NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB001 Maths Part 2 (Business) Examination

2013-14

MARK SCHEME

Notice to markers.

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A4. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate-or otherwise produce-a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks.

When this happens, write ECF next to the ticks.

M=Method
A=Answer

If a student has answered more than the required number of questions, credit should only be given for the first n answers, in the order that they are written in the student's answer booklet (n being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Question A1	
a)	The mode = 39 **A1**
b)	Ordered data: 33,34,35,35,37,38,39,39,39,41 **M1** Q3=39 and Q1= 35 **A1** IQR=39-35=4 mins **A1**

Question A2

$\frac{6\times420+3\times450}{9}$ = **M1** for combined total number of items

M1 for combined total number of machines

$\frac{3870}{9}$ = 430 items per hour **A1**

Question A3	
a)	$\mathrm{E}[\mathrm{X}]= \mathrm{E}[\frac{3Y-20}{10}]$ $= \frac{3\mathrm{E}[Y]-20}{10}$ **M1** $=\frac{3\times2.9-20}{10}$ = -1.13 **A1**
b)	$V(X)$ $= V(\frac{3Y-20}{10})$ **M1** $=\left(\frac{3}{10}\right)^2 V(Y)$ **M1** $=\frac{9}{100}(1.7)$ =0.153 **A1**

Question A4

Let X be the number of people who own a bicycle where X is B(12, $\frac{2}{7}$)

Then

$p(X \geq 3) = 1 - p(x < 3)$ $=1 - (\, p(0) + p(1) + p(2))$ **M1**

$=1 - (\left(\frac{5}{7}\right)^{12} + \binom{12}{1}\left(\frac{2}{7}\right)^1\left(\frac{5}{7}\right)^{11} + \binom{12}{2}\left(\frac{2}{7}\right)^2\left(\frac{5}{7}\right)^{10}$ **M2**

=0.711 to 3 significant figures. **A1**

Question A5	
a)	$k = 1 - \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{8}\right) = \frac{1}{2}$ **A1**
b)	$E[4X] = 4E[X]$ **M1** $= 4(4 \times \frac{1}{4} + 5 \times \frac{1}{8} + 6 \times \frac{1}{8} + 7 \times \frac{1}{2})$ **M1** =23.5 **A1**

Question A6

$u = 3e^{\cos x}$ $\quad v = (2x^4 - 9)$

$u' = -3\sin x e^{\cos x}$ $\quad v' = 8x^3$

$$y' = \frac{(2x^4 - 9)(-3\sin x e^{\cos x}) - 3e^{\cos x}(8x^3)}{(2x^4 - 9)^2}$$

$$= \frac{-3e^{\cos x}[(2x^4 - 9)(\sin x) + (8x^3)]}{(2x^4 - 9)^2}$$

A1 for u'
A1 for v'
M1 for putting it together
A1 for answer
some simplification attempted.

Question A7

$$\frac{(x-2)}{(x-3)^2(x-4)} = \frac{A}{(x-3)} + \frac{B}{(x-3)^2} + \frac{C}{(x-4)}$$

$(x-2) = A(x-3)(x-4) + B(x-4) + C(x-3)^2$

Let $x = 4$

$2 = C$

Let $x = 3$

$1 = -B \Rightarrow B = -1$

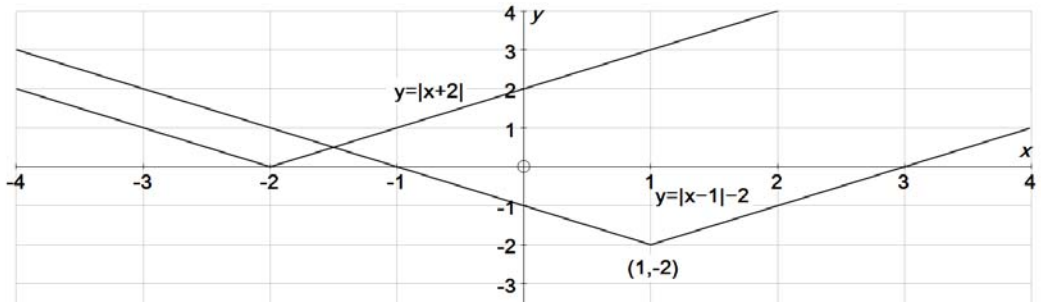
Eq. coeff

$x^2 : 0 = A + C \Rightarrow A = -2$

M1 for the split
A3 for the correct answer for each of A, B and C

Question A8

a)



A1 shape
A1 vertex co-ordinate (1,-2)
A1 y-intercept either -1 or (0,-1)
A1 x-intercept either 3 or (3,0)

b) Translation of 3 units parallel to the x-axis in the +ve direction. **A1**
Translation of 2 units parallel to the y-axis in the –ve direction. **A1**

For each mark must state "translation" **and** correct direction.

Question A9

$8 = 4x + 2y + 6xy$

When $x = -1$

$8 = -4 + 2y - 6y$

$12 = -4y \Rightarrow y = -3$

$0 = 4 + 2\frac{dy}{dx} + 6(y + x\frac{dy}{dx})$

$0 = 4 + 2\frac{dy}{dx} + 6(-3 + -1\frac{dy}{dx})$

$0 = -14 - 4\frac{dy}{dx}$

$\frac{14}{-4} = \frac{dy}{dx} \Rightarrow -3.5 \text{ or } \frac{7}{-2}$

Tangent eq.

$y = -3.5x + c$

$-3 = 3.5 + c \Rightarrow -6.5$

$y = -3.5x - 6.5$ or equivalent

A1 for the value of y

A1 for $4 + 2\frac{dy}{dx}$ and

A1 for $6(y + x\frac{dy}{dx})$

A1 for value of $\frac{dy}{dx}$

M1

A1

Section B
Answer 4 questions. This section carries 60 marks.

Question B1

If ε={the natural numbers which are less than 21}
A={2,8,11,13,15}
B={the odd numbers between 1 and 13 including 1 and 13}
C={the square numbers which are less than 21}

a)	W=6, X=4, Y=15 **A3** (1 for each)	
b)	i.	$A \cap B = \{11,13\}$ **A1**
	ii.	$B \cup C = \{1,3,4,5,7,9,11,13,16\}$ **A2** only award full marks for all elements correct
	iii.	$n((B \cup C)') = 20 - 9 = 11$ **A1**
c)	i.	Given that on 90 percent of occasions the drying times are less than 52 minutes $p(x < 52) = 0.9$ $p\left(z < \frac{52-45}{\sigma}\right) = 0.9$ **M1** From tables $p(z < 1.28) = 0.9$ Therefore $\frac{52-45}{\sigma} = 1.28$ **M1** $\sigma = \frac{52-45}{1.28} = 5.47$ **A1**
	ii.	$p(x > 60)$ $=p\left(z > \frac{60-45}{5.47}\right)$ **M1** $= p(z > 2.74) = 1 - p(z < 2.74)$ **M1** =1-0.9969 =0.0031 **A1**
	iii.	$\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$ $40.7 \pm 1.96\frac{5.47}{\sqrt{30}}$ **M1** (38.74,42.66) mins **A1**

Question B2	
a)	Using $y - \bar{y} = \frac{s_{xy}}{s_{x^2}}(x - \bar{x})$ where $s_{xy} = \frac{\sum xy}{n} - \bar{x}\bar{y}$ $s_{x^2} = \frac{\sum x^2}{n} - \bar{x}^2$ $s_{xy} = \frac{3897}{9} - \left(\frac{45}{9}\right)\left(\frac{625}{9}\right) = 85.777778$ **M1** $s_{x^2} = \frac{285}{9} - \left(\frac{45}{9}\right)^2 = 6.666667$ **M1** $y - \frac{625}{9} = \frac{85.777778}{6.666667}\left(x - \left(\frac{45}{9}\right)\right)$ **M1** $y = 12.87x + 5.11$ or any appropriately rounded coefficients **A1**
b)	The value of b represents the gradient of the line. **A1** **Or** The value of b represents the rate of change of y for a unit change in x.
c)	The value x=30 is quite far beyond the range of the recorded values of x, hence the equation would not provide a reliable estimate of y for a value of x = 30. **A1** Or any other well-justified response.
d)	Using the equation, when x= 6 $y = 12.87(6) + 5.11 = 82.33$ **A1**
e)	For calculating a difference between the estimate of y and the observed value of y $eg.\ 83 - 82.33 = 0.67$ **A1** For any appropriate comment which relates to the suitability (or otherwise) of the regression equation found such as "There is only a small difference therefore the regression equation is a good fit" **A1**
f)	Pearson's Product Moment Correlation Coefficient, r. $r = \frac{s_{xy}}{s_x s_y}$ where $s_{y^2} = \frac{\sum y^2}{n} - y^2$ $= \frac{53825}{9} - \left(\frac{625}{9}\right)^2 = 1158.024691$ **M1** $r = \frac{85.777778}{\sqrt{6.666667}\sqrt{1158.024691}}$ **M1** =0.976 **A1**
g)	The value of r indicates strong positive correlation **A1** as indicated by the appearance of the points in the scatter diagram lying very close to a straight line which has positive gradient **A1**
h)	The two variables may both have a dependence upon another (a third) variable **A1**

Question B3		
a)	"The peaks and troughs are decreasing over time and each set of quarterly variations within a year appear to decrease proportionally." **1 mark** for recognising the change of size of the seasonal variations over time and **1 mark** for recognising a proportional change.	
b)	A=(30+35+92+79)/4=59 **A1** B=(62.25+57.75)/2=60 **A1** C=92/58=1.59 **A1**	
c)	D=0.5 **A1** E=(0.48+0.43+0.5)/3=0.47 **A1** F=(1.14+1.18)/2=1.16 **A1**	
d)	A total of 3 marks are available for this section: A seasonal effect of 1.72 for Quarter 1 means that the sales in Quarter 1 are on average 72 percent above the general trend **A1** and 0.6 for Quarter 4 means that the sales in Quarter 4 are on average 40 percent below the general trend. (or are only equivalent to 60 percent of the general trend value) **A1** if the proportionality is accounted for in the description as for example with the words "72 percent..." **A1**	
e)	i.	quarter 1 of year 5 x=17 Trend $y = -2.86(17) + 91.7 = 43.08$ (£43,080) **A1** Seasonal Ratio 1.72 Projected value $43.08 \times 1.72 = 74.1$ (£74,100) **A1**
	ii.	quarter 3 of year 5 x=19 Trend $y = -2.86(19) + 91.7 = 37.36$ (£37,360) **A1** Seasonal Ratio 0.47 Projected value $37.36 \times 0.47 = 17.6$ (£17,600) **A1**

Question B4

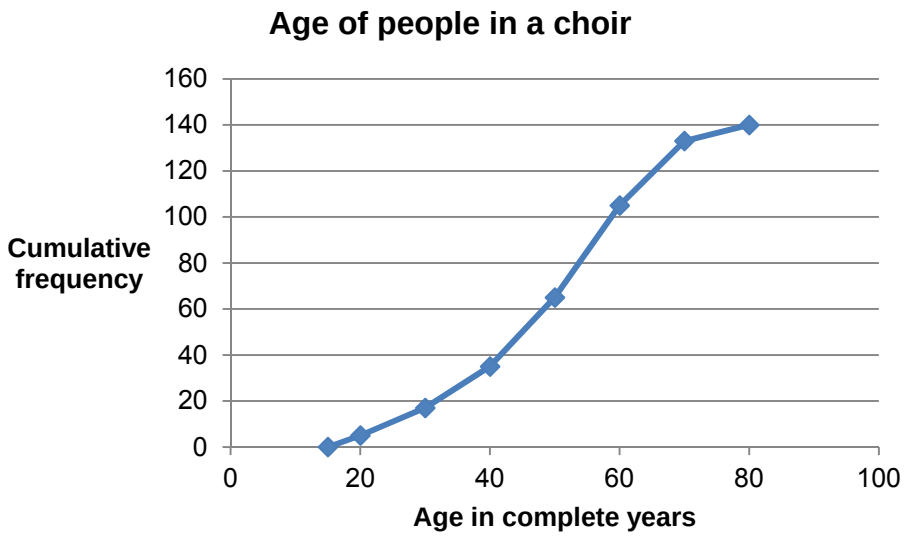
a) Negative skew **A1**

b)

Age	C.F.
15	0
20	5
30	17
40	35
50	65
60	105
70	133
80	140

M1 for upper class boundaries, **M1** for the cumulative frequency totals. (At least 6) correct in each case.

c)



M1 scale, **M1**both labels, **M1** for joining points with straight lines,
M2 for the CF curve being closed at the beginning (15,0). and at least 6 points are plotted correctly based on upper bounds found by the student)
.{**-1** if graph paper not used}

d)

Median =	52
Q1=	41
Q3=	62

M1 each from their graph. (**M3** in total)

e)

Q2-Q1=	11
Q3-Q2=	10

	M1 for the calculations **M1** for comment symmetrical data and therefore histogram and cumulative frequency chart are not from the same choir.
f)	About 50 people. **M1** for use of graph. **A1** for their answer.

Question B5

a) i. $\Delta = 4\begin{vmatrix}-1 & 3\\ 1 & -3\end{vmatrix} - 5\begin{vmatrix}1 & 3\\ -2 & -3\end{vmatrix} + 2\begin{vmatrix}1 & -1\\ -2 & 1\end{vmatrix}$ **M1**

$\Delta = 4(3-3) - 5(-3+6) + 2(1-2)$ **M1**

$\Delta = -15 - 2 = -17$ **M1**

Must show working, if no working then only 1 mark they have used their calculator.

ii. Using $A^{-1}A=I$

$$\frac{1}{-17}\begin{bmatrix}0 & 17 & 17\\ -3 & a & -10\\ b & c & -9\end{bmatrix}\begin{bmatrix}4 & 5 & 2\\ 1 & -1 & 3\\ -2 & 1 & -3\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$ **M1**

$$\frac{1}{-17}(-12 + a = 20) = 0$$

$8 + a = 0 \rightarrow a = -8$ **M1**

$$\frac{1}{-17}(4b + c + 18) = 0$$

$4b + c = -18$ eq1

$$\frac{1}{-17}(5b - c - 9) = 0$$

$5b - c = 9$ eq2

eq1+eq2 gives $9b = -9 \rightarrow b = -1 \ \therefore c = -14$ **M1, A1** for both answers

b) Hence,

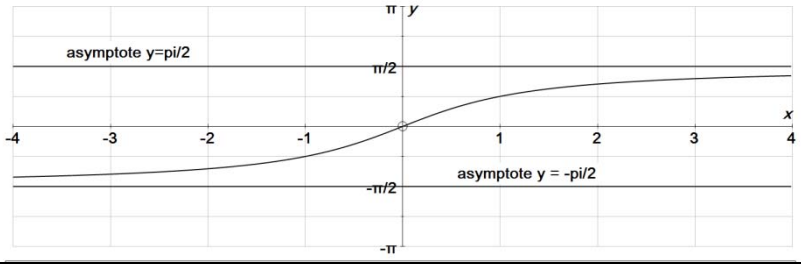
$$\begin{bmatrix}4 & 5 & 2\\ 1 & -1 & 3\\ -2 & 1 & -3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}3\\ 6\\ -4\end{bmatrix}$$ **M1**

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{-17}\begin{bmatrix}0 & 17 & 17\\ -3 & -8 & -10\\ -1 & -14 & -9\end{bmatrix}\begin{bmatrix}3\\ 6\\ -4\end{bmatrix}$$ **M1**

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{-17}\begin{bmatrix}34\\ -17\\ -51\end{bmatrix}$$ **M1**

$x = -2, y = 1, z = 3$ **M1**

c)



A1 shape
A1 domain $-\infty$ to ∞
A1 range $-\frac{\pi}{2} < f(x) < \frac{\pi}{2}$
Asymptotes at $y=-\frac{\pi}{2}$ and $y=\frac{\pi}{2}$ **A1**

Question B6

a)

$$\int e^x \cos x dx$$

$u = e^x$ $\quad v' = \cos x$

$u' = e^x$ $\quad v = \sin x$

$$\int e^x cosx\, dx = e^x sinx - \int e^x (sinx) dx \qquad (1)$$

$$\int e^x (sinx) dx$$

$u = e^x$ $\quad v' = sinx$

$u' = e^x$ $\quad v = -cosx$

$$\int e^x (sinx) dx = -e^x (cosx) + \int e^x (cosx) dx$$

$$\int e^x cosx\, dx = e^x sinx - \left[-e^x (cosx) + \int e^x (cosx) dx\right]$$

$$2\int e^x cosx\, dx = e^x sinx + e^x cosx$$

$$= \frac{1}{2} e^x (sinx + cosx) + c$$

M1 for u'
M1 for v

M1 for sub into (1)

A1 for answer

b)

$$\int_{-2}^{0} 5x(x+3)^6\, dx$$

x	u=x+3
0	3
-2	1

$u = x + 3 \rightarrow x = u - 3 \therefore 5x = 5u - 15$

$\frac{du}{dx} = 1 \rightarrow \frac{dx}{du} = 1$

$$\int_1^3 (5u - 15)u^6 (1) du = \int_1^3 (5u^7 - 15u^6) du$$

$$= \left[\frac{5}{8}u^8 - \frac{15}{7}u^7\right]_1^3$$

$$= \left(\frac{5}{8}(3)^8 - \frac{15}{7}(3)^7\right) - \left(\frac{5}{8}(1)^8 - \frac{15}{7}(1)^7\right)$$

$$= -584.285 = -584(awrt)$$

M1 for $5x = 5u - 15$

M1 for $\frac{dx}{du} = 1$

M1 for change of limits

M1 for $\int_1^3 (5u^7 - 15u^6)$

M1 for the integration

A1 awrt=answer will round to

c)

$$V = \pi\int_0^3 y^2\,dx = \pi\int_0^3 \mathrm{y} = \left(e^x + 3\right)^2 dx = \pi\int_0^3 e^{2x} + 6e^x + 9dx$$

$$= \pi\left[\frac{1}{2}e^{2x} + 6e^x + 9x\right]_0^3$$

$$= \pi\left[\left(\frac{1}{2}e^6 + 6e^3 + 9\times 3\right) - \left(\frac{1}{2}e^0 + 6e^0 + 9\times 0\right)\right]_0^3$$

$$= \pi(342.73) = 343\pi \text{ cubic units (awrt)}$$

M1 $\pi\int_0^3 e^{2x} + 6e^x + 9dx$

M1 for the integration

M1 for substitution of limits
A1 for awrt must be in terms of π
A1 for cubic units.