbol
        from sympy import integrate

In [4]: def Bx_hat(X, i):
            N = np.shape(X)[0]
            EX = np.mean(X)
            abs_i = abs(i)
            val = 0
            for n in range(0, N-abs(i)):
                val += (X[n+abs_i]-EX)*(X[n]-EX)
            val /= N
            return val

In [3]: np.random.seed(10)
        X = np.random.rand(2000)
        print(X[0:50])

[ 0.77132064  0.02075195  0.63364823  0.74880388  0.49850701  0.22479665
  0.19806286  0.76053071  0.16911084  0.08833981  0.68535982  0.95339335
  0.00394827  0.51219226  0.81262096  0.61252607  0.72175532  0.29187607
  0.91777412  0.71457578  0.54254437  0.14217005  0.37334076  0.67413362
  0.44183317  0.43401399  0.61776698  0.51313824  0.65039718  0.60103895
  0.8052232   0.52164715  0.90864888  0.31923609  0.09045935  0.30070006
  0.11398436  0.82868133  0.04689632  0.62628715  0.54758616  0.819287
  0.19894754  0.8568503   0.35165264  0.75464769  0.29596171  0.88393648
  0.32551164  0.1650159 ]


In [4]: plt.figure()
        amounts, _, _ = plt.hist(X)
        plt.title('hist of data')
```

```
plt.ylabel('amount')
plt.xlabel('t')
plt.show()
```




```
In [5]: data = np.zeros((3,10))
        data[0,:] = 200
        data[1,:] = amounts
        data[2,:] = np.absolute(amounts-data[0,:])/data[0,:]
        df = pd.DataFrame(data)
        df.index = ['theoretical value', 'practical value', 'error']
        df.columns = df.columns + 1
        df
```

```
Out[5]:                         1       2        3       4       5       6       7  \
        theoretical value  200.00  200.00  200.000  200.00  200.00  200.00  200.00
        practical value    204.00  214.00  211.000  186.00  182.00  190.00  196.00
        error                0.02    0.07    0.055    0.07    0.09    0.05    0.02

                               8        9     10
        theoretical value  200.0  200.000  200.0
        practical value    200.0  217.000  200.0
        error                0.0    0.085    0.0
```

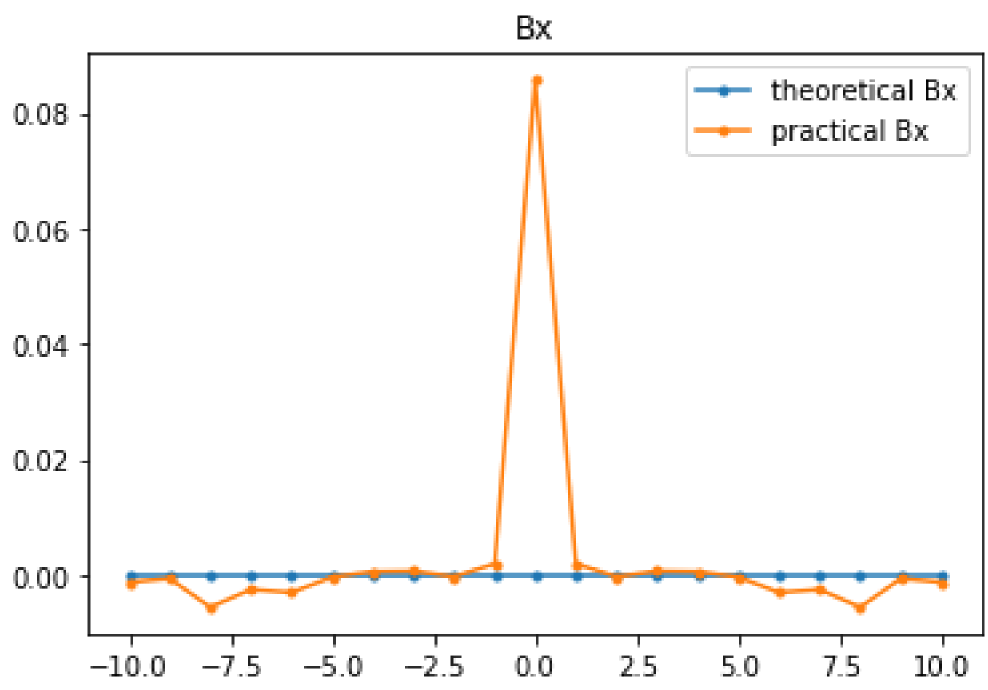
```
In [6]: print('theoretical mean: ', 0.5)
        print('practical mean: ', np.mean(X))
```

```
theoretical mean:  0.5
practical mean:  0.498595175369
```

```
In [7]: print('theoretical variance: ', 1/12)
        print('practical variance: ', np.var(X))
```

```
theoretical variance:  0.0833333333333333
practical variance:  0.0856778615785
```

```
In [8]: Bx_h = np.zeros((21, 1))
        x = np.arange(-10, 10+1, 1)
        for i, v in enumerate(x):
            Bx_h[i] = Bx_hat(X, v)

        plt.plot(x, np.zeros_like(x), '.-')
        plt.plot(x, Bx_h, '.-')
        plt.legend(['theoretical Bx', 'practical Bx'])
        plt.title('Bx')
        plt.show()
```




```
In [9]: pd.set_option('display.max_columns',100)
        pd.set_option('display.precision',5)
        data = np.zeros((2,np.shape(x)[0]))
```

```
data[0,:] = [0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0]
data[1,:] = np.squeeze(Bx_h)
df = pd.DataFrame(data)
df.index = ['theoretical Bx', 'practical Bx']
df.columns = x
df
```

Out[9]:

```
                    -10      -9      -8      -7      -6      -5     -4  \
theoretical Bx  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
practical Bx   -0.00122 -0.00059 -0.00554 -0.00247 -0.00298 -0.00035  0.0006

                     -3      -2      -1        0        1        2        3  \
theoretical Bx  0.00000  0.00000  0.00000  1.00000  0.00000  0.00000  0.00000
practical Bx    0.00068 -0.00029  0.00194  0.08568  0.00194 -0.00029  0.00068

                     4        5        6        7        8        9       10
theoretical Bx  0.0000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000
practical Bx    0.0006 -0.00035 -0.00298 -0.00247 -0.00554 -0.00059 -0.00122
```

2 Experiment 2

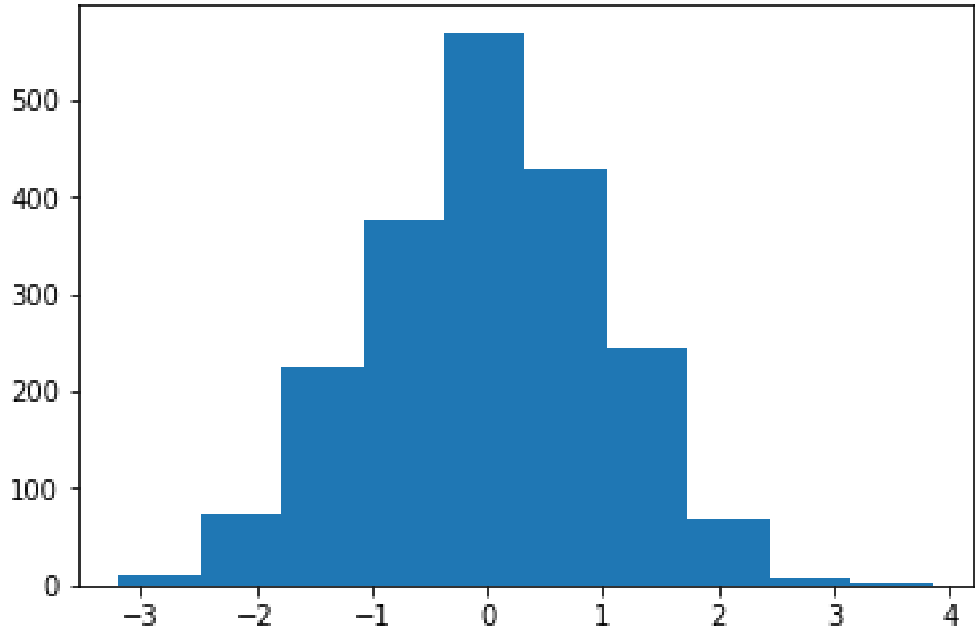
In [10]:

```
np.random.seed(55)
X = np.random.rand(25000)
# Y = np.random.randn(2000)
Y = np.zeros(2000)
for i in range(0,2000):
    Y[i] = np.sum(X[12*i:12*i+12]-0.5)
print(Y[0:50])
```

```
[-0.78580149  0.91558336  0.26679742  1.03427115  0.01360813 -1.43302117
  0.02838285  0.93930686 -0.27297461  0.71313708 -0.67394771 -1.18851013
 -1.03358154 -0.97556141 -0.05937913 -1.0852331   0.61210618 -0.49002067
 -1.15550763  0.31011511  0.10845409  0.83946053 -0.14980455 -0.56878991
  0.94269235  0.1884211   0.34181886 -1.59142244  1.04672648 -0.11098559
 -1.61946593  0.3348359   1.38315576 -0.36999028 -0.33928221  0.01447092
 -1.02190267 -0.24710888 -2.32144219 -0.401806    1.70194589  0.49332755
  0.1851737  -0.8163923   0.07346427  1.42283872  1.07912717  0.62912514
  1.23192427 -0.6240719 ]
```

In [11]:

```
plt.figure()
amounts, _, _ = plt.hist(Y)
plt.show()
```




```
In [12]: def gauss(x, mu1, sigma1):
             return 1/(s.sqrt(2*s.pi)*sigma1)*s.exp(-(x-mu1)**2/(2*sigma1**2))

In [13]: x = Symbol('x')
         y = gauss(x, 0, 1)
         lis_ther = []
         for i in ([-s.oo, -3], [-3, -2], [-2, -1], [-1, 0], [0, 1], [1, 2], \
                   [2, 3], [3, s.oo]):
             lis_ther.append(int(integrate(y, (x, i[0],i[1])).evalf()*2000))
         lis_pret = []
         lis_pret.append(np.sum(Y<-3))
         for i in ([-3, -2], [-2, -1], [-1, 0], [0, 1], [1, 2], [2, 3]):
             lis_pret.append(np.sum((Y>i[0])&(Y<i[1])))
         lis_pret.append(np.sum(Y>3))

         # pd.set_option('display.max_colwidth',100)
         arr_ther = np.array(lis_ther)
         arr_pret = np.array(lis_pret)
         data = np.zeros((3,arr_ther.shape[0]))
         data[0,:] = arr_ther
         data[1,:] = arr_pret
         data[2,:] = np.abs(arr_pret-arr_ther)/arr_ther * 100
         df = pd.DataFrame(data)
         df.index = ['theoretical amounts', 'practical amounts', 'error(%)']
         df.columns = ['(-oo,-3]','(-3,-2]','(-2,-1]','(-1,0]','(0,1]','(1,2]',\
```

```
                   '(2,3]','(3,oo)']
        df
```

Out[13]:

```
                      (-oo,-3]   (-3,-2]    (-2,-1]     (-1,0]      (0,1]  \
theoretical amounts        2.0  42.00000  271.00000  682.00000  682.00000
practical amounts          2.0  45.00000  284.00000  668.00000  667.00000
error(%)                   0.0   7.14286    4.79705    2.05279    2.19941

                         (1,2]     (2,3]  (3,oo)
theoretical amounts  271.00000  42.00000     2.0
practical amounts    287.00000  45.00000     2.0
error(%)               5.90406   7.14286     0.0
```

In [14]:

```
data = np.zeros((2,10))
data[0,:] = 200
data[1,:] = amounts
df = pd.DataFrame(data)
df.index = ['theoretical value', 'practical value']
df.columns = df.columns + 1
df
```

Out[14]:

```
                       1      2      3      4      5      6      7      8  \
theoretical value  200.0  200.0  200.0  200.0  200.0  200.0  200.0  200.0
practical value     10.0   73.0  225.0  375.0  569.0  428.0  243.0   67.0

                       9     10
theoretical value  200.0  200.0
practical value      9.0    1.0
```

In [15]:

```
print('theoretical mean value: ', 0.0)
print('practical mean value: ', np.mean(Y))
```

```
theoretical mean value:  0.0
practical mean value:  0.00705865846988
```

In [16]:

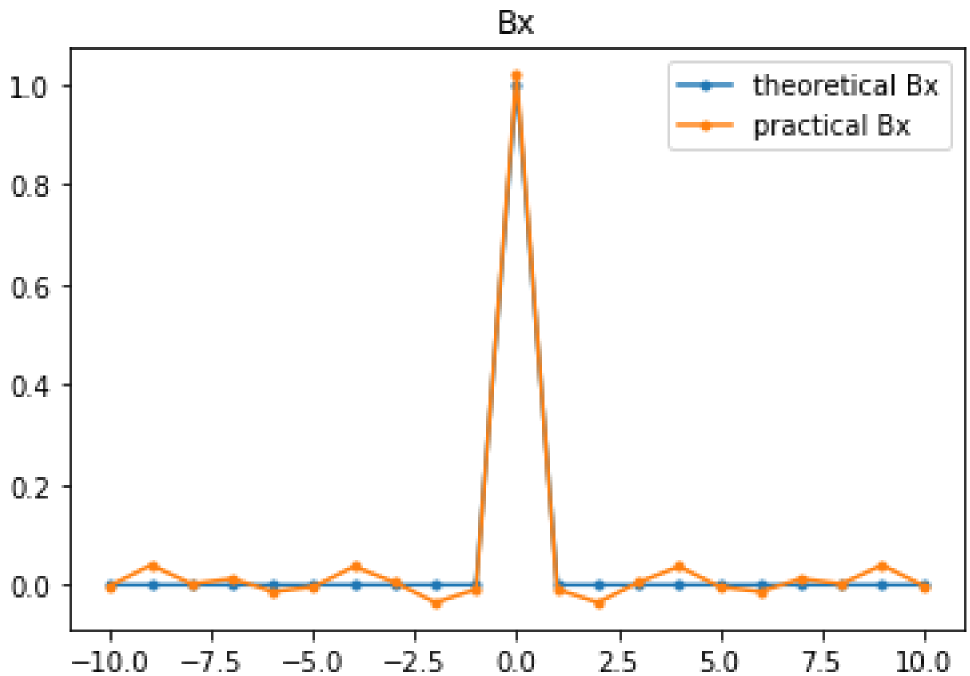
```
print('theoretical variance: ', 1)
print('practical variance: ', np.var(Y))
```

```
theoretical variance:  1
practical variance:  1.01861058877
```

In [17]:

```
Bx_h = np.zeros((21, 1))
x = np.arange(-10, 10+1, 1)
for i, v in enumerate(x):
    Bx_h[i] = Bx_hat(Y, v)
plt.figure()
plt.plot(x, [0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0], '.-')
```

```
plt.plot(x, Bx_h, '.-')
plt.legend(['theoretical Bx', 'practical Bx'])
plt.title('Bx')
plt.show()
```



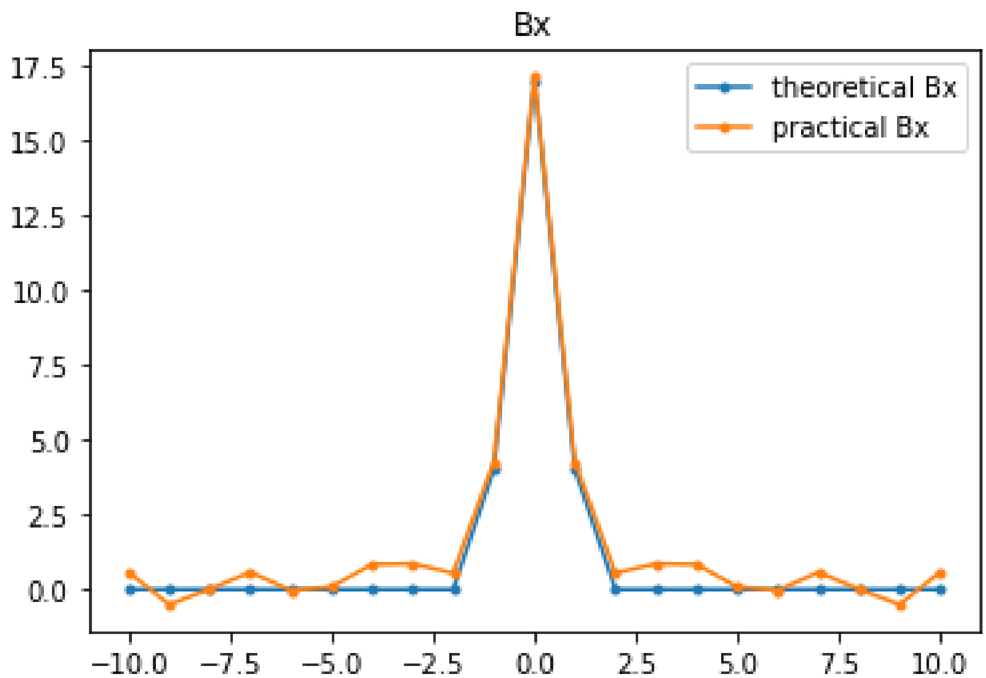
3 Experiment 3

```
In [18]: cosi = np.random.randn(2000)
         X = np.zeros(1000)
         for i in range(1000):
             X[i] = cosi[i+1] + 4*cosi[i]

In [19]: print('X theoretical mean:', 0.0)
         print('X practical mean: ', np.mean(X))

X theoretical mean: 0.0
X practical mean:  0.038527382835


In [20]: print('X^2 theoretical mean: ', 17)
         print('X^2 practical mean: ', np.mean(X**2))

X^2 theoretical mean:  17
X^2 practical mean:  17.1254809939
```

```
In [21]: print('X theoretical variance: ', 17)
         print('X practical variance: ', np.mean(X**2)-np.mean(X)**2)
```

```
X theoretical variance:  17
X practical variance:  17.1239966347
```

```
In [22]: Bx_h = np.zeros((21, 1))
         x = np.arange(-10, 10+1, 1)
         for i, v in enumerate(x):
             Bx_h[i] = Bx_hat(X, v)

         plt.figure()
         plt.plot(x, [0,0,0,0,0,0,0,0,0,4,17,4,0,0,0,0,0,0,0,0,0], '.-')
         plt.plot(x, Bx_h, '.-')
         plt.legend(['theoretical Bx', 'practical Bx'])
         plt.title('Bx')
         plt.show()
```



4 Expriment 4

```
In [23]: cosi = np.random.randn(2000)
         k = 10
         X = np.zeros(k)
```

```
        X[0] = np.random.randn()
        for i in range(1, k):
            X[i] = cosi[i] - 0.707*X[i-1]
```

```
In [24]: print('X theoretical mean: ', np.power(-0.707, k)*X[0])
         print('X practical mean: ', np.mean(X))
```

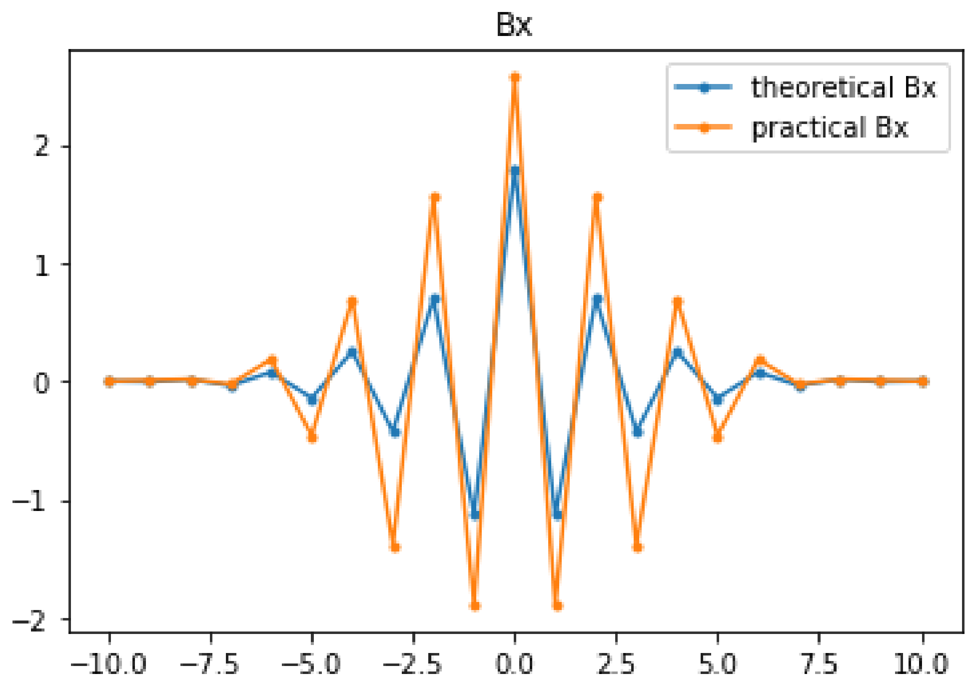
```
X theoretical mean:  -0.000248772231688
X practical mean:  -0.0943930641225
```

```
In [25]: print('X theoretical variance: ', np.power(-0.707, k)*X[0])
         print('X practical variance ', np.var(X))
```

```
X theoretical variance:  -0.000248772231688
X practical variance  2.57313200231
```

```
In [26]: def f_gamma_x(X, m, a):
             X_0 = X[0]
             N = np.shape(X)[0]
             EX = np.mean(X)
             abs_m = abs(m)
             val = 0
             for k in range(1, N-abs_m+1):
                 s = k + abs_m
                 val += np.power((-a),k+s)*X_0**2+(np.power(-a,np.abs(k-s))-\
                                                   np.power(-a,np.abs(k+s)))/(1-a**2)
             val /= N
             return val
```

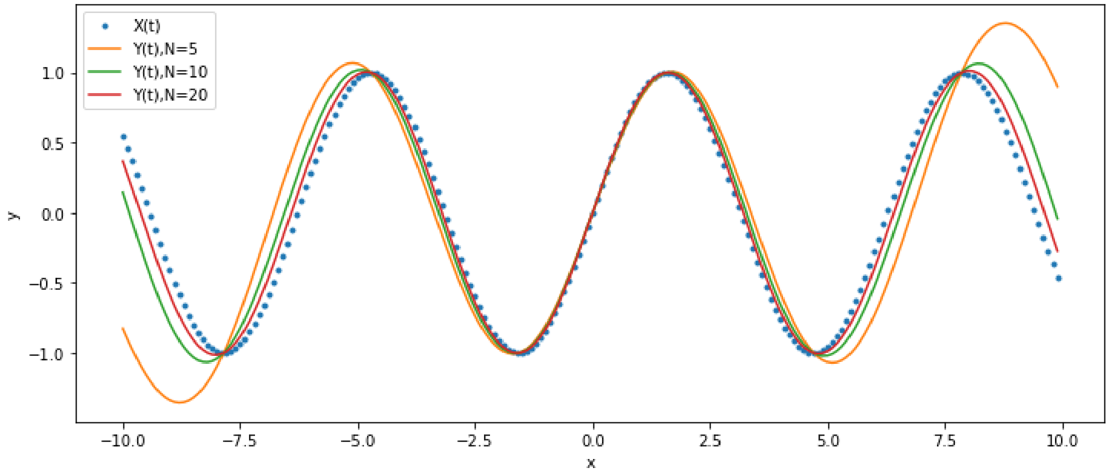
```
In [27]: Bx_h = np.zeros((21, 1))
         Bx_ex = np.zeros((21, 1))
         x = np.arange(-10, 10+1, 1)

         for i, v in enumerate(x):
             Bx_ex[i] = f_gamma_x(X, v, 0.707)

         for i, v in enumerate(x):
             Bx_h[i] = Bx_hat(X, v)

         plt.figure()
         plt.plot(x, Bx_ex, '.-')
         plt.plot(x, Bx_h, '.-')
         plt.legend(['theoretical Bx', 'practical Bx'])
         plt.title('Bx')
         plt.show()
```



5 Experiment 5

```
In [28]: def func(n):
             X = np.arange(-n, n+1, 1)
             X = np.sin(X*np.pi/2)
             return X

In [29]: def X_t(t):
             val = np.sin(t)
             return val
         def Y_t(t, N):
             val = 0
             for n in range(-N, N+1):
                 val += X_t(n*np.pi/2) * np.sin(t-n*np.pi/2) / (t-n*np.pi/2)
             return val

In [30]: x = np.arange(-10,10,0.1)
         x_t = X_t(x)
         plt.figure(figsize=[12,5])
         plt.plot(x, x_t, '.')
         for N in [5, 10, 20]:
             y_t = Y_t(x, N)
             plt.plot(x, y_t)
         plt.legend(['X(t)', 'Y(t),N=5', 'Y(t),N=10', 'Y(t),N=20'])
```

```
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```



6 Experiment 6

```
In [31]: n = 20
         A = np.zeros((n*2,n+1))
         B = np.zeros((n*2,n))
         Alfa = np.zeros((n,1))
         Beta = np.zeros((n,1))
         A[0,:] = np.arange(1,n+1+1)
         # B[0,:] = np.where(np.arange(0,n)%2==0,-np.arange(2,n+1+1),np.arange(2,n+1+1))
         B[0,:] = np.zeros((1,n))
         B[0,-1] = 1
         for i in range(0,n):
             for j in range(i,n,2):
                 A[i*2+1,j] = A[i*2,j+1]
             for j in range(i,n-1,2):
                 B[i*2+1,j] = A[i*2,j+1]
             if A[i*2+1,i] <= 0:
                 print('Instability')
                 break
             Alfa[i,0] = A[i*2,i]/A[i*2+1,i]
             Beta[i,0] = B[i*2,i]/B[i*2+1,i]
             if i!=n-1:
                 for j in range(i+1,n,2):
                     A[i*2+2,j] = A[i*2,j]
                     A[i*2+2,j+1] = A[i*2,j+1] - Alfa[i]*A[i*2+1,j+1]
                 for j in range(i+1,n-1,2):
```

```
                    B[i*2+2,j] = B[i*2,j]
                    B[i*2+2,j+1] = B[i*2,j+1] - Beta[i]*B[i*2+1,j+1]
```

Instability

In [32]:
```
pd.DataFrame(A[:i*2+1+1,:])
```

Out[32]:
```
     0    1    2    3    4    5    6    7     8     9    10    11    12    13  \
0  1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0   9.0  10.0  11.0  12.0  13.0  14.0
1  2.0  0.0  4.0  0.0  6.0  0.0  8.0  0.0  10.0   0.0  12.0   0.0  14.0   0.0
2  0.0  2.0  1.0  4.0  2.0  6.0  3.0  8.0   4.0  10.0   5.0  12.0   6.0  14.0
3  0.0  1.0  0.0  2.0  0.0  3.0  0.0  4.0   0.0   5.0   0.0   6.0   0.0   7.0
4  0.0  0.0  1.0  0.0  2.0  0.0  3.0  0.0   4.0   0.0   5.0   0.0   6.0   0.0
5  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0   0.0   0.0   0.0   0.0   0.0   0.0

     14    15    16    17    18    19    20
0  15.0  16.0  17.0  18.0  19.0  20.0  21.0
1  16.0   0.0  18.0   0.0  20.0   0.0   0.0
2   7.0  16.0   8.0  18.0   9.0  20.0  21.0
3   0.0   8.0   0.0   9.0   0.0  21.0   0.0
4   7.0   0.0   8.0   0.0   9.0 -22.0   0.0
5   0.0   0.0   0.0   0.0 -22.0   0.0   0.0
```

In [33]:
```
pd.DataFrame(B[:i*2+1+1,:])
```

Out[33]:
```
     0    1    2    3    4    5    6    7     8    9    10   11    12   13  \
0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0   0.0  0.0   0.0  0.0   0.0  0.0
1  2.0  0.0  4.0  0.0  6.0  0.0  8.0  0.0  10.0  0.0  12.0  0.0  14.0  0.0
2  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0   0.0  0.0   0.0  0.0   0.0  0.0
3  0.0  1.0  0.0  2.0  0.0  3.0  0.0  4.0   0.0  5.0   0.0  6.0   0.0  7.0
4  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0   0.0  0.0   0.0  0.0   0.0  0.0
5  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0   0.0  0.0   0.0  0.0   0.0  0.0

     14   15    16   17    18   19
0   0.0  0.0   0.0  0.0   0.0  1.0
1  16.0  0.0  18.0  0.0  20.0  0.0
2   0.0  0.0   0.0  0.0   0.0  0.0
3   0.0  8.0   0.0  9.0   0.0  0.0
4   0.0  0.0   0.0  0.0   0.0  0.0
5   0.0  0.0   0.0  0.0 -22.0  0.0
```

7 Experiment 7

In [5]:
```
n = 6
A = np.zeros((n*2,n+1))
B = np.zeros((n*2,n+1))
Alfa = np.zeros((n,1))
```

```python
Beta = np.zeros((n,1))
A[0,:] = np.array([1,0.5,0.58,-0.01,-0.0119,0.00005,0.00006])
B[0,:] = np.array([0,0,   0,    0,      0,      1,  -0.55])
flag = 0
for i in range(0,n):
    A[i*2+1,0:n+1-i] = A[i*2,0:n+1-i][::-1]
    B[i*2+1,:] = A[i*2+1,:]
    if A[i*2,0] <= 0:
        print('Instability')
        flag = 1
        break
    Alfa[i,0] = A[i*2+1,0]/A[i*2,0]
    Beta[i,0] = B[i*2,n-i]/B[i*2+1,n-i]
    if i!=n-1:
        for j in range(0,n-i):
            A[i*2+2,j] = A[i*2,j] - Alfa[i]*A[i*2+1,j]
        for j in range(0,n-i):
            B[i*2+2,j] = B[i*2,j] - Beta[i]*A[i*2+1,j]
if flag != 1:
    print('Stability')
```

```
Stability
```

In [6]:
```python
pd.DataFrame(A[:i*2+1+1,:])
```

Out[6]:

	0	1	2	3	4	5	6
0	1.000000	0.500000	0.580000	-0.010000	-0.011900	0.00005	0.00006
1	0.000060	0.000050	-0.011900	-0.010000	0.580000	0.50000	1.00000
2	1.000000	0.500000	0.580001	-0.009999	-0.011935	0.00002	0.00000
3	0.000020	-0.011935	-0.009999	0.580001	0.500000	1.00000	0.00000
4	1.000000	0.500000	0.580001	-0.010011	-0.011945	0.00000	0.00000
5	-0.011945	-0.010011	0.580001	0.500000	1.000000	0.00000	0.00000
6	0.999857	0.499881	0.586929	-0.004039	0.000000	0.00000	0.00000
7	-0.004039	0.586929	0.499881	0.999857	0.000000	0.00000	0.00000
8	0.999841	0.502251	0.588948	0.000000	0.000000	0.00000	0.00000
9	0.588948	0.502251	0.999841	0.000000	0.000000	0.00000	0.00000
10	0.652926	0.206404	0.000000	0.000000	0.000000	0.00000	0.00000
11	0.206404	0.652926	0.000000	0.000000	0.000000	0.00000	0.00000

In [7]:
```python
pd.DataFrame(B[:i*2+1+1,:])
```

Out[7]:

	0	1	2	3	4	5	6
0	0.000000	0.000000	0.000000	0.000000	0.0000	1.000	-0.55
1	0.000060	0.000050	-0.011900	-0.010000	0.5800	0.500	1.00
2	0.000033	0.000028	-0.006545	-0.005500	0.3190	1.275	0.00
3	0.000020	-0.011935	-0.009999	0.580001	0.5000	1.000	0.00
4	0.000007	0.015244	0.006204	-0.745001	-0.3185	0.000	0.00
5	-0.011945	-0.010011	0.580001	0.500000	1.0000	0.000	0.00

```
6  -0.003797  0.012056  0.190935 -0.585751  0.0000  0.000  0.00
7  -0.004039  0.586929  0.499881  0.999857  0.0000  0.000  0.00
8  -0.006163  0.355899  0.483782  0.000000  0.0000  0.000  0.00
9   0.588948  0.502251  0.999841  0.000000  0.0000  0.000  0.00
10 -0.291131  0.112880  0.000000  0.000000  0.0000  0.000  0.00
11  0.206404  0.652926  0.000000  0.000000  0.0000  0.000  0.00
```

```
In [8]: Alfa
```

```
Out[8]: array([[  6.00000000e-05],
               [  2.00000001e-05],
               [ -1.19448000e-02],
               [ -4.03917349e-03],
               [  5.89041668e-01],
               [  3.16122127e-01]])
```

```
In [9]: Beta
```

```
Out[9]: array([[-0.55      ],
               [ 1.275     ],
               [-0.3185    ],
               [-0.58583443],
               [ 0.48385875],
               [ 0.17288375]])
```

```
In [17]: A[:,0]
```

```
Out[17]: array([  1.00000000e+00,   6.00000000e-05,   9.99999996e-01,
                  2.00000000e-05,   9.99999996e-01,  -1.19448000e-02,
                  9.99857318e-01,  -4.03859718e-03,   9.99841005e-01,
                  5.88948014e-01,   6.52926085e-01,   2.06404383e-01])
```

```
In [16]: A[:,0][::2]
```

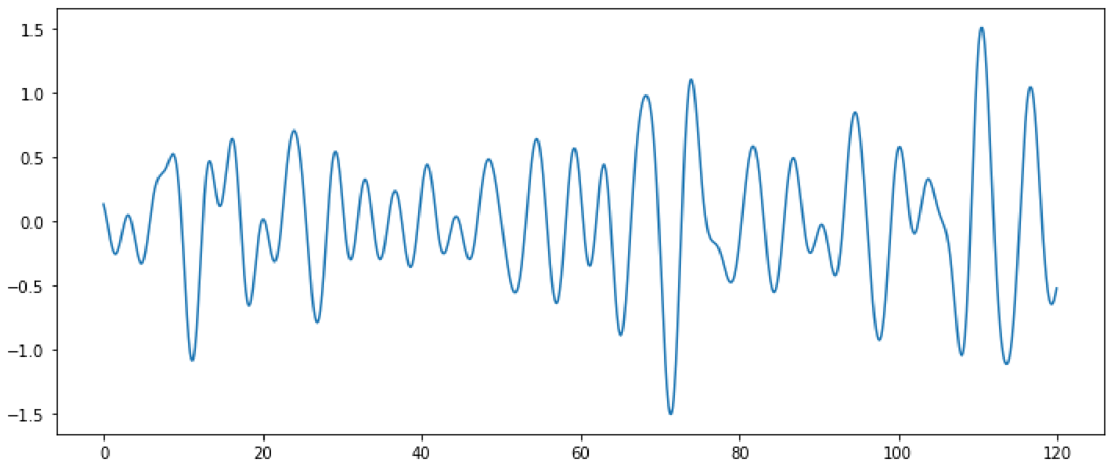
```
Out[16]: array([ 1.        ,  1.        ,  1.        ,  0.99985732,  0.99984101,
                 0.65292608])
```

```
In [1]: In = 1/np.power(A[0,0],n)*np.sum(Beta**2/A[:,0][::2])
        print(In)
```

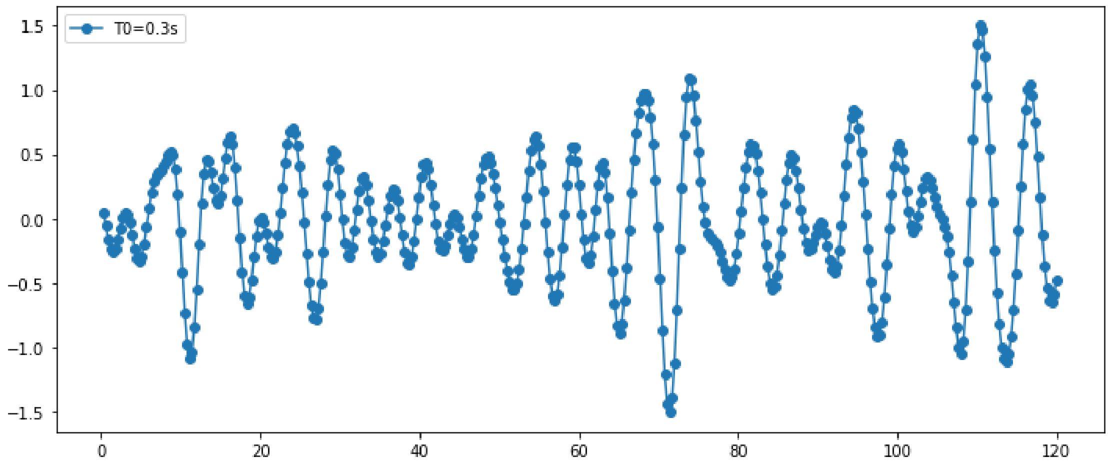
```
        ---------------------------------------------------------------------------

        NameError                                 Traceback (most recent call last)

        <ipython-input-1-63ecede5a6f1> in <module>()
    ----> 1 In = 1/np.power(A[0,0],n)*np.sum(Beta**2/A[:,0][::2])
          2 print(In)


        NameError: name 'np' is not defined
```

8 Experiment 8

In [40]:
```
A = 0.78
B = 3.11/4
N = 40
np.random.seed(12)
epsilon = np.random.rand(N+1)*2*np.pi-np.pi
w = 0.05*np.arange(0, 41)

def sqrt_calculate(w_n, w_n_1):
    if w_n==0:
        w_n=1e-8
    if w_n_1==0:
        w_n_1=1e-8
    val = np.exp(-B*np.power(w_n, -4))-np.exp(-B*np.power(w_n_1,-4))
    val = np.sqrt(2*A/(4*B)*val)
    return val

def cosi(t):
    val = 0
    for i in range(1, N+1):
        w_n = w[i]
        w_n_1 = w[i-1]
        val += sqrt_calculate(w_n, w_n_1)*np.cos(w_n*t+epsilon[i])
    return val

t = np.arange(0, 120, 0.1)
val = cosi(t)
plt.figure(figsize=[12,5])
plt.plot(t, val, '-')
plt.show()
```

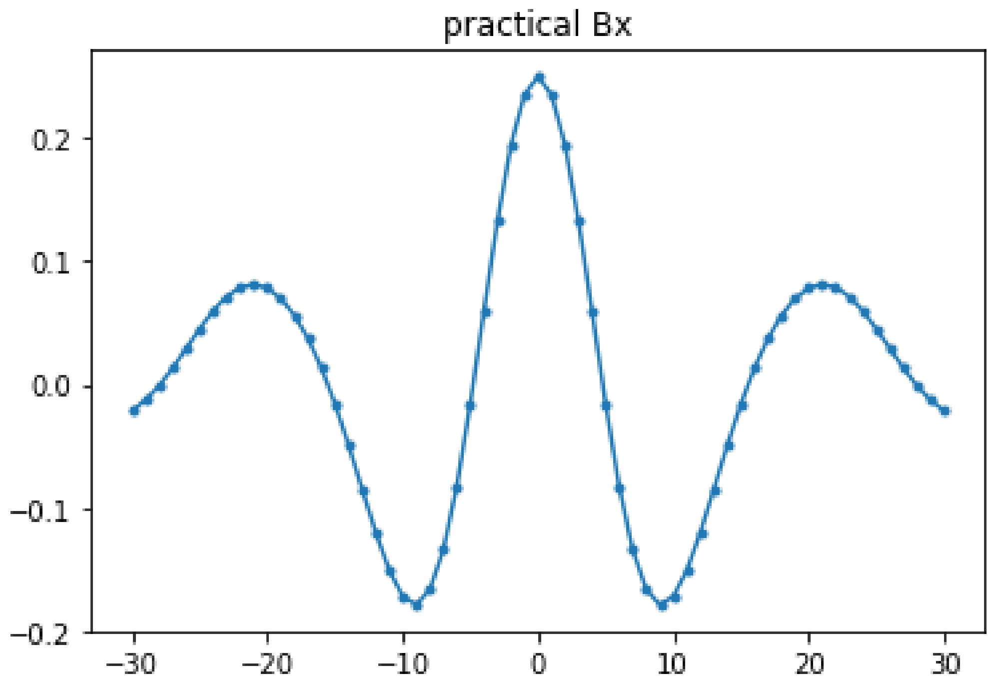


```
In [41]: T0 = 0.3
         nT0 = T0*np.arange(1,400+1,1)
         lis = cosi(nT0)
         plt.figure(figsize=[12,5])
         plt.plot(nT0, lis, '-o')
         plt.legend(['T0=0.3s'])
         plt.show()
```



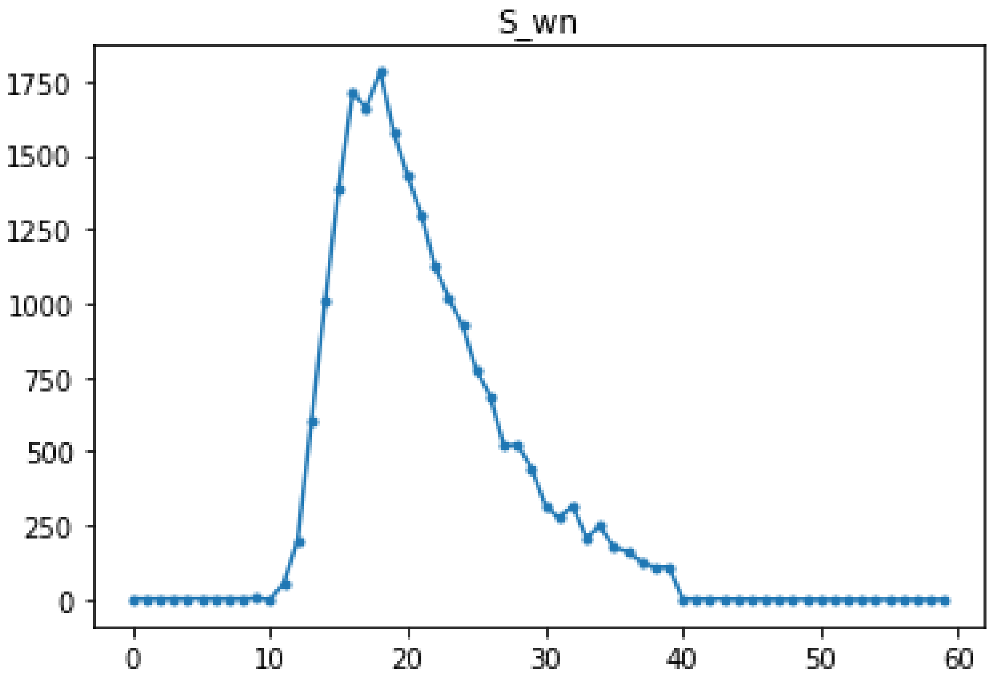

```
In [42]: print('mean: ', np.mean(lis))

mean:  -0.00436546577416
```

```
In [43]: Bx_h = np.zeros((61, 1))
         x = np.arange(-30, 30+1, 1)
         for i, v in enumerate(x):
             Bx_h[i] = Bx_hat(lis, v)

         plt.figure()
         plt.plot(x, Bx_h, '.-')
         plt.title('practical Bx')
         plt.show()
```

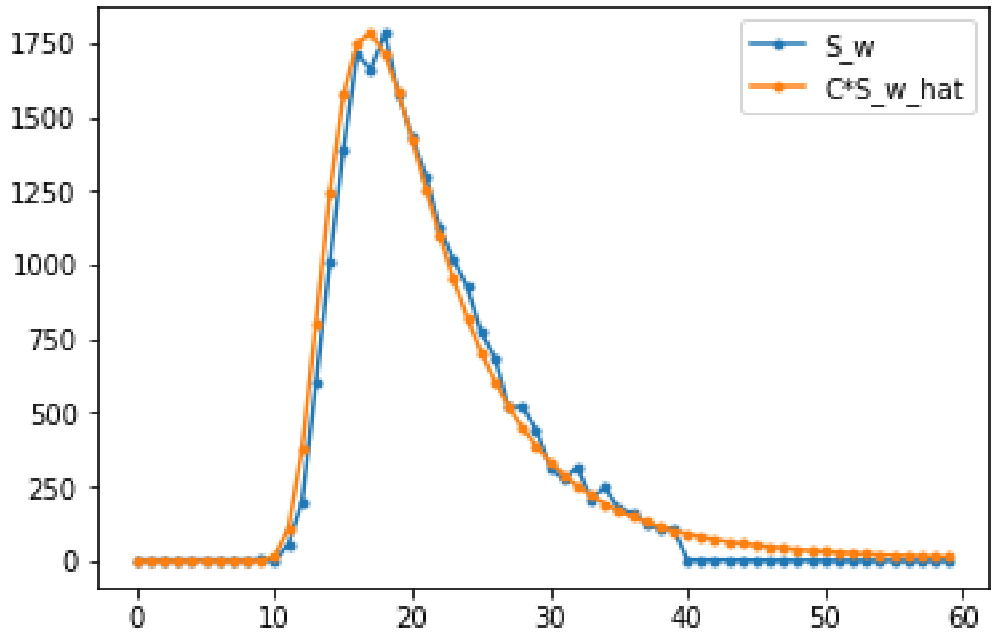



```
In [44]: def S_cosi_hat(w):
             T0 = 0.3
             N = 400
             val = 0
             for m in range(1,N+1):
                 val += cosi(m*T0)*np.exp(np.complex(0,-w*m*T0))
             return val.imag**2+val.real**2
```

```
In [45]: S_cosi_w_n_hat = np.zeros((60,1))
         for i,v in enumerate(range(1, 61)):
             w_n = 0.05*v
             S_cosi_w_n_hat[i] = S_cosi_hat(w_n)
         plt.plot(S_cosi_w_n_hat, '.-')
         plt.title('S_wn')
         plt.show()
```




```
In [46]: def S_cosi(w):
             return A/np.power(w,5)*np.exp(-B/np.power(w,4))
         S_cosi_w_n = np.zeros((60,1))
         for i,v in enumerate(range(1, 61)):
             w_n = 0.05*v
             S_cosi_w_n[i] = S_cosi(w_n)

         C = np.max(S_cosi_w_n_hat)/np.max(S_cosi_w_n)

In [47]: plt.plot()
         plt.plot(S_cosi_w_n_hat, '.-')
         plt.plot(S_cosi_w_n*C, '.-')
         plt.legend(['S_w', 'C*S_w_hat'])
         plt.show()
```


S_w
C*S_w_hat
1750
1500
1250
1000
750
500
250
0
0
10
20
30
40
50
60