Homework problems 2

The answers to the homework problems should be returned to the corresponding folder on the robotics course ELEC-C1320 MyCourses-platform. The answer files should be named as "solutions_homework problems 2_Firstname_Lastname.pdf". Deadline for returning the solutions is Thursday 26.10, 12:00 Noon. You can use Finnish, Swedish or English in your solutions.

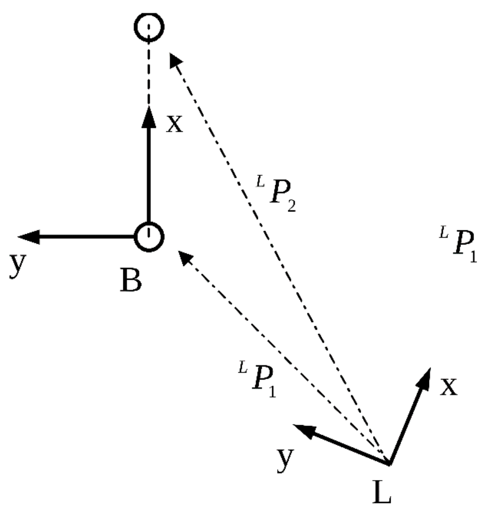
Note: Please, return <u>one</u> pdf-file for your solutions. <u>Mark clearly your name and student number on the answer sheet!</u>

1. The positions of two landmarks are measured by a laser range meter. The position vectors given with respect to frame {L} of the laser range finder are denoted as ${}^{L}\boldsymbol{P}_1$ ja ${}^{L}\boldsymbol{P}_2$ in the figure below.

An orthonormal right-handed coordinate frame {B} is formed (w.r.t frame {L}) based on the positions of the landmarks. For that, we have the following rules:

- The origin of frame {B} (*w.r.t frame* {L}) is assigned to the location of the first landmark
- The x-axis of frame {B} is pointing from the first landmark towards the second landmark along the xy-plane of frame {L} (*i.e. the unit vector of the x-axis of frame* {B} *is perpendicular to the z-axis of frame* {L})
- The z-axis of frame {B} is pointing upwards (along the z-direction of frame {L}) and the y-axis comes from the right hand rule.

The mathematical steps to form a 4x4 homogenous transformation matrix ${}^{L}\boldsymbol{T}_B$ describing the pose (position and orientation) of frame {B} with respect to frame {L} as a function of vector components of the position vectors ${}^{L}\boldsymbol{P}_1$ ja ${}^{L}\boldsymbol{P}_2$ are as follows:



$${}^{L}P_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} \qquad {}^{L}P_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix}$$

So, the task is to apply the given rules to determine the homogenous transformation matrix ${}^{L}\boldsymbol{T}_B$

$$ {}^{L}\boldsymbol{T}_B = \begin{bmatrix} {}^{L}\boldsymbol{X}_B & {}^{L}\boldsymbol{Y}_B & {}^{L}\boldsymbol{Z}_B & {}^{L}\boldsymbol{P}_B \\ 0 & 0 & 0 & 1 \end{bmatrix} $$ where

${}^{L}\boldsymbol{P}_B = {}^{L}\boldsymbol{P}_1$, i.e. the origin of frame {B} is at the position P_1

and the direction of the x-axis of frame *B* (*given w.r.t. frame L*) points from P_1 towards P_2, parallel to the xy-plane of frame *L*

$$ {}^{L}\boldsymbol{X}_B = \begin{bmatrix} \frac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} & \frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} & 0 \end{bmatrix}^T $$

and further, we know that the z-axis of frame {B} is pointing to the same direction as the z-axis of frame {L}

$$ {}^{L}\boldsymbol{Z}_B = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T $$

and finally, the direction of the y-axis can be determined by applying the cross-product rule (Corke, p. 31)

${}^{L}\boldsymbol{Y}_B = {}^{L}\boldsymbol{Z}_B \times {}^{L}\boldsymbol{X}_B$ (*see Corke's text book p. 43 for a method for the calculation of the cross-product by utilizing the skew-symmetric matrix form of the first vector in the calculations*)

An alternative way of determining the ${}^{L}\boldsymbol{X}_B$ and ${}^{L}\boldsymbol{Y}_B$ direction vectors:

First determine the rotation angle θ of frame {B} around the z-axis of frame {L} by considering the direction of the x-axis of frame B w.r.t the x-axis of frame {L}.

$$ \theta = atan2(y_2 - y_1, x_2 - x_1) $$

then ${}^{L}\boldsymbol{R}_B$ becomes $$ {}^{L}\boldsymbol{R}_B = \begin{bmatrix} \cos\theta & -sin\,\theta & 0 \\ \sin\theta & cos\,\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

And now we have all the required vector/matrix components for forming the homogenous transformation matrix ${}^{L}\boldsymbol{T}_B$

a) Calculate the numerical from of ${}^{L}\boldsymbol{T}_B$ when the position vectors are

${}^{L}\boldsymbol{P}_1$=[9 15 3 1]^T and ${}^{L}\boldsymbol{P}_2$=[9 10 5 1]^T (*10 points*)

Hint:

You solution should look like: : $${}^{L}\boldsymbol{T}_B = \begin{bmatrix} 0 & \ldots & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) Let's assume that the first position vector ${}^{L}\boldsymbol{P}_1$ is the same as in part **a)**. About the 2nd position vector ${}^{L}\boldsymbol{P}_2$ we know that its x-coordinate is the same as the x-coordinate of the first position vector, i.e. $x_1 = x_2$ and the y-coordinate of the second position vector has a larger value than the y-coordinate of the first position vector, i.e. $y_2 > y_1$. Determine the numerical form of ${}^{L}\boldsymbol{T}_B$.

(*10 points*)

Hint:

You solution should look like: $${}^{L}\boldsymbol{T}_B = \begin{bmatrix} 0 & \ldots & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Integration of rotational motion when using rotation matrix for representing orientation.
Imagine that an Inertial Measurement Unit (IMU) has been mounted on board of a mobile robot. The gyroscopes and accelerometers of the IMU are constantly measuring rotational speed and linear acceleration of the robot. The problem consists of two parts:

a) First, the initial orientation of the mobile robot (or the IMU) needs to be determined. The initial orientation is described with three consecutive rotations, first θ_y (*yaw angle*) around z-axis followed by θ_p (*pitch angle*) around y-axis and θ_r (*roll angle*) around x-axis (compare Corke's text book, p. 84). Let's assume that the heading of the robot, i.e. rotation around z-axis of the fixed navigation reference frame, corresponds to zero *yaw-angle*, θ_y=0.

The tilt angles of the robot, θ_p and θ_r, are determined by measuring the direction of gravitational acceleration seen by the acceleration sensors of the IMU. The tilt angles can be computed by utilizing the equations (3.20) and (3.21) of Corke's text book (p. 84). As an answer give the calculated pitch and roll angles (in degrees) and the corresponding 3x3 orthonormal rotation matrix (which is calculated from the roll, pitch and yaw angles as **R_initial** = rotz(yaw==0.0)*roty(pitch)*rotx(roll);) (*10 points*).

$\boldsymbol{a}_{IMU}$=[a_x a_y a_z]^T =[−1.7035 4.8305 8.3666]^T; % accelerometer measurements before robot starts to move [m/s^2]

g = 9.81 m/s^2 % gravitational acceleration

b) Then, compute an update of the rotation matrix **R_initial** calculated in **part a**. The rotation matrix describes the orientation of the robot w.r.t. the reference (inertial) frame. An IMU sensor is fixed to the robot body. The sensor measures continuously the angular velocities, $\boldsymbol{\omega}$, and the linear accelerations $\boldsymbol{a}$ experienced by the robot. Compute an update of the rotation matrix by means of the measured angular velocities.– *Hint: Implement the method in Matlab. Part of the required Matlab code can be found from the similar example problem, presented at the end of Lecture slides_3.*

To solve the problem, you can use Eq. (3.7) of Corke's text book. With the equation you can compute an approximation of the rotation matrix at a given time instant δ_t into the future (here 60 ms).

The sum of an orthonormal matrix and another matrix is **not** an orthonormal matrix, but if the added term is small then it will be "close" and we can normalize it. You can use the equations, presented on slide 19 (of Lecture_slides_3) to do the normalization.

Repeat the step 5 times, **with and without** normalizing the updated rotation matrix after each of the five steps. As an answer to the problem, give the numerical forms of the final rotation matrix (*acquired after repeating the update step 5 times*) with and without doing the normalization after each update step.

What can you say about the difference between the final rotation matrices corresponding to the two cases (i.e. with and without matrix normalization step)? *Hint: You can, for example, compare the determinants of the two rotation matrices.*

(*10 points*)

The numerical value of the angular velocity vector $\boldsymbol{\omega}_{IMU}$ is

$\boldsymbol{\omega}_{IMU} = [\omega_x \ \ \omega_y \ \ \omega_z]^T = [0.7 \ \ 0.8 \ \ 0]^T$; % gyroscope measurements in [radians/sec]

Assume that the gyro measurements of the angular velocity remain constant during the 5 integration/update steps.

3. In this problem you are asked to analyze a 4 point linear path with parabolic blends (LSBP) trajectory. For the analysis, you are given two files. In "linear segments with parabolic blends example.pdf" the calculations of key parameters of the trajectory are explained. In "linear_segment_with_parabolic_blend_via_points.m" Matlab code to implement the trajectory generation algorithm is given.

The task for you consists of two parts, a) and b):

a) The basic trajectory setup is determined as follows: *path points **are 0, 30, 70 and 20 degrees**, motion time between the path points is **2 seconds**, the initial and final velocities for the path are **0 deg/s** and the acceleration/deceleration for the blend sections is **±50 deg/s***. The basic trajectory has been implemented in the Matlab code file "linear_segment_with_parabolic_blend_via_points.m". Your task is to:

- Determine the lengths of the seven time segments of the trajectory. The path points have been given indices 1, 2, 3 and 4. So, the time segments to be determined have names: t_1, t_{12}, t_2, t_{23}, t_3, t_{34}, t_4. See "linear segments with parabolic blends example.pdf" file for more details.
- Plot the trajectory (separate plots for the position, velocity and acceleration) and attach images of the curves with your solution.

5p

b) Change *the acceleration/deceleration for the blend sections to **±25 deg/s**. What are the lengths of the seven time segments of the trajectory now**?** How does the lower acceleration/deceleration for the blend sections affect the trajectory**?*** *5p*