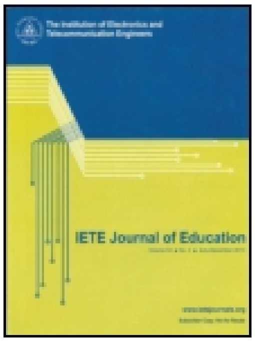

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On the Visualization of the Weaver's "Third Method" for SSB Generation

Murali Krishna Parekkat & Deepti Das Krishna

On the Visualization of the Weaver's "Third Method" for SSB Generation

Murali Krishna Parekkat[1] and Deepti Das Krishna[2]

[1]SPA Group, Naval Physical Oceanographic Laboratories (NPOL), Cochin 682021, India, [2]Department of Electronics and Communication, Rajagiri School of Engineering & Technology, Rajagiri Valley, Cochin 682039, India

ABSTRACT

The Weaver technique or the third method of generation of single sideband modulation with suppressed carrier (SSB-SC) amplitude modulation (AM) signals is one such algorithm which looks deceptively simple when compared with the more analytically tractable phase-shift method. With the advent of digital signal processing (DSP) techniques such as the CORDIC (coordinate rotation digital computer) algorithm, an elegant low-complex implementation of the Weaver technique is now feasible, which necessitates a better understanding of the underlying principle. However, despite its popularity, it is not explained in detail in conventional text books on communication engineering and in general literature. Using the background theory of complex signals (I and Q), in this paper, we have depicted the entire operation graphically. Combined with necessary mathematics and implementation ideas, this will be a comprehensive tutorial on the third method of SSB-SC generation and should motivate students to look at the underlying mathematical theory of problems in the area of communications and signal processing for similar interpretations.

Keywords:
CORDIC, Digital up-convertor, Quadrature signals, SSB generation, Weaver method.

INTRODUCTION

The single sideband modulation with suppressed carrier (SSB-SC) is a refinement of the amplitude modulation (AM) in terms of improved bandwidth and power efficiency. In order to generate the SSB-SC, three methods are generally described in the literature [1–3]. The first two methods are the filter method and the phasing method. The main drawback of the filter method is that it requires a filter which should have an ideal behaviour and in the case of the phasing method, it requires a broadband phase shifting circuit which used to be difficult to realize with an analogue implementation.

The third method, which was originally described by Donald K Weaver in 1956 [4], is termed the "Weaver's Method". Though similar to the phasing technique, this method uses quadrature signals generated at a fixed frequency which is less complex than implementing quadrature phase shift across the entire audio bandwidth. Also, the identical low-pass filters used are in the audio band which further reduces the implementation complexity. Moreover, techniques like the CORDIC (coordinate rotation digital computer) algorithm can be used to efficiently realize this method with a low-cost fixed point hardware [5, 6].

It is precisely the advent of such implementation options that have necessitated a relook for an improved understanding of the Weaver's method. We feel that with such an elucidation, there will be similar efforts to describe other algorithms used in modern communication engineering.

2. THE WEAVER ARCHITECTURE

The Weaver architecture for the SSB-SC generation is shown in the Figure 1. It consists of four balanced modulators, two carrier signal generators, two audio low-pass filters, and two 90° phase-shift networks. In practice, the phase-shift networks can be replaced by using a pair of quadrature oscillators. In the first stage, it makes an IQ modulation with f_o at the centre of the audio band. The filtered output of the mixers are given by the signals

$$V_1(t) = \text{low pass }(\text{Sin}2\pi f_m t \cdot \text{Sin}2\pi f_o t) = \frac{1}{2}\text{Cos}[2\pi(f_o - f_m)t] \tag{1}$$

$$V_2(t) = \text{low pass }(\text{Sin}2\pi f_m t \cdot \text{Cos}2\pi f_o t) = \frac{1}{2}\text{Sin}[2\pi(f_o - f_m)t]. \tag{2}$$

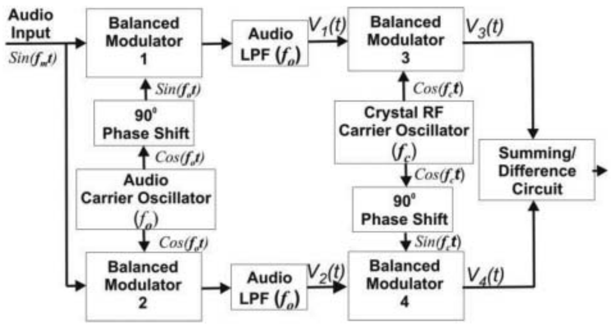

Figure 1: The Weaver method block diagram.

In the second stage, the two mixer outputs given in Eqs. (1) and (2) are IQ modulated with the RF carrier and is followed by a summing/difference circuit which generates the desired sideband signal,

$$V_3(t) = \frac{1}{2}(\mathrm{Cos}[2\pi(f_o - f_m)t]\cdot \mathrm{Cos}2\pi f_c t)$$
$$= \frac{1}{4}\mathrm{Cos}[2\pi(f_c + (f_o - f_m))t] + \frac{1}{4}\mathrm{Cos}[2\pi(f_c - (f_o - f_m))t] \tag{3}$$

$$V_4(t) = \frac{1}{2}(\mathrm{Sin}[2\pi(f_o - f_m)t]\cdot \mathrm{Sin}2\pi f_c t)$$
$$= \frac{1}{4}\mathrm{Cos}[2\pi(f_c + (f_o - f_m))t] - \frac{1}{4}\mathrm{Cos}[2\pi(f_c - (f_o - f_m))t]. \tag{4}$$

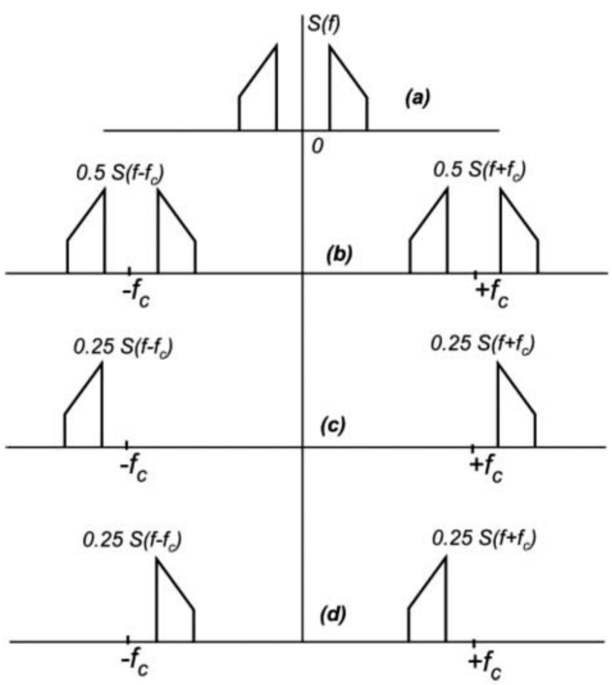

Figure 2: Amplitude spectrum of (a) Baseband signal. (b) Double sideband (DSB) signal. (c) Upper single sideband (USB) signal. (d) Lower single sideband (LSB) signal.

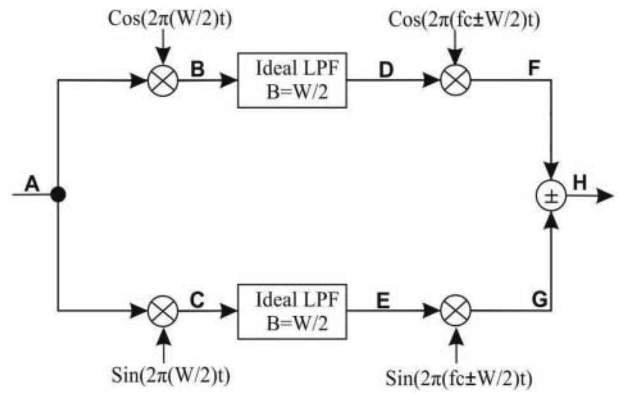

Figure 3: Signal flow graph for the Weaver's method.

From (3) and (4), it is evident that an addition operation will yield the upper sideband, while a subtraction will yield the lower sideband .

3. ANALYTICAL REPRESENTATION OF THE SSB-SC SIGNALS

Analytically single sideband has the form of a quadrature amplitude modulation in the special case where one of the baseband waveforms is derived from the other, instead of being independent messages [6] as shown below

$$s_{ssb}(t) = s(t)\cdot \mathrm{Cos}(2\pi f_c t) - \bar{s}(t)\cdot \mathrm{Sin}(2\pi f_c t), \tag{5}$$

where *s(t)* is the message, $\bar{s}(t)$ is its Hilbert transform, and f_c is the radio carrier frequency. Since *s(t)* is real-valued, its Fourier transform, *S(f)* is Hermitian symmetrical about the $f = 0$ axis as shown in Figure 2(a). Double sideband (DSB) modulation of *s(t)* to frequency

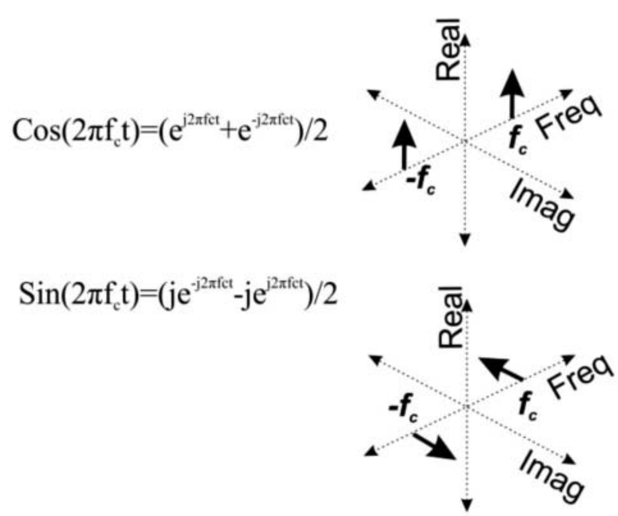

Figure 4: Complex frequency domain representation of sine and cosine functions.

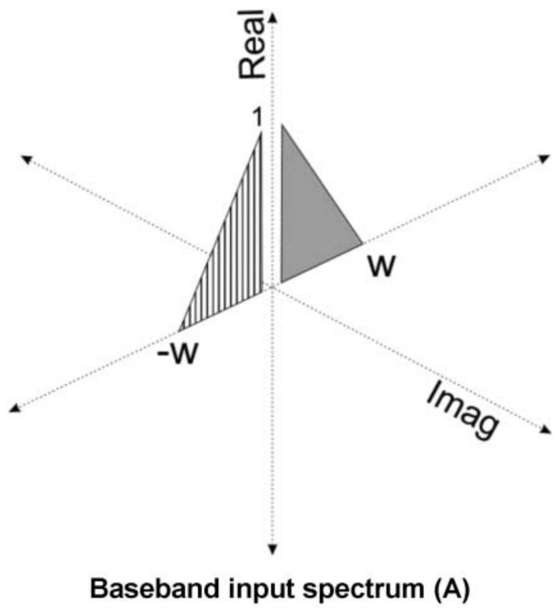

Figure 5: Baseband input spectrum (A).

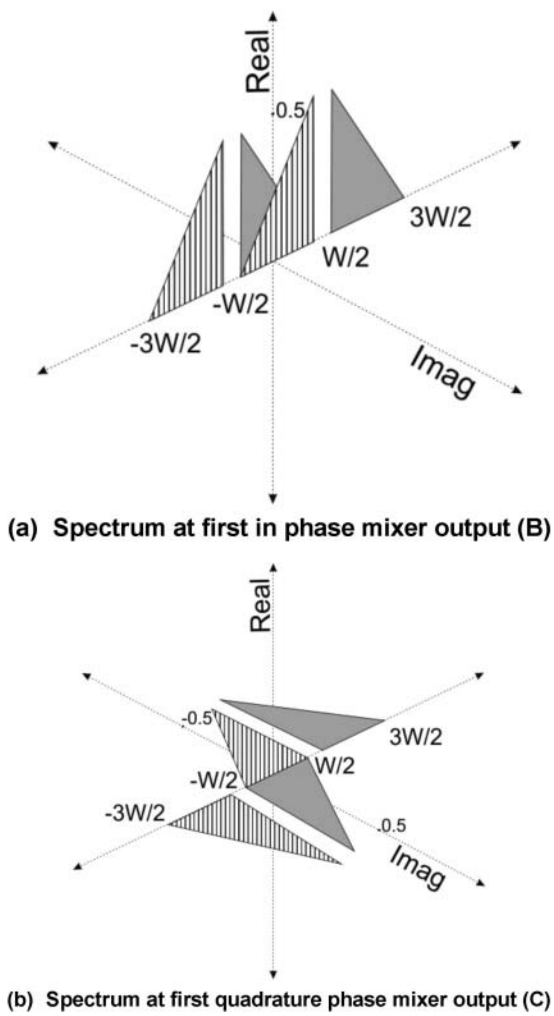

Figure 6 (a): Spectrum at first in phase mixer output (B). (b): Spectrum at first quadrature phase mixer output (C).

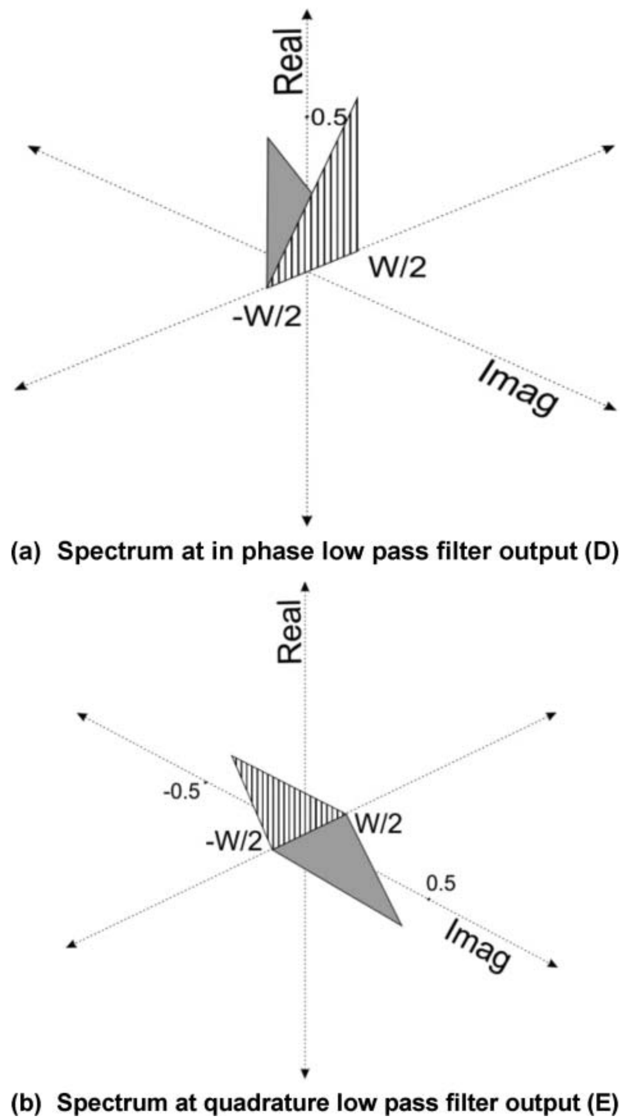

Figure 7 (a): Spectrum at in phase low pass filter output (D). (b): Spectrum at quadrature low pass filter output (E).

f_c moves the axis of symmetry to $f = \pm f_c$ and the two sides of each axis are called sidebands as shown in Figure 2(b). Single sideband modulation eliminates one sideband of each axis, while preserving *s(t)* as shown in Figure 2(c) and 2(d).

There are several manner of ways in which this can be achieved with the simplest technique being generating a DSB signal using a balanced modulator and then removing the undesired sideband with a band pass filter (filter method). The other technique is to implement Eq. (5) using the phase-shifting technique (phasing method). This will call for a broadband phase shifter, which can be implemented using the Hilbert transform, apart from a quadrature oscillator to generate the cosine and sine carriers.

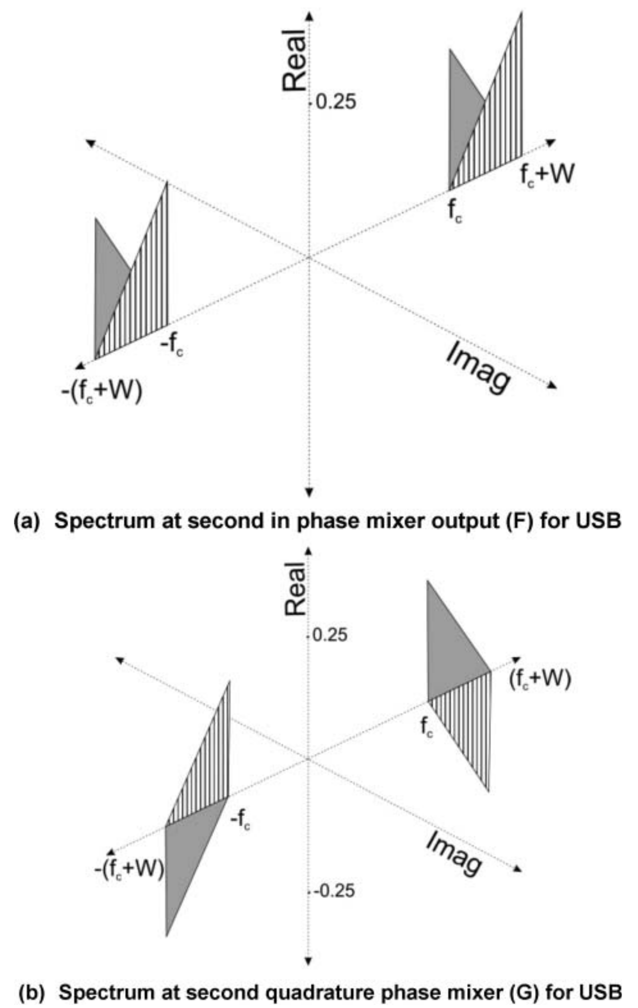

Figure 8 (a): Spectrum at second in phase mixer output (F) for USB. (b): Spectrum at second quadrature phase mixer (G) for USB.

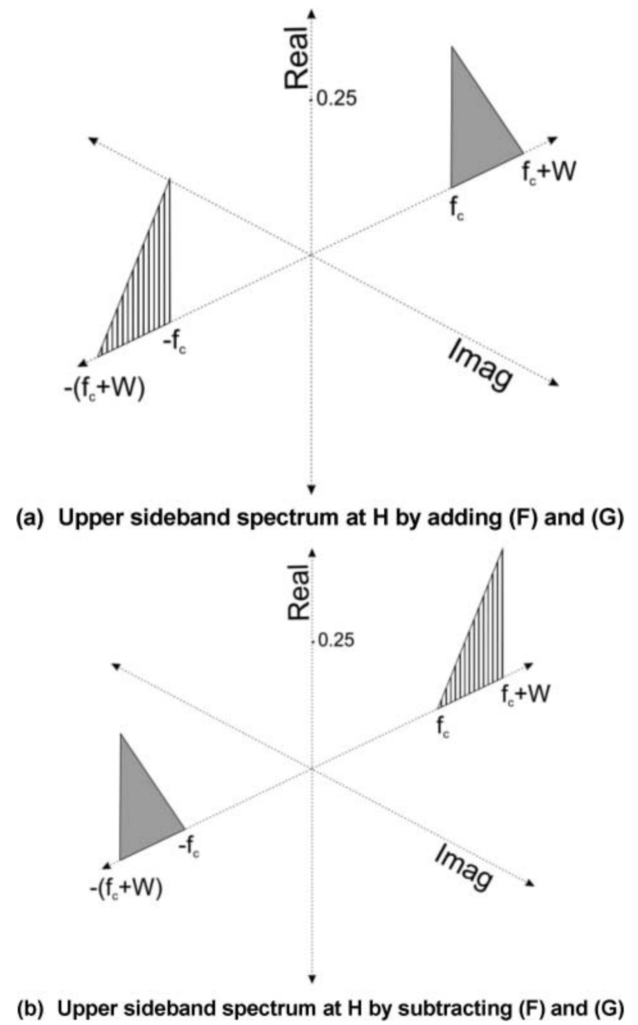

Figure 9 (a): Upper sideband spectrum at H by adding (F) and (G). (b): Upper sideband spectrum at H by subtracting (F) and (G).

While the first technique is very simple to explain and the second more analytically involved, the third method or the Weaver's method which was proposed as an alternative to both is elegant in the sense that it interweaves both the basic quadrature signal theory and a simple implementation. A solid understanding of the basic theory of quadrature signals can be obtained from [7]. In this paper, we make use of a similar approach to describe in a simple graphical manner the SSB-SC generation by means of the Weaver's method.

4. GRAPHICAL INTERPRETATION OF WEAVER

The Weaver method is illustrated in Figure 3 as a signal flow graph with labels at locations where the spectrum is described. The key to understand the entire process is to focus on the properties of quadrature signals. A pair of periodic signals are said to be in "quadrature" when they differ in phase by 90 degrees. The "in-phase" or reference signal is referred to as "I," and the signal that is shifted by 90 degrees (the signal in quadrature) is called "Q". An example of this is the sine wave and the cosine wave and its complex frequency domain representation is shown in Figure 4, where by convention the real part and imaginary parts are termed as I and Q components, respectively [7].

Keeping Eqs. (1)−(4) as reference, we can draw the spectra at each point in the signal flow graph. Starting with the point A, let the input to the modulator be a band limited real signal with symmetric Fourier transform as indicated in Figure 5. Note that the positive and negative bands are shaded differently in order to help in the illustration.

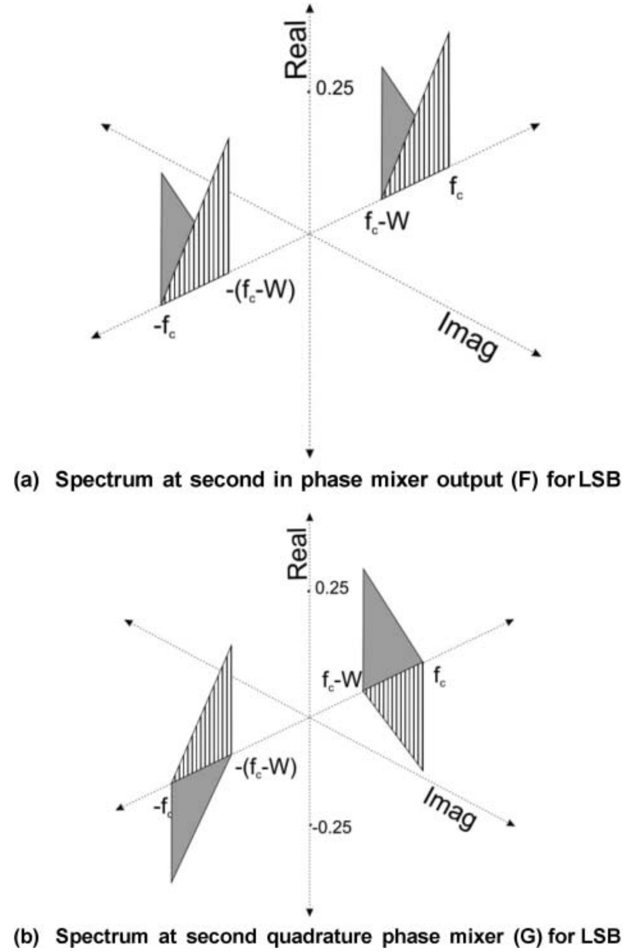

Figure 10 (a): Spectrum at second in phase mixer output (F) for LSB. (b): Spectrum at second quadrature phase mixer (G) for LSB.

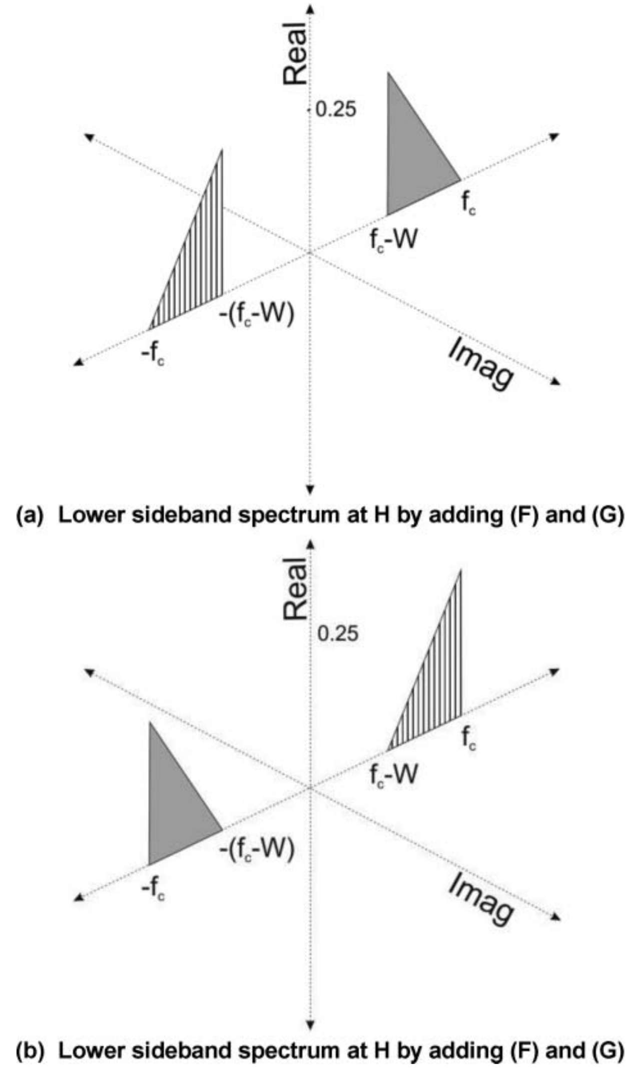

Figure 11 (a): Lower sideband spectrum at H by adding (F) and (G). (b): Lower sideband spectrum at H by subtracting (F) and (G).

At point B, in the signal flow graph, (the output of the first balanced modulator), the input band is shifted in both directions due to the multiplication with a cosine function and is indicated in Figure 6(a). A similar shift is also manifested at C due to the multiplication with a sine function which Figure 6(b) illustrates. Note that these two shifts are occurring in their respective axis (I and Q) only. The ideal low-pass filter will band limit the shifted signals to within $\pm W$ Hz.

Figure 7(a) and 7(b), respectively, illustrate the resultant spectra at points D and E. The second pair of quadrature oscillators and mixers is responsible for the final translation of the input spectrum to the sub bands around the desired carrier frequency. At this stage, it is critical to understand how the respective values of the mixing frequencies affect the overall spectral output.

In the analysis given by (3) and (4), the idea conveyed is that the two mixer frequencies (f_0 and f_c) are independent. This is in fact slightly misleading since the values of the two mixer frequencies are related as indicated in Figure 3. If the message signal is band limited to ±W Hz as indicated, then the first mixer must be at W/2 Hz and the second mixer must be at ($f_c \pm W/2$) Hz in order to achieve single-sideband modulation. It is this aspect which is clearly brought out in this paper. Following with the example, consider a situation where the upper sideband needs to be generated.

Figure 8(a) shows the spectral shift occurring as a result of the multiplication with the second cosine function at ($f_c + W/2$). It is the second multiplication with a sine function which is the critical step. From Figure 4, it can be seen that multiplication by $\mathrm{Sin}2\pi f_c t$ will cause the spectra to rotate by 90° at $-f_c$ and by $-90°$ at f_c, as given in Figure 8(b).

It is evident from the graphs in Figure 8(a) and 8(b) that either the addition or subtraction of these resultant spectra will yield upper sideband as given in Figure 9 (a) and 9(b). The only difference in the result is the spectral orientation of the sidebands. Note that for demodulation, the additive result will require a mixing by Cos $2\pi f_c t$ while the difference result will require a mixing by Cos $2\pi(f_c+W)t$.

In a similar manner, the lower sideband generation can also be illustrated. Figure 10(a) shows the spectral shift occurring as a result of the multiplication with the second cosine function at $(f_c - W/2)$. Similar to the case of the upper sideband, the multiplication with the second sine function will orient the spectra along the real axis as given in Figure 10(b).

From the graphs in Figure 10(a) and 10(b), it is clear that either on addition or subtraction, these spectra will yield the lower sideband as given in Figure 11(a) and 11(b) with the difference only in the spectral orientation. For demodulation, the difference in result needs to be mixed with Cos $2\pi f_c t$ while the additive result is to be mixed with Cos $2\pi(f_c-W)t$.

With reference to Figures 1 and 3 and the graphical analysis that has been conducted, one can conclude that the generation of the lower or upper sideband is not connected with the addition or subtraction of the resultant spectra. If the mixing frequencies are chosen as mentioned, either operation will give rise to an upper or lower sideband modulated spectrum. This fact remains hidden in the mathematical analysis and requires a graphical approach to make it visible.

5. CONCLUSION

In this paper, we have, through a graphical analysis technique, explored the Weaver method for the generation of the SSB signal. Starting with the basic theory of quadrature signals, we have attempted to demonstrate an alternate way in understanding and appreciating the underlying structure of this popular but very little understood technique. We hope that this example will motivate practitioners of DSP to relook at similar cases and try to arrive at graphical interpretations for the same. To conclude, we quote the verses of T. S. Eliot [8] "*We shall not cease from exploration, and the end of all our exploring will be to arrive where we started and know the place for the first time*".

REFERENCES

1. S. Haykin, *Communication Systems*, 3rd ed. Singapore: John Wiley & Sons, 2008.
2. B. P. Lathi, and Z. Ding, *Modern Digital and Analog Communication Systems*, 4th ed. Oxford: Oxford University Press, 2010.
3. H. Taub, and D. Schilling, *Principles of Communications Systems*, 3rd ed. Delhi: Tata McGraw Hill, 2008.
4. D. K. Weaver, "A third method of generation and detection of single-sideband signals," in *Proceedings of the IRE*, 1956, pp. 1703–5.
5. P. K. Meher, J. Valls, T.-B. Juang, K. Sreedharan, and K. Maharatna, "50 years of CORDIC: Algorithms, architectures, and applications," *IEEE Transactions on Circuits and Systems*, Vol. 56, no. 9, pp. 1893–1907, Sep. 2009.
6. Y. Huang, "The implementation of a digital SSB modulator based on folding transformation," *Applied Mechanics and Materials*, Vol. 543–547, pp. 524–527, Mar. 2014.
7. R. Lyons, *Understanding Digital Signal Processing*, 3rd ed. Prentice Hall Publishing, 2011.
8. T. S. Eliot, "Little Gidding," *Four Quartets.* London: Faber & Faber, 1944.

Authors

Murali Krishna Parekkat received the BTech degree in electronics engineering from Cochin University of Science & Technology (CUSAT), Cochin in 1996, the ME degree in communication engineering from PSG College of Technology, Coimbatore in 1999 and the PhD degree in electrical engineering from Indian Institute of Technology, Bombay in 2003. Since then he has been working with the Naval Physical Oceanographic Laboratory at Cochin under the Defence Research & Development Organisation as a scientist. His areas of interest include digital signal processing, wireless communications and applied linear algebra.

E-mail: mk.npol@gmail.com

Deepti Das Krishna received the BSc (Hons) degree, the MSc degree in electronics and the MTech degree in microwave technology in 1999, 2001, and 2002, respectively, from the University of Delhi South Campus, and the PhD degree in microwave technology in 2011 from Cochin University of Science & Technology (CUSAT). She is currently an associate professor with the Department of Electronics, Rajagiri School of Engineering & Technology, Cochin. Her areas of interest include antennas, microwaves, and wireless communications.

E-mail: deeptidk@rajagiritech.ac.in

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