Create a single degree of freedom trajectory generator for a path with multiple via points

The **path** should be made **smooth** by utilizing "**linear segment with parabolic blends**"- type of motion when moving from one path point to the next one. See Craig's text book pp. 210-216 for a more detailed discussion of the method.

For the problem, the values of the joint angle in the **path points are 0, 30, 70 and 20 degrees**. **Motion time** between the path points is **2 seconds**. The **initial and final velocities** for the path are **0 deg/s**. The **acceleration/deceleration** for the blend sections is **±50 deg/s^2**

The method is described in Craig's text book but we will also summarize the key equations here. A smooth path with via points utilizing linear segments with parabolic blends approach is described in the figure (from Craig, p. 214). *(Note that from the robot control perspective the location of θ_i, i.e. the location of the start of the motion, is actually on the vertical axis marked with the left most black dot. It is also drawn in the middle of the blend section for the purpose of determination of the constant velocity segment. The same is true also for the last section of the trajectory although the right most black dot is not drawn in the figure*).

Now we sum up the equations based on which the trajectory can be generated for the multisegment path:

The equations for the interior path points are a bit different than for the first and last segment (Eq. 7.24 in the book).

So, for the interior path segments we calculate the velocity for the linear portion of the segment

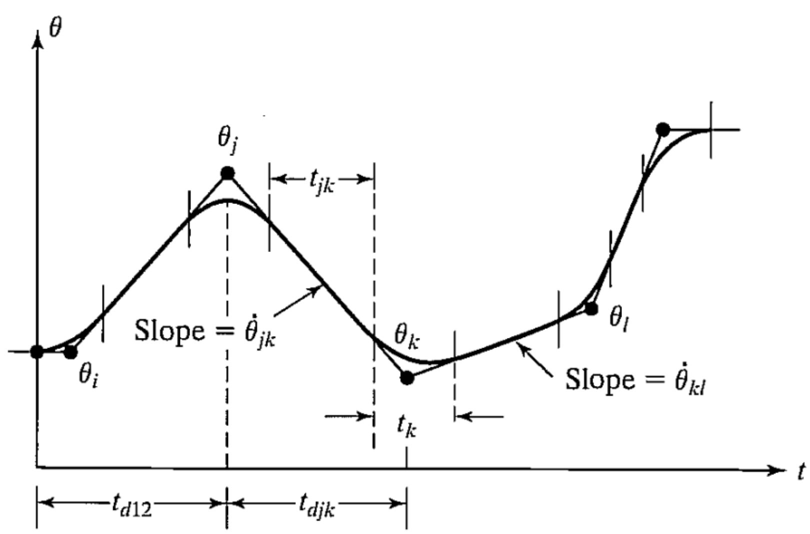

FIGURE 7.9: Multisegment linear path with blends.

$$\dot{\theta}_{jk} = \frac{\theta_k - \theta_j}{t_{djk}}$$ (7.24, Craig p. 213)

where t_{djk} is total duration of the path segment *jk*.

The acceleration/deceleration for the blend part of the motion is ± the given value, where the sign is determined as $sign(\dot{\theta}_{kl} - \dot{\theta}_{jk})$

The acceleration/ deceleration time, i.e. the duration of the blend section is determined as

$$t_k = \frac{\dot{\theta}_{kl} - \dot{\theta}_{jk}}{\ddot{\theta}_k}$$

and the duration of the linear motion as

$$t_{jk} = t_{djk} - \frac{1}{2}t_j - \frac{1}{2}t_k$$

For the first segment of the path we have (note that in the figure indices of the 1st and 2nd path points are *i* and *j* respectively)

$$\frac{\theta_2 - \theta_1}{t_{d12} - \frac{1}{2}t_1} = \ddot{\theta}_1 t_1$$ (7.25, Craig p. 214)

where t_1 is the blend time to accelerate from the initial path point to the beginning of the constant speed section

Whether we have acceleration or deceleration for the blend section is determined with the sign-function: $sign(\theta_2 - \theta_1)$, i.e. we have $\ddot{\theta}_1 = sign(\theta_2 - \theta_1)|\ddot{\theta}_1|$

$$t_1 = t_{d12} - \sqrt{t^2{}_{d12} - \frac{2(\theta_2 - \theta_1)}{\ddot{\theta}_1}}$$ (7.26, Craig p. 214)

$$\dot{\theta}_{12} = \frac{\theta_2 - \theta_1}{t_{d12} - \frac{1}{2}t_1}$$

$$t_{12} = t_{d12} - t_1 - \frac{1}{2}t_2$$

And finally for the last segment of the path we have

(Note that in the figure, the index of the last path point would be *m*)

$$\frac{\theta_{n-1} - \theta_n}{t_{d(n-1)n} - \frac{1}{2} t_n} = \ddot{\theta}_n t_n$$

from where we get

$$\ddot{\theta}_n = sign(\theta_{n-1} - \theta_n)\left|\ddot{\theta}_n\right|$$

$$t_n = t_{d(n-1)n} - \sqrt{t^2_{d(n-1)n} + \frac{2(\theta_n - \theta_{n-1})}{\ddot{\theta}_n}}$$

(7.27, Craig p. 214)

$$\dot{\theta}_{(n-1)n} = \frac{\theta_n - \theta_{n-1}}{t_{d(n-1)n} - \frac{1}{2} t_n}$$

(7.28, Craig p. 215)

$$t_{(n-1)n} = t_{d(n-1)n} - t_n - \frac{1}{2} t_{n-1}$$

As a reminder, note that in the equations above t_x indicates the duration of acceleration/deceleration phase *x*, i.e. the blend region to pass path point *x* smoothly, whereas t_{xy} gives the duration of the constant speed, i.e. linear section, connecting path points *x* and *y*. The corresponding value of the constant speed is given as $\dot{\theta}_{xy}$.