NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME001 Mathematics
Part 2 Examination (Science & Engineering)

Mark Scheme

Notice to markers.

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A9. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the candidate to calculate-or otherwise produce-a piece of information that is to be used later in the question, a marker should consider the possibility of error carried forward. A careless error early in the question may make it impossible for a candidate to answer the remainder of the question correctly. Where a candidate has been careless with initial data, but has gone on to demonstrate knowledge of the correct method, they should be awarded marks for the method only.

When this happens, write ECF next to the ticks.

M=Method
A=Answer

Section A

Question A1	
Correct use of Quotient Rule	**[M1]**
$\frac{dy}{dx} = \frac{3x^2(1 + x^2) - x^3(2x)}{(1 + x^2)^2}$	
Whole expression correct	**[1]**
Substitute in $x = 2$	**[M1]**
Giving $\frac{28}{25}$ (or equivalent- decimal is acceptable)	**[1]**

Question A2	
The function has an inverse because it is a one-one function. (Any other form of words can be permitted as long as the candidate has shown clear understanding.)	**[1]**
$y = \dfrac{3x + 2}{5}$ thus $x = \dfrac{5y - 2}{3}$. Therefore $f^{-1}(x) = \dfrac{5x - 2}{3}$	
Correct rearranging	**[M1]**
Correct answer in terms of x	**[1]**

Question A3	
$2(1 - \cos^2\theta) - \cos\theta = 2$ Using $\sin^2\theta + \cos^2\theta = 1$	**[M1]**
$2 - 2\cos^2\theta - \cos\theta - 2 = 0$	
$2\cos^2\theta + \cos\theta = 0$ Quadratic equation in $\cos\theta$ set equal to zero	**[M1]**
$\cos\theta\,(2\cos\theta + 1) = 0$ Factorising	**[M1]**
$\cos\theta = 0$ or $-½$ so $\theta = \pi/2,\ 3\pi/2,\ 2\pi/3,\ 4\pi/3$. (Any 2 correct – 1 mark) (All 4 correct – 2 marks)	**[2]**
(Ignore any answers outside the range)	

Question A4	
$\dfrac{du}{dx} = 3x^2$	**[1]**
Integral becomes $\dfrac{1}{3}\displaystyle\int_1^2 \dfrac{1}{u}\,du$ Attempt to transform completely into u	**[M1]**
$= \dfrac{1}{3}\Big[\ln u\Big]_1^2$	**[1]**
$= \dfrac{1}{3}[\ln 2 - \ln 1]$ Substitutes in limits and subtracts the right way round	**[M1]**
$= \dfrac{1}{3}\ln 2$	**[1]**

Question A5	
Volume = $\pi \int_1^2 (x^2 + 3)^2 \, dx$	**[M1]**
$= \pi \int_1^2 (x^4 + 6x^2 + 9) \, dx \quad = \pi \left[x^5/5 + 2x^3 + 9x \right]_1^2$ (1 term correct – 1) (All correct – 2)	**[2]**
$= \pi \, [(\frac{32}{5} + 16 + 18) - (\frac{1}{5} + 2 + 9)]$ Substituting in limits and subtracting the right way round	**[M1]**
$= \pi \, (40\frac{2}{5} - 11\frac{1}{5}) = 29\frac{1}{5}\pi$ or $\frac{146}{5}\pi$.	**[1]**

Question A6	
If **a** . **b** = 0, they are perpendicular.	**[1]**
$(3 \times -2) + (-3 \times 5) + (7 \times p) = 0$ (If this is written without the statement above being made, the first 2 marks can be given)	**[1]**
$-6 - 15 + 7p = 0$, thus $p = 3$.	**[1]**

Question A7	
$y \, dy = (x + 1) \, dx$ (separating variables – integral signs not needed)	**[M1]**
$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + C$	**[1]**
Substituting $x = 1$ and $y = 1$ and finding a value of C.	**[M1]**
$\frac{1}{2} = \frac{1}{2} + 1 + C$ so $C = -1$ giving $\frac{1}{2}y^2 = \frac{1}{2}x^2 + x - 1$	
Thus $y = \sqrt{(x^2 + 2x - 2)}$	**[1]**

Question A8	
$\frac{dx}{dy} = \frac{\sec^2 y}{\tan y}$ Attempt to use Chain Rule	**[M1]**
Must be in terms of y	**[1]**
Substitutes in $y = \frac{1}{4}\pi$ and inverts	**[M1]**
$\frac{dy}{dx} = \frac{1}{2}$.	**[1]**

Question A9	
$f'(x) = 3x^2$	**[1]**
Correct use of formula	**[M1]**
($x_1 = 2.9259$) x_2 = any number rounding to 2.924	**[1]**
Writing the answer to 2.924 (to 4 significant figures)	**[1]**

Question A10	
Mean = 48 ÷ 6 = 8 Mode = 7 Median = 8 (One mark for each)	**[3]**

Section B

Question B1			
a)	i.	Using implicit differentiation	[M1]
		$2x + 2x\frac{dy}{dx} + 2y + 3y^2\frac{dy}{dx} = 0$ Correct use of product rule	[M1]
		Correct expression	[1]
		$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 3y^2}$ Factorising	[M1]
		Correct answer	[1]
	ii.	$\frac{dx}{dy} = \frac{2x + 3y^2}{-2x - 2y}$	[1]
	iii.	$-\frac{18}{10}$ (or equivalent)	[1]
b)	i.	$4^2 - 17 = -1$ Substituting in values	[M1]
		$4.3^2 - 17 = 1.49$ One correct answer	[1]
		Change of sign, so there is a root between 4.0 and 4.3	
		Both answers correct, with reason and conclusion	[1]
	ii.	$x = ½(x + \frac{17}{x})$ giving $2x = x + \frac{17}{x}$ (Multiplying by 2)	[M1]
		$2x^2 = x^2 + 17$	
		$x^2 - 17 = 0$ (Multiplying by x and bringing everything to one side)	[1]
	iii.	Substituting $x_0 = 4.3$	[M1]
		$x_1 = 4.1267$	[1]
		$x_2 = 4.1231$	[1]

Question B2			
a)	i.	Correct shape (Stretch of scale factor 2 in y-direction) Crossing at (-2, 0) and (6, 0) Maximum at (1, 14)	[1] [1] [1]
	ii.	Correct shape (Translation of 3 units in positive x-direction) Crossing at (1, 0) and (9, 0) Maximum at (4, 7)	[1] [1] [1]
	iii.	Correct shape (Reflection in y-axis) Crossing at (-6, 0) and (2, 0) Maximum at (-1, 7)	[1] [1] [1]
b)	i.	h(2) = 5 g(5) = 22	[1] [1]
	ii.	$(5x - 3)^2 + 1 = 65$ $(5x - 3) = \pm 8$ (Correct rearranging and showing 2 possible answers) $x = \frac{11}{5}$ or -1 (1 mark each) **<u>Alternatively</u>** $25x^2 - 30x + 9 + 1 = 65$ (First mark) $25x^2 - 30x - 55 = 0$ giving $(5x - 11)(x + 1) = 0$ (M1 mark) $X = \frac{11}{5}$ or -1 (1 mark each)	[1] [M1] [2]

Question B3			
a)	i.	$\tan 15 = \tan(60 - 45) = \dfrac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} = \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}}$ (1 mark) (1 mark) (1 mark)	**[3]**
	ii.	cot 165 = 1/tan 165 Realising tan 165 = - tan 15 $= -\dfrac{1 + \sqrt{3}}{\sqrt{3} - 1}$	**[M1]** **[M1]** **[1]**
b)	i.	$\cos 2x = (1 - \sin^2 x) - \sin x$ $= 1 - 2\sin^2 x$	**[M1]** **[1]**
	ii.	$(1 - 2\sin^2 x) + 2 \sin x \cos x = 1$ (1 for each correct part on LHS) $2 \sin x(\sin x - \cos x) = 0$ (Factorising) $\sin x = 0$ or $\sin x = \cos x$ (1 for each correct answer) x = 0, 180, 360, 45, 225 degrees (Any 2 correct – 1 mark; 2 marks for all 5 correct) (Ignore solutions outside the range)	**[M2]** **[M1]** **[2]** **[2]**

Question B4			
a)	i.	Writing integrand as $\sec^2 x$ $= \tan x + C$	**[M1]** **[1]**
	ii.	$\frac{du}{dx} = 1$ Integral becomes $\int_1^2 \frac{u-1}{u}\,du$ (Writing in terms of u) $= \left[u - \ln u\right]_1^2$ (Correct breaking up and integrating) $= (2 - \ln 2) - (1 - \ln 1)$ (Substituting in limits and subtracting the right way round) $= 1 - \ln 2$	**[1]** **[M1]** **[M1]** **[M1]** **[1]**
b)	i.	$5x = A(x^2 + 1) + Bx(x - 2) + C(x - 2)$ $A = 2$; $B = -2$; $C = 1$. (1 mark each)	**[M1]** **[3]**
	ii.	Writing the integrand as partial fractions $= 2\ln(x - 2) - \ln(x^2 + 1) + \tan^{-1}x + C$ (1 mark for any one correct term; 2 marks for any two correct terms; 3 marks for all terms correct and + C. If the + C is not shown and the mark for omitting this has already been lost in a) i, then there is no penalty for this second omission).	**[M1]** **[3]**

Question B5			
a)	i.	$\mathbf{r} = (2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) + t(3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$ **or** $\mathbf{r} = (5\mathbf{i} + \mathbf{j} - 3\mathbf{k}) + t(-3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k})$	**[1]**
	ii.	Vector AB = $(3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$ and vector AC = $(2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ (Both correct) $(3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}) \cdot (2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 6+4+6 = 16$ (Using their AB and AC) $AB = \sqrt{(3^2 + 2^2 + (-6)^2)} = 7$ $AC = \sqrt{(2^2 + 2^2 + (-1)^2)} = 3$ (Using their AB and AC) $16 = 7 \times 3 \times \cos\theta$ where θ denotes angle CAB So $\cos CAB = \dfrac{16}{21}$	 **[1]** **[M1]** **[M1]** **[1]**
	iii.	$\text{Sin}^2\theta = 1 - \left(\dfrac{16}{21}\right)^2 = \dfrac{185}{441}$ So $\sin CAB = \sqrt{185}/21$. Alternatively, M1 for drawing a right-angled triangle with A or θ marked, 21 shown as the hypotenuse, 16 as the adjacent and $\sqrt{185}$ as the opposite. 1 for final answer.	**[M1]** **[1]**
	iv.	Area = $\frac{1}{2} \times 7 \times 3 \times \sqrt{185}/21$ (Using their previous answers) $= \frac{1}{2}(\sqrt{185})$	**[M1]** **[1]**
b)	**1.** $1 - 2t = 2 + s$ **2.** $3 + 3t = -1 - s$ **3.** $-2 + t = 4 - 2s$ Solving any two of these equations. M1 for correct approach; 1 for first correct solution; M1 for substituting; 1 for second correct solution. If **1** and **2** solved, $t = -3$ and $s = 5$ If **1** and **3** solved, $t = -\dfrac{8}{3}$ and $s = \dfrac{13}{3}$ If **2** and **3** solved, $t = -\dfrac{14}{5}$ and $s = \dfrac{22}{5}$ Confirming solutions do not satisfy third equation (A statement is sufficient)		**[1]** **[M1]** **[1]** **[M1]** **[1]** **[1]**

Question B6			
a)	i.	Any sensible example	**[1]**
	ii.	For the mean to be 6, readings must add up to 42. The six given readings add up to 43, so k = -1.	**[M1]** **[1]**
	iii.	Mean = 54 ÷ 6 = 9 Sum of squares comes to 568 Standard deviation = $\sqrt{\left(\frac{568}{6} - 9^2\right)}$ (Using their sum of squares and their mean) = √(13.6) ≈ 3.7	**[1]** **[M1]** **[1]**
b)	i.	Frequency / Mid-value (x) / fx / interval width / freq. density: 20 / 50 / 1000 / 20 / 1.0 42 / 65 / 2730 / 10 / 4.2 44 / 72.5 / 3190 / 5 / 8.8 40 / 77.5 / 3100 / 5 / 8.0 30 / 85 / 2550 / 10 / 3.0 24 / 105 / 2520 / 30 / 0.8 Totals: 200 / / 15090 / / Mean = 15090 ÷ 200 = 75.45 (1 mark for correct mid-values; 1 mark for correct fx column; 1 mark for correct estimated mean.)	**[3]**
	ii.	Modal interval is 70 < t ≤ 75	**[1]**
	iii.	Dividing frequency by interval width Correct frequency densities	**[M1]** **[1]**
	iv.	Correct histogram. 1 mark lost (up to a maximum of 3 marks) for each incorrect rectangle.	**[3]**

Table for b) i.:

Frequency	Mid-value (x)	fx	interval width	freq. density
20	50	1000	20	1.0
42	65	2730	10	4.2
44	72.5	3190	5	8.8
40	77.5	3100	5	8.0
30	85	2550	10	3.0
24	105	2520	30	0.8
200		15090		