Homework problems 1

The answers to the homework problems should be returned to the corresponding folder on the robotics course ELEC-C1320 MyCourses-platform. The answer files should be named as "solutions_homework problems 1_Firstname_Lastname.pdf". Deadline for returning the solutions is Thursday 5.10, 12:00 Noon. You can use Finnish, Swedish or English in your solutions.

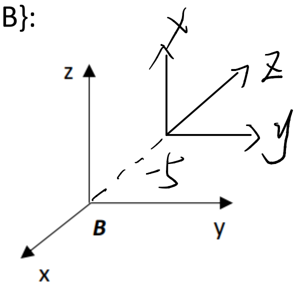
Note: Please, return <u>one</u> pdf-file for your solutions. <u>Mark clearly your name and student number on the answer sheet!</u>

1. The homogenous transformation matrix ***T*** describes the position and orientation of a new coordinate frame {N} with respect to the base frame {B}:

$\sin(-\frac{\pi}{2})$

$${}^{B}\boldsymbol{T}_N = \begin{bmatrix} {}^{B}\boldsymbol{R}_N & \boldsymbol{t} \\ 0_{1x3} & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$-\sin(-\frac{\pi}{2})$



a) Illustrate the new coordinate frame {N} in relation to the base frame {B} (i.e the position of the origin and the directions of the coordinate axes of the new frame {N} in relation to the base frame {B} shown in the drawing above). (4 points)

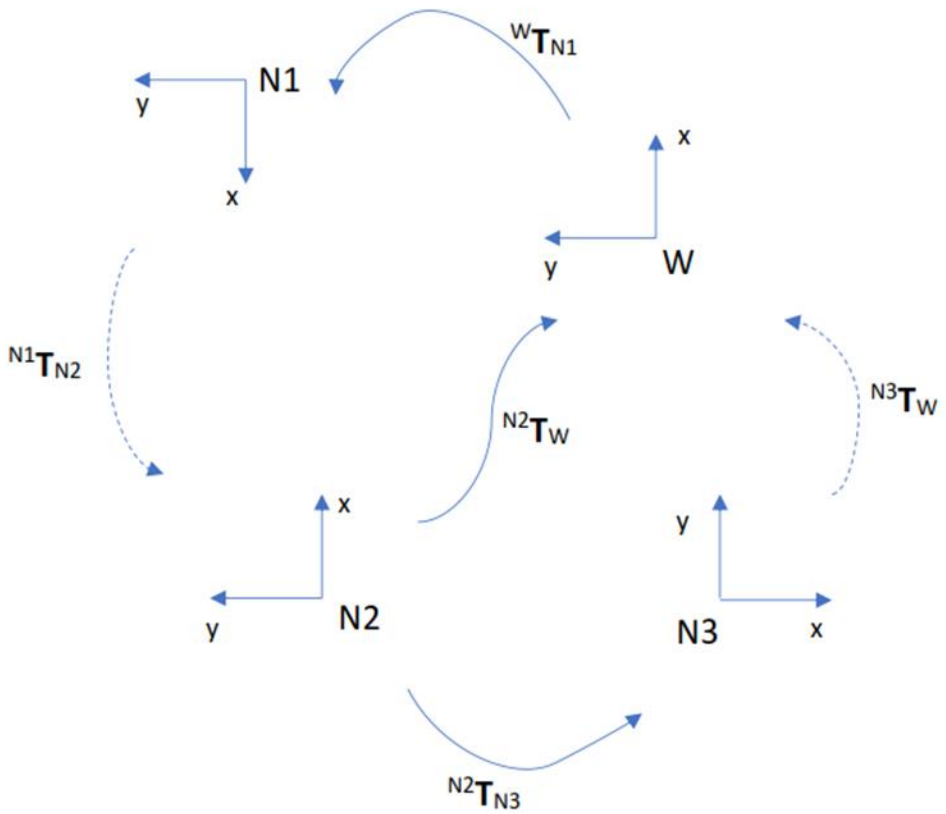
b) Does ***T*** describe <u>a right-handed coordinate frame</u>? If it does or does not, explain how did you come to the conclusion? (2 points)

rotation matrix is $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

The determinant of it is 1

so it describe a right-handed coordinate frame.

2. The task is to solve the unknown relative locations of coordinate frames, marked with the dashed arrow lines, by utilizing the known relative locations marked with the solid arrow lines. The setup is illustrated in the figure below.



a) Determine first, on the matrix symbol level, the equations for the unknown transformation matrices $^{N1}\mathbf{T}_{N2}$ and $^{N3}\mathbf{T}_{W}$. (*7 points*)

$^{N_1}T_{N_2} = (^{N_2}T_W \, ^WT_{N_1})^{-1}$ $\quad$ $^{N_3}T_W = (^{N_2}T_{N_3})^{-1} \, ^{N_2}T_W$

b) Thereafter calculate the numerical forms of the 4x4 homogenous transformation matrices $^{N1}\mathbf{T}_{N2}$ and $^{N3}\mathbf{T}_{W}$. (*7 points*)

$$^{N_1}T_{N_2} = \left(\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1}$$

The known transformation matrices are:

$$^{W}\mathbf{T}_{N1} = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \left(\begin{bmatrix} -1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1}$$

$$^{N2}\mathbf{T}_{W} = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 & 9 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
{}^{N2}\mathbf{T}_{N3} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Hint: in order to calculate the inverse of a numerical form homogenous transformation matrix, we must use the equation (2.25) of Corke's text book:

$$
\boldsymbol{T}^{-1} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{R}^T & -\boldsymbol{R}^T\boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \qquad (2.25)
$$

$$
{}^{N_3}T_W = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 1 & -6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0 & -1 & 0 & -7 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

3. 3D-frame {B} is located initially coincident with the frame {A}. We first translate the origin of frame {B} **3 units** in the direction of its **Y**-axis. Then we rotate the translated frame {B} about its **X**-axis by **90 degrees**. And finally, we translate the origin of the rotated frame {B} **7 units** in the direction of its **Z**-axis.

a) Give the 4x4 homogenous transformation matrix that describes the position and orientation of frame {B} with respect to frame {A}. (5 points)

b) The coordinates of a target point with respect to frame {B} are ^{B}**P1**= [x=0,y=9,z=0]. What are the coordinates of the target point given with respect to frame {A} ? (5 points)

c) The coordinates of another target point, given with respect to frame {A}, are ^{A}**P2**= [x=0,y=9,z=0]. What are the coordinates of this target point given with respect to frame {B} ? (5 points)

a) $trans(X=0, Y=3, Z=0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad rot\_X(90°) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$trans(X=0, Y=0, Z=7) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$
{}^{A}T_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

b) $A_{P_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 9 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ 9 \\ 1 \end{bmatrix}$

c) $B_{P_2} = B_{P_1} (A_{P_1})^{-1} A_{P_2} = (A_{T_B})^{-1} A_{P_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 9 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -13 \\ 1 \end{bmatrix}$

4. We are supposed to create a new coordinate frame {B} by rotating **30** degrees around the **x**-axis of the original coordinate frame {A}.

Instead of using the "rotations about principal axis-formulas of slide 15, p. 34 Corke's text book" use the ***Rodrigues' rotation formula*** (slide 33, p. 42-43 of Corke's text book) to form the corresponding 3x3 rotation matrix ${}^{A}\boldsymbol{R}_{B}$. (*5 points*)

取 $V_x = (1, 0, 0)^T$

$$AR_B = I_{3\times3} + \sin\theta [\hat{v}]_\times + (1-\cos\theta)[\hat{v}]_\times^2$$

$$= I_{3\times3} + \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} + \frac{2-\sqrt{3}}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

5. **a)** For the same **30** degrees rotation around the **x**-axis as in problem 4, create the corresponding unit quaternion by applying the formula (2.22). (compare slide 36 and pp. 44-45 of Corke's text book). Check if you get the same result with the Robotics toolbox (by using the UnitQuaternion constructor of the Corke's Robotics toolbox) (*5 points*)

b) The task is to compound two consecutive rotations expressed with UnitQuaternions. You are supposed to compound the two rotations by using the matrix-vector multiplication formula (see p. 44 of the text book or lecture slide 37).

Use the UnitQuaternion, constructed in part **a)** and compound it with itself to create a rotation of **60°** around the **x**-axis of the original frame.

Check with the UnitQuaternion constructor of Corke's Matlab robotics toolbox if you get the same result. (*5 points*)

*Hint: If you face problems with the toolbox, try using the commands **R**= rotx(angle), UnitQuaternion(**R**) to check your result with the toolbox. Other ways of creating the same matrix **R** may not, for some unknown reasons, necessarily give the correct result with the UnitQuaternion(**R**) command.*

a) $q = [w, x, y, z]$ $\theta = \frac{\pi}{6}$

$q = [\cos(\frac{\theta}{2}), \sin(\frac{\theta}{2}), 0, 0]$

$= [0.9659, 0.2588, 0, 0]$

```
q = UnitQuaternion.Rx(pi/6);
display(q);
```

```
q =

0.96593 < 0.25882, 0, 0 >
```

b) $q_1 = q_2$

$q_1 \oplus q_2 = \begin{bmatrix} 2\cos^2(\frac{\pi}{12}) - 1 \\ 2\sin(\frac{\pi}{12})\cos(\frac{\pi}{12}) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.8660 \\ 0.5 \\ 0 \\ 0 \end{bmatrix}$

```
q = UnitQuaternion.Rx(pi/6);
q = q*q;
display(q);
```

```
q =

0.86603 < 0.5, 0, 0 >
```