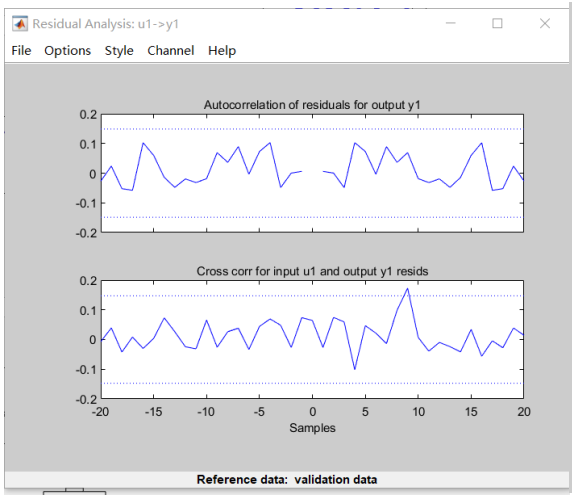
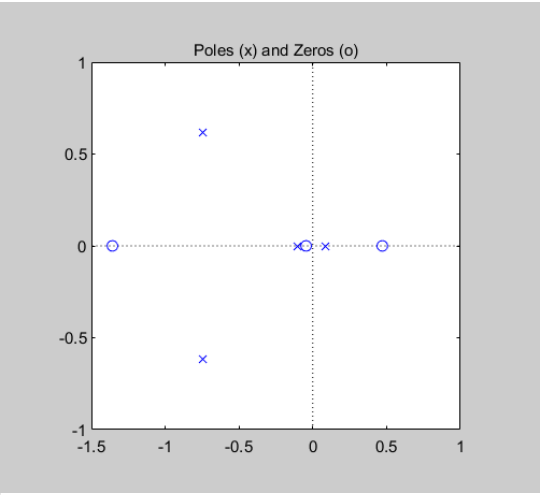
System1

```
>> data1 = iddata(y1,u1,1);
nk1 = delayest(data1);
disp(nk1);
     1
```

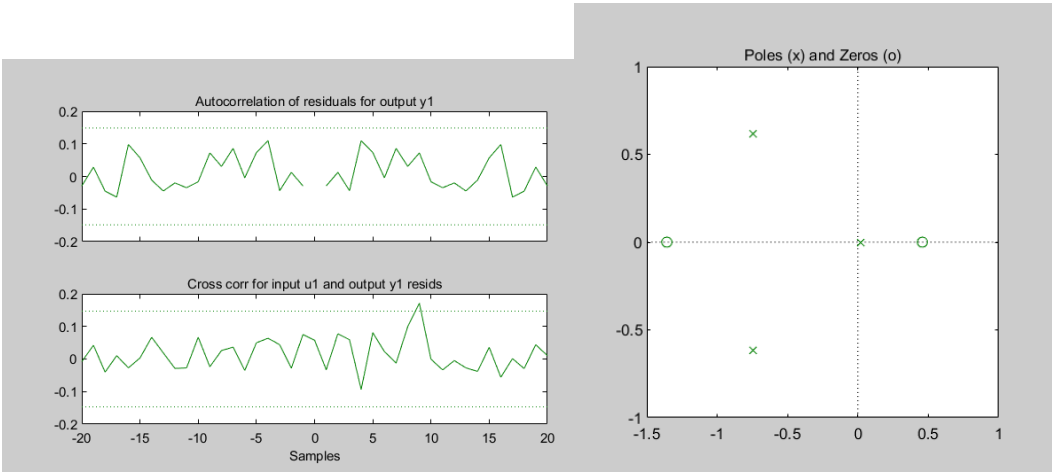
Begin with ARX441





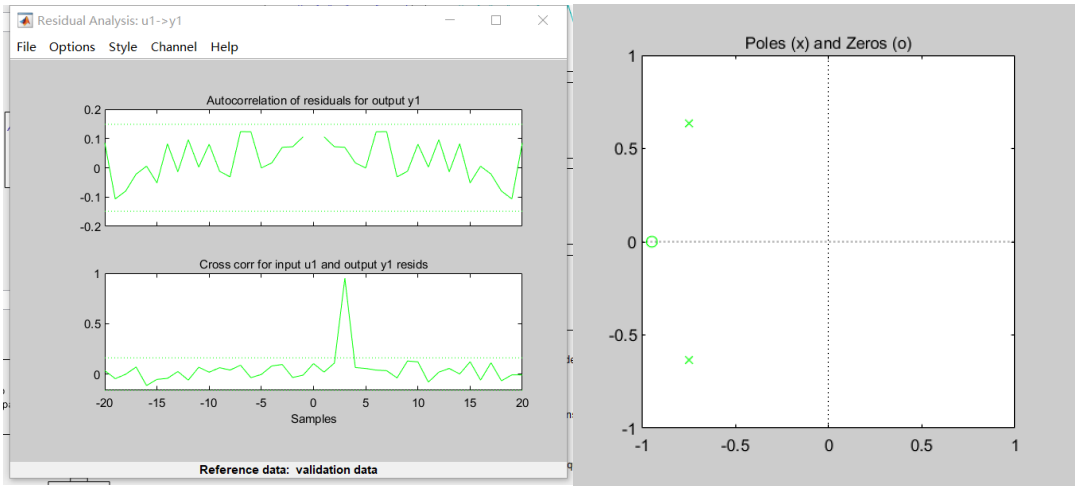

```
arx441 =
Discrete-time ARX model: A(z)y(t) = B(z)u(t) + e(t)
  A(z) = 1 + 1.505 (+/- 0.03821) z^-1 + 0.9463 (+/- 0.0569) z^-2 - 0.001237 (+/- 0.03687) z^-3 - 0.008212 (+/- 0.008207) z^-4

  B(z) = -1.423 (+/- 0.007231) z^-1 - 1.327 (+/- 0.05487) z^-2 + 0.8596 (+/- 0.05092) z^-3 + 0.04101 (+/- 0.03661) z^-4

名称: arx441
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   na=4   nb=4   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARX on time domain data "identification data".
Fit to estimation data: 97.59% (prediction focus)
FPE: 0.03665, MSE: 0.03542
More information in model's "Report" property.
```

The residual analysis is good， but the poles and zeros need to modify, so I reduce the ARX order to [3 3 1]




```
arx331 =
Discrete-time ARX model: A(z)y(t) = B(z)u(t) + e(t)
  A(z) = 1 + 1.473 (+/- 0.008348) z^-1 + 0.9061 (+/- 0.0126) z^-2 - 0.01902 (+/- 0.008169) z^-3

  B(z) = -1.422 (+/- 0.007221) z^-1 - 1.281 (+/- 0.01403) z^-2 + 0.8904 (+/- 0.01364) z^-3

名称: arx331
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   na=3   nb=3   nk=1
   Number of free coefficients: 6
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARX on time domain data "identification data".
Fit to estimation data: 97.58% (prediction focus)
FPE: 0.03646, MSE: 0.03553
More information in model's "Report" property.
```

Still exist some problems., so I change the ARX model to [2 2 1]



```
arx221 =
Discrete-time ARX model: A(z)y(t) = B(z)u(t) + e(t)
  A(z) = 1 + 1.5 (+/- 0.006591) z^-1 + 0.9635 (+/- 0.006455) z^-2

  B(z) = -1.437 (+/- 0.03283) z^-1 - 1.362 (+/- 0.03435) z^-2

名称: arx221
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   na=2   nb=2   nk=1
   Number of free coefficients: 4
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Estimated using ARX on time domain data "identification data".
Fit to estimation data: 88.96% (prediction focus)
FPE: 0.754, MSE: 0.7412
More information in model's "Report" property.
```

I think the final model is ARX221

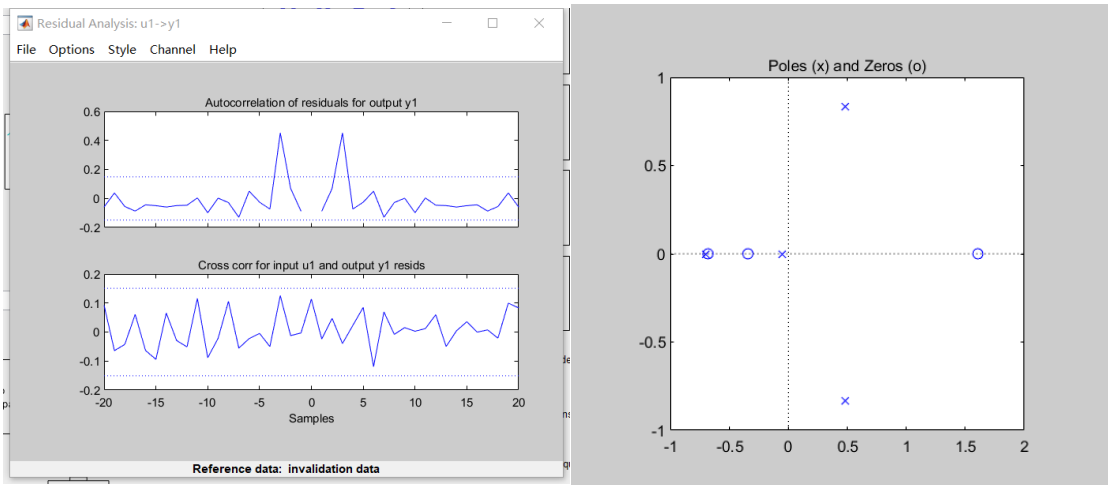
Alternative model:

I don't think there has any alternative models based on the process of the model structure and order selection strategy. The Residual analysis is good, so I just need to select model from ARX.

System2

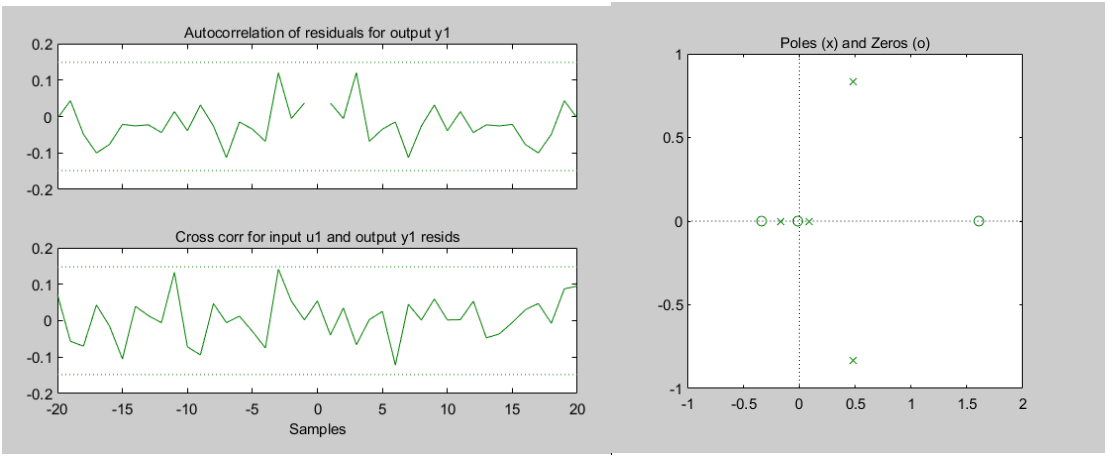
```
>> data2 = iddata(y2,u2,1);
nk2 = delayest(data2);
disp(nk2);
     1
```

Start with the model ARX[4 4 1]



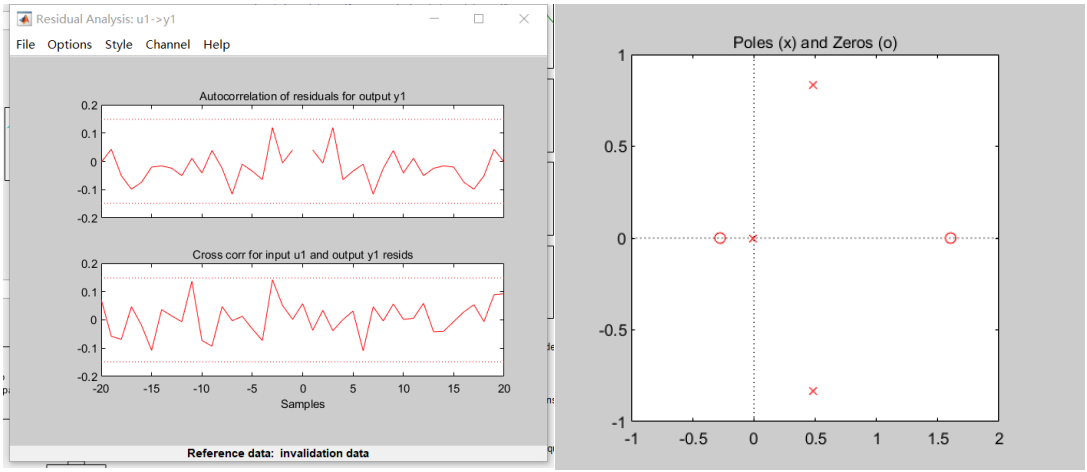
Obviously the residual analysis is not good , so we need to test the model

BJ[4 2 2 4 1]



```
bj42241 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -1.158 (+/- 0.007522) z^-1 + 1.456 (+/- 1.296) z^-2 + 0.6512 (+/- 1.722) z^-3 + 0.008761 (+/- 0.5823) z^-4

  C(z) = 1 + 1.666 (+/- 0.1763) z^-1 + 0.6781 (+/- 0.1762) z^-2

  D(z) = 1 + 1.743 (+/- 0.1571) z^-1 + 0.7558 (+/- 0.1569) z^-2

  F(z) = 1 - 0.886 (+/- 1.119) z^-1 + 0.8376 (+/- 1.071) z^-2 + 0.08448 (+/- 1.029) z^-3 - 0.01288 (+/- 0.01602) z^-4

名称: bj42241
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   nb=4   nc=2   nd=2   nf=4   nk=1
   Number of free coefficients: 12
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: Near (local) minimum, (norm(g) < tol)..
Number of iterations: 6, Number of function evaluations: 15

Estimated using PEM on time domain data "identification data".
Fit to estimation data: 96.33% (prediction focus)
FPE: 0.03953, MSE: 0.0382
More information in model's "Report" property.
```

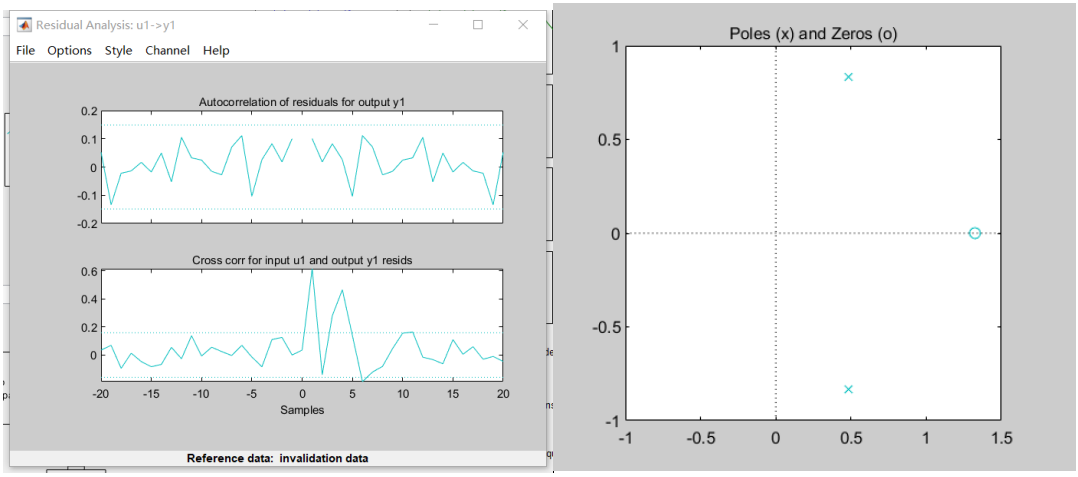
Seems the parameter test is bad, and no similar things now. So I reduce the order of BJ moder to [3 2 2 3 1]




```
bj32231 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -1.159 (+/- 0.007495) z^-1 + 1.541 (+/- 0.02647) z^-2 + 0.5169 (+/- 0.03316) z^-3

  C(z) = 1 + 1.665 (+/- 0.1742) z^-1 + 0.6743 (+/- 0.1739) z^-2

  D(z) = 1 + 1.742 (+/- 0.155) z^-1 + 0.7537 (+/- 0.1545) z^-2

  F(z) = 1 - 0.9584 (+/- 0.01597) z^-1 + 0.9211 (+/- 0.01545) z^-2 + 0.004078 (+/- 0.01484) z^-3

名称: bj32231
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   nb=3   nc=2   nd=2   nf=3   nk=1
   Number of free coefficients: 10
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: Near (local) minimum, (norm(g) < tol)..
Number of iterations: 7, Number of function evaluations: 17

Estimated using PEM on time domain data "identification data".
Fit to estimation data: 96.32% (prediction focus)
FPE: 0.03935, MSE: 0.03824
More information in model's "Report" property.
```

Still the variance is too big in C(z) and not enough similar,so I reduce the order

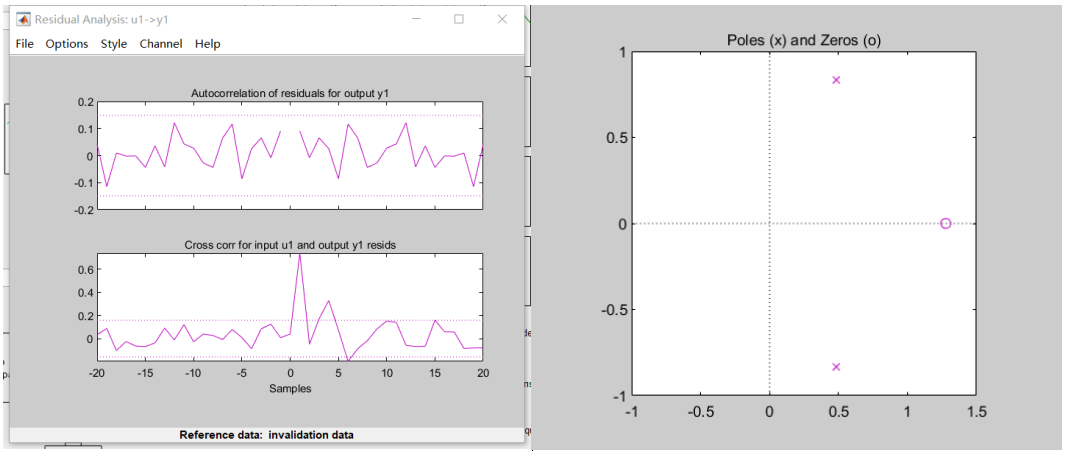
again to BJ[2 2 2 2 1]




```
bj22221 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = -1.492 (+/- 0.0135) z^-1 + 1.98 (+/- 0.01348) z^-2

  C(z) = 1 - 0.9101 (+/- 0.05881) z^-1 + 0.7511 (+/- 0.05816) z^-2

  D(z) = 1 - 0.9319 (+/- 0.04134) z^-1 + 0.8895 (+/- 0.04023) z^-2

  F(z) = 1 - 0.9721 (+/- 0.0008976) z^-1 + 0.9305 (+/- 0.0008999) z^-2

名称: bj22221
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   nb=2   nc=2   nd=2   nf=2   nk=1
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: Maximum number of iterations reached..
Number of iterations: 20, Number of function evaluations: 42

Estimated using PEM on time domain data "identification data".
Fit to estimation data: 90.35% (prediction focus)
FPE: 0.2696, MSE: 0.2636
More information in model's "Report" property.
```

I think the C and D are similar enough , I will test the OE without reducing order

OE[2 2 1]




```
oe221 =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
  B(z) = -1.559 (+/- 0.01014) z^-1 + 1.997 (+/- 0.0102) z^-2

  F(z) = 1 - 0.9677 (+/- 0.0005219) z^-1 + 0.9292 (+/- 0.0005191) z^-2

名称: oe221
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   nb=2   nf=2   nk=1
   Number of free coefficients: 4
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: Near (local) minimum, (norm(g) < tol)..
Number of iterations: 3, Number of function evaluations: 7

Estimated using PEM on time domain data "identification data".
Fit to estimation data: 90.43%
FPE: 0.2619, MSE: 0.2589
More information in model's "Report" property.
```

I think the model now is good enough , so the final model is the OE[2 2 1]

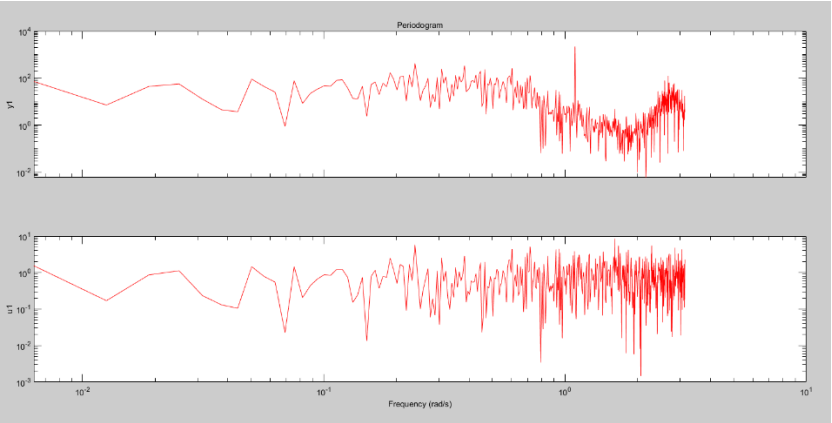
Alternative model:

I think BJ[2 2 2 2 1] is also fit for the data, I think although the zeros are out of the unit circle, but it's hard to eliminate, maybe it's the characteristics of this dataset.

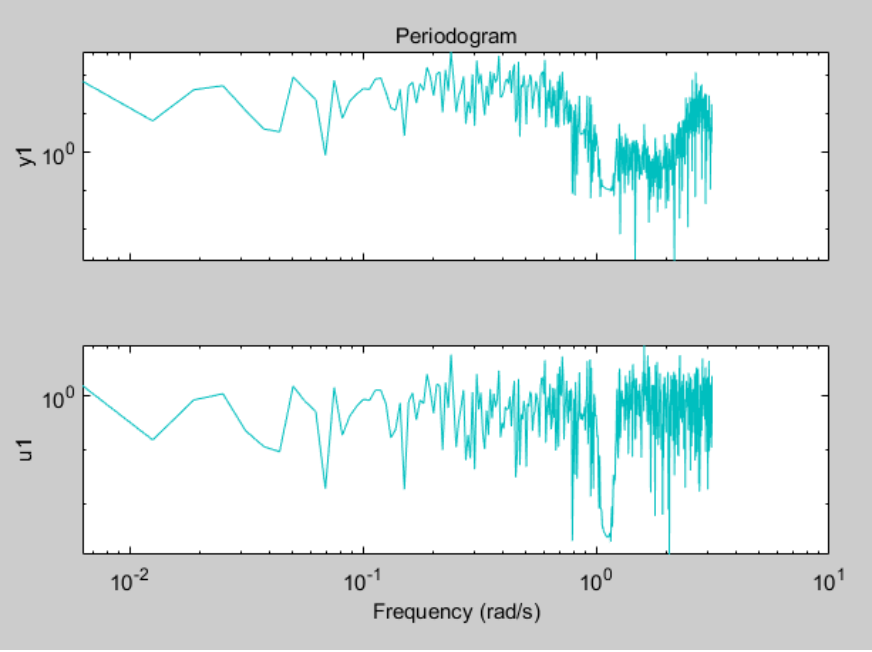
System 3

```
>> data3 = iddata(y3,u3,1);
nk3 = delayest(data3);
disp(nk3);
     3
```

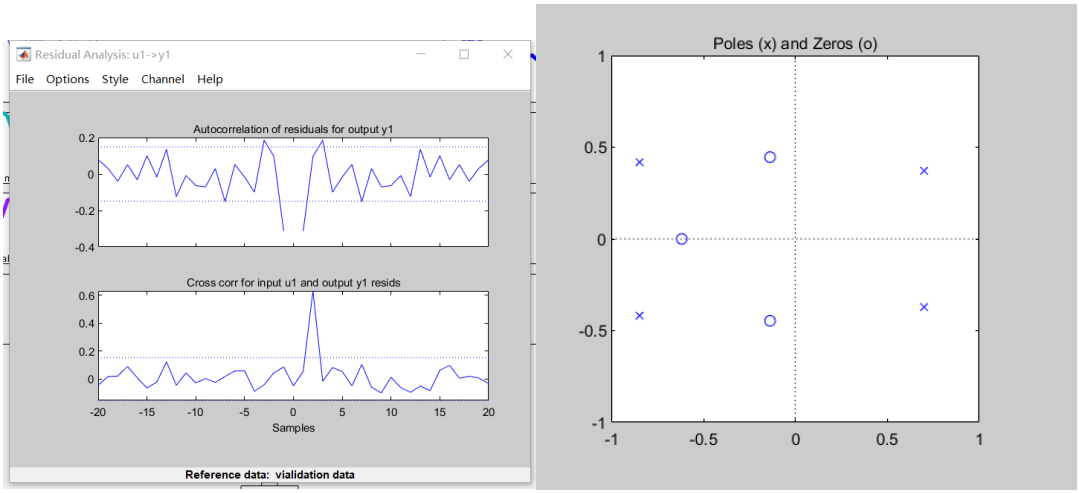
There are some differences in system 3 , you can find a peak in the data spectra



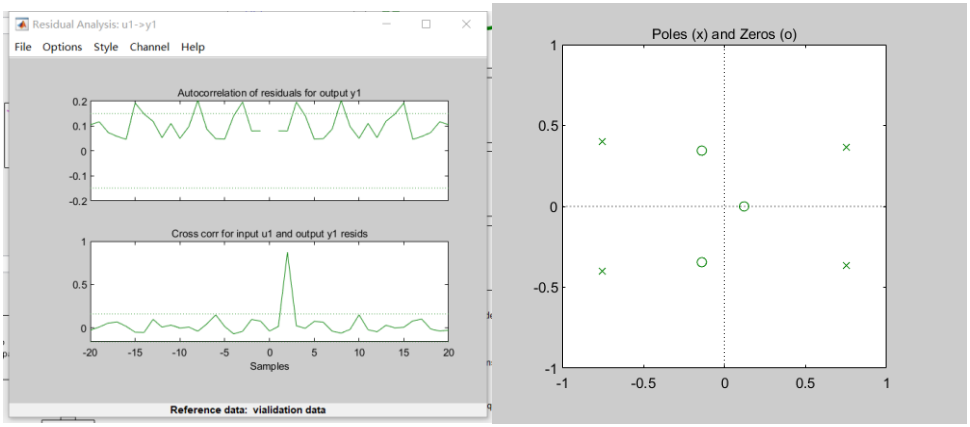
So I need to filter it After filter, the data spectra looks like below:



Now I begin with the ARX[4 4 3]

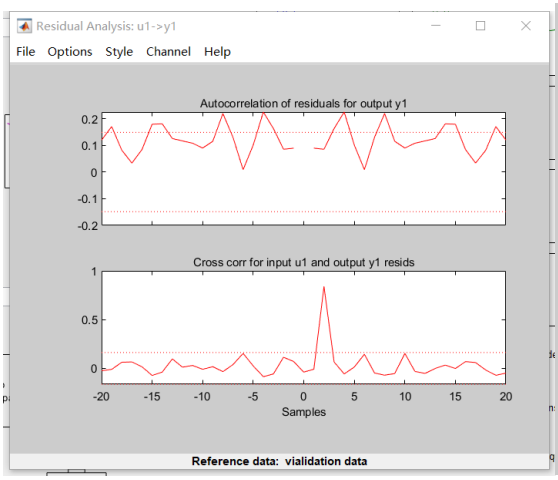


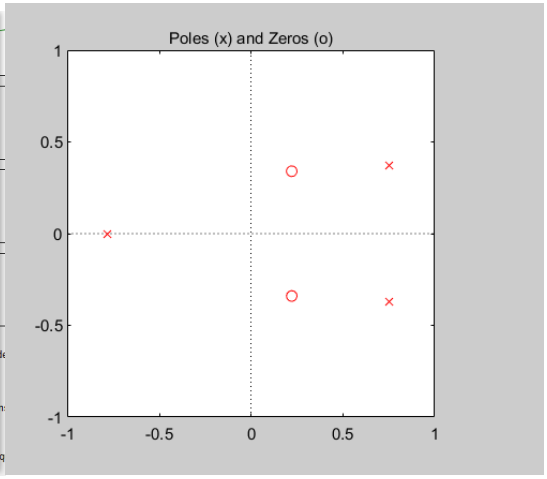
The residual analysis is bad , so I will test BJ [4 2 2 4 3]




```
bj42243 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 3.691 (+/- 0.09159) z^-3 + 0.574 (+/- 0.4166) z^-4 + 0.3844 (+/- 0.2368) z^-5 - 0.06252 (+/- 0.1074) z^-6

  C(z) = 1 - 0.2403 (+/- 0.1207) z^-1 - 0.129 (+/- 0.1039) z^-2

  D(z) = 1 + 1.544 (+/- 0.08435) z^-1 + 0.755 (+/- 0.07343) z^-2

  F(z) = 1 + 0.002701 (+/- 0.1042) z^-1 - 0.839 (+/- 0.0792) z^-2 - 0.04231 (+/- 0.07096) z^-3 + 0.5121 (+/- 0.06292) z^-4

名称: bj42243
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   nb=4   nc=2   nd=2   nf=4   nk=3
   Number of free coefficients: 12
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: No improvement along the search direction with line search..
Number of iterations: 8, Number of function evaluations: 159
```

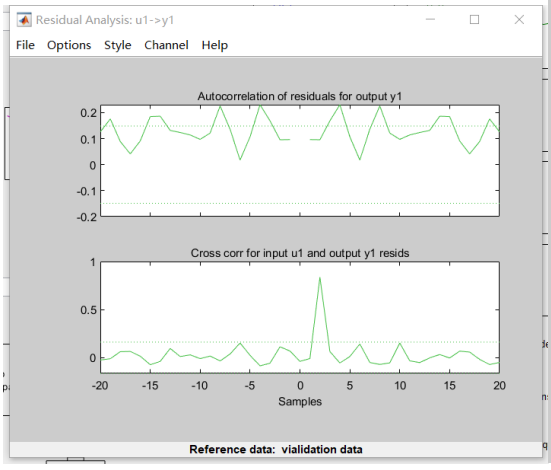
The residual analysis almost satisfied the requirement but the parameter variance is so big. So I need to reduce the order to BJ[3 2 2 3 3]

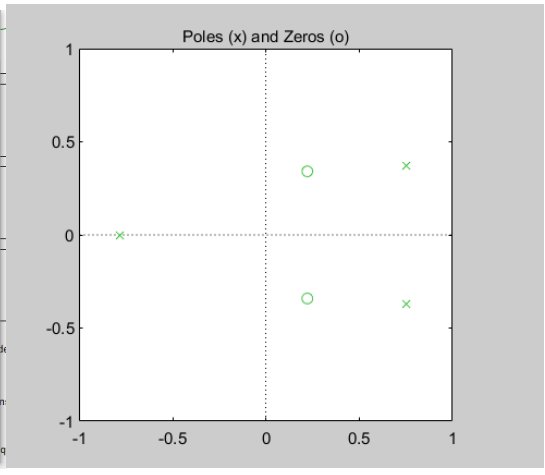





```
bj32233 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 3.579 (+/- 0.07945) z^-3 - 1.595 (+/- 0.1593) z^-4 + 0.5917 (+/- 0.09586) z^-5

  C(z) = 1 - 0.6036 (+/- 0.187) z^-1 + 0.003476 (+/- 0.1491) z^-2

  D(z) = 1 + 1.122 (+/- 0.1562) z^-1 + 0.3377 (+/- 0.1287) z^-2

  F(z) = 1 - 0.7234 (+/- 0.05124) z^-1 - 0.4756 (+/- 0.07719) z^-2 + 0.552 (+/- 0.03577) z^-3

名称: bj32233
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   nb=3   nc=2   nd=2   nf=3   nk=3
   Number of free coefficients: 10
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: No improvement along the search direction with line search..
Number of iterations: 9, Number of function evaluations: 163

Estimated using PEM on time domain data "identification data".
Fit to estimation data: 73.69% (prediction focus)
FPE: 1.254, MSE: 1.219
```

The variance still too high in c , so I try the BJ[3 1 2 3 3]



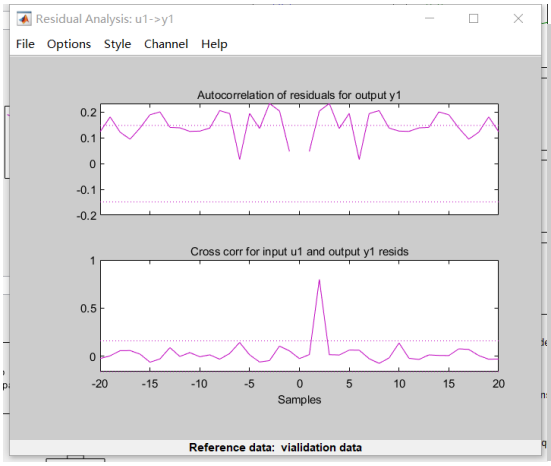


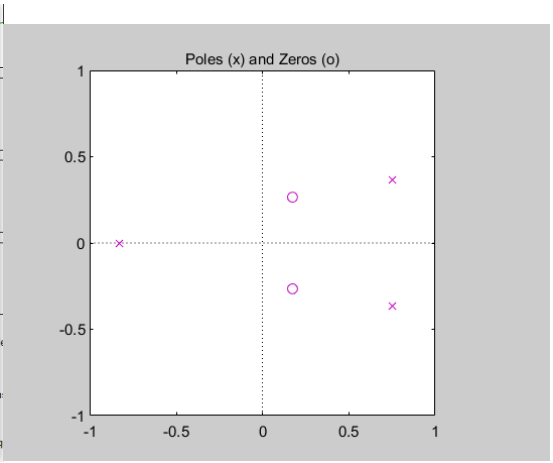
```
bj31233 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 3.589 (+/- 0.07045) z^-3 - 1.605 (+/- 0.1457) z^-4 + 0.5973 (+/- 0.0861) z^-5

  C(z) = 1 - 0.6133 (+/- 0.0601) z^-1

  D(z) = 1 + 1.121 (+/- 0.05866) z^-1 + 0.3363 (+/- 0.05216) z^-2

  F(z) = 1 - 0.7214 (+/- 0.05039) z^-1 - 0.4784 (+/- 0.0759) z^-2 + 0.5533 (+/- 0.03507) z^-3

名称: bj31233
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   nb=3   nc=1   nd=2   nf=3   nk=3
   Number of free coefficients: 9
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: Near (local) minimum, (norm(g) < tol)..
Number of iterations: 10, Number of function evaluations: 130

Estimated using PEM on time domain data "identification data".
Fit to estimation data: 73.69% (prediction focus)
FPE: 1.251, MSE: 1.22
More information in model's "Report" property.
```

Both requirements are met, so I think the optimal model is BJ[3 1 2 3 3]

Alternative model

BJ[3 1 1 3 3] can also satisfy the requirements.






```
bj31133 =
Discrete-time BJ model: y(t) = [B(z)/F(z)]u(t) + [C(z)/D(z)]e(t)
  B(z) = 3.471 (+/- 0.06649) z^-3 - 1.213 (+/- 0.1313) z^-4 + 0.3506 (+/- 0.07909) z^-5

  C(z) = 1 - 0.7441 (+/- 0.04108) z^-1

  D(z) = 1 + 0.8199 (+/- 0.03955) z^-1

  F(z) = 1 - 0.6765 (+/- 0.04736) z^-1 - 0.5474 (+/- 0.07125) z^-2 + 0.5817 (+/- 0.033) z^-3

名称: bj31133
采样时间: 1 seconds

Parameterization:
   Polynomial orders:   nb=3   nc=1   nd=1   nf=3   nk=3
   Number of free coefficients: 8
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: No improvement along the search direction with line search..
Number of iterations: 9, Number of function evaluations: 167
```