Tesi

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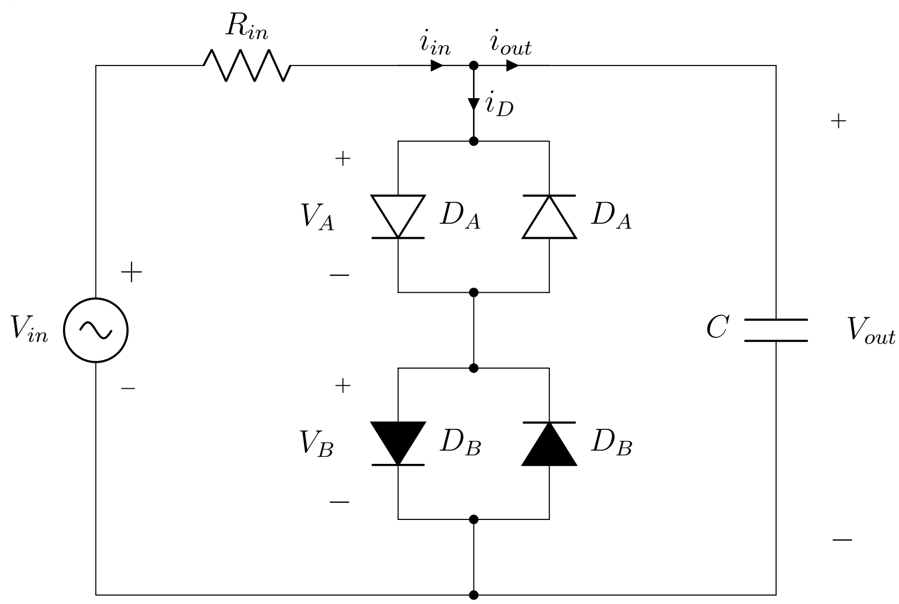
Circuito



$$i_D = \beta_A \left(e^{\alpha_A V_A(t)} - 1\right)$$
$$i_D = \beta_B \left(e^{\alpha_B V_B(t)} - 1\right)$$

Tensione V_A

$$\begin{aligned}
\beta_A\left(e^{\alpha_A V_A(t)}-1\right)-\beta_A\left(e^{-\alpha_A V_A(t)}-1\right) &= \beta_B\left(e^{\alpha_B V_B(t)}-1\right)-\beta_B\left(e^{-\alpha_B V_B(t)}-1\right)\\
\beta_A\left(e^{\alpha_A V_A(t)}-e^{-\alpha_A V_A(t)}\right) &= \beta_B\left(e^{\alpha_B V_B(t)}-e^{-\alpha_B V_B(t)}\right)\\
\cancel{2}\beta_A\left(\frac{e^{\alpha_A V_A(t)}-e^{-\alpha_A V_A(t)}}{2}\right) &= \cancel{2}\beta_B\left(\frac{e^{\alpha_B V_B(t)}-e^{-\alpha_B V_B(t)}}{2}\right)\\
\beta_A\sinh(\alpha_A V_A(t)) &= \beta_B\sinh(\alpha_B V_B(t))\\
\sinh(\alpha_A V_A(t)) &= \frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\\
\alpha_A V_A(t) &= \operatorname{arcsinh}\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)\\
V_A(t) &= \frac{1}{\alpha_A}\operatorname{arcsinh}\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)
\end{aligned} \tag{1}$$

Equazione differenziale

$$\begin{aligned}
V_{out}(t) &= V_{in}(t)-R_{in}\left[i_{out}+i_D\right]\\
V_{out}(t) &= V_{in}(t)-R_{in}\left[C\frac{\mathrm{d}V_{out}(t)}{\mathrm{d}t}+i_D\right]\\
V_{out}(t) &= V_{in}(t)-R_{in}\left[C\frac{\mathrm{d}V_A(t)}{\mathrm{d}t}+C\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}+i_D\right]\\
V_A(t)+V_B(t) &= V_{in}(t)-R_{in}\left[C\frac{\mathrm{d}V_A(t)}{\mathrm{d}t}+C\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}+\beta_B\left(e^{\alpha_B V_B(t)}-e^{-\alpha_B V_B(t)}\right)\right]\\
V_A(t)+V_B(t) &= V_{in}(t)-R_{in}\left[C\frac{\mathrm{d}V_A(t)}{\mathrm{d}t}+C\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}+2\beta_B\sinh(\alpha_B V_B(t))\right]\\
\frac{V_A(t)+V_B(t)}{R_{in}} &= \frac{V_{in}(t)}{R_{in}}-C\frac{\mathrm{d}V_A(t)}{\mathrm{d}t}-C\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}-2\beta_B\sinh(\alpha_B V_B(t))\\
C\frac{\mathrm{d}V_A(t)}{\mathrm{d}t}+C\frac{\mathrm{d}V_B(t)}{\mathrm{d}t} &= \frac{V_{in}(t)-V_A(t)-V_B(t)}{R_{in}}-2\beta_B\sinh(\alpha_B V_B(t))\\
\frac{\mathrm{d}V_A(t)}{\mathrm{d}t}+\frac{\mathrm{d}V_B(t)}{\mathrm{d}t} &= \frac{1}{C}\left(\frac{V_{in}(t)-V_A(t)-V_B(t)}{R_{in}}-2\beta_B\sinh(\alpha_B V_B(t))\right)
\end{aligned} \tag{2}$$

Derivo V_a

$$
\begin{aligned}
\frac{\mathrm{d}V_a(t)}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\alpha_A}\operatorname{arcsinh}\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)\right) \\
&= \frac{1}{\alpha_A}\cdot\frac{\mathrm{d}}{\mathrm{d}t}\left(\arcsin\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)\right) \\
&= \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)}{\alpha_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}} \\
&= \frac{\beta_B}{\beta_A}\cdot\frac{\frac{\mathrm{d}}{\mathrm{d}t}(\sinh(\alpha_B V_B(t)))}{\alpha_A\sqrt{1+(\sinh(\alpha_B V_B(t)))^2}} \\
&= \frac{\beta_B\cosh(\alpha_B V_B(t))\frac{\mathrm{d}}{\mathrm{d}t}(\alpha_B V_B(t))}{\alpha_A\beta_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}} \\
&= \frac{\alpha_B\beta_B\cosh(\alpha_B V_B(t))}{\alpha_A\beta_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}\cdot\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}
\end{aligned}
\tag{3}
$$

Sostituisco V_B e $\frac{\mathrm{d}V_B}{\mathrm{d}t}$ nella (2)

$$\frac{\alpha_B\beta_B\cosh(\alpha_B V_B(t))}{\alpha_A\beta_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}\cdot\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}+\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}=$$

$$=\frac{1}{C}\left(\frac{1}{R_{in}}\left(V_{in}(t)-\frac{1}{\alpha_A}\operatorname{arcsinh}\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)-V_B(t)\right)-2\beta_B\sinh(\alpha_B V_B(t))\right)$$

$$\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}\left(\frac{\alpha_B\beta_B\cosh(\alpha_B V_B(t))}{\alpha_A\beta_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}+1\right)=$$

$$=\frac{1}{C}\left(\frac{1}{R_{in}}\left(V_{in}(t)-\frac{1}{\alpha_A}\operatorname{arcsinh}\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)-V_B(t)\right)-2\beta_B\sinh(\alpha_B V_B(t))\right)$$

$$\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}\left(\frac{\alpha_B\beta_B\cosh(\alpha_B V_B(t))+\alpha_A\beta_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}{\alpha_A\beta_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}\right)=$$

$$=\frac{1}{C}\left(\frac{1}{R_{in}}\left(V_{in}(t)-\frac{1}{\alpha_A}\operatorname{arcsinh}\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)-V_B(t)\right)-2\beta_B\sinh(\alpha_B V_B(t))\right)$$

$$\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}=\frac{\alpha_A\beta_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}{\alpha_B\beta_B\cosh(\alpha_B V_B(t))+\alpha_A\beta_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}\cdot$$

$$\cdot\frac{1}{C}\left(\frac{1}{R_{in}}\left(V_{in}(t)-\frac{1}{\alpha_A}\operatorname{arcsinh}\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)-V_B(t)\right)-2\beta_B\sinh(\alpha_B V_B(t))\right)$$

$$\frac{\mathrm{d}V_B(t)}{\mathrm{d}t}=\frac{\alpha_A\beta_A}{C}\int\frac{\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}{\alpha_B\beta_B\cosh(\alpha_B V_B(t))+\alpha_A\beta_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}\cdot$$

$$\cdot\left(\frac{1}{R_{in}}\left(V_{in}(t)-\frac{1}{\alpha_A}\operatorname{arcsinh}\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)-V_B(t)\right)-2\beta_B\sinh(\alpha_B V_B(t))\right)\mathrm{d}t \tag{4}$$

Discretizzo la (4)

Metodo di Eulero all'indietro: $\dfrac{\mathrm{d}y}{\mathrm{d}x} \sim \dfrac{y_n - y_{n-1}}{h}$ quindi $\frac{y_n - y_{n-1}}{h} = f(x_n, y_n)$ e si ottiene $y_n = y_{n-1} + hf(x_n, y_n)$. Sia $T = h$ allora

$$
\begin{aligned}
\widehat{V}_B[n] = {} & \frac{\alpha_A \beta_A T}{C} \left(\frac{\sqrt{1 + \left(\frac{\beta_B}{\beta_A} \sinh(\alpha_B V_B[n])\right)^2}}{\alpha_B \beta_B \cosh(\alpha_B V_B[n]) + \alpha_A \beta_A \sqrt{1 + \left(\frac{\beta_B}{\beta_A} \sinh(\alpha_B V_B[n])\right)^2}} \cdot \right. \\
& \left. \cdot \left(\frac{1}{R_{in}} \left(V_{in}(t) - \frac{1}{\alpha_A} \operatorname{arcsinh}\left(\frac{\beta_B}{\beta_A} \sinh(\alpha_B V_B[n]) \right) - V_B[n] \right) - 2\beta_B \sinh(\alpha_B V_B[n]) \right) \right) + \\
& + \widehat{V}_B[n-1]
\end{aligned}
\tag{5}
$$

Algoritmo di punto fisso

$$
\begin{aligned}
x^{(\lambda+1)} &= x^{(\lambda)} - K(x^{(\lambda)})(x^{(\lambda)} - g(x^{(\lambda)})) \\
&= x^{(\lambda)} - \sum_{l=0}^{L} \left(J_f(x^{(\lambda)})\right)^l (x^{(\lambda)} - f(x^{(\lambda)}))
\end{aligned}
\tag{6}
$$

poiché $K^{(L)}(x) = \sum_{l=0}^{L} \left(J_f(x)\right)^l$

Applico l'algoritmo di punto fisso

$$
\begin{aligned}
x^{(\lambda+1)} = x^{(\lambda)} - \sum_{l=0}^{L} \left(J_f(x^{(\lambda)})\right)^l (x^{(\lambda)} - f(x^{(\lambda)})) \Rightarrow \\
\Rightarrow \widetilde{V}_B^{(\lambda+1)} = \widetilde{V}_B^{(\lambda)} - \sum_{l=0}^{L} \left(J_{V_B}(\widetilde{V}_B^{(\lambda)})\right)^l (\widetilde{V}_B^{(\lambda)} - \widehat{V}_B[\widetilde{V}_B^{(\lambda)}])
\end{aligned}
\tag{7}
$$

Calcolo la matrice jacobiana della funzione V_B

$$J_{V_B}(t) = \frac{\mathrm{d}V_B(t)}{\mathrm{d}t} = \frac{\alpha_A\beta_A}{C}\left(\frac{\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}{\alpha_B\beta_B\cosh(\alpha_B V_B(t)) + \alpha_A\beta_A\sqrt{1+\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right)^2}}\cdot\right.$$
$$\left.\cdot\left(\frac{1}{R_{in}}\left(V_{in}(t) - \frac{1}{\alpha_A}\operatorname{arcsinh}\left(\frac{\beta_B}{\beta_A}\sinh(\alpha_B V_B(t))\right) - V_B(t)\right) - 2\beta_B\sinh(\alpha_B V_B(t))\right)\right) \tag{8}$$