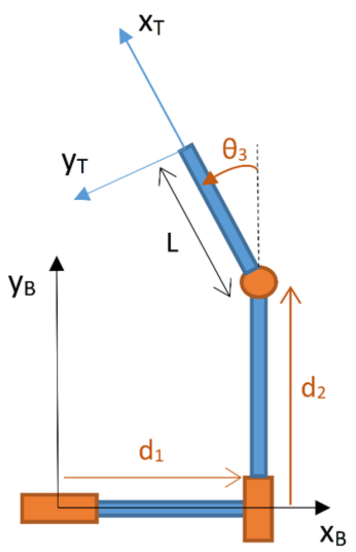
Homework problems 3

The answers to the homework problems should be returned to the corresponding folder on the robotics course ELEC-C1320 MyCourses-platform. The answer files should be named as "solutions_homework problems 3_Firstname_Lastname.pdf". Deadline for returning the solutions is Thursday 9.11, 12:00 Noon. You can use Finnish, Swedish or English in your solutions.

Note: Please, return one pdf-file for your solutions. Mark clearly your name and student number on the answer sheet!

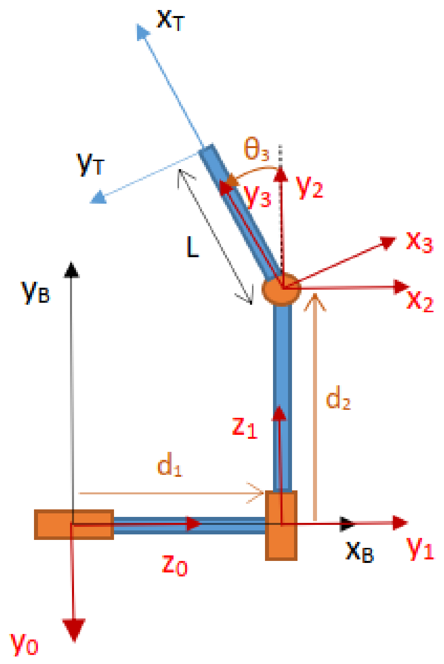
1. In the figure a planar PPR robot is shown. The orientation of the last link when the joint variable θ_3 is zero is indicated with the dashed line. The direction of positive rotation of θ_3 is indicated with the arrow in the figure.

Solve the forward kinematics problem of the manipulator to describe the tool frame (T) with respect to the robot base frame (B). In other words, assign the link frames in a figure and provide the corresponding Denavit Hartenberg-parameters in a table table (as well as base and tool transformation matrices). You should use here the **Standard DH-parameter convention**. (10 points)



Hint:

To help you solving the problem the illustration of the link frames, which is part of the asked solution is given to you below:



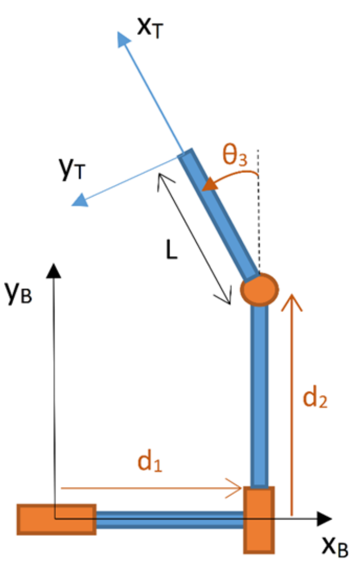
Also, the Denavit Hartenberg parameter table is given to you with the joint control variables already filled in:

Link	Θ_i	d_i	a_i	α_i	σ_i (joint type symbol:P-prismatic/R-rotational)
1		d_1			P
2		d_2			P
3	Θ_3				R

So, your remaining task is to fill in the parameter table and give the **Base** and **Tool** 4x4 homogenous transformation matrices.

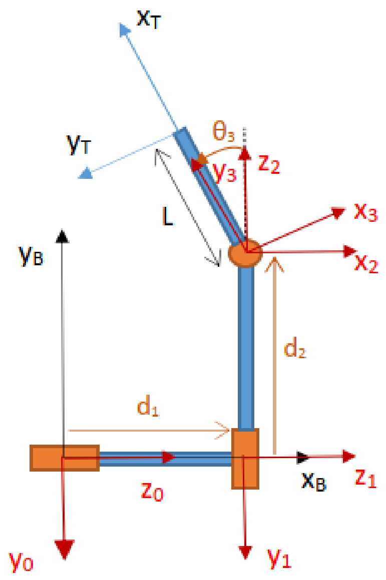
2. In the figure a planar PPR robot is shown. The orientation of the last link when the joint variable θ_3 is zero is indicated with the dashed line. The direction of positive rotation of θ_3 is indicated with the arrow in the figure.

Solve the forward kinematics problem of the manipulator to describe the tool frame (T) with respect to the robot base frame (B). In other words, assign the link frames in a figure and provide the corresponding DenavitHartenberg-parameters in a table table (as well as base and tool transformation matrices). You should use here the **Modified DH-parameter convention**. (10 points) (*NOTE: this is actually the same task as in problem 1 except that you should apply here the alternative DH parameter convention*)



Hint:

To help you solving the problem the illustration of the link frames, which is part of the asked solution is given to you below:



Also, the Denavit Hartenberg parameter table is given to you with the joint control variables already filled in:

Link	α_{i-1}	a_{i-1}	d_i	Θ_i	σ_i
1			d_1		P
2			d_2		P
3				Θ_3	R

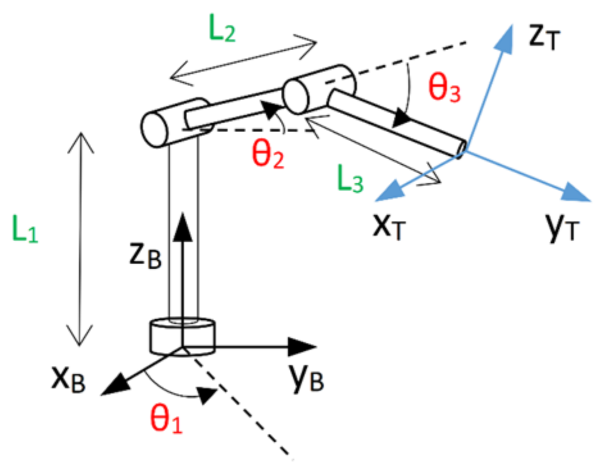
So, your remaining task is to fill in the parameter table and give the **Base** and **Tool** 4x4 homogenous transformation matrices.

3. In the figure a 3-axes RRR-type manipulator is shown. When all the joint angles have a value zero, the upper arm (i.e. "shoulder" and "elbow" links) is oriented horizontally above the X_B axis. In the joint configuration, shown in the figure, the angle θ_1 has a positive value, the angle θ_2 (angle of the shoulder link with respect to the X_BY_B-plane) has also a positive value but the angle θ_3 which rotates the last link with respect to the "shoulder" link has a negative value.

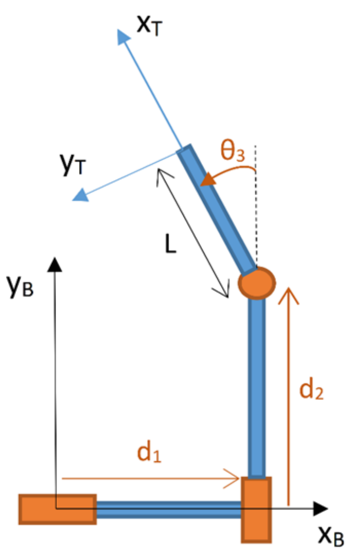
Solve the forward kinematics problem of the manipulator to describe the tool frame {T} with respect to the robot base frame {B}. In other words, assign the link frames in the figure and provide the corresponding DenavitHartenberg-parameters in a table (as well as base and tool transformation matrices).

a) Use the **Standard** Denavit-Hartenberg (DH) parameter convention for your solution. (10 points)

b) Use the **Modified** Denavit-Hartenberg (DH) parameter convention for your solution. (10 points)



4. In the figure a planar PPR robot is shown. The orientation of the last link when the joint variable θ_3 is zero is indicated with the dashed line. The direction of positive rotation of θ_3 is indicated with the arrow in the figure. Determine the **inverse kinematics solution** for the mechanism by using the **geometric approach**. **I.e find the equations for the robot joint control variables as a function of the Cartesian task space position and orientation of the robot tool frame (T) with respect to the robot base frame (B).** More specifically, find the equations $d_1=f(x,y,\phi)$, $d_2=f(x,y,\phi)$, $\theta_3=f(x,y,\phi)$ where (x,y,ϕ) is the position and orientation of the tool frame (T) with respect to the base frame (B) and (d_1,d_2,θ_3) are the joint control variables of the robot. The orientation of the tool frame, ϕ, is defined as the angle of x_T-axis with respect to x_B-axis (see the second figure below) (10 points)



The motion range of the joints of the mechanism are:

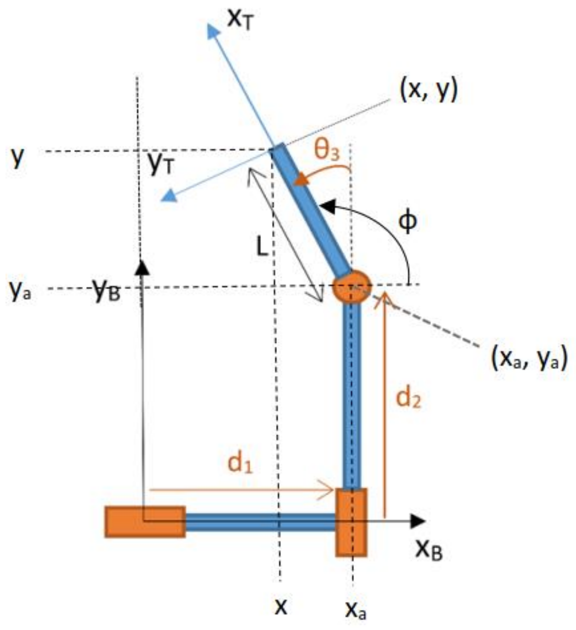
$d_{1min} < d_1 < d_{1max}$

$d_{2min} < d_2 < d_{2max}$

$-180 < \theta_3 \leq 180$

In this case, it is a good idea to calculate a new task coordinate frame position which is located coincident with the last link frame. This adjusted task space target position is marked in the figure below with (x_a,y_a). These adjusted coordinates will serve as the starting point to solve the equations for the first two degrees of freedom, d_1 and d_2.

In the figure below, also the task space tool frame orientation angle ϕ is illustrated.



Hint:

First solve θ_3 by studying the figure above. From the figure, we can see that θ_3 and ϕ have a 90˚ mutual offset, i.e. when θ_3 equals zero ϕ is 90˚. So we get for θ_3:

$\theta_3 = \phi - 90^\circ$, Which corresponds to $\theta_3 = f(x,y,\phi)$ and is therefore already one third of your solution!

And for the adjusted task space target position we get the following equations:

$x_a = x + \sin(\theta_3)L = \ldots$

$y_a = y - \cos(\theta_3)L = \ldots$

So, your task remains to finalize the solution by creating the equations $d_1 = f(x,y,\phi)$, $d_2 = f(x,y,\phi)$