NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME001 Mathematics
Part 2 Examination

Version 2 2012-13

Mark Scheme

Section A
Answer ALL questions. This section carries 40 marks.

Notice to markers.

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A4. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the candidate to calculate-or otherwise produce-a piece of information that is to be used later in the question, a marker should consider the possibility of error carried forward. A careless error early in the question may make it impossible for a candidate to answer the remainder of the question correctly. Where a candidate has been careless with initial data, but has gone on to demonstrate knowledge of the correct method, they should be awarded marks for the method only.

When this happens, write ECF next to the ticks.

M=Method
A=Answer

Question A1	
Showing correct use of Product Rule.	**[M1]**
$\frac{dy}{dx}$ **=** $-3e^{-3x}\ln(\sin x) + e^{-3x} \times \frac{\cos x}{\sin x}$ (1 for each correct part)	**[2]**

Question A2	
Any clear method which leads to finding the range.	**[M1]**
Any investigation of a turning point in the domain	**[M1]**
$-1 \le f(x) \le 8$ (1 for each correct limit)	**[2]**
(Accept $-1 \le y \le 8$ but not $-1 \le x \le 8$. Accept also any suitable wording or range is in interval [-1 , 8]).	

Question A3	
Finding cos A by either using a sketch of a right-angled triangle with angle A shown, 5 as the opposite and 13 as the hypotenuse, or using the $\sin^2 A + \cos^2 A = 1$ formula.	**[M1]**
	[1]

Cos A = $-\frac{12}{13}$.	[M1]
Using sin 2A = 2 sin A cos A	[1]
Giving $2 \times \frac{5}{13} \times -\frac{12}{13} = -\frac{120}{169}$.	

Question A4	
Using integration by parts in the right direction $\left[x\left(\tfrac{1}{2}e^{2x}\right) \right]_0^1 - \int_0^1 \tfrac{1}{2}e^{2x}\,dx$	[M1]
$\left[x\left(\tfrac{1}{2}e^{2x}\right) \right]_0^1 - \left[\tfrac{1}{4}e^{2x} \right]_0^1$ (1 for each correct part)	[2]
Substituting in limits and subtracting the right way round.	[M1]
$= (\tfrac{1}{2}e^2 - 0) - (\tfrac{1}{4}e^2 - \tfrac{1}{4}) = \tfrac{1}{4}e^2 + \tfrac{1}{4}$.	[1]
Answer given to 4 sf 2.097	[1]

Question A5	
$\mathbf{a} - \mathbf{b} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$	[M1]
Magnitude of $\mathbf{a} - \mathbf{b} = \sqrt{(2^2 + (-3)^2 + 6^2)} = 7$ (using their $\mathbf{a} - \mathbf{b}$)	[M1]
Dividing their $\mathbf{a} - \mathbf{b}$ by their magnitude	[M1]
$= \frac{1}{7}(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$ or $\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$.	[1]

Question A6	
Using Quotient Rule correctly	[M1]
$\frac{dy}{dx} = \frac{(1 + x^2) \quad - x(2x)}{(1 + x^2)^2} = \frac{1 - x^2}{(1 + x^2)^2}$ (but no need to simplify)	
(1 for correct top line and 1 for correct bottom line)	[2]
Substitute in $x = 3$ and invert	[M1]
Giving $-\frac{100}{8}$ (or equivalent)	[1]

Question A7	
$x_1 = 2.2929$ $x_2 = 2.2368$ $x_3 = 2.2361$	**[3]**
(1 for each correct answer which must be rounded to at least 4 decimal places)	

Question A8	
Integrating $\pi \sec^2 x$ with limits $\pi/4$ and $\pi/3$.	**[M1]**
giving tan x with limits $\pi/4$ and $\pi/3$.	**[1]**
Substituting in the limits and subtracting the right way round	**[M1]**
giving $\pi(\sqrt{3} - 1)$ (≈ 2.3).	**[1]**

Question A9	
$\frac{1}{y}\,dy = \frac{1}{1+x^2}\,dx$ Separating the variables (integral signs not needed)	**[M1]**
Integrating both sides	**[M1]**
$\ln y = \tan^{-1}x + C$	
1 mark for either ln y or arctanx; 2 marks for both correct and + C.	**[2]**

Question A10	
The ages of the seven children add up to $7 \times 11 = 77$	**[M1]**
The ages of the eight children add up to $8 \times 10 = 80$	**[M1]**
The child who joins the group is 3 years old.	**[1]**

Section B
Answer <u>4</u> questions. This section carries 60 marks.

Question B1			
a)	i.	Substituting in values f(2) = 4 and f(3) = -7 There has been a change of sign, so a root lies between 2 and 3. (One answer correct – 1 mark Both answers correct, with reason and conclusion – 2 marks)	**[M1]** **[2]**
	ii.	$f'(x) = 3x^2 - 12x$ (Attempt to find f′(x)) $x_1 = 2.3 - \frac{0.427}{-11.73}$ (Substituting into correct formula) = any number rounding to 2.336.	**[M1]** **[M1]** **[1]**
b)	i.	(M1 for correct implicit differentiation) $6x + 2y^2 + 2x(2y\frac{dy}{dx}) + \frac{dy}{dx} = 0$ (M1 for correct Product Rule) (Final mark for correct answer) Factorise and rearrange Giving $\frac{dy}{dx} = \frac{-6x - 2y^2}{4xy + 1}$	**[M1]** **[M1]** **[1]** **[M1]** **[1]**

	ii.	Substituting in $x = 1$ and $y = 2$ Inverting 2 = (their normal) × 1 + c or (y − 2) = (their normal) (x − 1) $y = \frac{9}{14}x + \frac{19}{14}$ (or equivalent)	**[M1]** **[M1]** **[M1]** **[1]**

Question B2			
a)	i.	Either drawing a right-angled triangle with 60 degrees shown, adjacent 1, opposite √3 and hypotenuse 2 giving sin 60 = $\frac{\sqrt{3}}{2}$ or using the formula $\cos^2 x + \sin^2 x = 1$ and reaching the stage $\sin^2 x = 1 - ¼ = ¾$ (M mark) giving sin 60 = $\frac{\sqrt{3}}{2}$ (final mark)	**[M1]** **[1]**
	ii.	Cos 15 = cos (60 − 45) = cos 60 cos 45 + sin 60 sin 45 $= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$ (M mark) (Final mark)	**[M1]** **[M1]** **[1]**
b)	i.	Using $\cos^2 x + \sin^2 x = 1$ Dividing by $\sin^2 x$ giving $1 + \cot^2 x = \csc^2 x$.	**[M1]** **[M1]** **[1]**
	ii.	$2 \csc^2 x - 5 \cot x = 5$ $2(1 + \cot^2 x) - 5 \cot x = 5$ (using identity)	 **[M1]**

		$2\cot^2 x - 5\cot x - 3 = 0$ (forming a 3-term quadratic = 0)	[M1]
		$(2\cot x + 1)(\cot x - 3) = 0$ (factorising)	[M1]
		$\cot x = -\frac{1}{2}$ or 3, giving $\tan x = -2$ or $\frac{1}{3}$ (1 for each correct answer)	[2]
		x = answers rounding to 116.6, 296.6, 18.4 and 198.4 degrees (1 mark for any two correct; 2 marks for all 4 correct)	[2]

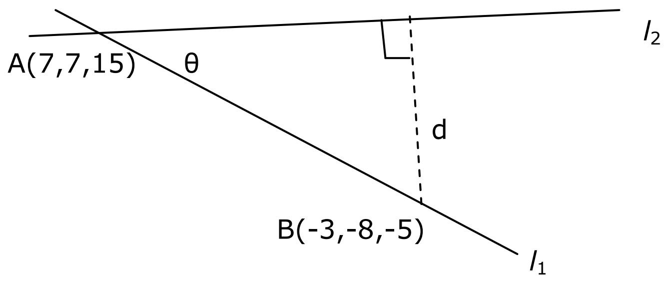
Question B3			
a)	i.	Correct V-shape which touches the positive x-axis and crosses the y=axis.	[1]
		(3,0) and (0,6) marked (1 mark for each)	[2]
	ii.	Solve $2x - 6 = 9$	[M1]
		Solve $2x - 6 = -9$	[M1]
		Giving $x = 7\frac{1}{2}$ and $-1\frac{1}{2}$. (Both correct)	[1]
		Alternatively, square the equation to give $(2x - 6)^2 = 81$	
		$4x^2 - 24x - 45 = 0$ (for reaching a 3-term quadratic = 0)	[M1]
		$(2x - 15)(2x + 3) = 0$ (factorising)	[M1]
		$x = 7\frac{1}{2}$ or $-1\frac{1}{2}$.	[1]
b)	i.	g (3) = 5 and (substituting their g value into f)	[M1]
		$f(5) = e^{16}$.	[1]
	ii.	$y = e^{3x+1}$ $\ln y = 3x + 1$ (correct use of logs)	[M1]
		$x = (\ln y - 1) \div 3$ (correct rearrangement and exchanging x and y)	[M1]
		$f^{-1}(x) = (\ln x - 1) \div 3$	[1]
	iii.	Parabola correct way up with minimum in fourth quadrant	[M1]

		Crosses x-axis at (0, 0) and (4, 0); minimum at (2, -8) (1 mark for each. If (0, 0) has not been shown, but the parabola clearly passes through the origin then this mark can still be given.)	**[3]**

Question B4			
a)	i.	Use one of the identities for cos $2x$	**[M1]**
		Writing integral as $\frac{1}{2}\int \cos 2x + 1 \; dx$	**[1]**
		$= \frac{1}{2}(\frac{1}{2}\sin 2x + x) + C$.	**[2]**
		(One part of the answer correct – 1 mark; whole answer correct with + C – 2 marks).	
	ii.	$\frac{du}{dx} = 3x^2$ (Differentiating u)	**[M1]**
		Integral becomes $\int 2u^7 \; du$ (writing integral in terms of u)	**[M1]**
		$= \frac{1}{4}u^8 + C$	**[1]**
		$= \frac{1}{4}(1 + x^3)^8 + C$.	**[1]**
		(The + C must be shown; however if the mark for its omission has already been lost in part i, then the mark is not lost again here).	
b)	i.	$4 = A(1 - x) + B(3 + x)$ (or equivalent correct method)	**[M1]**
		A = 1 and B = 1 (1 mark each)	**[2]**
	ii.	Integrate the partial fractions between -2 and -1	**[M1]**
		giving $\left[\ln(3 + x) - \ln(1 - x)\right]_{-2}^{-1}$	**[1]**

		-2 = (ln 2 – ln 2) – (ln 1 – ln 3) (substitute in the limits and subtract the right way round) = ln 3	**[M1]** **[1]**

Question B5		
a)	$1 + 2t = -3 + 5s$ $-2 + 3t = 1 + 3s$ $3 + 4t = 7 + 4s$ (Forming three equations in t and s) Solving any two of these equations $t = 3$ Substituting correctly $s = 2$ Confirming the values of t and s satisfy the third equation or substituting t and s into both vector equations A lies at (7, 7, 15)	 **[M1]** **[1]** **[M1]** **[1]** **[M1]** **[1]**
b)	Using the scalar product for vectors 2**i** + 3**j** + 4**k** and 5**i** + 3**j** + 4**k**. Magnitudes are $\sqrt{29}$ and $\sqrt{50}$ respectively (both must be correct) Their scalar product is 35. $35 = \sqrt{29} \times \sqrt{50} \times \cos\theta$ where θ is the angle between the vectors $\cos\theta = 35 \div (\sqrt{29}\sqrt{50})$ (using their values for the magnitudes and scalar product) Giving $\theta = 23.2^{\circ}$.	**[M1]** **[1]** **[1]** **[M1]** **[1]**
c)	$k = -5$	**[1]**

d)		
	AB = $\sqrt{(10^2 + 15^2 + 20^2)} = \sqrt{725}$ (using their A and B)	**[M1]**
	d = 'their AB' sin 'their angle θ'	**[M1]**
	= $\sqrt{725}\sin 23.2 \approx 10.6$	**[1]**
	Alternative solution to part d).	
	Let the point where the shortest distance meets l_2 be P.	
	Coordinates of P are $(-3 + 5s,\ 1 + 3s,\ 7 + 4s)$.	
	Vector BP is defined as	
	$(-3 + 5s - (-3))\mathbf{i} + (1 + 3s - (-8))\mathbf{j} + (7 + 4s - (-5))\mathbf{k}$.	
	$= 5s\mathbf{i} + (3s + 9)\mathbf{j} + (4s + 12)\mathbf{k}$. (An expression for BP must be reached)	**[M1]**
	Using the scalar product,	
	$(5s\mathbf{i} + (3s + 9)\mathbf{j} + (4s + 12)\mathbf{k}).(5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) = 0$.	
	$25s + 9s + 27 + 16s + 48 = 0$, giving $s = -1\frac{1}{2}$. (Finding a value for s)	**[M1]**
	Length of BP is $\sqrt{((-10\frac{1}{2} + 3)^2 + (-3\frac{1}{2} + 8)^2 + (1 + 5)^2)}$.	
	This comes to $\sqrt{112.5} \approx 10.6$.	**[1]**

Question B6			
a)	i.	Mean = 14 ÷ 7	**[M1]**
		= 2	**[1]**
	ii.	$3^2 + 0^2 + (-1)^2 + 2^2 + 0^2 + 4^2 + 6^2 = 66$	**[M1]**
		Standard deviation = $\sqrt{(66 \div 7 - 2^2)}$	**[M1]**
		≈ 2.33	**[1]**
b)	i.	<table><tr><th>Number of apples</th><th>mid-value (x)</th><th>frequency (f)</th><th>fx</th><th>cum fr</th></tr><tr><td>80 – 90</td><td>85</td><td>4</td><td>340</td><td>4</td></tr><tr><td>91 – 100</td><td>95.5</td><td>6</td><td>573</td><td>10</td></tr><tr><td>101 – 110</td><td>105.5</td><td>12</td><td>1266</td><td>22</td></tr><tr><td>111 – 120</td><td>115.5</td><td>18</td><td>2079</td><td>40</td></tr><tr><td>121 – 130</td><td>125.5</td><td>34</td><td>4267</td><td>74</td></tr><tr><td>131 – 140</td><td>135.5</td><td>36</td><td>4878</td><td>110</td></tr><tr><td>141 – 150</td><td>145.5</td><td>10</td><td>1455</td><td>120</td></tr><tr><td></td><td></td><td>120</td><td>14858</td><td></td></tr></table>	
		Estimated mean = 14858 ÷ 120 ≈ 123.8	**[3]**
		(M1 – finding mid-values and correct construction of fx column)	

		M1 – adding up fx column and dividing by 120 1 – correct mean)	
	ii.	Correct final column	**[1]**
	iii.	Correct plotting with upper interval values (1 mark for 2 or less errors) Curve drawn	**[2]** **[1]**
	iv.	Correct median from curve (about 126) Correct quartiles, subtracting to give interquartile range (LQ about 116; UQ about 134 and IQR about 18)	**[M1]** **[M1]**
	v.	**B** (negatively skewed)	**[1]**