



## MULTI-OBJECTIVE IDENTIFICATION OF INDUCTION MACHINE MODELS

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**Abstract**— In this study the characterization of induction machines is formulated as a nonlinear estimation problem. The solution is obtained with a multi-objective genetic algorithm. The data used in the characterization correspond to the quantities that can be measured in laboratory tests. The characterization obtained with the proposed technique is evaluated by simulation and with the data provided by the machine manufacturer. The results obtained either by simulation or based on manufacturer's data demonstrate the correctness as well as the feasibility of the proposed approach.

**Keywords**— induction machines, nonlinear estimation, parameter estimation, multi-objective identification.

**Resumo**— Neste trabalho a caracterização de máquinas de indução é formulada como um problema de estimação não linear. A solução é obtida mediante a aplicação de um algoritmo genético multi-objetivo. Os dados utilizados na caracterização correspondem às grandezas que podem ser medidas em testes laboratoriais. A caracterização obtida com a técnica proposta é avaliada por simulação e utilizando-se dados disponibilizados pelo fabricante da máquina elétrica. Os resultados dos testes simulados e dos testes realizados com os dados do fabricante comprovam a corretude, bem como a aplicabilidade da abordagem proposta.

**Palavras-chave**— induction machines, nonlinear estimation, parameter estimation, multi-objective identification.

### 1 Introduction

Induction motors are widely used to drive mechanical loads in commercial and industrial systems due to their low cost and reliability. In order to design an induction motor drive system and for prediction purposes it is necessary to determine induction motor equivalent circuit model parameters (Chapman, 2005). The equivalent circuit is used, for instance, to predict the system behavior, compute efficiency, and controller design.

The equivalent circuit model parameters are, generally, determined via the classical locked-rotor and no-load tests as prescribed in the IEEE Standard 112 (*IEEE Standard Test Procedure for Polyphase Induction Motors and Generators*, 2004). The parameters obtained from the classical tests data did not represent the machine behavior in the entire slip range ( $s \in [0, 1]$ ) (Lima et al., 1997). This is because the induction machine parameters are not constant over the entire slip range variation. So, it is necessary to modify the initial parameters obtained from the classical tests in order to improve the model description and reduce the difference between the real and the estimated performances.

The use of system identification to estimate these parameters is not a new solution (Kumar et al., 2014; Lima et al., 1997). The equations relating the phase current, the slip, the parameters, the input power and the electromagnetic torque are nonlinear and involve many parameters. The equivalent circuit model parameters estimation can be formulated as a nonlinear least

squares minimization problem and solved by traditional optimization methods or by genetic algorithms (Kumar et al., 2014).

Usually, in the nonlinear least squares minimization approach all the parameters are estimated by using a single cost function or objective function (Single Objective Optimization Problem - SOO). Such objective function is obtained by the aggregation of all the existing objectives and the most common aggregation method being the weighted sum. Despite its simplicity, the weighted sum approach has several drawbacks and limitations (Das and Dennis, 1997; Messac et al., 2000). The most clear limitations are related with the choice of the weights which must be defined a priori; the priori selection does not guarantee that the final solution will be acceptable; the aggregate function leads to one solution; and the trade-offs between objectives cannot be easily evaluated (Das and Dennis, 1997).

Another way to solve the induction machine estimation problem is to consider a collection of objectives functions and thus adopt a multi-objective optimization (MOO) approach. In this case, each objective function may have a different optimal solution and, if this is true, the objective functions are known as conflicting. When the objective functions are conflicting the problem results in a solution set instead of a solution point as in the SOO problem formulation. The solutions in the solution set fit a predetermined definition for an optimum. They are often called Pareto optimal and constitute the Pareto set.

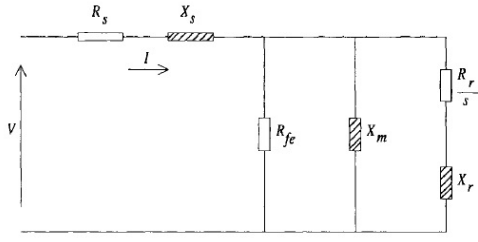


Figure 1: Equivalent circuit of an induction machine.

Table 1: Equivalent circuit parameters and variables.

$R_s$	stator resistance
$X_s$	stator leakage reactance
$R_{fe}$	iron loss resistance
$X_m$	magnetizing reactance
$R_r$	rotor resistance
$X_r$	rotor leakage reactance
$s$	machine slip

Genetic algorithms are natural candidates in order to solve multi-objective problems. These algorithms perform global search, can be executed in parallel and are suited to solve problems where the space of potential solutions is quite large. They also do not involve many mathematical requirements about the problems and can handle different kinds of objective functions and constraints.

In this paper it is proposed an estimation technique for induction machine equivalent circuit model parameters formulated as a multi-objective optimization problem and solved using a genetic algorithm. All the parameters are estimated and no one is considered known a priori. The objectives considered are the current-slip, torque-slip, power-slip and power-factor-slip curves. The constraints are the parameters minimum and maximum bounds. It is also evaluated the robustness of the estimation algorithms.

This paper is organized as follows: in Section 2 it is described the induction machine equivalent circuit model and it is presented the system equations; in Section 3 it is formulated the multi-objective optimization problem and it is described the genetic algorithms; in Section 4, the parameter estimation problem is formulated as a multi-objective optimization problem; in Section 5, the simulations results are presented; in Section 6 are presented pseudo-experimental test results; and in Section 7, the conclusions are provided.

## 2 Machine model

The equivalent circuit model per phase for a three-phase symmetrical voltage source is given in Figure 1. The variables are referred to the stator reference frame and they are listed in Table 1.

From Figure 1 it is possible to derive the equations relating the model parameters with the desired measured variable. The chosen variables are the stator current  $I(s, \theta)$ , the input power  $P(s, \theta)$  and the electromagnetic torque  $T(s, \theta)$ ; this notation is used to emphasize its dependence with the machine slip  $s$ . The equations for  $I(s, \theta)$ ,  $P(s, \theta)$  and  $T(s, \theta)$  can be found in (Lima et al., 1997). These equations are revisited, the power factor  $PF(s, \theta)$  equation is added and they are defined as follows:

$$I(s, \theta) = V \sqrt{\frac{C^2 + D^2}{A^2 + B^2}} \quad (1)$$

$$P(s, \theta) = 3V^2 \frac{AC - BD}{A^2 + B^2} \quad (2)$$

$$T(s, \theta) = 3V^2 \frac{p}{\omega} \frac{R_{fe}^2 R_r}{(A^2 + B^2)s} \quad (3)$$

$$PF(s, \theta) = \frac{DB - CA}{\sqrt{E^2 + F^2}} \quad (4)$$

where  $\omega$  is the stator angular frequency,  $p$  is the number of poles pairs and

$$A = R_s \left( \frac{R_r}{s} + R_{fe} \left( 1 + \frac{X_r}{X_m} \right) \right) + X_s \left( \frac{R_{fe} R_r}{s X_m} - X_r \right) + \frac{R_{fe} R_r}{s} \quad (5)$$

$$B = X_s \left( \frac{R_r}{s} + R_{fe} \left( 1 + \frac{X_r}{X_m} \right) \right) - R_s \left( \frac{R_{fe} R_r}{s X_m} - X_r \right) + R_{fe} X_r \quad (6)$$

$$C = \frac{R_r}{s} + R_{fe} \left( 1 + \frac{X_r}{X_m} \right) \quad (7)$$

$$D = \frac{R_{fe} R_r}{s X_m} - X_r \quad (8)$$

$$E = DB - CA \quad (9)$$

$$F = CB - DA. \quad (10)$$

The equivalent circuit model considering the iron loss resistance  $R_{fe}$  there is no direct Park equivalent model. So in order to translate de induction machine model to its Park equivalent model it is possible to neglect the iron losses. The current, power, power factor equations are the same, but the torque is now given by

$$T(s, \theta) = 3V^2 \frac{p}{\omega} \frac{R_r}{(A^2 + B^2)s} \quad (11)$$

and the terms are

$$A = R_s \left( 1 + \frac{X_r}{X_m} \right) + \frac{R_r}{s} \left( 1 + \frac{X_s}{X_m} \right) \quad (12)$$

$$B = X_r + X_s \left( 1 + \frac{X_r}{X_m} \right) - R_s \left( \frac{R_r}{s X_m} \right)$$

$$C = 1 + \frac{X_r}{X_m} \quad (13)$$

$$D = \frac{R_r}{s X_m} \quad (14)$$

### 3 Multi-objective optimization

From equations (1)-(4) it is possible to see they are nonlinear and involve many parameters. If all the parameters need to be estimated, two objective functions (current-slip and torque-slip curves, for instance), the parameters estimation problem becomes a multi-objective optimization problem. The general multi-objective optimization problem is defined as follows:

$$\min F(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x}), \dots, F_K(\mathbf{x})]^T \quad (15)$$

subject to

$$\begin{aligned} g_j(\mathbf{x}) &\leq 0, & j &= 1, 2, \dots, M \\ h_l(\mathbf{x}) &= 0, & l &= 1, 2, \dots, E \end{aligned}$$

where  $\mathbf{x} = [x_1, x_2, \dots, x_N]^T \in \mathbf{X}$  is the decision or design variable vector,  $N$  is the number of decision or design variables,  $F(\mathbf{x})$  is the vector of objective functions,  $K$  is the number of objective functions,  $M$  is the number of inequality constraints,  $E$  is the number of equality constraints.

#### 3.1 Genetic algorithm

There are several techniques to solve multi-objective optimization problems. There are two major classes: classical techniques and intelligent techniques (Ngatchou et al., 2005). Classical techniques convert multi-objective problems into single objectives problems by aggregation or by optimizing one objective and treating the others as constraints. This single objective problem is solved using the traditional scalar optimization techniques and results a single solution. In case of aggregation, it is necessary to determine the weights between the objectives before the problem be solved. On the other hand, the intelligent techniques solve the multi-objective problem by optimizing the individual objectives simultaneously. Population-based algorithms have the advantage of evaluating multiple potential solutions in a single iteration (Ngatchou et al., 2005). Genetic algorithms, evolutionary algorithms and evolutionary strategies are on of these methods.

Genetic algorithms can treat any kind of objective function and constraint as they do not require gradient information. They are also parallel algorithms and are suited to solve problems where the space of potential solutions is large. It uses the microbiology language and genetic operations in developing new potential solutions. A population represents a group of potential solution points, a generation represents the iteration, a chromosome is comparable to a design point and a gene is comparable to a component of the design vector.

The multi-objective genetic algorithm used is a variant of the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) that uses a controlled elitist genetic algorithm. In non-elitist algorithms,

the genetic operator may destroy some of the non-dominated solutions to explore the design space. Elitism can speed up the performance of the genetic algorithm significantly, also it helps to prevent the loss of good solutions once they have been found (Deb et al., 2002; Afzal and Kim, 2014).

#### 3.2 Parameter estimation

Equations (1)-(4) express measurable variables issued from the equivalent circuit model. Using the multi-objective framework, it is possible to use all these measurable variables as objective functions. In this work, the multi-objective estimation problem is stated as

$$\min F(\theta) = [F_1(\theta), F_2(\theta), F_3(\theta), F_4(\theta)]^T \quad (16)$$

where

$$F_j(\theta) = \sum_{i=1}^N [\Gamma_m(s_i) - \Gamma_c(s_i, \theta)]^2 \quad (17)$$

where  $j = 1, \dots, K = \dim(F)$ ,  $\Gamma_m(s_i)$  denote the experimentally collected variables,  $\Gamma_c(s_i, \theta)$  are the calculated quantities by using equations (1)-(4) for a given parameter  $\theta$ . For each  $j$ ,  $\Gamma_m(s_i)$  and  $\Gamma_c(s_i, \theta)$  represent a specific pair of an electrical quantities, i.e.,  $j = 1$  stands for  $I(s_i)$  and  $I_c(s_i, \theta)$ ;  $j = 2$  stands for  $T(s_i)$  and  $T_c(s_i, \theta)$ ;  $j = 3$  stands for  $P(s_i)$  and  $P_c(s_i, \theta)$ ;  $j = 4$  stands for  $PF(s_i)$  and  $PF_c(s_i, \theta)$ . The parameter vector  $\theta$  is defined as

$$\theta = [R_r, X_r, X_s, R_s, X_m, R_{fe}]^T \quad (18)$$

and the optimization problem is subject to the minimum and maximum limits as

$$\theta_{min}^l \leq \theta^l \leq \theta_{max}^l, l = 1, \dots, L = \dim(\theta),$$

and when the iron loss is not considered the reduced parameter vector is given by

$$\theta_r = [R_r, X_r, X_s, R_s, X_m]^T \quad (19)$$

### 4 Simulation study

The multi-objective estimation based on genetic algorithms was studied for a 1.5kW three-phase squirrel cage induction motor. The motor data are given in Table 2.

Equations (1)-(4) have been used to generate 50 the data points for each variable. A pseudo-random Gaussian number generator was used to simulate the measurement noise (zero mean and variance equal to one tenth of the nominal value of the respective quantity). The NSGA-II was employed to estimate the parameters. The population sizes was 100 and the initial population was randomly generated for each of the parameter vector  $\theta$  element ( $\theta \in \mathbb{R}^{100 \times 6}$ ). The crossover fraction is 0.8, the Pareto-front population fraction is 0.85 and the function tolerance is  $10^{-6}$ .

Table 2: Induction motor electrical data.

Power	1.5kW
Voltage	220V
Current	5.8A
Frequency	60Hz
Number of poles	2
Torque	8.0Nm
$R_s$	1.93
$X_s$	1.658
$R_{fe}$	310
$X_m$	38.7
$R_r$	3.84
$X_r$	6.789

Table 3: Case 1 - Estimated parameters for  $\theta_r$ .

	True	MGA	[%]	WNLS	[%]
$R_s$	1.93	1.859	3.68	1.080	44.04
$X_s$	1.658	1.668	0.6	2.034	22.68
$X_m$	38.7	39.079	0.98	39.715	2.62
$R_r$	3.84	3.864	0.62	3.982	3.69
$X_r$	6.789	6.794	0.07	6.816	0.39

#### 4.1 Case 1. Single objective function

In this first case it was used just on objective function  $F_1(\theta)$  (stator current objective), see equation (16). As there is just one objective, it was used the single objective genetic algorithm (GA). In order to compare the results, it was used the non-linear least squares algorithm (NLS). The initial parameters used for the NLS algorithm are the ones obtained from the classical tests. The estimated parameters and the absolute value for the error are listed in Table 3; the reduced parameter vector  $\theta_r$  was used. In Figure 2 it is shown the resultant curves. Both the methods results in good estimates for the current-slip curve.

In Figure 3 it is shown the power-slip curves based on the estimated parameters. It is possible to see that this curve is not good using any of the methods. The power objective function was not used in the optimization problem.

For the parameter vector  $\theta$  the results obtained before for the current-slip and power-slip

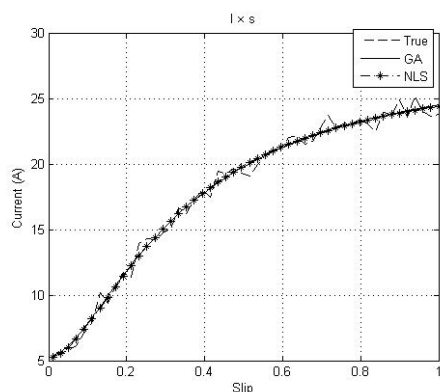


Figure 2: Case 1 - Estimated GA and NLS stator current-slip curves.

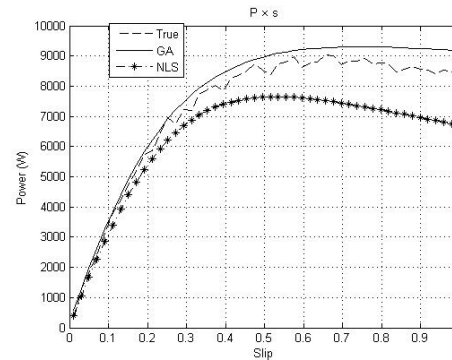


Figure 3: Case 1 - Estimated GA and NLS power-slip curves.

curves are the same. Both methods do not provide good results.

#### 4.2 Case 2. Two objective functions

In the second case it was used two objective functions  $F_1(\theta)$  (stator current objective) and  $F_3(\theta)$  (power objective), see equation (16). As there are two objectives, the problem is a MOO problem. In order to compare with the multi-objective genetic algorithm (MGA), it was used a weighted nonlinear least squares algorithm (WNLS). The cost function used for the WNLS is a weighted sum of the current and power functions. The initial parameters used for the WNLS algorithm are the ones obtained from the classical tests. When the reduced vector is considered the results are the ones listed in Table 4. In Figure 4 it is shown the current-slip curves and in Figure 5 it is shown the power-slip curves based on the estimated parameters. Based on Figure 4 and 5 it is possible to see that both methods provide good estimates for the current and power characteristic curves. But looking at the parametric errors, the WNLS estimates are unacceptable.

Table 4: Case 2 - Estimated parameters for  $\theta_r$ .

	True	MGA	[%]	WNLS	[%]
$R_s$	1.93	2.098	8.70	2.012	4.24
$X_s$	1.658	1.591	4.04	3.921	136.49
$X_m$	38.7	39.079	0.98	29.715	23.21
$R_r$	3.84	3.878	0.99	3.346	12.86
$X_r$	6.789	6.951	2.38	3.826	43.64

#### 4.3 Case 3. Two objective functions (with $R_{fe}$ )

The estimated parameters are listed in Table 5. In Figure 6 it is shown the current-slip curves and in Figure 7 it is shown the power-slip curves based on the estimated parameters. The results obtained with the reduced parameter vector (without the iron loss resistance) yield current and power curves quite well fitted, but the estimated

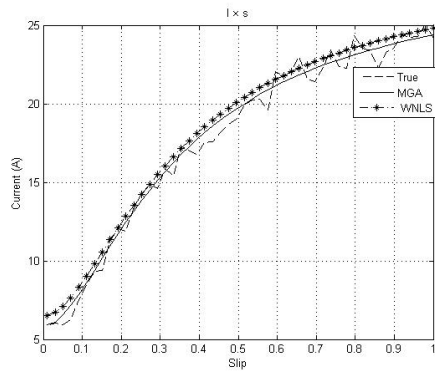


Figure 4: Case 2 - Estimated MGA and WNLS stator current-slip curves.

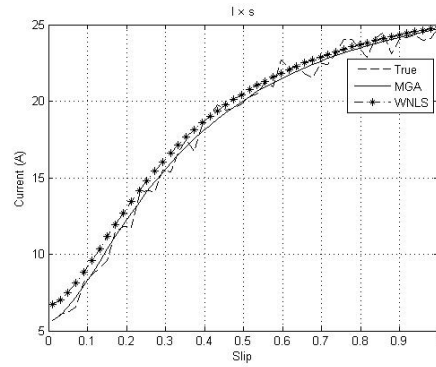


Figure 6: Case 3 - Estimated MGA and WNLS stator current-slip curves.

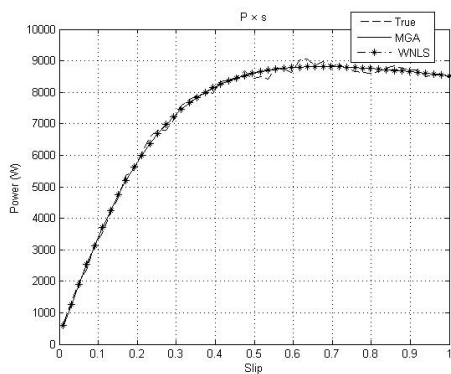


Figure 5: Case 2 - Estimated MGA and WNLS power-slip curves.

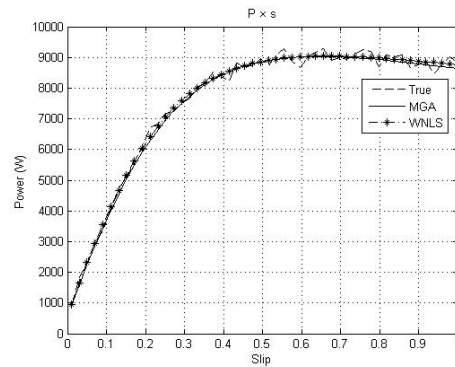


Figure 7: Case 3 - Estimated MGA and WNLS power-slip curves.

parameters with the WNLS method result in large parametric errors.

Table 5: Case 3 - Estimated parameters  $\theta$ .

	True	MGA	[%]	WNLS	[%]
$R_s$	1.93	1.884	2.38	1.787	7.41
$X_s$	1.658	1.489	10.19	4.102	147.41
$X_m$	38.7	37.955	1.92	34.711	10.31
$R_r$	3.84	3.987	3.82	3.604	6.14
$X_r$	6.789	7.140	5.17	6.342	6.58
$R_{fe}$	310	319.194	2.96	403.561	30.18

#### 4.4 Robustness test

To evaluate the robustness of the estimation, 100 independent runs have been executed. Only Case 2 for the robustness test. The results are summarized in Table 6.

It is clear that the multi-objective genetic algorithm presents better results than the weighted nonlinear least squares algorithm. The mean values are closer to the true ones and these can be confirmed by the mean errors ( $|\%|$ ) and standard deviation ( $std$ ) values.

## 5 Pseudo-experimental tests

In this section pseudo-experimental tests results are discussed. The pseudo prefix comes from the fact that the data used in the tests have been collected at the machine manufacturer web site which is publicly available (<http://ecatalog.weg.net/>). For the tests a three-phase induction motor of 30kW, 220V, 60Hz, 1 pole-pair has been selected; its nominal current and torque are  $I_n = 102A$  and  $T_n = 161Nm$ , respectively. The characterization has been performed with three different methods. The first one was a genetic algorithm with a weighted sum cost function (GA) for the single-objective optimization problem. For the multi-objective optimization it was used the multi-objective genetic algorithm (MGA) and the weighted nonlinear least squares algorithm (WNLS). The estimated parameters are listed in Table 7 and the current-slip and torque-slip curves follow in Figures 8 and 9.

## 6 Conclusion

The pseudo-experimental test results were satisfactory and confirmed what was observed in the simulation studies. The adoption of a multi-objective optimization approach to address the characterization of a three-phase squirrel cage in-



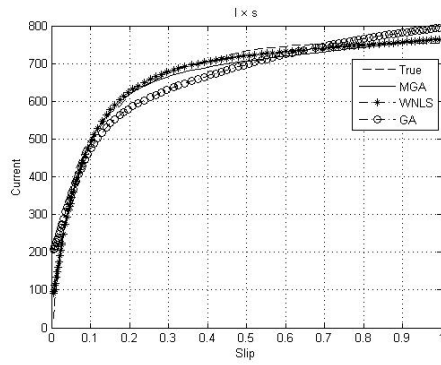


Figure 8: Pseudo-experimental test results - current-slip curves.

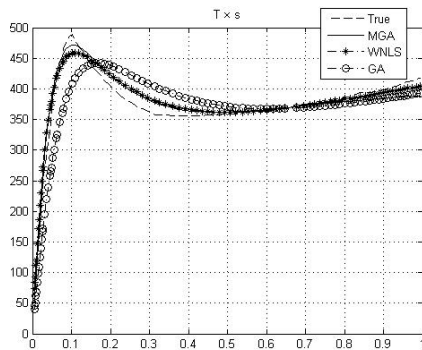


Figure 9: Pseudo-experimental test results - torque-slip curves.

Table 6: Mean value, error and deviation.

	MGA	[%]	std	WNLS	[%]	std
$\bar{R}_s$	1.91	0.7	0.17	1.89	1.6	0.26
$\bar{X}_s$	1.45	11.9	0.14	4.28	158.2	0.52
$\bar{X}_m$	38.13	1.4	1.31	37.95	1.9	10.48
$\bar{R}_r$	3.90	1.7	0.07	3.34	12.8	0.08
$\bar{X}_r$	7.06	4.0	0.17	3.44	49.1	0.47

Table 7: Pseudo-experimental test results - estimated parameters.

	GA	MGA	NLS
$R_s$	0.064	0.102	0.1416
$X_s$	0.045	0.095	0.006
$X_m$	1.33	2.477	3.968
$R_r$	0.106	0.098	0.096
$X_r$	0.266	0.125	0.171
$R_{fe}$	3.352	5.656	7.235

duction motor has allowed to obtain a better set of parameters than the one determined by using standard single objective techniques, e.g., weighted nonlinear least squares algorithm. The basic advantage of using a multi-objective genetic algorithm is its convergence properties when compared to the traditional search techniques has also been

verified. The proposed method is intended to be used together with the classical tests to improve the quality of the electrical parameter as more measurement data of the steady-state characteristic curves become available in the entire slip range. The use of equality constraints, like, for instance, the maximum torque and a detailed robustness study are currently under investigation and have not been included due to space limitations.

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