Aalto University
School of Electrical Engineering

ELEC-E8103 Modelling, Estimation and Dynamic Systems

4. Simulink

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Learning objectives

- Modelling of dynamic systems
  - First-principles modelling
    - Use theoretical equations to describe the system
    - Newton's 2nd law ($ma = \sum F$), Kirchhoff's law ($\sum I = 0$), etc.
  - Data-driven modelling
    - Use experimental data to describe the system
    - Regression, identification, etc.

Next weeks

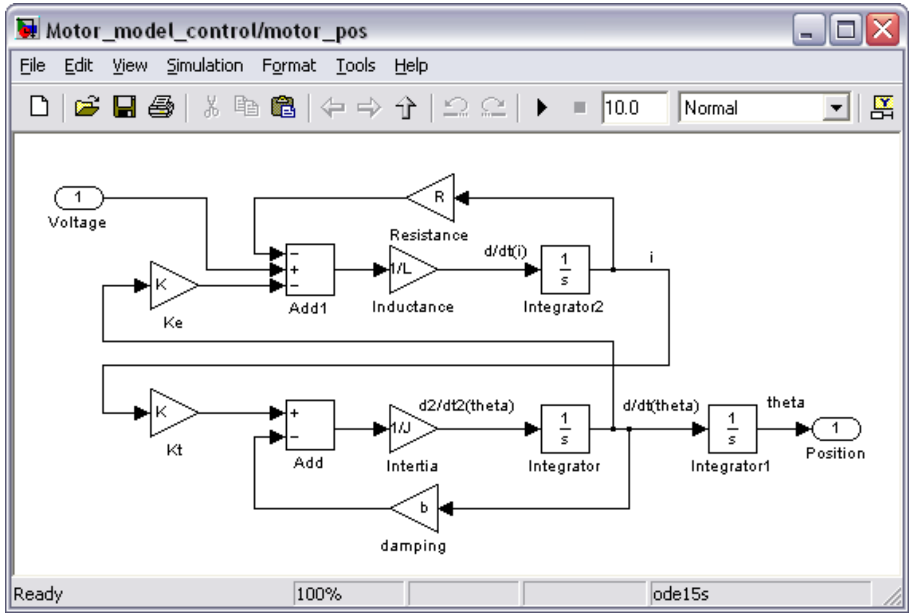
Learning objectives

- Modelling of dynamic systems
  - First-principles modelling
    - Use theoretical equations to describe the system
    - Newton's 2nd law ($ma = \sum F$), Kirchhoff's law ($\sum I = 0$), etc.
  - Differential equation(s) representing the dynamics of the system (e.g., $m\ddot{z} = -mg$) 微分方程
    - Solve these equations to simulate the system's response
      - Reformulate the equations (transfer function, state-space, etc.)
      - Numerical integration (Euler 1st order, Runge-Kutta, etc.)

Scripts to solve the ODEs (yesterday's lecture)

Simulink

Simulink



- What is Simulink?
  - A toolbox of MATLAB
  - A powerful tool to simulate systems
  - Graphical environment (block diagram system)

- Why Simulink?
  - Graphical environment (better visualisation/understanding)
  - Excellent integration with MATLAB (obviously)
  - Linear/non-linear systems

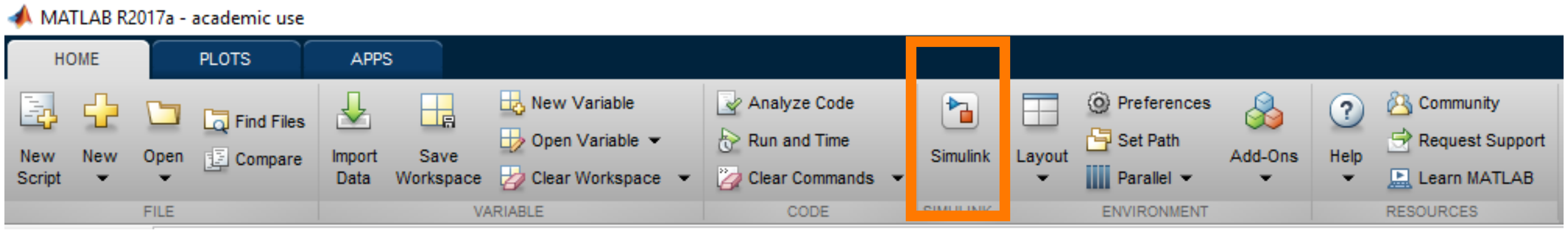
Simulink

- Content of this lecture
  - Implementation of different systems
  - Differential equations → Simulink block diagram

- What systems will be modelled today?
  - Pendulum system
  - Chemical reactor
  - RLC circuit
  - Mass-spring-damper system

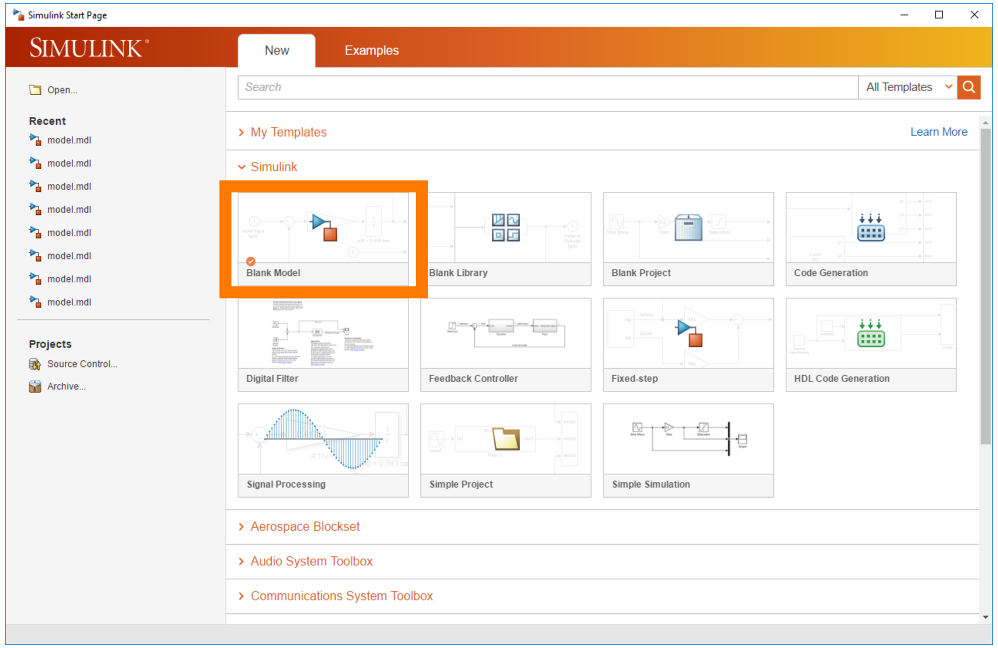
Simulink - Overview

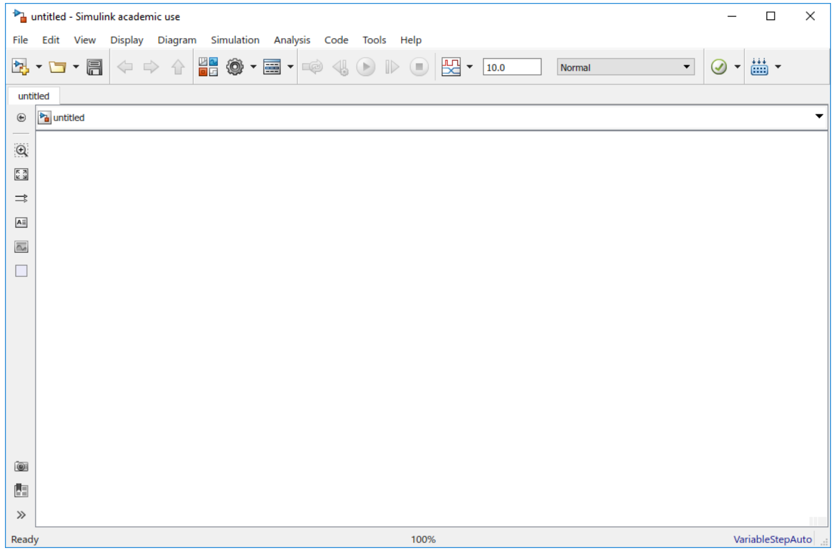
- How to start Simulink?
  - Start MATLAB
  - Type “Simulink” in the command window
  - Or select the Simulink button in the toolbar



Simulink - Overview

- How to create a new model?
  - Click on “Blank Model”





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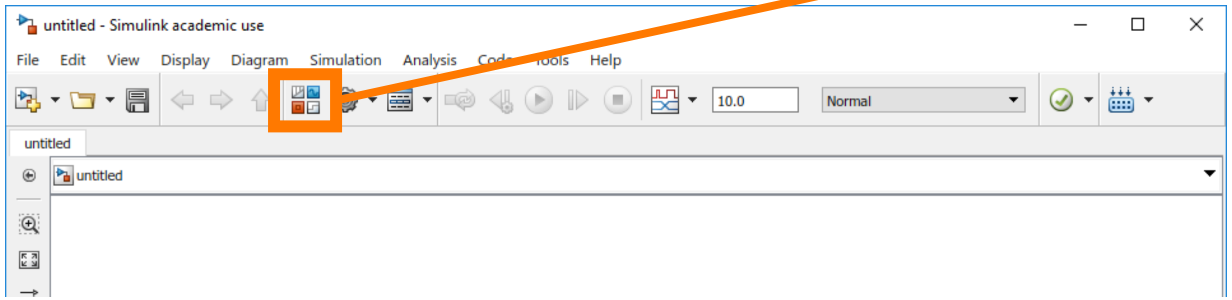
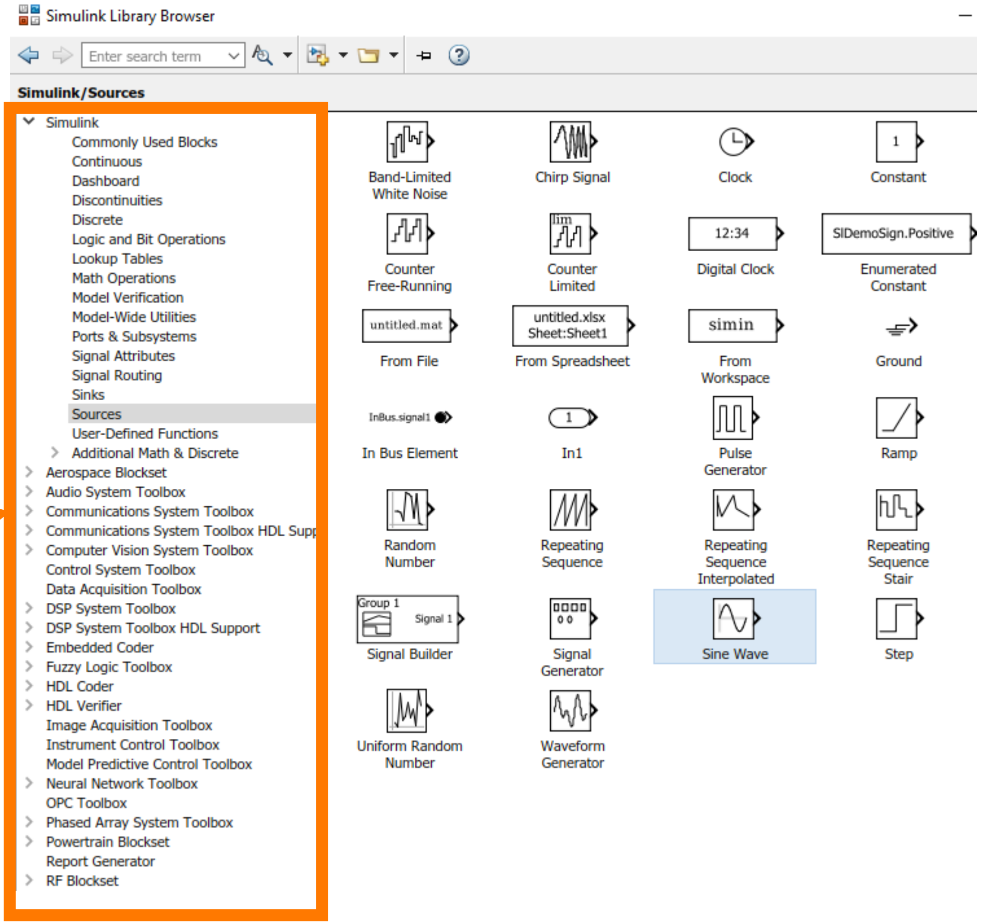
Simulink - Overview

- How to create a new system?
  - Open the Simulink Library Browser
  - Drag and drop the desired block into the workspace
  - Double-click the block to edit its properties





Modelling of a pendulum system

- Pendulum (motor + rigid bar attached to it)

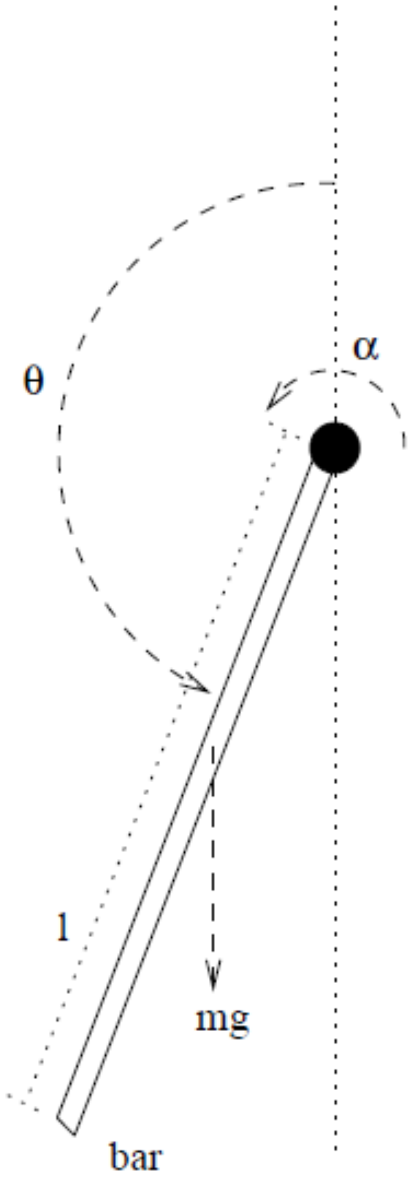
$$J\ddot{\theta} = mlg\sin(\theta) - (b + \frac{K^2}{R})\,\dot{\theta} + K_m\alpha$$

  – **First step:** Isolate the highest order
    - (already done)

  – **Second step:** Identify the blocks

$$J\ddot{\theta} = \boxed{mlg\sin(\theta)} - \boxed{(b + \frac{K^2}{R})\,\dot{\theta}} + \boxed{K_m\alpha}$$

  – **Last step:** Create the Simulink diagram



Modelling of a pendulum system

- What blocks are needed?
  - $J\ddot{\theta} = mlg\sin(\theta) - \left(b + \frac{K^2}{R}\right)\dot{\theta} + K_m\alpha$
  - Constant gain ( )
  - Mathematical function ( )
  - Integration/derivation ( )
  - Input ( )
  - Output ( )

- Simulink
  - Commonly Used Blocks
  - Continuous
  - Dashboard
  - Discontinuities
  - Discrete
  - Logic and Bit Operations
  - Lookup Tables
  - Math Operations
  - Matrix Operations
  - Messages & Events
  - Model Verification
  - Model-Wide Utilities
  - Ports & Subsystems
  - Signal Attributes
  - Signal Routing
  - Sinks
  - Sources
  - String
  - User-Defined Functions
  - Additional Math & Discrete
  - Quick Insert

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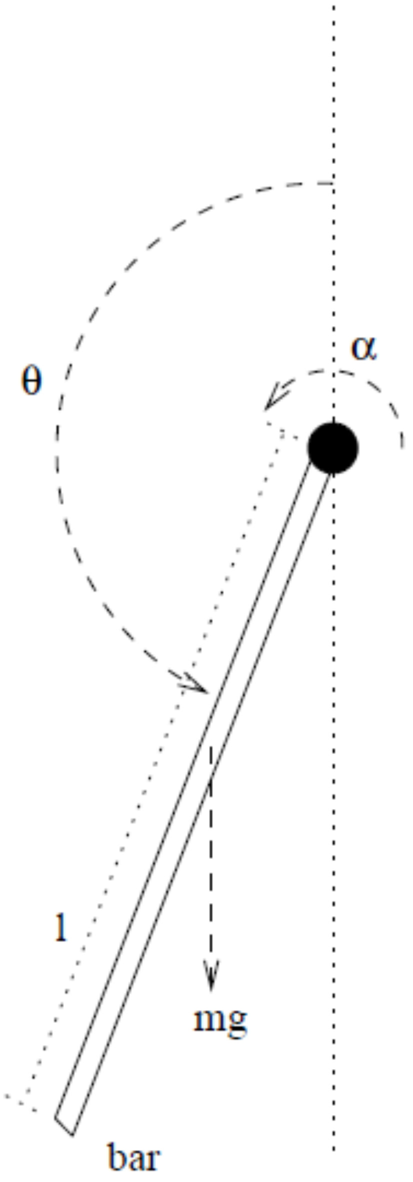
Modelling of a pendulum system

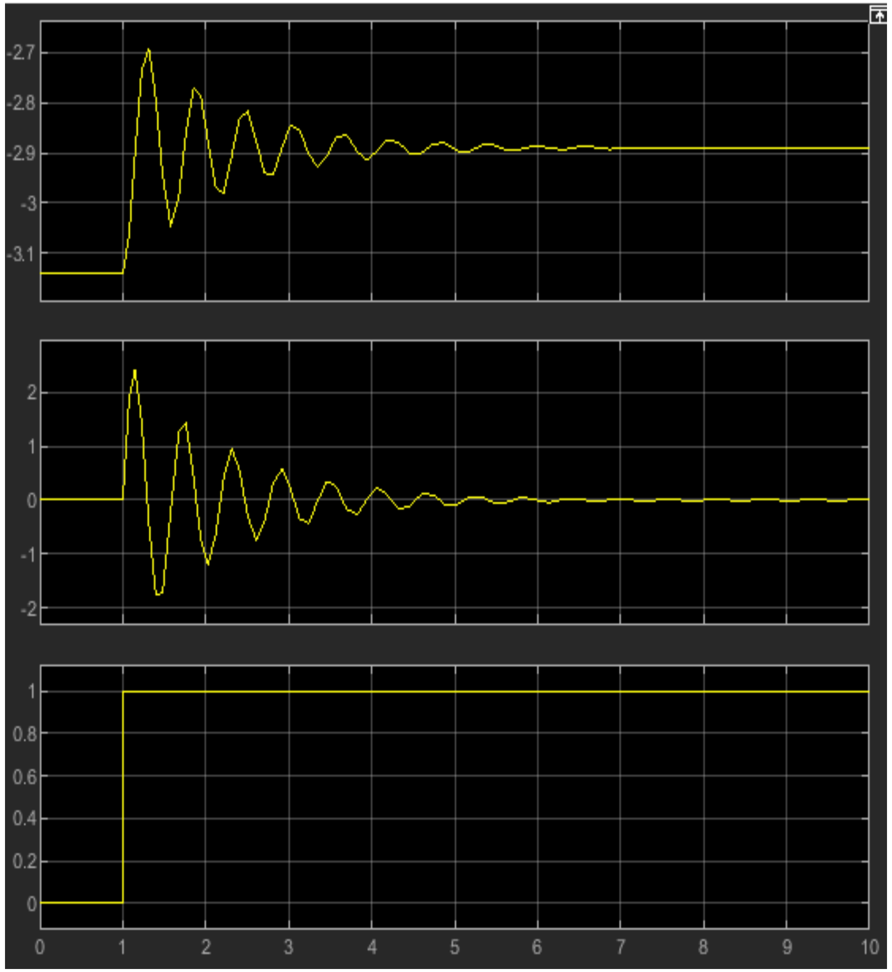
- System parameters:
  - $\alpha: 0 \rightarrow 1$ (Step input)
  - $m = 0.055$
  - $l = 0.042$
  - $g = 9.81$
  - $J = 1.9098 \times 10^{-4}$
  - $b = 3 \times 10^{-6}$
  - $K = 53.6 \times 10^{-3}$
  - $R = 9.50$
  - $K_m = {}^{K}/_{R}$
  - $\dot{\theta}(0) = 0$
  - $\theta(0) = \pi$ (Pointing down)



Modelling of a pendulum system

- Output of the simulation
  - Not smooth
  - Poorly accurate
- How to improve it?
  - Try a different solver
  - Decrease tolerances
  - Decrease a simulation step
  - A trade-off between accuracy and computation time!

(So, do not try to always get the perfect output, which is anyway an approximation)



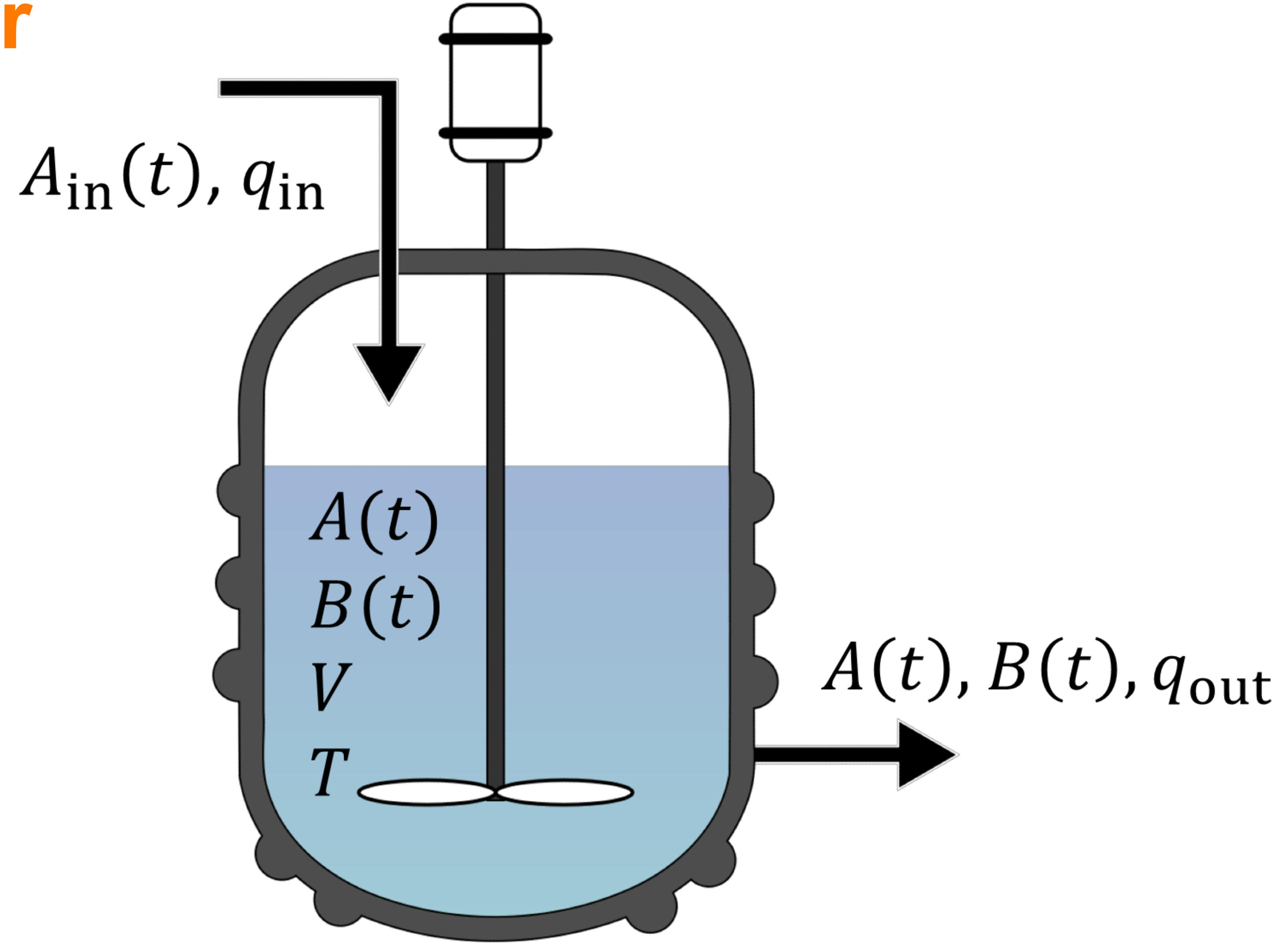
Modelling of a reactor

- Chemical reactor
  - Concertation rates of the reactant A and product B:

$$\begin{cases} V\dot{A} = -A(q + Vk) + A_{\text{in}}q \\ V\dot{B} = AVk - Bq \end{cases}$$

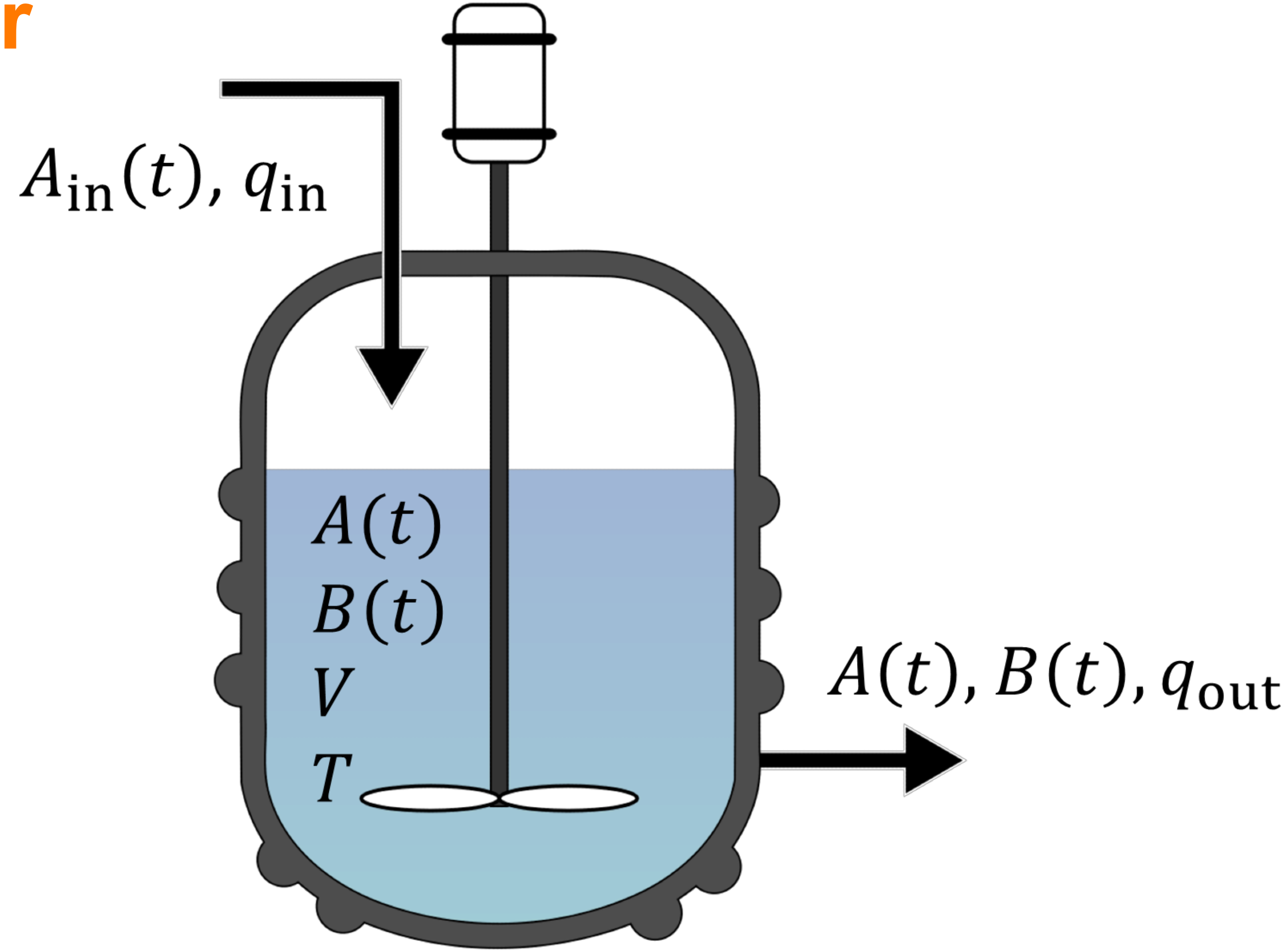
  - **Outputs:** A and B
  - **Input:**

$$A_{\text{in}} = \frac{1}{0.3\sqrt{\pi}} e^{-\left(\frac{t}{0.3}\right)^2}$$



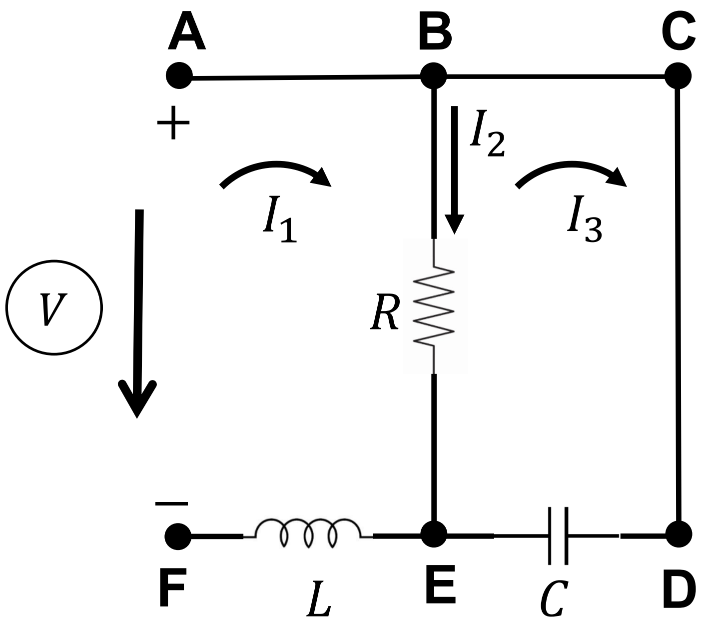
Modelling of a reactor

- System parameters:
  - $k = k_0 e^{-\frac{E_a}{RT}}$
  - $q_{\text{in}} = q_{\text{out}} = 0.25$
  - $V = 10$
  - $k_0 = 9.4$
  - $E_a = 2500$
  - $R = 831$
  - $T = 293$
  - $A(0) = 0$
  - $B(0) = 0$



Modelling of an RLC circuit

- Kirchhoff's laws:
  - **FABEF:** $V - I_2 R - L\frac{dI_1}{dt} = 0$
  - **BCDEB:** $-\frac{q}{C} + I_2 R = 0$
  - **Node B:** $I_1 = I_2 + I_3$



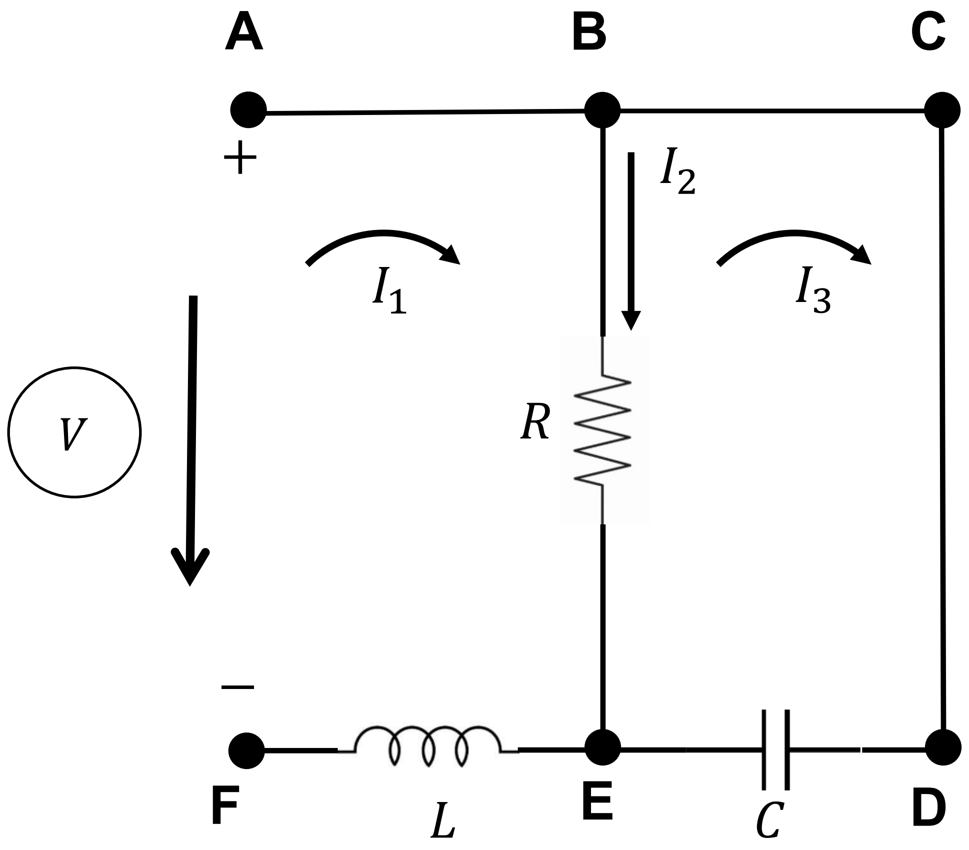
- **Outputs:** q and I_1; **Input:** V

- $$\begin{cases} V - \left(I_1 - \frac{dq}{dt}\right)R - L\frac{dI_1}{dt} = 0 \\ -\frac{q}{C} + \left(I_1 - \frac{dq}{dt}\right)R = 0 \\ I_3 = \frac{dq}{dt} \end{cases}$$

$$\boxed{\begin{cases} L\frac{dI_1}{dt} = V - I_1 R + R\frac{dq}{dt} \\ R\frac{dq}{dt} = -\frac{q}{C} + I_1 R \end{cases}}$$

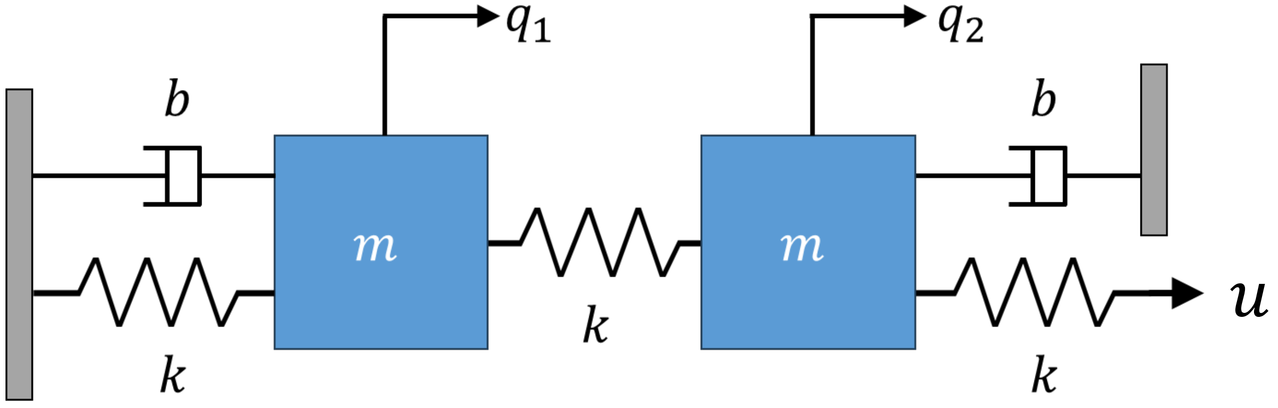
Modelling of an RLC circuit

- System parameters:
  - $V = 220 \cdot \sin(100 \cdot \pi \cdot t)$
  - $R = 1$
  - $L = 2$
  - $C = 1$
  - $I_1(0) = 0$
  - $q(0) = 0$



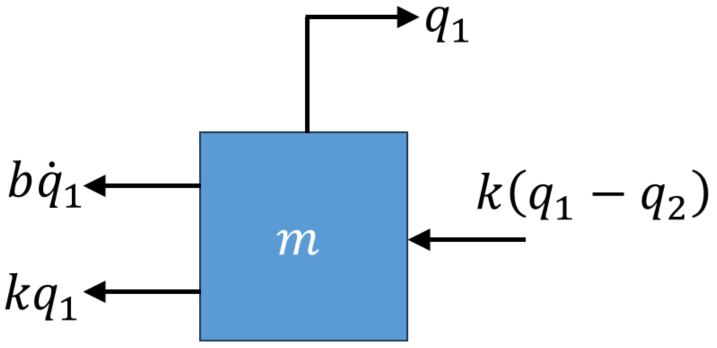
Modelling of a mass-spring-damper system

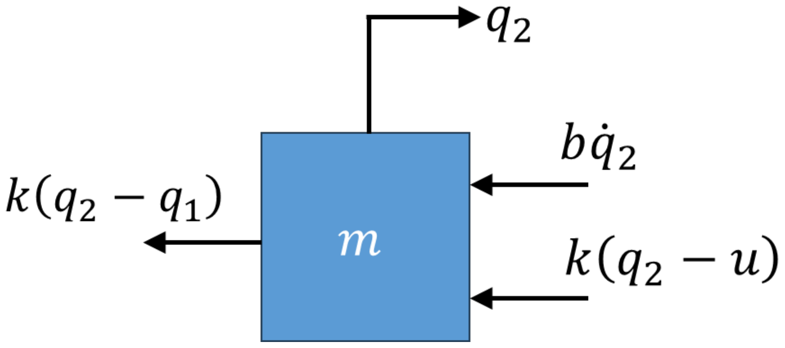
- **Input:** $u = A \sin \omega t$
- **Output:** q_2



- Dynamics of q_1 and q_2:

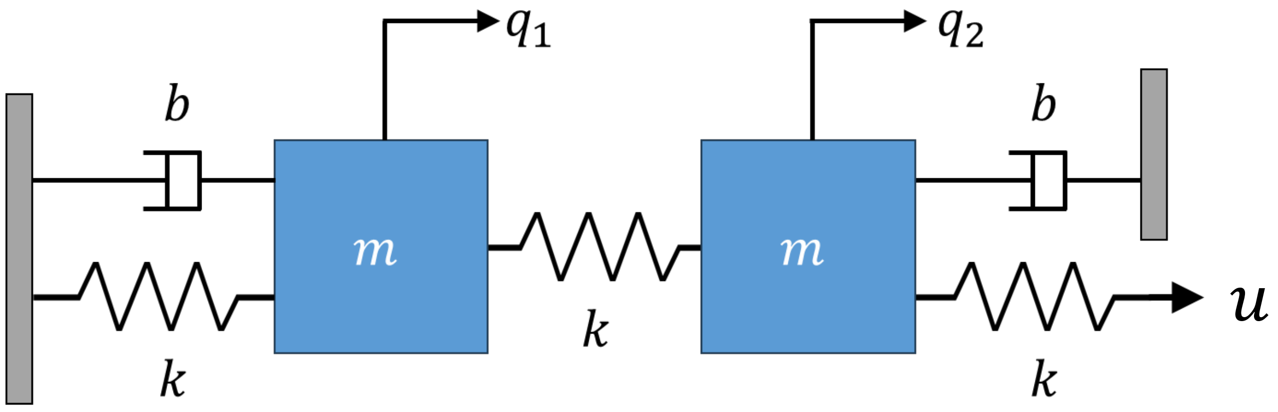
$$\begin{cases} m_1\ddot{q}_1 = -2kq_1 - b\dot{q}_1 + kq_2 \\ m_2\ddot{q}_2 = kq_1 - 2kq_2 - b\dot{q}_2 + ku \end{cases}$$





Modelling of a mass-spring-damper system

- System parameters:
  - $m_1 = m_2 = 250$
  - $A = 0.01$
  - $k = 50$
  - $b = 50$
  - $\omega = 1$
  - $q_1(0) = 0$
  - $q_2(0) = 0$
  - $\dot{q}_1(0) = 0$
  - $\dot{q}_2(0) = 0$



Questions?