



THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME001 Mathematics Part 2 Examination (Science & Engineering)

Examination Session
Semester Two

Time Allowed
2 Hours 10 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

- SECTION A** **Answer ALL questions. This section carries 40% of the exam marks.**
- SECTION B** **Answer 4 questions. This section carries 60% of the exam marks.**

The marks for each question are indicated in square brackets [].

Your School or College will provide a Formula Booklet.

- **Answers must not be written during the first 10 minutes.**
- Write your Candidate Number clearly on the answer books in the space provided.
- Write the answers in the answer books provided. Additional sheets will be provided on request.
- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.
- **No** written material is to be brought into the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Full marks will only be given for **full and detailed answers**.

Section A

Answer ALL questions. This section carries 40 marks.

Question A1

A curve is defined as $y = \frac{x^3}{1+x^2}$

Find the gradient of the curve when $x = 2$.

[4]**Question A2**

A function is defined as $f(x) = \frac{3x+2}{5}$.

Explain why $f(x)$ has an inverse, and write down an expression for $f^{-1}(x)$.

[3]**Question A3**

Solve the equation $2 \sin^2 \theta - \cos \theta = 2$ for $0 \leq \theta \leq 2\pi$.

Give your answers as exact multiples of π .

[5]**Question A4**

Use the substitution $u = 1 + x^3$ to evaluate $\int_0^1 \frac{x^2}{1+x^3} dx$

[5]**Question A5**

The curve $y = x^2 + 3$ is rotated about the x -axis. Find the exact volume of the solid formed between $x = 1$ and $x = 2$.

[5]**Question A6**

Vectors \mathbf{a} and \mathbf{b} are defined as $\mathbf{a} = 3\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 5\mathbf{j} + p\mathbf{k}$.

Find the value of p if \mathbf{a} is perpendicular to \mathbf{b} .

[3]

Question A7

Solve the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$

given $y = 1$ when $x = 1$. In your answer, express y in terms of x .

[4]**Question A8**

Given that $x = \ln(\tan y)$, find the value of $\frac{dy}{dx}$ when $y = \frac{\pi}{4}$.

[4]**Question A9**

The function f is defined as $f(x) = x^3 - 25$.

An approximate solution to the equation $f(x) = 0$ is $x_0 = 3$.

Use the Newton-Raphson method, **twice**, to give a better approximation.

[4]

Give your answer to four significant figures.

In this question one mark will be awarded for the correct use of significant figures.

Question A10

The ages of 6 children are 7, 12, 10, 9, 7, 3.

Find the mean, mode and median.

[3]

Section B
Answer 4 questions. This section carries 60 marks.

Question B1

a) A curve C has equation $x^2 + 2xy + y^3 = 13$.

i. Find $\frac{dy}{dx}$ in terms of x and y . **[5]**

ii. Write down an expression for $\frac{dx}{dy}$ in terms of x and y . **[1]**

iii. Find the value of $\frac{dx}{dy}$ at the point (3,2). **[1]**

b) i. Show that the equation $x^2 - 12 = 0$ has a root between **[3]**
 $x = 4.0$ and $x = 4.3$

ii. Show that $x = \frac{1}{2} \left[x + \frac{17}{x} \right]$ can be written as $x^2 - 17 = 0$. **[2]**

iii. A better approximation to $x^2 - 17 = 0$ can be obtained by using the
 iterative formula $\frac{1}{2} \left[x_n + \frac{17}{x_n} \right]$

Starting with $x_0 = 4.3$ find x_1 and x_2 **[3]**

Question B2

a)

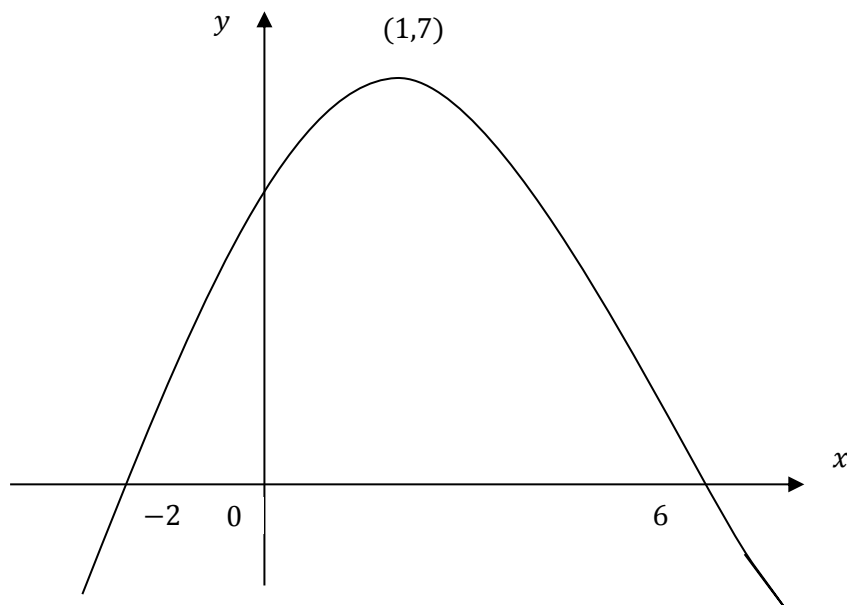


Figure 1

Figure 1 shows the curve $y = f(x)$. It crosses the x -axis at $(-2, 0)$ and $(6, 0)$. It also has a maximum at $(1, 7)$.

On separate axes, sketch the following: (On each sketch show clearly the coordinates where the curve crosses the x -axis, and the coordinates of the maximum point.)

i. $y = 2f(x)$ [3]

ii. $y = f(x - 3)$ [3]

iii. $y = f(-x)$ [3]

b) The functions $g(x)$ and $h(x)$ are defined as follows:

$$g(x) = 5x - 3 \text{ and } h(x) = x^2 + 1$$

i. Evaluate $g(h(2))$. [2]

ii. Solve the equation $h(g(x)) = 65$. [4]

Question B3

- a) i. Given $\tan 60^\circ = \sqrt{3}$ and $\tan 45^\circ = 1$, use a trigonometric formula [3]
to find the exact value of $\tan 15^\circ$.
(You must show each stage of your working.)
- ii. Write down the exact value of $\cot 165^\circ$. [3]
- b) i. Given $\cos 2x = \cos^2 x - \sin^2 x$ show that $\cos 2x = 1 - 2 \sin^2 x$. [2]
- ii. Solve the equation $\cos 2x + \sin 2x = 1$ for $0^\circ \leq x \leq 360^\circ$. [7]

Question B4

- a) i. Find $\int \frac{1}{\cos^2 x} dx$ [2]
- ii. Use the substitution $u = 1 + x$ to evaluate $\int_0^1 \frac{x}{1+x} dx$. [5]
- b) i. Express $\frac{5x}{(x-2)(x^2+1)}$ in the form $\frac{A}{x-2} + \frac{Bx}{x^2+1} + \frac{C}{x^2+1}$ [4]
- ii. Find $\int \frac{5x}{(x-2)(x^2+1)} dx$. [4]

Question B5

- a) i. Find the vector equation of the line l_1 which joins $A(2, -1, 3)$ and $B(5, 1, -3)$. [1]

The line l_2 passes through $A(2, -1, 3)$ and $C(4, 1, 2)$.

- i. Show that the cosine of acute angle CAB is $\frac{16}{21}$. [4]
- iii. Find the exact value of the sine of angle CAB . [2]
- iv. Find the exact area of triangle ABC . [2]
- b) Two lines have vector equations $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + t(-2\mathbf{i} + 3\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} + s(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$.
Show that these lines do **not** intersect. [6]

Question B6

- a) i. Give an example of discrete data. [1]
- ii. Seven readings are recorded: 7, 10, k , 3, 2, 9, 12.
Find k if the mean of these readings is 6. [2]
- iii. Find the mean and standard deviation of 3, 7, 8, 10, 11, 15. [3]
- b) 200 students took part in solving a Mathematics problem. The times that it took them to complete it are shown below.

Time t (seconds)	Frequency
$40 \leq t \leq 60$	20
$60 < t \leq 70$	42
$70 < t \leq 75$	44
$75 < t \leq 80$	40
$80 < t \leq 90$	30
$90 < t \leq 120$	24

(You may wish to copy and extend the table to help you answer some of the questions below.)

- i. Estimate the mean. [3]
- ii. State the modal interval. [1]
- iii. Work out the frequency densities. [2]
- iv. Draw a histogram to represent the data. [3]