

Probability Final Project Report

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Section: L3

Github Link: <https://github.com/Maaaz-Shaheen/Probability-Final-Project-Random-Walker>

App link: <https://prob-project-ms05176.netlify.app/>

Task1:

Our model for this task was used to calculate the expected distance from the starting point after a number of steps. The default values for our model are 10000, each of 100 steps, with step size being a discrete random variable with values -1 and 1, each with equal probabilities. As the number of simulations increased, the expected value of the mean got closer to 0, as can be seen in the histogram Fig.1. below. If the probabilities are changed to favour one direction, the histogram shifts in the particular direction(Fig 2). The correctness of my implementation can be verified by looking at the absolute distance from the initial point. For the absolute distance with equal probabilities, we can use the formula in Fig 3 and we get the value 7.97... mathematically. If we calculate the expected absolute distance, then as we increase the number of simulations, we get closer to 7.9.

Task2:

One assumption made in this task is that the dots meet not when they are at the same point only, but when they have crossed each other too. The probabilities and step sizes are the same as task 1. The distance between the points is assumed to be 5 here. If we increase the absolute distance between them, the time for them to meet increases, however, except for a few outliers, they generally meet around the region. If the probabilities are increased in one direction, it does not seem to have an effect on the meeting times. But shifts the meeting place in the direction of the greater probability. The number of simulations is also kept at a 1000 here.

Task 3:

The particle was placed in a Cartesian plane at origin. It's step size is discrete from 0, 0.5 and 1. The orientation was to be taken as discrete from $[0 - 2\pi]$ so the values 0, $(\frac{1}{2})\pi$, π , and $(\frac{3}{2})\pi$ were assumed, each with an equal probability. The motion of the particle was decided using vectors, with a random value being chosen from the step size and the orientation and then converted to Cartesian values, with each value being added to the particles previous x and y position. The model of re-entry was decided to be in a manner similar to the Snake game, where exiting from the circular region at x,y would place you at the corresponding -x,-y region. Here the circular region was assumed to be spherical in thought, which means a particle existing the circle is not actually exiting but is moving on a spherical object, where the next point it is transferred to is just represented on the opposite end of the circle. Keeping in line

with this analogy, the correct distance the point the particle needed to be transferred to must also keep in mind the distance it has travelled already. Which has been factored for in the code.

Task 4:

Not much was changed except the step size was chosen from a continuous range from $[0 - 1]$. This is verified in a similar degree as task 1 also.

Task 5/6:

Not much was changed except the step size was chosen from a continuous range from $[0 - 1]$ and angle was chosen from a range of $[0 - 2\pi]$. 2π is not included as the orientation of 2π and 0 are the same, leading to an unfair bias in that direction, removing uniformity.

Task 7:

Not much was changed except the step size was chosen from a discrete range from $[0 - 1]$ i.e. -1 , 0 , and 1 , while orientation was kept continuous.

Task 8:

The majority of the details were carried over from task 5. However, the initial position of the nodes was carried out using the formulas in fig 4. This is due to the fact that we wished to assume a truly uniform randomness for the initial point of the two nodes. Just simply choosing a random value from $[0, 1]$ and from $[0, 2\pi]$ would result in more values closer to the center, which is why these formulas were used. As this simulation could run for very long per iteration, an upper limit of 100,000 steps was bound. And any simulation that did not have the nodes meet within this time was discarded. And the remaining expected value is calculated from the results that bore fruit. I could not find a way to verify this. The paper that I did find (reference 3 below) was extremely dense and required topics not studied yet. However, I believe the simulation does run correctly and would yield the correct result.

Task 9:

For this task, I extended the current application to study the spread of the corona epidemic. The charts on the TV all say “flatten the curve”, so I chose to simulate two types of scenario.

There are 200 dots denoting people on the screen. There is one infected person at the start. An infected person can infect another by coming within 12 units of another person. The simulation stops when all of the people are infected.

I ran the simulation in two scenarios. One with everyone having uniform probability to move in any direction with magnitudes $[0, 1]$. This was to simulate the quarantined situation, where people would not move much from their original position. In this case, on a simulation it took a long time for everyone to be infected, as seen in the figure 4 i.e. around 400 days.

In the other simulation, every dot tends to keep moving in the direction they were initialised in. When they hit another particle, they switch directions with each other. This was to simulate the environment where people were interacting. In this case it took an abysmally small time to infect everyone, around 20 days, showing that indeed, we should work to flatten the curve, otherwise a health care system will not be able to handle it.

Figures:

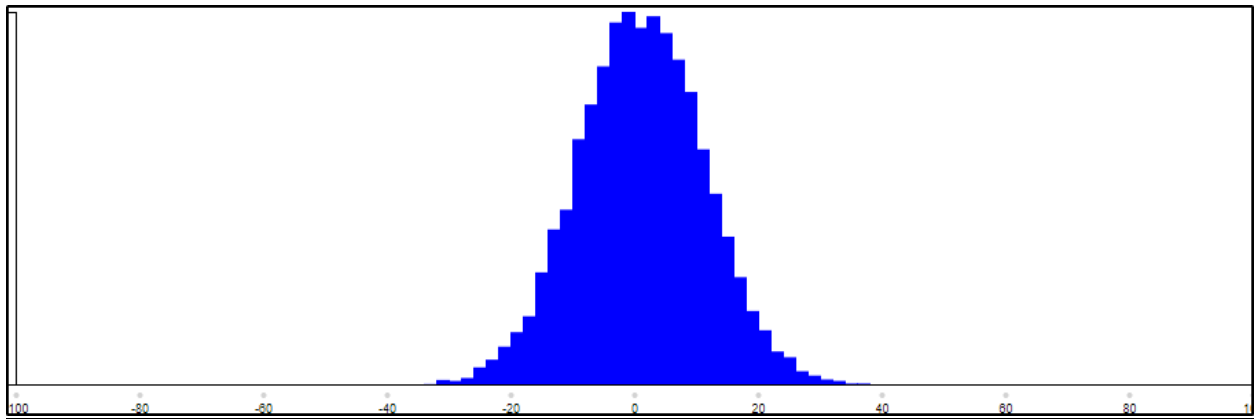


Fig. 1 – Even probability walks

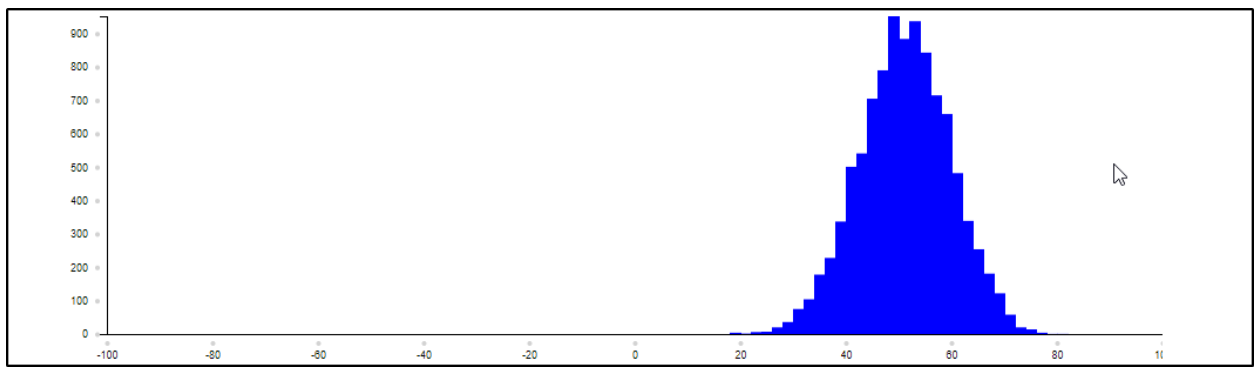


Fig. 2 – Uneven probability walks

$$\langle d_N \rangle \sim \sqrt{\frac{2N}{\pi}},$$

Fig. 3 – Absolute distance formula

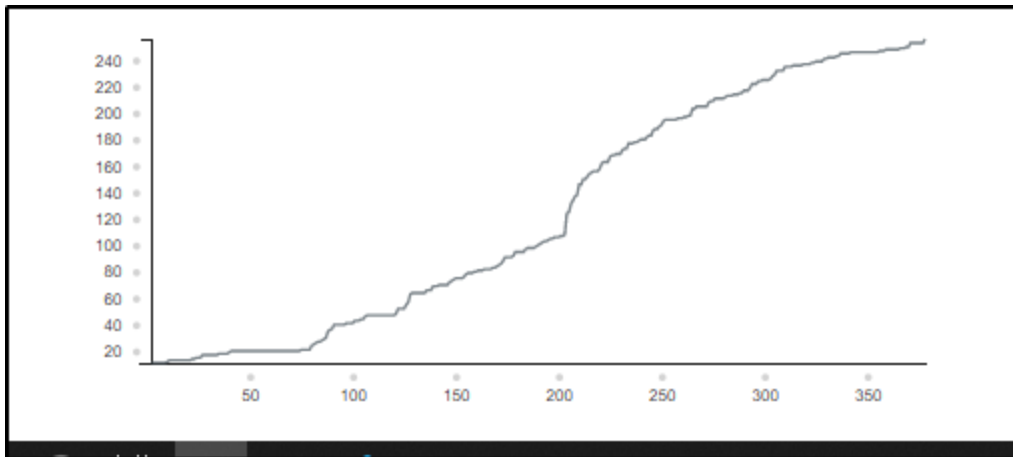


Fig. 4 – Quarantine spread

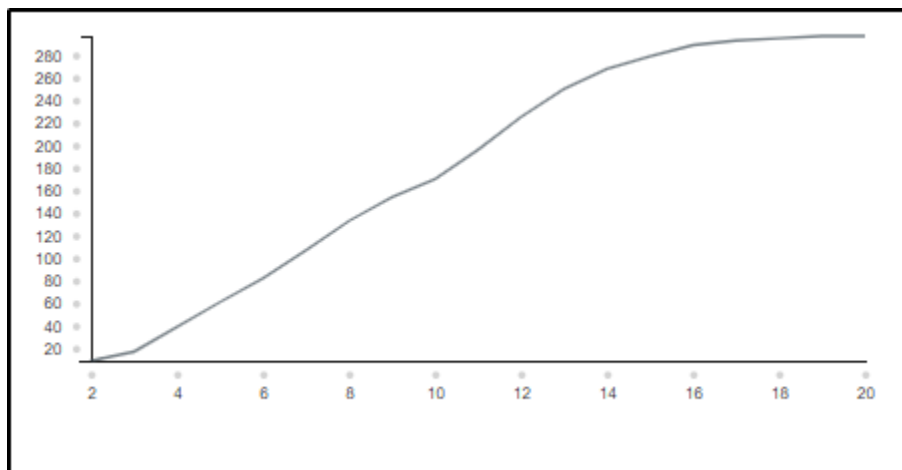


Fig 5 – No quarantine spread

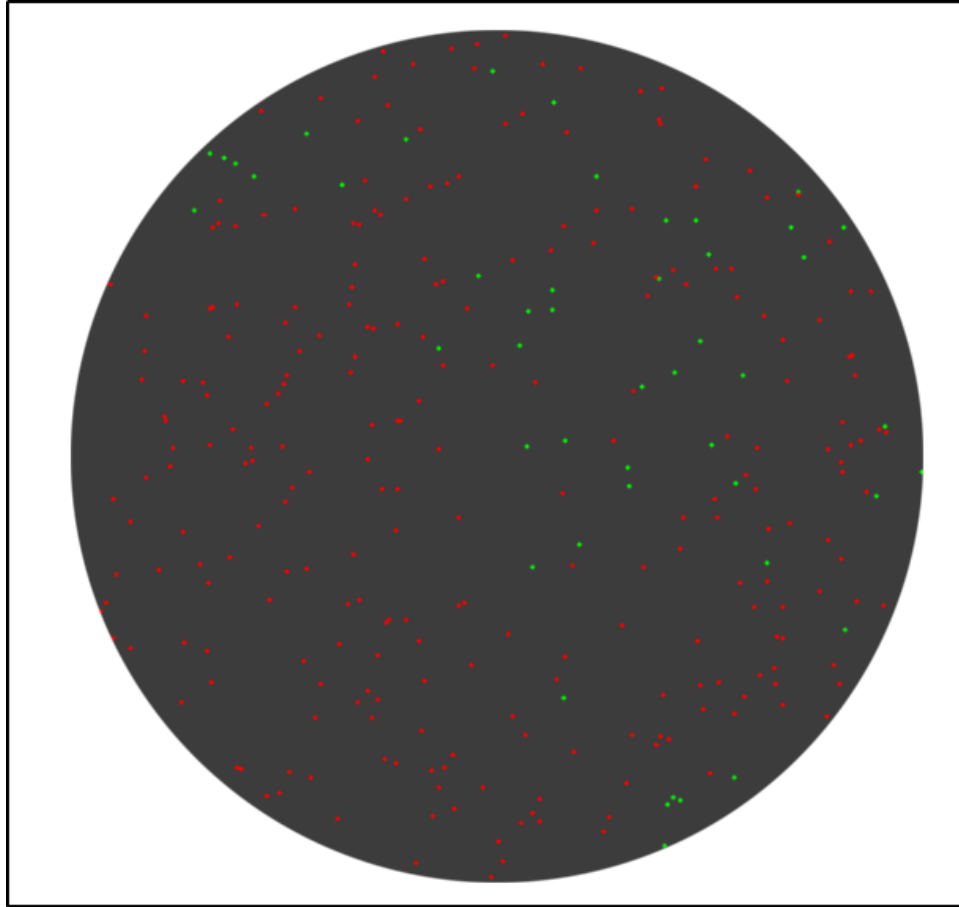


Fig 6 – The Spread of the Epidemic

References:

1. <https://mathworld.wolfram.com/RandomWalk1-Dimensional.html>
2. <https://stackoverflow.com/a/50746409>
3. https://www.researchgate.net/figure/Encounter-probability-of-two-random-walkers-initially-placed-at-x-1-x-2-where-x-1_fig3_231110695
4. <https://www.washingtonpost.com/graphics/2020/world/corona-simulator/>