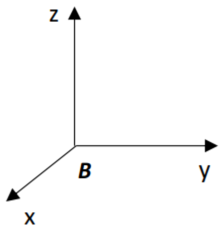
ELEC-C1320/ELEC-D1320 – Robotiikka, Exam 12.12.2019 (3 hours)

It is allowed to use a calculator in the exam.

You can use Finnish, English or Swedish in your solutions. Tehtävänannot on esitetty suomeksi sinisellä värillä. The problem definitions are given in Finnish in blue color.

1. The homogenous transformation matrix $\boldsymbol{T}$ describes the position and orientation of a new coordinate frame {N} with respect to the base frame {B}:
Homogeeninen muunnosmatriisi $\boldsymbol{T}$ kuvaa uuden koordinaatiston {N} paikkaa ja asentoa peruskoordinaatiston {B} suhteen:

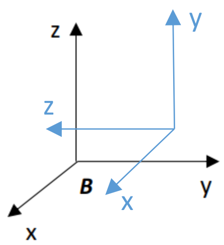
$$ {}^{B}\boldsymbol{T}_N = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



Illustrate the new coordinate frame {N} in relation to the base frame {B} (*i.e the position of the origin and the directions of the coordinate axes of the new frame {N} in relation to the base frame {B} shown in the drawing above*).
Esitä kuvan avulla uuden koordinaatiston {N} sijainti peruskoordinaatiston {B} suhteen (toisin sanoen, piirrä uuden koordinaatiston {N} origin paikka ja akselien suunnat suhteessa peruskoordinaatistoa {B} esittävään kuvaan, joka on annettu yllä)

(5 points)

Solution:



2. 3D-frame {B} is located initially coincident with the frame {A}. We first rotate frame {B} about its y-axis by 90 degrees. Then we translate the origin of the rotated frame {B} 5 units in the direction of its z-axis. 3D-koordinaatisto {B} on aluksi samassa paikassa ja asennossa koordinaatiston {A} kanssa. Ensimmäisessä vaiheessa koordinaatiston {B} asentoa muutetaan kiertämällä sitä oman y-akselinsa ympäri 90 astetta. Tämän jälkeen kiertyneen koordinaatiston {B} origon paikkaa siirretään 5 yksikköä oman z-akselinsa suuntaan.

a) Give the 4x4 homogenous transformation matrix, which describes the position and orientation of frame {B} with respect to frame {A}. Määritä 4x4 homogeeninen muunnosmatriisi, joka kuvaa koordinaatiston {B} paikkaa ja asentoa koordinaatiston {A} suhteen. (7 points)

b) The coordinates of a point **P** with respect to frame {B} be are [x=3,y=0,z=0]. What are the coordinates of point **P** given with respect to frame {A}? Pisteen **P** koordinaatit koordinaatiston {B} suhteen ovat [x=3,y=0,z=0]. Mitkä ovat pisteen **P** koordinaatit koordinaatiston {A} suhteen? (7 points)

Solution:

a) We first form two transformation matrices one for the translation and one for the rotation.

$$\text{transl}(x=0,y=0,z=5)=\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 5\\0 & 0 & 0 & 1\end{bmatrix},$$

$$\text{rot\_y}(90^o)=\begin{bmatrix}\cos(90) & 0 & \sin(90) & 0\\0 & 1 & 0 & 0\\-\sin(90) & 0 & \cos(90) & 0\\0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\0 & 1 & 0 & 0\\-1 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

Then we form the matrix equation by placing the matrices of individual transformations, in the correct order, in the equation (starting from the left with the first transformation):

$${}^{A}\boldsymbol{T}_{B}=\begin{bmatrix}0 & 0 & 1 & 0\\0 & 1 & 0 & 0\\-1 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 5\\0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 5\\0 & 1 & 0 & 0\\-1 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

b) And now we can calculate the coordinates of point **P** w.r.t. frame {A}:

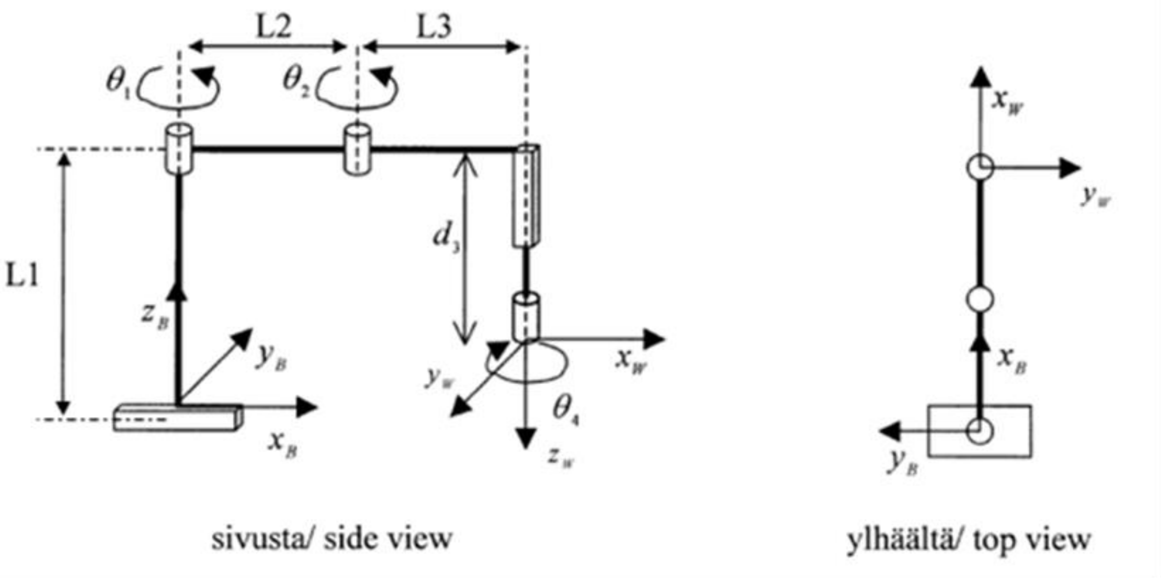
$${}^{A}\mathbf{P}={}^{A}\boldsymbol{T}_{B}*{}^{B}\mathbf{P}=\begin{bmatrix}0 & 0 & 1 & 5\\0 & 1 & 0 & 0\\-1 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}3\\0\\0\\1\end{bmatrix}=\begin{bmatrix}5\\0\\-3\\1\end{bmatrix}$$

3. In the figure below the kinematic structure of a four degree-of-freedom SCARA (*Selective Compliance Assembly Robot Arm*) robot is shown. The first two joints are rotational (shoulder, θ_1, and elbow, θ_2, joints move the tool on a plane), then a prismatic joint, d_3 follows, which moves the tool up and down. And finally, in the kinematic chain, a rotational joint, θ_4, adjusts the orientation of the tool, for example, to grasp objects, which are laying on a pallet, oriented parallel to the xy-plane of the B-frame. In the figure, the manipulator is shown in its home/zero position (i.e. when all the rotational joint control variables are zero, the upper arm is oriented horizontally above the x_B-axis and x_w is codirectional with X_B, Y_w with $-Y_B$ and Z_w with $-Z_B$).

Alla olevassa kuvassa on esitetty neljän liikevapausasteen SCARA (*Selective Compliance Assembly Robot Arm*) robotin kinemaattinen rakenne. Kaksi ensimmäistä liikevapausastetta ovat kiertyvä niveliä (olkanivel, θ_1, ja kyynärnivel, θ_2, liikuttavat robotin työkalua tasossa), niitä seuraa prismaattinen vapausaste d_3, joka liikuttaa robotin työkalua ylös/alas suunnassa. Viimeisenä liikevapausasteena on kiertonivel θ_4, jonka avulla voidaan ohjata robotin työkalu haluttuun asentoon tasolla. Kuvassa manipulaattori on esitetty koti/nolla asennossaan (eli kun robotin kiertyville niveliile annetaan nolla ohjausarvoiksi, yläkäsivarsi on vaakasuorassa asennossa x_B-akselin yläpuolella ja ranne-/työkalukoordinaatiston x_w-akseli on samansuuntainen X_B-akselin kanssa, Y_w $-Y_B$:n kanssa ja Z_w $-Z_B$:n kanssa.)

Give in a table the link parameters and variables (i.e. Denavit-Hartenberg parameters) required for constructing the forward kinematic transformation of the manipulator for describing the tool/wrist frame (**W**) with respect to the robot base frame (**B**). For this, give also the required base and tool transformation matrices. It is your choice to use either the Standard or Modified DH-parameter convention. Also, number and mark in the figure the corresponding link-frames.

Anna taulukossa robottimekanismin suoraa kinemaattista muunnosta vastaavat linkkiparametrit ja -muuttujat (ts. Denavit-Hartenberg-parametrit), jotka määrittävät työkalu-/rannekoordinaatiston **W** paikan ja asennon robotin peruskoordinaatiston **B** suhteen. Anna myös kuvausta varten tarvittavat perusmuunnos- ja työkalumuunnosmatriisit. Tämän lisäksi, numeroi ja merkitse kuvaan mekanismin suoraa kinemaattista muunnosta vastaavat linkkikoordinaatistot. (18 points)



Solution Standard DH:

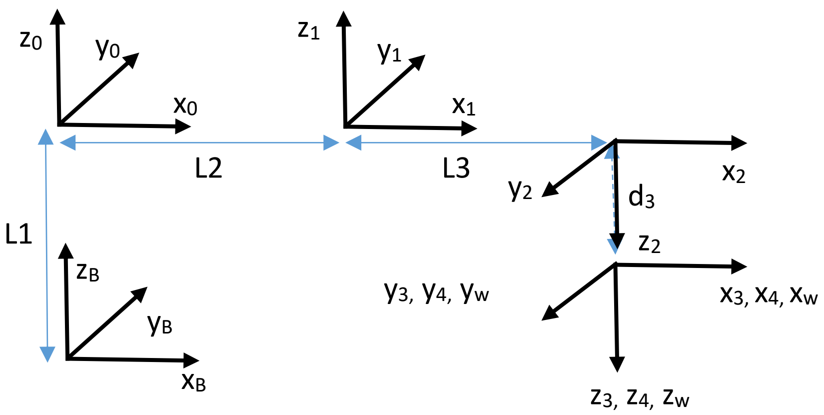
The Standard DH-parameters for the four axes RRPR manipulator mechanism are as follows:

Link	Θ_i	d_i	a_i	α_i	σ_i
1	Θ_1	0	L2	0	R
2	Θ_2	0	L3	180	R
3	0	d_3	0	0	P
4	Θ_4	0	0	0	R

In addition we will need a base transformation to contribute for the height of the “shoulder link” above the origin of the *B*-frame, L1 (along the z_B axis)

base transformation = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(*Note that alternatively we could have considered the L1-distance by giving the d_1 parameter a constant value of L1 (first row of the DH-parameter table). –* *this is because the translation L1 and Θ_1 take place along/around the very same axis*)



Note: The parameter d_3 is assumed to comprise of a minimum value and a controllable extension range. The figures show the situation when the extension part equals zero length. The 4th –frame and W-frame are the same in the drawing above. The 3rd and 4th frame are also the same in case θ_4 is zero.

Solution Modified DH:

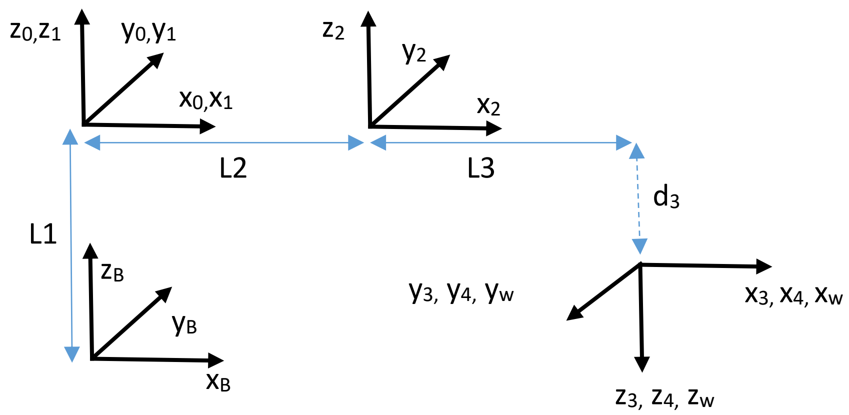
The Modified DH-parameters for the four axes RRPR manipulator mechanism are as follows:

Link	α_{i-1}	a_{i-1}	d_i	Θ_i	σ_i
1	0	0	0	Θ_1	R
2	0	L2	0	Θ_2	R
3	180	L3	d_3	0	P
4	0	0	0	Θ_4	R

In addition we will need a base transformation to contribute for the height of the "shoulder link" above the origin of the *B*-frame, L1 (along the z_B axis)

$$\text{base transformation} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(*Note that alternatively we could have considered the L1-distance by giving the d_1 parameter a constant value of L1 (first row of the DH-parameter table). – this is because the translation L1 and Θ_1 take place along/around the very same axis*)



Note: The parameter d_3 is assumed to comprise of a minimum value and a controllable extension range. The figures show the situation when the extension part equals zero length. The 4th –frame and W-frame are the same in the drawing above. The 3rd and 4th frame are also the same in case θ_4 is zero.

4. The task is to form an **orthonormal, right-handed** coordinate frame by means of three reference points (3x1 point vectors with x-, y- and z-coordinates given w.r.t. the world frame), $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$. Rules for creating the coordinate frame are as follows:

1. The origin of the coordinate frame should be located at point $\mathbf{P}_1$.
2. The x-axis of the orthogonal coordinate frame should be parallel to the vector pointing from $\mathbf{P}_1$ to $\mathbf{P}_2$.
3. The z-axis should be the parallel to the normal of the plane the basis of which is formed from the $\mathbf{P}_2$-$\mathbf{P}_1$ and $\mathbf{P}_3$-$\mathbf{P}_1$ vectors (i.e. the vectors $\mathbf{P}_2$-$\mathbf{P}_1$ and $\mathbf{P}_3$-$\mathbf{P}_1$ are parallel to the plane)
4. The y-axis is determined with right hand-rule to form an orthogonal coordinate frame.

Show the different steps and calculations to create an orthonormal **homogenous 4x4 transformation matrix** that describes the position and orientation of the new coordinate frame with respect to the reference frame. So, describe what kind of operations to do in which order to create the requested 4x4 homogenous transformation matrix by means of the three 3D-point vectors.

Tehtävänä on muodostaa suorakulmainen oikeakätinen koordinaatisto annetun kolmen referenssipisteen avulla (kutakin referenssipistettä vastaa 3x1-paikkavektori, johon on talletettu referenssipisteen x-, y- ja z-koordinaatit maailmankoordinaatiston suhteen), $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$. Koordinaatisto tulee luoda seuraavien sääntöjen mukaisesti:

1. Koordinaatiston origon tulee sijaita pisteessä $\mathbf{P}_1$.
2. Koordinaatiston x-akselin tulee olla samansuuntainen pisteestä $\mathbf{P}_1$ pisteeseen $\mathbf{P}_2$ viritetyn vektorin kanssa.
3. Koordinaatiston z-akselin tulee olla vektorien $\mathbf{P}_2$-$\mathbf{P}_1$ ja $\mathbf{P}_3$-$\mathbf{P}_1$ virittämän tason normaalin suuntainen (ts. *tason virittävät vektorit* $\boldsymbol{P}_2$-$\boldsymbol{P}_1$ *ja* $\boldsymbol{P}_3$-$\boldsymbol{P}_1$ *ovat tason itsensä suuntaisia*)
4. Koordinaatiston y-akseli määritetään oikean käden kiertosäännon avulla.

Esitä menetelmän eri vaiheet ortonormeeratun koordinaatiston paikkaa ja asentoa kuvaavan **4x4 homogeenisen muunnosmatriisin** muodostamiseksi kolmen 3D-referenssipisteen avulla. Kuvaa menetelmän eri vaiheet järjestyksessä. (13 points)

Solution:

The homogenous transformation matrix that describes the position and orientation of the new coordinate frame with respect to the reference frame is

$$\begin{bmatrix} \mathbf{X} & \mathbf{Y} & \mathbf{Z} & \mathbf{P} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$\mathbf{P} = \mathbf{P}_1$

$\mathbf{V}_1 = \mathbf{P}_2 - \mathbf{P}_1$

$\mathbf{V}_2 = \mathbf{P}_3 - \mathbf{P}_1$

$\mathbf{X} = \mathbf{V}_1 / \mathrm{norm}(\mathbf{V}_1)$

$\mathbf{Z} = \mathbf{V}_1 \times \mathbf{V}_2 / \mathrm{norm}(\mathbf{V}_1 \times \mathbf{V}_2)$

$\mathbf{Y} = \mathbf{Z} \times \mathbf{X}$

5. In the figure below a 3-axes RPP manipulator mechanism is illustrated. When the angle of the first joint, θ_1, is zero the upper arm is oriented parallel to the y_B-axis. An external force vector (wrench) ***W*** is exerted at the origin of the tool frame {T}. The force is marked with the red arrow in the figure. Alla olevassa kuvassa on esitetty 3-akselisen RPP robottimekanismin kinemaattinen rakenne. Kun ykkösnivelen ohjauskulma, θ_1, saa arvon nolla yläkäsivarsi asemoituu y_B-koordinaatiakselin yläpuolelle sen suuntaisesti. Ulkoinen voima, jota merkitään symbolilla ***W*,** kohdistetaan työkalukoordinaatiston {T} origoon. Ulkoista voimaa kuvaa punainen nuoli kuvassa.

The 3x3 Jacobian matrix to calculate the linear velocity of the origin of the tool frame expressed with respect to the base frame {B} as a function of the joint velocities, ***J***, is given in the equation below / Työkalukoordinaatiston origon lineaarinopeuksien laskemiseksi peruskoordinaatiston {B} akselien suunnissa nivelnopeuksien funktiona käytettävä 3x3 jakobiaanimatriisi, ***J***, on annettu alla olevassa yhtälössä

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \boldsymbol{J}\dot{\boldsymbol{q}} = \begin{bmatrix}\frac{dx}{d\theta_1} & \frac{dx}{dd_2} & \frac{dx}{dd_3}\\ \frac{dy}{d\theta_1} & \frac{dy}{dd_2} & \frac{dy}{dd_3}\\ \frac{dz}{d\theta_1} & \frac{dz}{dd_2} & \frac{dz}{dd_3}\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{d}_2\\ \dot{d}_3\end{bmatrix} = \begin{bmatrix}-\cos(\theta_1)\,d_3 & 0 & -\sin(\theta_1)\\ -\sin(\theta_1)\,d_3 & 0 & \cos(\theta_1)\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{d}_2\\ \dot{d}_3\end{bmatrix}$$

The value of the external force vector (or wrench) ***W*** is / Ulkoisen voimavektorin ***W*** arvo on

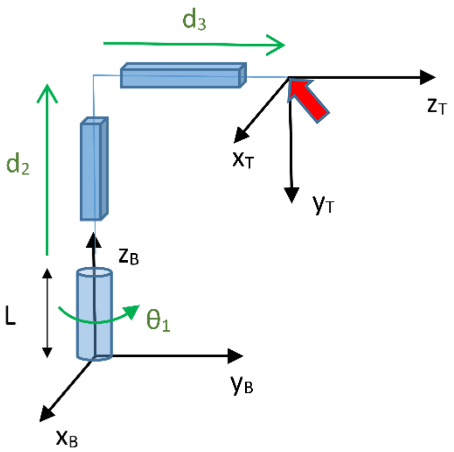
$${}^{B}\boldsymbol{W} = \begin{bmatrix}-5N\\ -5N\\ 0\end{bmatrix}$$

The values of the joint variables for the calculations are / Nivelohjauksien arvot laskentaa varten ovat

$\theta_1 = 0.0°$, $d_2 = 0.5m$, $d_3 = 0.7m$

The value for the constant base height of the mechanism is L=0.3m / Mekanismin rungon vakiokorkeusmitta L=0.3m

The task is to calculate torques and forces affecting joints 1, 2 and 3 due to the external force in the given configuration of the manipulator arm. To solve the problem **you must utilize the Jacobian matrix** of the manipulator. Tehtävänä on laskea ulkoisen voiman vaikutuksesta syntyvät mekanismin nivelmomentit ja –voimat nivelille 1, 2, ja 3. **Tehtävä on ratkaistava mekanismin Jakobiaani-matriisin avulla.** (10 points)



Solution:

First calculate the transpose of the Jacobian matrix

$$J^T = \begin{bmatrix} -\cos(\theta_1)\,d_3 & -\sin(\theta_1)\,d_3 & 0 \\ 0 & 0 & 1 \\ -\sin(\theta_1) & \cos(\theta_1) & 0 \end{bmatrix}$$

and the Jacobian transpose with numerical parameter values becomes

$$J^T = \begin{bmatrix} -\cos(0.0) * 0.7m & -\sin(0.0) * 0.7m & 0 \\ 0 & 0 & 1 \\ -\sin(0.0) & \cos(0.0) & 0 \end{bmatrix} = \begin{bmatrix} -0.7m & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Now we can calculate the joint torques/forces caused by the external force (wrench)

$$\boldsymbol{Q} = \begin{bmatrix} -0.7m & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -5N \\ -5N \\ 0 \end{bmatrix} = \begin{bmatrix} 3.5Nm \\ 0 \\ -5N \end{bmatrix}$$

ELEC-C1320 Robotiikka - Equations

Link parameters and the corresponding elementary transformations as well as the symbolic form of the link matrix according to the **standard** Denavit and Hartenberg parameter convention:

$$^{j-1}\xi_j\left(\theta_j, d_j, a_j, \alpha_j\right) = \mathscr{R}_z\left(\theta_j\right) \oplus \mathscr{T}_z\left(d_j\right) \oplus \mathscr{T}_x\left(a_j\right) \oplus \mathscr{R}_x\left(\alpha_j\right)$$

$$^{j-1}\boldsymbol{A}_j = \begin{pmatrix} \cos\theta_j & -\sin\theta_j\cos\alpha_j & \sin\theta_j\sin\alpha_j & a_j\cos\theta_j \\ \sin\theta_j & \cos\theta_j\cos\alpha_j & -\cos\theta_j\sin\alpha_j & a_j\sin\theta_i \\ 0 & \sin\alpha_j & \cos\alpha_j & d_j \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Link parameters and the corresponding elementary transformations as well as the symbolic form of the link matrix according to the **modified** Denavit and Hartenberg parameter convention:

$$^{j-1}\xi_j\left(\alpha_{j-1}, a_{j-1}, d_j, \theta_j\right) = \mathscr{R}_x\left(\alpha_{j-1}\right) \oplus \mathscr{T}_x\left(a_{j-1}\right) \oplus \mathscr{T}_z\left(d_j\right) \oplus \mathscr{R}_z\left(\theta_j\right)$$

$$^{j-1}\boldsymbol{A}_j = \begin{bmatrix} \cos\theta_j & -\sin\theta_j & 0 & a_{j-1} \\ \sin\theta_j\cos\alpha_{j-1} & \cos\theta_j\cos\alpha_{j-1} & -\sin\alpha_{j-1} & -\sin\alpha_{j-1}\,d_j \\ \sin\theta_j\sin\alpha_{j-1} & \cos\theta_j\sin\alpha_{j-1} & \cos\alpha_{j-1} & \cos\alpha_{j-1}\,d_j \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Elementary rotation transformations (i.e. rotations about principal axis by Θ):

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse of a 4x4 transformation matrix:

$$\boldsymbol{T}^{-1} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{t} \\ \boldsymbol{0}_{1\times3} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{R}^T & -\boldsymbol{R}^T\boldsymbol{t} \\ \boldsymbol{0}_{1\times3} & 1 \end{pmatrix} \tag{2.25}$$

Derivation of trigonometric functions:

Dsinx = cosx

Dcosx = - sinx

Definition of (manipulator) Jacobian matrix:

If $\boldsymbol{y} = F(\boldsymbol{x})$ and $\boldsymbol{x} \in \mathbb{R}^n$ and $\boldsymbol{y} \in \mathbb{R}^m$ then the Jacobian is the $m \times n$ matrix

$$J = \frac{\partial F}{\partial \boldsymbol{x}} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

Jacobian transpose transforms a wrench (a vector of forces and torques) applied at the end-effector, ${}^0\boldsymbol{W}$, to torques and forces experienced at the joints $\boldsymbol{Q}$:

$$\boldsymbol{Q} = {}^0\boldsymbol{J}(\boldsymbol{q})^T\, {}^0\boldsymbol{W} \tag{8.9}$$