NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB001 Maths Part 2 (Business) Examination

2013-14

MARK SCHEME

Notice to markers.

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A6. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate-or otherwise produce-a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks.

When this happens, write ECF next to the ticks.

M=Method
A=Answer

If a student has answered more than the required number of questions, credit should only be given for the first n answers, in the order that they are written in the student's answer booklet (n being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1	
a)	Mode = 2 **A1**
b)	Median number of times corresponds to $\frac{35+1}{2}$ th = 18^{th} cumulative frequency **M1** Therefore median = 2 **A1**

Question A2

a) P(that there are more than two glasses in a box that do not have pictures on them)

$\mathrm{p}(\mathrm{X} > 2) = 1 - \big(\mathrm{p}(\mathrm{X} = 0) + \mathrm{p}(\mathrm{X} = 1) + \mathrm{p}(\mathrm{X} = 2)\big)$ **M1**

$= 1 - \left(\binom{6}{0}(0.1)^0(0.9)^6 + \binom{6}{1}(0.1)^1(0.9)^5 + \binom{6}{2}(0.1)^2(0.9)^4 \right)$ **M1**

=1-0.98415

=0.01585 **A1**

b) For one box

$E[X] = np$

$6 \times 0.1 = 0.6$

Therefore in four boxes expect a total of

$4 \times 0.6 = 2.4$ glasses with no picture **M1, A1**

Question A3

$\bar{x} = \frac{\sum x}{n}$ therefore total of the times for the 45 employees is

$\sum x = n\bar{x} = 45 \times 39 = 1755$ mins

Total for all 47 employees is $1755 + 35 + 60$ = 1850 mins **M1**

Then the mean travelling time to work of the 47 employees is

$\frac{1850}{47} = 39.36$ mins **A1 (accept anything which rounds to 39.4 mins)**

Question A4

Given

$p(A) = 0.1,\ p(A \cup B) = 0.7\ ,\ p(B|A) = 0.3\ ,\ p(C) = 0.2\ ,\ p(C|A) = 0\ and\ p(C|B) = 0$

a) $p(A \cap B) = p(B|A)p(A) = 0.3 \times 0.1$ **M1**

=0.03 **A1**

b) Since $p(C|A) = 0\ and\ p(C|B) = 0$ then

$p(A \cup B \cup C) = p(A \cup B) + p(C) = 0.7 + 0.2$ **M1**

$= 0.9$ **A1**

Question A5	
a)	$k = 1 - (\frac{3}{5} + \frac{1}{10} + \frac{1}{10}) = \frac{2}{10} = \frac{1}{5}$ **A1**
b)	$E[3X + 2] = \sum(3x + 2)p(x)$ **M1** $= (3 \times 4 + 2)\frac{3}{5} + (3 \times 5 + 2)\frac{1}{10} + (3 \times 6 + 2)\frac{1}{5} + (3 \times 7 + 2)\frac{1}{10}$ **M1 for finding 3x+2 and M1 for finding p(x)** =8.4+1.7+4+2.3 = 16.4 **A1**
c)	$p(4 \leq X \leq 5.7))$ $= \frac{3}{5} + \frac{1}{10} = \frac{7}{10}$ **A1**

Question A6

If $f(x) = e^{2x}$ and $g(x) = x^2 + 3$ express $gf(x)$ as a single function $h(x)$.

$h(x) = gf(x) = g(e^{2x})$ **M1**

$h(x) = g(e^{2x}) = (e^{2x})^2 + 3$ **M1**

$h(x) = (e^{4x}) + 3$ **A1**

$h(x) = (e^{12}) + 3$=162757.7914=163000 **(3sf)** **M1, A1**

Question A7

Find the inverse of $f(x)=\dfrac{4}{x+2}$

$f(x)=\dfrac{4}{x+2}$

$y=\dfrac{4}{x+2}$ **Or**

$y(x+2)=4$ **M1** $x+2=\dfrac{4}{y}$

$xy+2y=4$

$xy=4-2y$

$x=\dfrac{4-2y}{y}$ **M1** $x=\dfrac{4}{y}-2$

$f^{-1}(x)=\dfrac{4}{x}-2$ **A1** $f^{-1}(x)=\dfrac{4}{x}-2$

Question A8

a)

$f(x)=x(x-3)=x^2-3x$

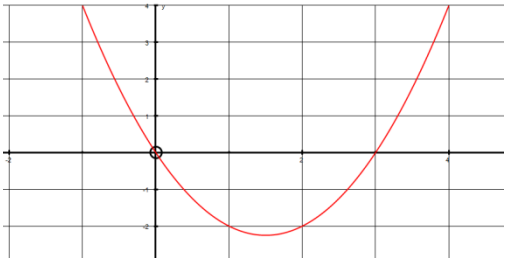
$f(-x)=-x(-x-3)$ **M1**

$x^2+3x\neq f(x)$ not even

$x^2+3x\neq -f(x)=-x^2+3x$ not odd **M1**

Therefore neither **A1**

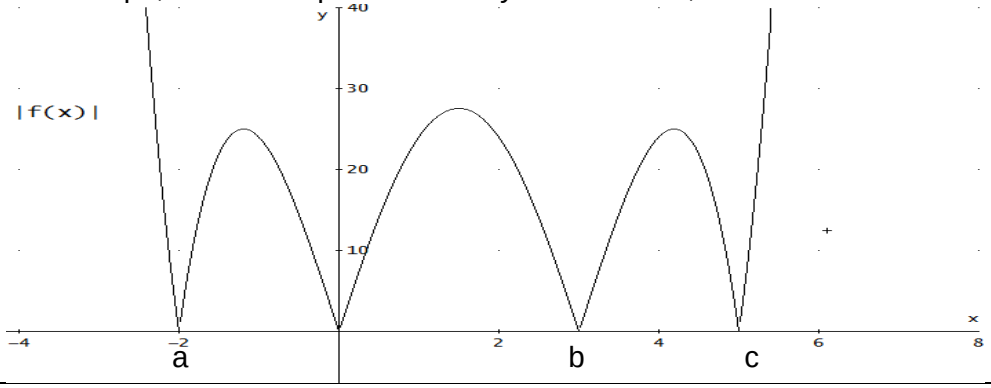
b)



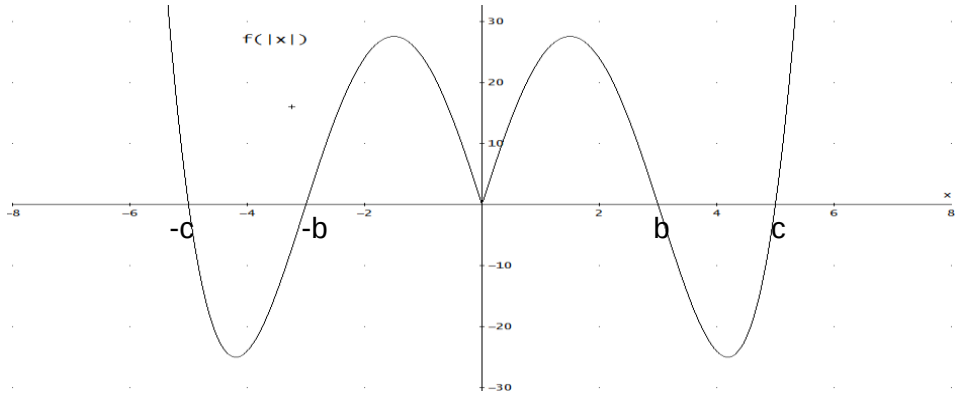
The graph is not symmetrical about the y-axis and is changed if rotated 180^0 about the origin.
Sketch **A1**
Not symmetrical **A1**
Can't rotate **A1**

Question A9

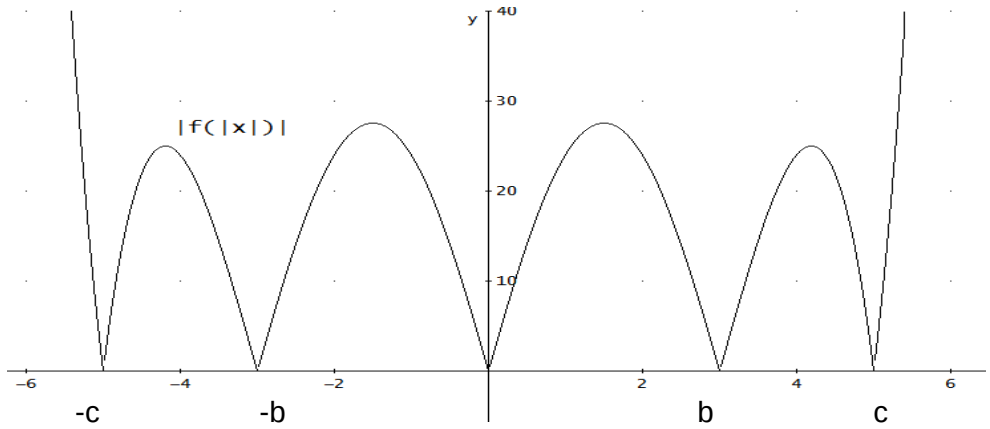
a) **A1** shape, **A1** for the points correctly labelled as a, b and c.



b) **A1** shape, **A1** for the points correctly labelled as –c,-b.b.and c



c) **A1** shape**,** **A1** for the points –c,-b, b and c.



Section B
Answer 4 questions. This section carries 60 marks.

Question B1		
a)	i.	$n(M) = 46 + 25 = 71$ **A1**
	ii.	$n(\bar{E} \cap M) = 46$ **A1**
	iii.	$n(\bar{E} \cup M) = 27 + 46 + 25 = 98$ **A1**
b)	i.	the set of males who are employed is $M \cap E$ **A1**
	ii.	the people who are unemployed who are not male is $\bar{E} \cap \bar{M}$ **A1**
	iii.	the set of males who are unemployed or people who are employed is $(M \cap \bar{E}) \cup E$ (or any equivalent set) **M1, A1**
c)	i.	If 3 per cent of carrots selected at random are rejected as too small then $p(X < K) = 0.03$ $p\left(Z < \frac{K-125}{\sqrt{20}}\right) = 0.03$ **M1** therefore by symmetry $p\left(Z > \frac{125-K}{\sqrt{20}}\right) = 0.03$ $p\left(Z < \frac{125-K}{\sqrt{20}}\right) = 0.97$ From Normal tables, $p(Z < 1.88) = 0.97$ **M1** Therefore $\frac{125-K}{\sqrt{20}} = 1.88$ **M1** $K = 125 - 1.88(\sqrt{20}) = 116.59 \ mm$ **A1**
	ii.	the probability that a randomly selected carrot has a length which is at least 130mm $p(X \geq 130)$ $=1 - p\left(Z < \frac{130-125}{\sqrt{20}}\right)$ **M1** $=1 - p(Z < 1.12)$ =1-0.8686 = 0.1314 **A1**
	iii.	$\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$ $68 \pm 1.96\frac{4}{\sqrt{25}}$ **M1** (66.43,69.57) or any suitably rounded answer such as (66.4,69.6) **A1**

Question B2

a) Using $y - \bar{y} = \frac{s_{xy}}{s_{x^2}}(x - \bar{x})$ where $s_{xy} = \frac{\sum xy}{n} - \bar{x}\bar{y}$ $s_{x^2} = \frac{\sum x^2}{n} - \bar{x}^2$

$s_{xy} = \frac{21405}{9} - \left(\frac{485}{9}\right)\left(\frac{552}{9}\right) = -926.8518519$

$s_{x^2} = \frac{31475}{9} - \left(\frac{485}{9}\right)^2 = 593.2098765$

$y - \frac{552}{9} = \frac{-926.8518519}{593.2098765}(x - \left(\frac{485}{9}\right))$ **M3**

$y = -1.56x + 145.53$ or any appropriately rounded coefficients **A1**

b) The value of b represents the gradient of the line. **A1**

Or

The value of b represents the rate of change of y for a unit change in x.

c) The value x=140 is quite far beyond the range of the recorded values of x, hence the equation would not provide a reliable estimate of y for a value of x = 140. **A1**

Or any other well-justified response.

d) If one of the observed pairs is x = 55 and y= 49

Then using the regression equation, when x= 55

$y = -1.56(55) + 145.53 = 59.73 = 59.7\ approx$ **A1**

e) For calculating a difference between the estimate of y and the observed value of y

$eg.$ 49-59.7=-10.7 **A1**

For any appropriate comment which relates to the suitability (or otherwise) of the regression equation found such as “There is quite a big difference the regression equation doesn’t work so well for that point (but r is high so that point may just be an outlier)” **A1**

f) Pearson's Product Moment Correlation Coefficient, r.

$r = \frac{s_{xy}}{s_x s_y}$

where

$s_{y^2} = \frac{\sum y^2}{n} - y^2$

$= \frac{47204}{9} - \left(\frac{552}{9}\right)^2 = 1483.111111$

$r = \frac{-926.8518519}{\sqrt{593.2098765}\sqrt{1483.111111}}$ **M2**

=-0.988 **A1**

g) The value of r indicates strong negative correlation **A1**

as indicated by the appearance of the points in the scatter diagram lying very close to a straight line which has negative gradient **A1**

h)	Any sensible comments which may include: “Measures of the two variables have been made over time. They may both, separately, have a dependence upon time.” “ A large value of the correlation coefficient does not PROVE a causal relationship.” **A1** (ie.for any appropriate comment such as that one does not necessarily cause the other.)

Question B3		
a)	"The peaks and troughs are decreasing over time and each set of quarterly variations within a year they appear to decrease proportionally." **A1** for recognising the change of size of the seasonal variations over time and **A1** for recognising a proportional change.	
b)	A=(143+49+41+97)/4=82.5 **A1** B=(77+79.75)/2=78.375 **A1** C=41/85.125=0.48 **A1**	
c)	D=1.82 **A1** E=(1.59+1.75+1.82)/3=1.72 **A1** F=(1.14+1.18)/2=1.16 **A1**	
d)	A seasonal effect of 0.47 for Quarter 1 means that the sales in Quarter 1 are on average 53 percent below the general trend **M1** and 0.6 for Quarter 4 means that the sales in Quarter 4 are on average 40 percent below the general trend. (or are only equivalent to 60 percent of the general trend value) **M1** **A1** Only award this mark if the proportionality is accounted for in the description of the figures.	
e)	i.	quarter 1 of year 2009 x=17 Trend $y = 2.86(17) + 45.9 = 94.52$ (£94,520) **M1** Seasonal Ratio 0.47 Projected value $94.52 \times 0.47 = 44.4244$ (£44,424) **A1** Accept £44,424.40
	ii.	quarter 3 of year 2009 x=19 Trend $y = 2.86(19) + 45.9 = 100.24$ (£100,240) **M1** Seasonal Ratio 1.72 Projected value $100.24 \times 1.72 = 172.41$ (£172,410) **A1**

Question B4

a) Symmetrical.

+ve skew

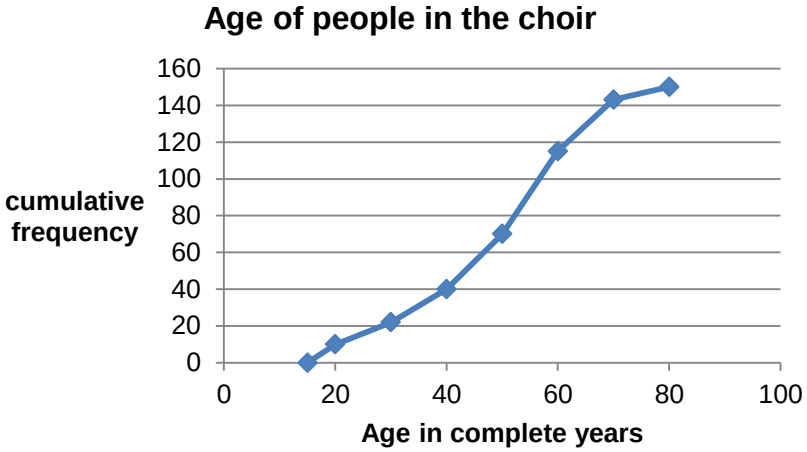
–ve skew A1 for one correct answer, **A2** if all are correct

b)

Age in complete years	Frequency	End points	cum freq
		15	0
15 to 19	10	20	10
20 to 29	12	30	22
30 to 39	18	40	40
40 to 49	30	50	70
50 to 59	45	60	115
60 to 69	28	70	143
70 to 79	7	80	150
Total	**150**		

M1 for the end points and **M1** for the cum. Freq. Column. (at least 6 in each correct to get mark).

c)



M1 scale
M1 for the both labels
A2 for the plots (1 mark **must be** for plotting the first point at their lower class bound on the horizontal axis and 1 mark if at least 6 points are plotted correctly based on upper bounds found by the student).**-1** if graph paper is not used.

d)

Median =	52
Q1=	38
Q3=	58

M1 each from their graph. **(3 marks in total)**

<table>
<tr><td>e)</td><td><table><tr><td>Q2-Q1=</td><td>14</td></tr><tr><td>Q3-Q2=</td><td>6</td></tr></table>
M1 for both cal. A1 for comment Negative skew therefore c.</td></tr>
<tr><td>f)</td><td>About 55 people. M1 for use of graph. A1 for their answer.</td></tr>
</table>

Question B5

a)i. $\Delta = 2\begin{vmatrix}3 & 2\\ 1 & 3\end{vmatrix} - 1\begin{vmatrix}-1 & 2\\ -2 & 3\end{vmatrix} + 2\begin{vmatrix}-1 & 3\\ -2 & 1\end{vmatrix}$...**M1**

$\Delta = 2(9-2) - 1(-3+4) + 2(-1+6)$....**M1**

$\Delta = 14 - 1 + 10 = 23$ ...**M1**

Must show working, if no working then only M1 they have used their calculator.

ii. Using $A^{-1}A=I$

$$\frac{1}{23}\begin{bmatrix}7 & -1 & -4\\ -1 & a & -6\\ b & c & 7\end{bmatrix}\begin{bmatrix}2 & 1 & 2\\ -1 & 3 & 2\\ -2 & 1 & 3\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$.....**M1**

$\frac{1}{23}(-2-a+12) = 0$

$10 - a = 0 \rightarrow a = 10$ **M1**

$\frac{1}{23}(2b - c - 14) = 0$

$2b - c = 14$ eq1

$\frac{1}{23}(b + 3c + 7) = 0$

$b + 3c = -7$ eq2

3eq1+eq2 gives $7b = 35 \rightarrow b = 5 \ \therefore c = -4$ .....**M1, A1** for both answers

iii. Hence,

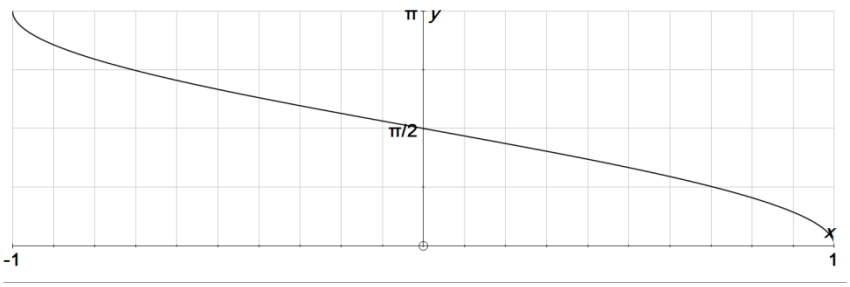
$$\begin{bmatrix}2 & 1 & 2\\ -1 & 3 & 2\\ -2 & 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}2\\ 1\\ 9\end{bmatrix}$$ **M1**

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{23}\begin{bmatrix}7 & -1 & -4\\ -1 & 10 & -6\\ 5 & -4 & 7\end{bmatrix}\begin{bmatrix}2\\ 1\\ 9\end{bmatrix}$$.....**M1**

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{23}\begin{bmatrix}-23\\ -46\\ 69\end{bmatrix}$$.....**M1**

$x = -1, y = -2, z = 3$ **M1**

b) i.



M1 shape
M1 domain -1 to 1
M1 range $0 \ to \ \pi$

ii. $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ **M1**

Question B6

a)

$$\int \frac{x^3-3x-5}{(x+2)(x-1)}\,dx = \int \frac{x^3-3x-5}{(x^2+x-2)}\,dx$$

$$= \int \frac{x(x^2+x-2)-(x^2+x-2)-7}{(x^2+x-2)}\,dx$$

$$= \int x-1+\frac{-7}{(x^2+x-2)}\,dx$$

$$= \frac{-7}{(x^2+x-2)} = \frac{-7}{(x+2)(x-1)}$$

$$= \frac{-7}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

$$= \frac{-7}{(x+2)(x-1)} = \frac{A(x-1)+B(x+2)}{(x+2)(x-1)}$$

$$= -7 = A(x-1)+B(x+2)$$

Let $x=1 \Rightarrow -7 = B(1+2)$

$$\Rightarrow B = \frac{-7}{3}$$

Let $x=-2 \Rightarrow -7 = A(-2-1)$

$$\Rightarrow A = \frac{7}{3}$$

$$\int x-1+\frac{-7}{(x^2+x-2)}\,dx = \int x-1+\frac{7}{3(x+2)}-\frac{7}{3(x-1)}\,dx$$

$$= x^2 - x + \frac{7}{3}\ln(x+2) - \frac{7}{3}\ln(x-1) + c$$

$$= x^2 - x + \frac{7}{3}\left(\ln\left(\frac{x+2}{x-1}\right)\right) + c$$

M1 for attempting the division

M1 for attempting Partial Fractions

A1 for B

A1 for A

M1 for the integration

A1 for the answer

b)

$$\int xe^{2-3x^2}\,dx$$

$$u = 2-3x^2$$

$$\frac{du}{dx} = -6x \Rightarrow \frac{dx}{du} = \frac{1}{-6x}$$

$$\int xe^{2-3x^2}\,dx = \int xe^u \frac{1}{-6x}du$$

$$= \int \frac{1}{-6}e^u du = \frac{1}{-6}\int e^u du = \frac{1}{-6}e^{2-3x^2} + c$$

M1 for $\frac{dx}{du} = \frac{1}{-6x}$

M1 for $\int xe^u \frac{1}{-6x}du$

M1 for $\frac{1}{-6}\int e^u du$

A1 mark for $\frac{1}{-6}e^{2-3x^2} + c$

must have $+c$

c)

$$V = \pi \int_{-1}^{1} y^2 dx = \pi \int_{-1}^{1} \left(\frac{1}{\cos(x)} \right)^2 dx = \pi \int_{-1}^{1} \frac{1}{(\cos(x))^2} dx$$

$$= \pi \int_{-1}^{1} \frac{1}{\cos^2(x)} dx$$

$$= \pi \left[\tan(x) \right]_{-1}^{1}$$

$$= \pi \left[\tan(1) - \tan(-1) \right]$$

$$= 9.785 \text{ cubic units}$$

$$= 9.79 \text{ cubic units}$$

M1 $\pi \int_{-1}^{1} \frac{1}{(\cos(x))^2} dx$

M1 for the integration

M1 for substitution of limits

A1 for answer which rounds to 10.0
A1 cubic units.