



THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME001 Mathematics Part 2 (Science & Engineering) Examination

Examination Session
Semester Two

Time Allowed
2 Hours 10 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A Answer ALL questions. This section carries 40% of the exam marks.

SECTION B Answer 4 questions. This section carries 60% of the exam marks.

The marks for each question are indicated in square brackets [].

Your School or College will provide a Formula Booklet.

- **Answers must not be written during the first 10 minutes.**
- Write your Candidate Number clearly on the answer books in the space provided.
- Write the answers in the answer books provided. Additional sheets will be provided on request.
- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.
- **No** written material is to be brought into the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Full marks will only be given for **full and detailed answers**.

Section A

Answer ALL questions. This section carries 40 marks.

Question A1

Differentiate $y = e^{-3x} \ln(\sin x)$ with respect to x .

[3]

(You do not need to simplify your answer.)

Question A2

The function $f(x)$ is given by $f(x) = x^2 - 8x + 15$ with domain $1 \leq x \leq 5$.

Find the range of $f(x)$.

[4]

Question A3

Angle A is obtuse and $\sin A = \frac{5}{13}$.

Without working out the value of angle A , find the exact value of $\sin 2A$.

You must show each stage of your working.

[4]

Question A4

Use integration by parts to find the exact value of $\int_0^1 x e^{2x} dx$ giving your answer to **four significant figures**.

[6]

In this question one mark will be awarded for the correct use of significant figures.

Question A5

Vectors \mathbf{a} and \mathbf{b} are defined as $\mathbf{a} = 8\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}$.

[4]

Write down the unit vector $\mathbf{a} - \mathbf{b}$.

Question A6

If $y = \frac{x}{1+x^2}$, find $\frac{dx}{dy}$ when $x = 3$.

[5]

Question A7

An iterative formula is defined as $x_{n+1} = \frac{1}{2} \left[x_n + \frac{5}{x_n} \right]$

Starting with $x_0 = 2.8$, work out the values of x_1 , x_2 and x_3 .

[3]**Question A8**

The curve $y = \sec x$ is rotated about the x-axis.

Find the volume formed between $x = \frac{1}{4} \pi$ and $\frac{1}{3} \pi$.

[4]**Question A9**

Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{1+x^2}$

[4]

(In your solution, you do not need to write y in terms of x .)

Question A10

The mean age of a group of 7 children is 11 years. When another child joins the group, the mean is now 10 years. Find the age of the child who joins the group.

[3]

Section B

Answer 4 questions. This section carries 60 marks.

Question B1

- a) i. The function f is defined by $f(x) = x^3 - 6x^2 + 20$.
- [3]
- Show that $f(x)$ has a root between $x = 2$ and $x = 3$.
- ii. Starting with the initial value $x_0 = 2.3$, use the Newton-Raphson Method **once** to give a better approximation to the root of the equation in part i.
- [3]
- Give your answer to **four** significant figures.
- b) A curve C has equation $3x^2 + 2xy^2 + y = 13$
- i. Find $\frac{dy}{dx}$ in terms of x and y .
- [5]
- ii. Find the equation of the normal to curve C at the point $(1, 2)$.
- [4]
- Give your answers to **one** decimal place.

Question B2

- a) Given that $\cos 60^\circ = \frac{1}{2}$ and $\cos 45^\circ = \frac{1}{\sqrt{2}}$
- i. Show that $\sin 60^\circ = \frac{\sqrt{3}}{2}$
- [2]
- ii. Find the exact value of $\cos 15^\circ$.
- [3]
- You must show each stage of your working.
- b) i. Use a trigonometric identity to show that $\csc^2 x = \cot^2 x + 1$.
- [3]
- ii. Solve the equation $2 \csc^2 x - 5 \cot x = 5$ for $0 \leq x \leq 360^\circ$.
- [7]
- Give your answers to one decimal place.

Question B3

- a) i. Sketch the graph of $y = |2x - 6|$ showing clearly the points where the graph crosses the x - and y -axes. [3]
- ii. Find the values of x satisfying the equation $|2x - 6| = 9$. [3]
- b) Functions f and g are defined as follows:
- $f(x) = e^{3x+1}$ and $g(x) = x^2 - 4$.
- i. Write down the exact value of $f[g(3)]$. [2]
- ii. Write down an expression for $f^{-1}(x)$. [3]

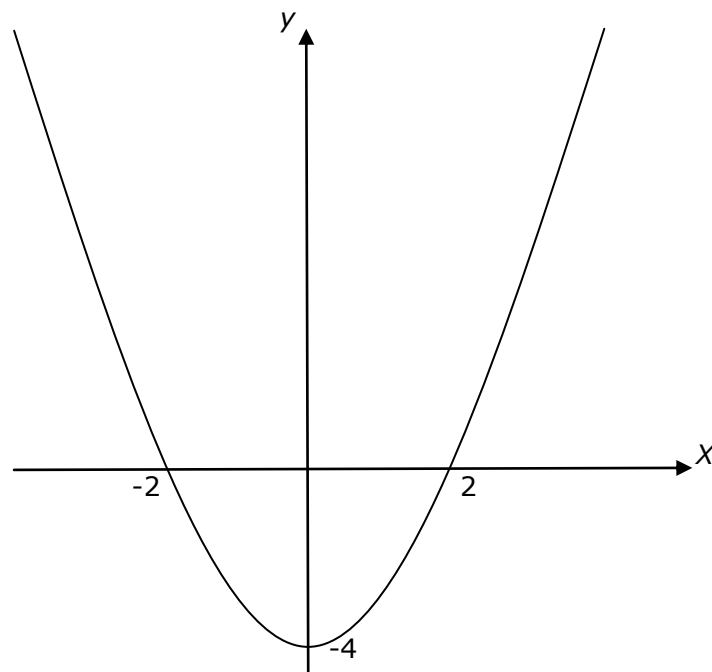


Figure 1

Figure 1 shows the graph of $y = g(x)$ (which is not drawn to scale).

- iii. Draw a sketch of $y = 2g(x - 2)$. On your sketch, show clearly the coordinates of the minimum point, and where the graph crosses the x - and y - axes. [4]

Question B4

- a) i. Use a trigonometric identity to find $\int \cos^2 x \, dx$. [4]
- ii. Use the substitution $u = 1 + x^3$ to find $\int 6x^2(1 + x^3)^7 \, dx$. [4]
- b) i. Express $\frac{4}{(3+x)(1-x)}$ in the form $\frac{A}{3+x} + \frac{B}{1-x}$ [3]
- ii. Show that the area beneath the curve $y = \frac{4}{(3+x)(1-x)}$ the lines $x = -2$ and $x = -1$ is $\ln 3$. [4]

Question B5

The line l_1 has equation $\mathbf{r} = (\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + t(2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$ and

the line l_2 has equation $\mathbf{r} = (-3\mathbf{i} + \mathbf{j} + 7\mathbf{k}) + s(5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$ where

t and s are scalars.

- a) Show that the lines l_1 and l_2 intersect, and find the coordinates of the point of intersection A. [6]
- b) Find the acute angle between the lines l_1 and l_2 . [5]
- Point B lies on the line l_1 and has coordinates $(-3, -8, k)$.
- c) Find the value of k . [1]
- d) Find the shortest distance between B and the line l_2 . [3]

Question B6

- a) The midnight temperatures were recorded in Manchester during one week in February. The readings were 3, 0, -1, 2, 0, 4, 6 °C.

i. Find the mean. **[2]**

ii. Find the standard deviation. **[3]**

- b) The numbers of apples were counted on 120 trees and the results are shown below.

Number of apples	Frequency
80 - 90	4
91 - 100	6
101 - 110	12
111 - 120	18
121 - 130	34
131 - 140	36
141 - 150	10

(You may want to copy and extend the table to help you answer some of the questions below)

i. Estimate the mean. **[3]**

ii. Work out the cumulative frequencies **[1]**

iii. On the graph paper provided draw a cumulative frequency curve. **[3]**

iv. Use your curve to estimate the median and interquartile range. **[2]**

v. The frequency distribution could be described as **[1]**

(choose one of the answers **A**, **B**, **C** or **D** below)

A Positively skewed

B Negatively skewed

C Uniform

D Bimodal