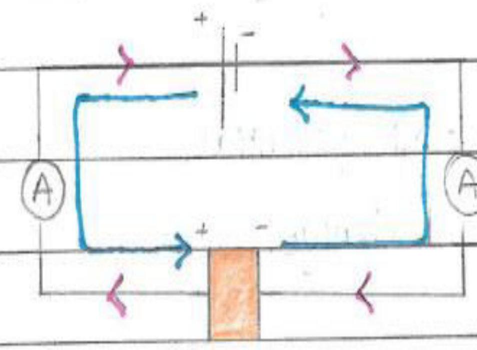
Capacitance

What is a capacitor?

A capacitor stores electrostatic energy (charge)



e⁻ flow

conventional current.

dielectric medium (insulator)

The negative terminal of the power supply pushes electrons onto one plate, making it negatively charged. Electrons are repelled from the other plate, making it positively charged. The capacitor is fully charged when the two plates have equal and opposite charges, and the two ammeters give the same reading. To discharge the capacitor, connect it's two leads together, and the electrons would flow back.

$$k = \frac{\varepsilon}{\varepsilon_0}$$

ε = permitivity of medium

ε_0 = permitivity of free space

k = di-electric constant / relative permitivity.

Why do capacitors store energy, and not charge?

When charging a capacitor, work is done to move electrons from one plate, and push them onto the other plate. The electrons that are deposited onto the other plate repel the further incoming electrons, so work has to be done. So the electrons have energy, ∴ energy is stored on a capaction.

The net charge on the two plates would be zero since they are equal and opposite, ∴ energy is stored.

Explain why the capacitor stores energy but not charge:

- Charges on plates are equal and opposite , hence there is no resultant charge; energy is stored because there is a charge separation

Uses of capacitors:

- blocking DC
- Smoothing the output of a rectifier
- time delay in electronic circuits
- tuning circuits in radio receivers.
- Store energy in electric circuits.

THE CAPACITANCE OF A CAPACITOR

The **capacitance** of a capacitor is the charge stored on one plate per unit of potential difference between the plates. Given by the equation:

$$C = \frac{Q}{V}$$

- *Q* is the magnitude of the charge on each of the capacitor's plates
- *V* is the potential difference across it

The capacitance of a capacitor is directly proportional to the area of the plates and inversely proportional to the distance between the plates

That is,

Parallel plate capacitor:

$$C = \frac{A \varepsilon_0 k}{d}$$

A = area of plates

k = dielectric constant

d = seperation of plates.

The unit of capacitance is the **FARAD (F)**

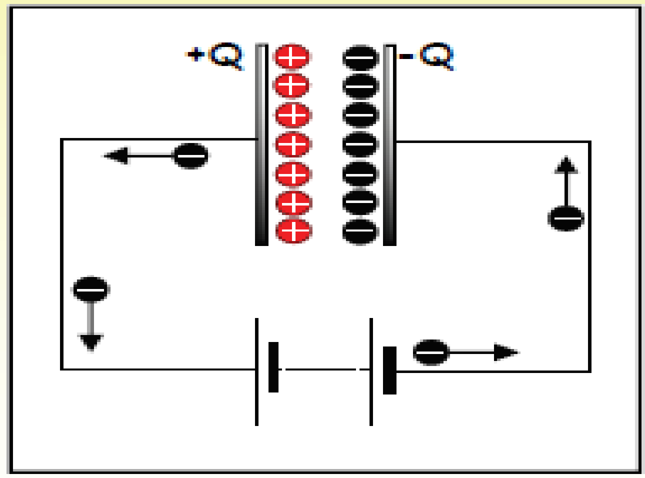
1 farad = 1 coulomb per volt

1 F = 1 C V^{-1}

CHARGING OF A PARALLEL-PLATE CAPACITOR

*Two parallel, metal plates placed close to each other form a capacitor. When such a capacitor is connected to a battery, one of the plates **gains electrons** and so becomes **negatively charged**.*

*This causes an equal number of electrons to be **repelled** from the other plate, which then becomes **positively** charged. The arrival of electrons at one plate and the repulsion of electrons from the other occurs simultaneously.*



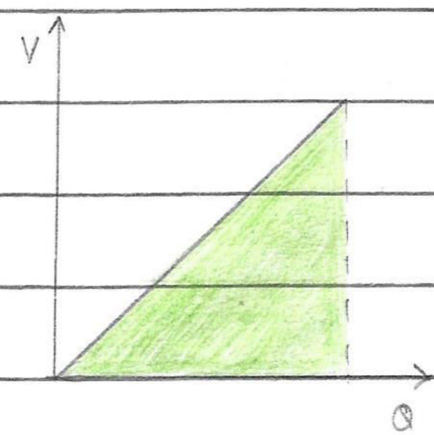
If one plate stores charge −Q, the other stores charge +Q and we say that charge Q is stored.

The amount of charge stored depends on the POTENTIAL DIFFERENCE (VOLTAGE) of the supply to which the plates are connected.

During charging of a capacitor, power supply is used to push electrons from one plate to another; power supply does work on the electrons, increasing their potential energy, which is recovered during discharging

Calculating work done/energy stored in a capacitor.

$$W = \tfrac{1}{2}QV$$
$$= \tfrac{1}{2}CV^2$$
$$= \tfrac{1}{2}\frac{Q^2}{C}$$



work done = area under V-Q graph.

- The area under a graph of p.d. against charge is equal to work done

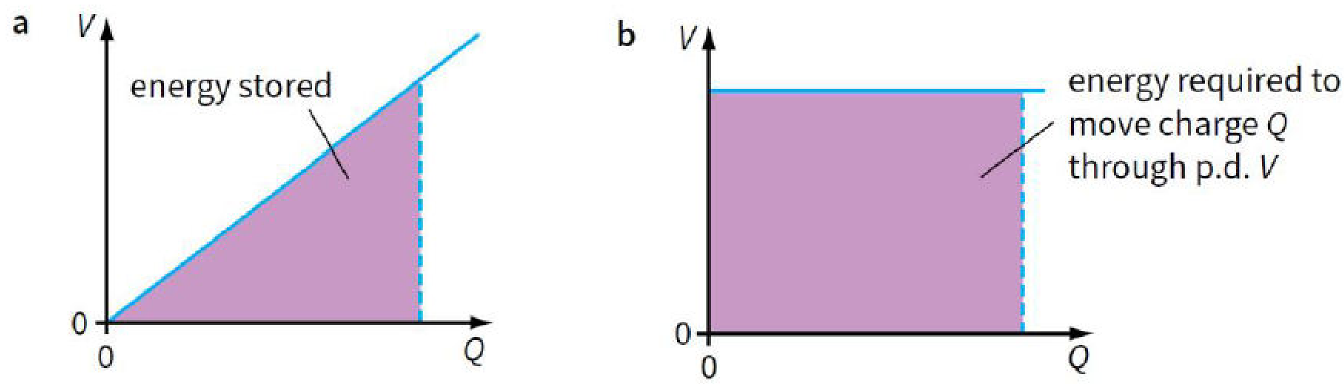


Figure 24.7 The area under a graph of voltage against charge gives a quantity of energy. The area in **a** shows the energy stored in a capacitor; the area in **b** shows the energy required to drive a charge through a resistor.

If the capacitor is charged from a battery (or similar source), the charge (Q) flows at constant p.d. (V). In this case, the energy drawn from the battery is equal to QV (i.e. twice the energy stored in the capacitor). The 'missing' energy is dissipated as heat in the connecting wires.

Capacitors in series and parallel.

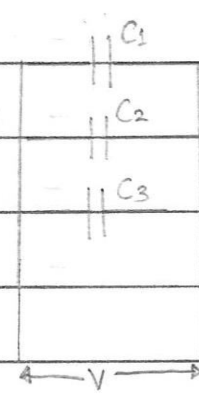
Parallel.

V is the same accross C_1, C_2 & C_3.

$Q = VC$

$CV = C_1V + C_2V + C_3V$

$C = C_1 + C_2 + C_3$



In parallel, capacitance adds up.

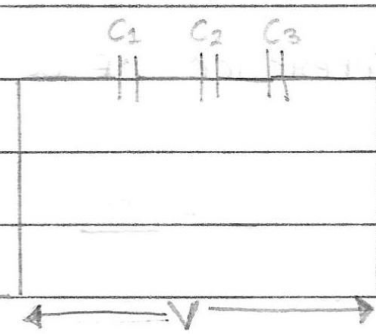
Series.

Q is the same accross C_1, C_2 + C_3

$$V = V_1 + V_2 + V_3$$

$$V = Q/C$$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

In series, capacitance reduces.

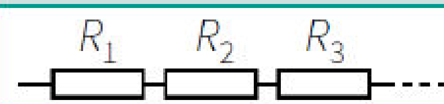
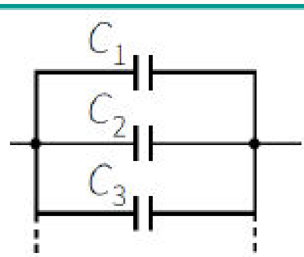
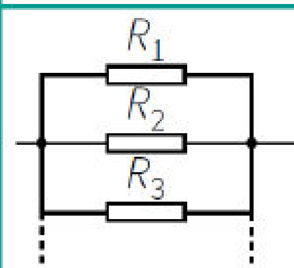
	Capacitors	**Resistors**
In series		
	store same charge	have same current
	$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$	$R_{total} = R_1 + R_2 + R_3 + \cdots$
In parallel		
	have same p.d.	have same p.d.
	$C_{total} = C_1 + C_2 + C_3 + \cdots$	$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

Table 24.3 Capacitors and resistors compared.

Capacitance of a Body

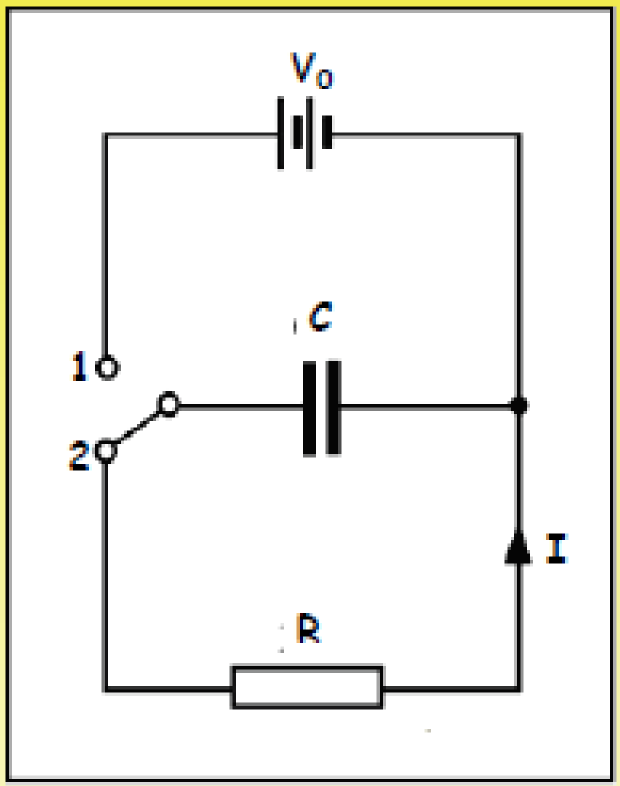
- Any isolated body can have a capacitance.
- Considering a sphere of radius r carrying charge Q, the potential at surface is

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

$$C = \frac{Q}{V} = \frac{Q}{Q/4\pi\varepsilon_o r} = 4\pi\varepsilon_o r$$

CAPACITOR DISCHARGE THROUGH A FIXED RESISTOR

- Consider the circuit shown opposite. With the switch in position (1), the capacitor is charged up by the supply.



- When the switch is moved to position (2) the capacitor gradually discharges through the fixed resistor (R). As the discharge occurs, the following quantities decrease :
  - The CHARGE stored, Q.
  - The CURRENT flowing, I.
  - The P.D. or VOLTAGE across the capacitor, V.

ANALYSIS OF CAPACITOR DISCHARGE GRAPHS

- When the charged capacitor is connected to the resistor (R), electrons start flowing around the circuit.

  If the Q stored is **high**, the V across the capacitor will be **high** and so the initial discharge I is also **high**.

  As Q decreases so too does V and I, but the rate of decay decreases with time.

- <u>MATHEMATICALLY</u> : $Q = CV$ and $V = IR$
  So $V \propto Q$ and $I \propto V$

Therefore, If Q is high, V is high and so I is high.

gradient of the Q/t graph = CURRENT (I)

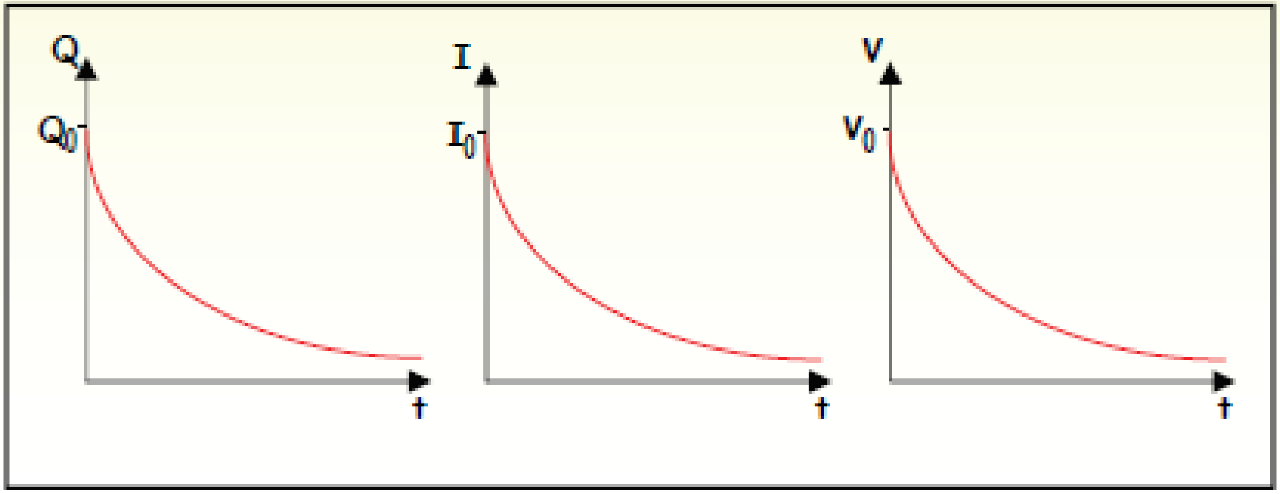
At first the gradient is steep, so the current is high.
Later, the gradient has decreased and so the current is less.

area enclosed by the I/t graph = charge (Q) that flowed from the capacitor

At first I is high, so the area under the graph is large, which means that a large amount of charge has flowed from the capacitor.

Later, I has decreased, so the area under the graph is smaller, which means that less charge has flowed out of the capacitor.

Q, I and V all decrease **EXPONENTIALLY** with time as the capacitor discharges. This is shown in the three graphs below.



CAPACITOR DISCHARGE EQUATIONS

- The Q/t, I/t and V/t graphs may be represented by the following discharge equations :

$$Q = Q_0 e^{-t/CR}$$

$$I = I_0 e^{-t/CR}$$

$$V = V_0 e^{-t/CR}$$

- These are all examples of the general equation :

$$x = x_0 e^{-t/CR}$$

- **Q, I, V** = Values of charge, current and p.d. at any time (t).
- **Q_0, I_0, V_0** = Values of charge, current and p.d. at time, t = 0.
- **C** = Capacitance of the capacitor.
- **R** = Resistance of the resistor through which the capacitor discharges.

TIME CONSTANT (T)

The quantity CR is called the TIME CONSTANT (T) of the circuit.

$$\underset{(s)}{T} = \underset{(F)}{C}\,\underset{(\Omega)}{R}$$

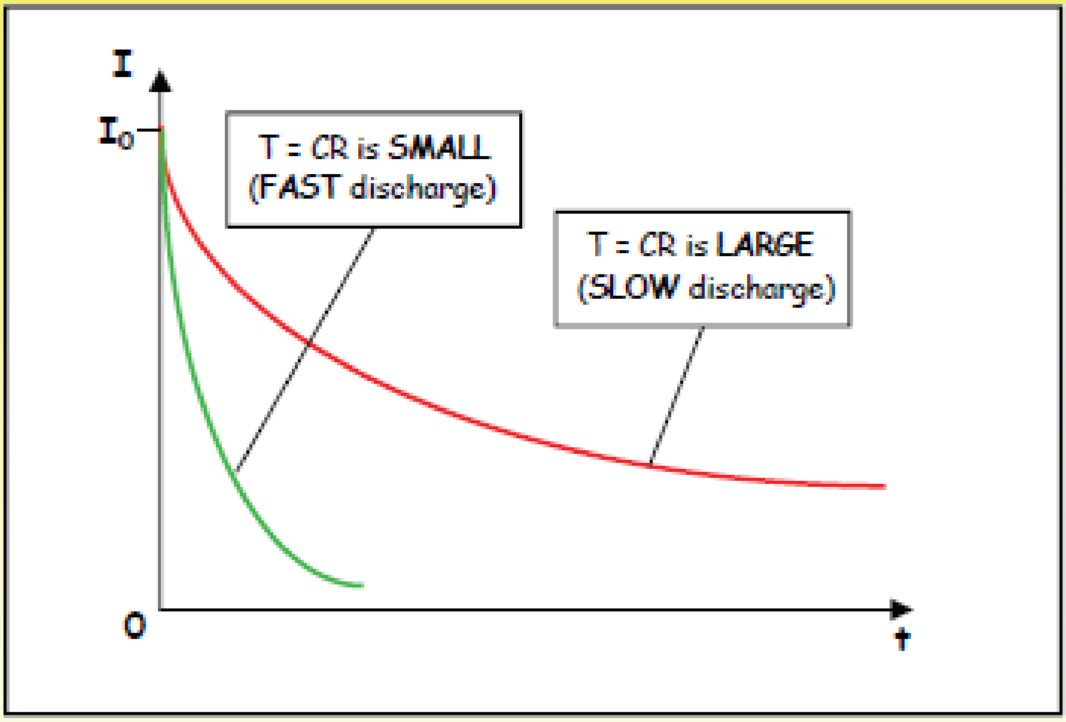
TIME CONSTANT (T) is the time taken for the current (I), charge (Q) or p.d. (V) to decrease to 1/e (≈ 37%) of its initial value.

- After $t = 2T$, the value falls to $1/e^2$ of its initial value.

  After $t = 3T$, the value falls to $1/e^3$ of its initial value.

- The value of the **TIME CONSTANT (T = CR)** can be increased by **increasing** the value of C or R or both C and R.

- If CR is **large**, the discharge will be **slow** and if it is **small** the discharge will be **fast**. This is shown in the diagram below.



- Circuits in which a capacitor discharges through a resistor are often used in **electronic timers**. At the start of a time interval, the capacitor is charged up. It gradually discharges, and eventually falls below a set (Q, I, or V) value. This triggers a switching circuit to make an alarm sound, or to have some other effect.