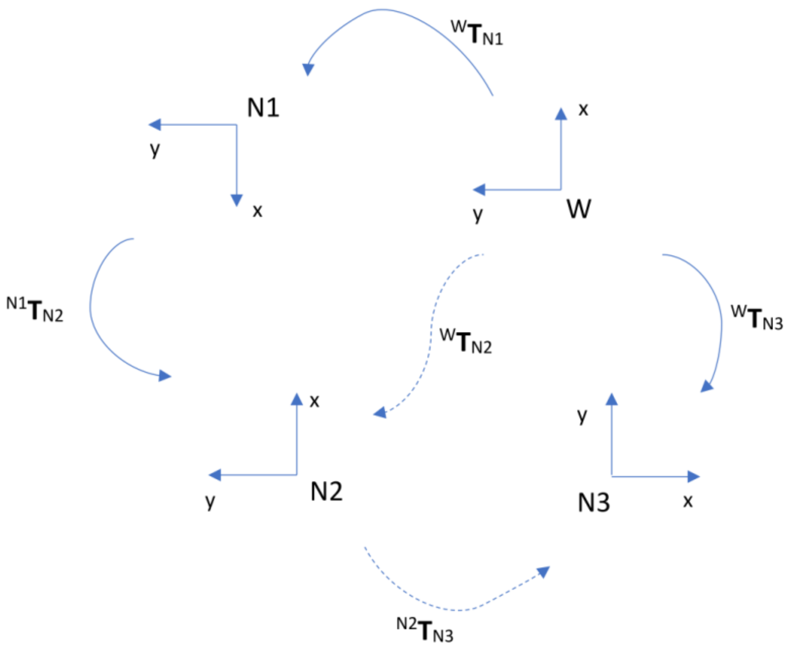
ELEC-C1320/ELEC-D1320 – Robotiikka, Exam 10.12.2019 (3 hours)

It is allowed to use a calculator in the exam.

You can use Finnish, English or Swedish in your solutions. Tehtävänannot on esitetty suomeksi sinisellä värillä. The problem definitions are given in Finnish in blue color.

1. The task is to solve, on the matrix symbol level, the unknown relative coordinate transformations ${}^{W}\mathbf{T}_{N2}$ and ${}^{N2}\mathbf{T}_{N3}$, marked with the dashed arrow lines in the figure, by utilizing the known relative coordinate transformations marked with the solid arrow lines. Tehtävänä on ratkaista matriisisymbolitasolla kuvassa katkoviivalla merkityt tuntemattomat suhteelliset koordinaatistomuunnokset, ${}^{W}\mathbf{T}_{N2}$ and ${}^{N2}\mathbf{T}_{N3}$, tunnettujen koordinaatistomuunnosten (merkitty kiinteällä nuoliviivalla) avulla. (5 points)



Solution:

Based on the figure above, we can directly form the equation for the unknown coordinate transformation ${}^{W}\mathbf{T}_{N2}$

$${}^{W}\mathbf{T}_{N2} = {}^{W}\mathbf{T}_{N1}\,{}^{N1}\mathbf{T}_{N2} \qquad (1)$$

To solve for the unknown coordinate transformation ${}^{N2}\mathbf{T}_{N3}$ we first form the equation

$${}^{W}\mathbf{T}_{N3} = {}^{W}\mathbf{T}_{N2}\,{}^{N2}\mathbf{T}_{N3}$$

and then assign the eq. of ${}^{W}\mathbf{T}_{N2}$ into it yielding

${}^{W}\mathbf{T}_{N3}= {}^{W}\mathbf{T}_{N1}\,{}^{N1}\mathbf{T}_{N2}\,{}^{N2}\mathbf{T}_{N3}$ (actually, we could have formed this eq. directly from the figure)

by multiplying the equation <u>from the left</u> with the inverse of ${}^{W}\mathbf{T}_{N1}\,{}^{N1}\mathbf{T}_{N2}$ we can separate the unknown transformation ${}^{N2}\mathbf{T}_{N3}$ on one side of the equation

$$({}^{N1}\mathbf{T}_{N2})^{-1}\,({}^{W}\mathbf{T}_{N1})^{-1}\,{}^{W}\mathbf{T}_{N3}= {}^{N2}\mathbf{T}_{N3} \qquad (2)$$

we could also replace the inverses of the original transformations with the "basic forms" of the corresponding matrices to get

$${}^{N2}\mathbf{T}_{N3} = {}^{N2}\mathbf{T}_{N1}\,{}^{N1}\mathbf{T}_{W}\,{}^{W}\mathbf{T}_{N3}$$

2. 3D-frame {B} is located initially coincident with the frame {A}. We first translate the origin of frame {B} 5 units in the direction of its x-axis. Then we translate the translated frame {B} 3 units in the direction of its z-axis. And finally we rotate the translated frame {B} about its y-axis by 90 degrees. 3D-koordinaatisto {B} on aluksi samassa paikassa ja asennossa koordinaatiston {A} kanssa. Koordinaatiston {B} asemaa muutetaan aluksi siirtämällä koordinaatiston {B} origon paikkaa 5 yksikköä oman x-akselinsa suuntaan. Tämän jälkeen siirretyn koordinaatiston {B} asemaa muutetaan siirtämällä koordinaatiston {B} origon paikkaa 3 yksikköä oman z-akselinsa suuntaan. Lopuksi siirretyn koordinaatiston {B} asemaa muutetaan kiertämällä sitä oman y-akselinsa ympäri 90 astetta.

a) Give the 4x4 homogenous transformation matrix which describes the position and orientation of frame {B} with respect to frame {A}. Määritä 4x4 homogeeninen muunnosmatriisi, joka kuvaa koordinaatiston {B} paikkaa ja asentoa koordinaatiston {A} suhteen. (7 points)

b) The coordinates of a point **P** with respect to frame {B} be are [x=0, y=0, z=9]. What are the coordinates of point **P** given with respect to frame {A}? Pisteen **P** koordinaatit koordinaatiston {B} suhteen ovat [x=0,y=0,z=9]. Mitkä ovat pisteen **P** koordinaatit koordinaatiston {A} suhteen? (7 points)

Solution:

a) We first form two transformation matrices one for the two consecutive translations and one for the rotation.

$$\text{transl(x=5,y=0,z=3)}=\begin{bmatrix}1 & 0 & 0 & 5\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 3\\ 0 & 0 & 0 & 1\end{bmatrix},$$

rot_y(90°)= $\begin{bmatrix} \cos(90) & 0 & \sin(90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(90) & 0 & \cos(90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Then we form the matrix equation by placing the matrices of individual transformations, in the correct order, in the equation (starting from the left with the first transformation):

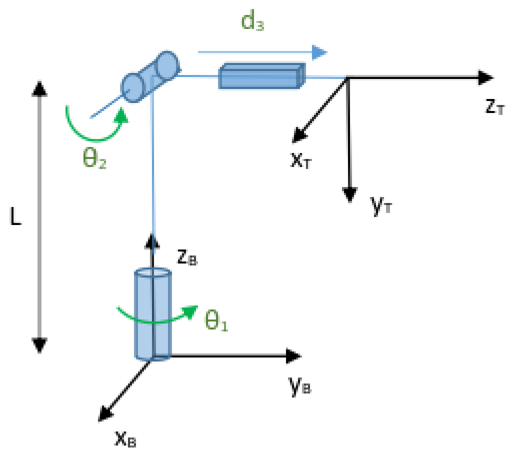
$$^{A}\boldsymbol{T}_{B}=\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) And now we can calculate the coordinates of point **P** w.r.t. frame {A}:

$$^{A}\mathbf{P}= {}^{A}\boldsymbol{T}_{B}*{}^{B}\mathbf{P}= \begin{bmatrix} 0 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

3. In the figure a 3-axes RRP-type manipulator is shown. <u>When the rotational joint angles, θ_1 and θ_2, have a value zero, the upper arm is oriented horizontally above the Y_B axis</u>. Kuvassa on esitetty 3-akselisen RRP-manipulaattorin kinemaattinen rakenne. <u>Kun kiertonivelten, θ_1 ja θ_2, nivelohjauskulmat ovat nollia, mekanismin ylempi varsi on vaakasuorassa asennossa Y_B-akselin yläpuolella.</u>

Solve the forward kinematics problem of the manipulator to describe the tool frame {T} with respect to the robot base frame {B}. In other words, <u>assign the link frames in the figure and provide the corresponding DenavitHartenberg-parameters in a table as well as the Base and Tool transformation matrices</u>. It is your choice to use either the Standard or Modified DH-parameter convention. Ratkaise manipulaattorin suora kinemaattinen muunnos, joka kuvaa työkalukoordinaatiston {T} paikkaa ja asentoa robotin peruskoordinaatiston {B} suhteen. <u>Toisin sanoen, merkitse kuvaan mekanismin nivel-/varsikoordinaatistot sekä esitä vastaavat DenavitHartenberg-parametrit taulukossa, anna myös tarvittavat perusmuunnos- ja työkalumuunnosmatriisit.</u> Voit vapaasti valita kumpaa DH-parametriesitystä käytät ratkaisussasi, eli vaihtoehtoina ovat "Standard"- tai "Modified"-parametrointitavat. (18 points)



Solution:

a) **Standard DH**-convention

Here base transformation is used to cover the height of the base of the robot, “L”, to move from frame-B to frame 0.

$$Base = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we start propagating from the 0-frame towards the tool frame T, link by link, by applying the **standard** Denavith-Hartenberg (DH) parameters. The description of the parameters is given in table 7.1, p. 197 of Corke’s text book. The parameters for the links/joints 1, 2 and 3 given in a table are

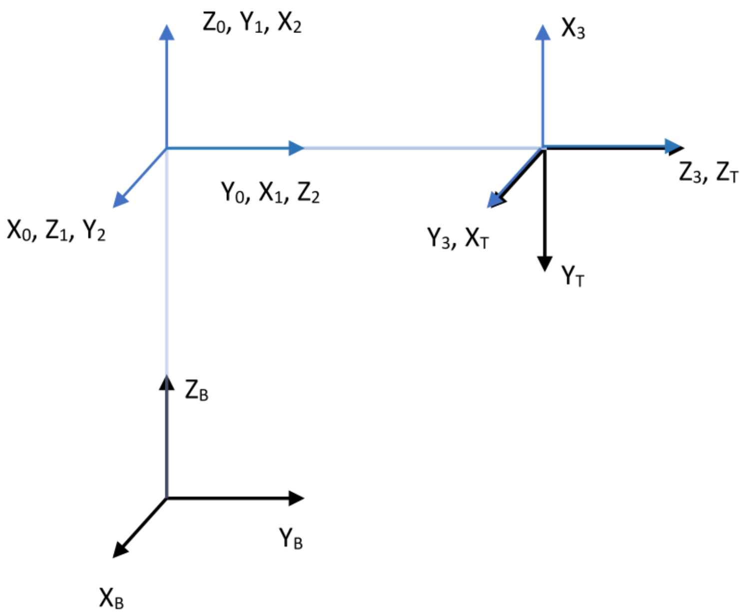
Link	Θ_i	d_i	a_i	α_i	σ_i
1	Θ_1+90	0	0	90	R
2	Θ_2+90	0	0	90	R
3	0	d_3	0	0	P

(Alternatively, the 90˚ offset added to Θ_1 in the table could have already been embedded to the base transformation matrix in which case it would not show up on the first line of the DH-parameter table. Consequently, the orientations of the 0-frame and 1-frame in the figure would also change correspondingly.)

With the tool transformation we need to describe the difference between the orientation of frame-3 and the tool frame T. In other words, the tool transformation describes the orientation of frame T in terms of the directions of the axis of frame 3. (*compare the "blue text box" on p.35 of Corke's text book*)

$$Tool = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And finally the link frames, acquired by applying each line of parameters from the DH-parameter table, are drawn into a figure:



b) Modified DH-convention

Also here base transformation is used to cover the height of the base of the robot, "L", to move from frame-B to frame 0.

$$Base = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

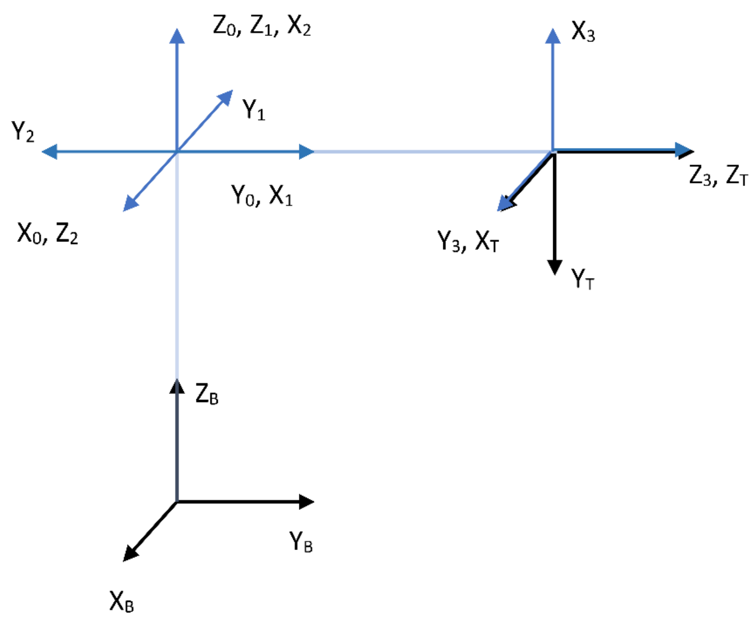
Now we start propagating from the 0-frame towards the tool frame T, link by link, by applying the **modified** Denavith-Hartenberg (DH) parameters. Compare equation (7.8) and figure 7.15, p. 219 of Corke's text book. The parameters for the links/joints 1, 2 and 3 given in a table are

Link	α_{i-1}	a_{i-1}	d_i	Θ_i	σ_i
1	0	0	0	Θ_1+90	R
2	90	0	0	Θ_2+90	R
3	90	0	d_3	0	P

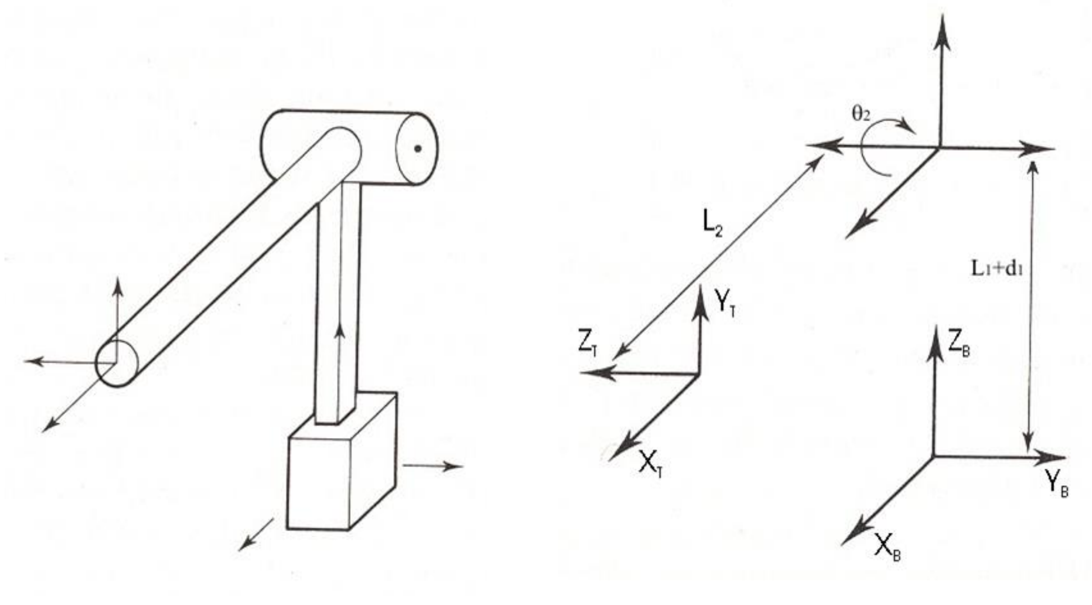
The position and orientation of frame-3 is the same as in part **a)** so the required tool transformation is the same

$$Tool = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And finally the link frames, acquired by applying each line of parameters from the DH-parameter table, are drawn into a figure:



4. Solve the inverse kinematics problem for the two degree of freedom PR manipulator shown in the figure. The first degree-of-freedom (dof) is prismatic (control of the height of the upper arm with respect to the horizontal plane, $L+d_1$) and the second dof is rotational (control of the angle of the upper arm with respect to the horizontal plane, θ_2). When the rotational joint is given a zero value, the upper arm is oriented horizontally above the X_B-axis. Also, the direction of positive rotation is marked in the figure.
Muodosta alla olevassa kuvassa esitetyn, kahden vapausasteen PR-robottimekanismin käänteinen kinemaattinen muunnos. Ensimmäinen liikevapausaste on prismaattinen (yläkäsivarren alkupisteen etäisyyden ohjaus robotin vaakasuoran kiinnitysalustan suhteen, $L+d_1$) ja toinen liikevapausaste on kiertyvä nivel (yläkäsivarren kierto vaakatason suhteen, θ_2). Kiertokulman arvolla nolla yläkäsivarsi on vaakatasossa X_B-akselin yläpuolella. Myös kiertokulman positiivinen kiertosuunta on merkitty kuvaan. (13 points)



Solution:

First solve θ_2 by means of the x-coordinate of the origin of the T-frame:

$x = L_2\,cos(\theta_2)$ => $cos(\theta_2) = \frac{x}{L_2}$ and further we know that $sin(\theta_2) = \pm\sqrt{1 - cos(\theta_2)^2}$

then $\theta_2 = atan2(sin(\theta_2), cos(\theta_2))$.

So a given X-coordinate can be reached with two different values of θ_2 (except for the two special cases when the upper arm is pointing horizontally or vertically).

One of the two values of θ_2 can be selected for example based on our preference of the orientation of the upper arm, select positive θ_2 if you prefer to have the upper arm inclined upwards or negative θ_2 if you prefer having the upper arm pointing in a downwards direction.

After selecting θ_2 we can calculate a value for d_1:

$z = L_1 + d_1 + L_2*\sin(\theta_2)$ => $d_1 = z - L_1 - L_2*\sin(\theta_2)$

5. In the figure below a 3-axes RPP manipulator mechanism is illustrated. When the angle of the first joint, θ_1, is zero the upper arm is oriented parallel to the y_B-axis. An external force **F** is exerted at the origin of the tool frame {T}. The force is marked with the red arrow in the figure. Alla olevassa kuvassa on esitetty 3-akselisen RPP robottimekanismin kinemaattinen rakenne. Kun ykkösnivelen ohjauskulma, θ_1, saa arvon nolla yläkäsivarsi asemoituu y_B-koordinaatiakselin yläpuolelle sen suuntaisesti. Ulkoinen voima, jota merkitään symbolilla **F,** kohdistetaan työkalukoordinaatiston {T} origoon. Ulkoista voimaa kuvaa punainen nuoli kuvassa.

The 3x3 Jacobian matrix for the linear velocity of the tool frame expressed with respect to the base frame {B} as a function of the joint velocities is / Työkalukoordinaatiston lineaarinopeutta peruskoordinaatiston {B} akselien suunnissa nivelnopeuksien funktiona kuvaa 3x3 jakobiaanimatriisi

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \boldsymbol{J}\dot{\boldsymbol{q}} = \begin{bmatrix}\frac{dx}{d\theta_1} & \frac{dx}{dd_2} & \frac{dx}{dd_3}\\ \frac{dy}{d\theta_1} & \frac{dy}{dd_2} & \frac{dy}{dd_3}\\ \frac{dz}{d\theta_1} & \frac{dz}{dd_2} & \frac{dz}{dd_3}\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{d}_2\\ \dot{d}_3\end{bmatrix} = \begin{bmatrix}-\cos(\theta_1)\,d_3 & 0 & -\sin(\theta_1)\\ -\sin(\theta_1)\,d_3 & 0 & \cos(\theta_1)\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{d}_2\\ \dot{d}_3\end{bmatrix}$$

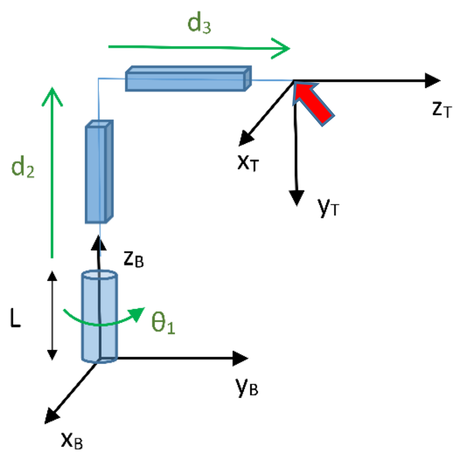
The value of the external force vector **F** is / Ulkoisen voimavektorin **F** arvo on

$$^{\boldsymbol{B}}\boldsymbol{F} = \begin{bmatrix}-5N\\ -5N\\ 3N\end{bmatrix}$$

The values of the joint variables for the calculations are / Nivelohjauksien arvot laskentaa varten ovat: $\theta_1 = 0.0°,\ d_2 = 0.5m,\ d_3 = 0.7m$

The value for the constant base height of the mechanism is L=0.3m / Mekanismin rungon vakiokorkeusmitta L=0.3m

The task is to calculate torques and forces affecting joints 1, 2 and 3 due to the external force in the given configuration of the manipulator arm. To solve the problem **you must utilize the Jacobian matrix** of the manipulator. Tehtävänä on laskea ulkoisen voiman vaikutuksesta syntyvät mekanismin nivelmomentit ja –voimat niveille 1, 2, ja 3. **Tehtävä on ratkaistava mekanismin Jakobiaani-matriisin avulla.** (10 points)



Solution:

First calculate the transpose of the Jacobian matrix

$$\boldsymbol{J}^T = \begin{bmatrix} -\cos(\theta_1)\, d_3 & -\sin(\theta_1)\, d_3 & 0 \\ 0 & 0 & 1 \\ -\sin(\theta_1) & \cos(\theta_1) & 0 \end{bmatrix}$$

and the Jacobian transpose with numerical parameter values becomes

$$\boldsymbol{J}^T = \begin{bmatrix} -\cos(0.0) * 0.7m & -\sin(0.0) * 0.7m & 0 \\ 0 & 0 & 1 \\ -\sin(0.0) & \cos(0.0) & 0 \end{bmatrix} = \begin{bmatrix} -0.7m & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Now we can calculate the joint torques/forces caused by the external force (wrench)

$$\boldsymbol{Q} = \begin{bmatrix} -0.7m & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -5N \\ -5N \\ 3N \end{bmatrix} = \begin{bmatrix} 3.5Nm \\ 3N \\ -5N \end{bmatrix}$$

ELEC-C1320 Robotiikka - Equations

Link parameters and the corresponding elementary transformations as well as the symbolic form of the link matrix according to the **standard** Denavit and Hartenberg parameter convention:

$$^{j-1}\xi_j\left(\theta_j, d_j, a_j, \alpha_j\right) = \mathscr{R}_z\left(\theta_j\right) \oplus \mathscr{T}_z\left(d_j\right) \oplus \mathscr{T}_x\left(a_j\right) \oplus \mathscr{R}_x\left(\alpha_j\right)$$

$$^{j-1}\boldsymbol{A}_j = \begin{pmatrix} \cos\theta_j & -\sin\theta_j \cos\alpha_j & \sin\theta_j \sin\alpha_j & a_j \cos\theta_j \\ \sin\theta_j & \cos\theta_j \cos\alpha_j & -\cos\theta_j \sin\alpha_j & a_j \sin\theta_i \\ 0 & \sin\alpha_j & \cos\alpha_j & d_j \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Link parameters and the corresponding elementary transformations as well as the symbolic form of the link matrix according to the **modified** Denavit and Hartenberg parameter convention:

$$^{j-1}\xi_j\left(\alpha_{j-1}, a_{j-1}, d_j, \theta_j\right) = \mathscr{R}_x\left(\alpha_{j-1}\right) \oplus \mathscr{T}_x\left(a_{j-1}\right) \oplus \mathscr{T}_z\left(d_j\right) \oplus \mathscr{R}_z\left(\theta_j\right)$$

$$
{}^{j-1}\boldsymbol{A}_j = \begin{bmatrix} \cos\theta_j & -\sin\theta_j & 0 & a_{j-1} \\ \sin\theta_j \cos\alpha_{j-1} & \cos\theta_j \cos\alpha_{j-1} & -\sin\alpha_{j-1} & -\sin\alpha_{j-1}\, d_j \\ \sin\theta_j \sin\alpha_{j-1} & \cos\theta_j \sin\alpha_{j-1} & \cos\alpha_{j-1} & \cos\alpha_{j-1}\, d_j \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Elementary rotation transformations (i.e. rotations about principal axis by Θ):

$$
R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}
$$

$$
R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}
$$

$$
R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

Inverse of a 4x4 transformation matrix:

$$
\boldsymbol{T}^{-1} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{R}^T & -\boldsymbol{R}^T\boldsymbol{t} \\ \boldsymbol{0}_{1\times 3} & 1 \end{pmatrix} \tag{2.25}
$$

Derivation of trigonometric functions:

Dsinx = cosx

Dcosx = - sinx

Definition of (manipulator) Jacobian matrix:

If $\boldsymbol{y} = F(\boldsymbol{x})$ and $\boldsymbol{x} \in \mathbb{R}^n$ and $\boldsymbol{y} \in \mathbb{R}^m$ then the Jacobian is the $m \times n$ matrix

$$
J = \frac{\partial F}{\partial \boldsymbol{x}} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}
$$

Jacobian transpose transforms a wrench (a vector of forces and torques) applied at the end-effector, ${}^{0}\boldsymbol{W}$, to torques and forces experienced at the joints $\boldsymbol{Q}$:

$$\boldsymbol{Q} = {}^{0}\boldsymbol{J}(\boldsymbol{q})^{T}\,{}^{0}\boldsymbol{W} \tag{8.9}$$