JEE MAINS

NEW SYLLABUS

MATRICES & DETERMINANTS

100 mins ka

JEE MAINS to ADVANCED : PLAYLISTS for DIFFERENT LEVELS

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CHAPTER HIGHLIGHTS: MATRICES & DETERMINANT

01	Average Number of Questions in JEE Main	**02**
02	Input Vs Output	• Low input high output • Questions based on direct concepts or properties/formula can be asked.
03	Prerequisites Knowledge	• ______
04	Easiest Topics	• Types of Matrices, **Algebra of Matrices**, Determinant Expansion • System of Linear Equations
05	Most Repeated Topics	• A^n form, Properties of Determinant • Transpose, **Adjoint** & Its Properties • System of Linear Equations
06	Key Highlights of Chapter	• This chapter is more of definition and properties centric. • Majorly easy to moderate **Doable** questions asked. • Properties are most important, Must do practice on properties of all topics

JEE MAIN : MATRICES & DETERMINANTS (NEW Syllabus)

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NEHA MA'AM'S ARMY

MATRICES & DETERMINANTS	2023 (24 Papers)	2022 (22 Papers)	2021 (26 Papers)	2020 (16 Papers)	2019 (16 Papers)
	47	48	62	31	31

TOPICS	Algebra of Matrices	Symmetric & Skew–Symmetric Matrices	System of Linear Equations	Adjoint & Its Properties	Determinants Expansion & Properties	Inverse of a Matrix	Misc. Problems
Number of Qs in JEE Main 2023	6	1	14	9	3	2	12
Number of Qs in JEE Main 2022	14	1	13	8	1	2	9
Number of Qs in JEE Main 2021	17	2	18	5	9	4	7

MATRICES & DETERMINANTS

- Matrices Introduction
- Types of Matrices
- Algebra of Matrices
- Trace of Matrix
- Transpose and its properties
- Some Special Types of Matrices
- Adjoint and Inverse of Matrices
- Elementary Operations
- Expansion of Determinant
- Minor & Cofactors
- Properties of Determinant
- Area of triangle Using Determinant
- Characteristic equation
- System of Linear Equations
- Some Special Determinant
- Differentiation & Integration of determinants
- Product of Determinant

D.I.Y

Q. If the system of equations

$x + y + z = 6,$

$x + 2y + \lambda z = 10$ and

$x + 2y + 3z = \mu$

has infinite solutions, then the value of $\lambda + 2\mu$ is equal to

A 20

B 22

C 23

D 25

MATRIX

Matrix is a rectangular arrangement of numbers, arranged in horizontal rows and vertical columns. Plural of matrix is matrices.

E.g. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$

MATRIX

Matrix is a rectangular arrangement of numbers, arranged in horizontal rows and vertical columns. Plural of matrix is matrices.

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Note:

1. Each entity or number in a matrix is called an element.
2. In any matrix Horizontal lines are called ROWS while the vertical lines are called COLUMNS.

ORDER of a MATRIX

If a matrix has m rows and n columns, then the order is written as $m \times n$.

Example: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}, \underset{1\times 3}{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}}, \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}_{3\times 2}$

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GENERAL MATRIX of ORDER m × n

A Matrix is an arrangement of various elements

a_{ij} → col no

row no

$$\begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \vdots & & & \vdots \\ & & a_{ij} & \\ a_{m1} & a_{m2} & \cdots\cdots & a_{mn} \end{bmatrix}$$

i^{th} row

j^{th} column

$m \times n$

rows

columns

$[a_{ij}]$ $m \times n$

Q. Construct a 2×3 $matrix A = [a_{ij}]$, whose elements are given by $a_{ij} = \frac{(i+2j)^2}{2}$

$a_{11} = \frac{(1+2(1))^2}{2} = \frac{9}{2}$

$a_{12} = \frac{(1+2(2))^2}{2} = \frac{25}{2}$

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times3}$

$a_{22} = \frac{(2+2(2))^2}{2} = \frac{36}{2} = 18$

Solution:

We have, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3}$

Since, $a_{ij} = \dfrac{(i+2j)^2}{2}$, therefore

$$a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2},\ a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2},$$

$$a_{13} = \frac{(1+6)^2}{2} = \frac{49}{2},\ a_{21} = \frac{(2+2)^2}{2} = 8,$$

$$a_{22} = \frac{(2+4)^2}{2} = 18 \text{ and } a_{23} = \frac{(2+6)^2}{2} = 32$$

Hence, the required matrix is $A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} & \frac{49}{2} \\ 8 & 18 & 32 \end{bmatrix}$.

Types of Matrices

1) Horizontal matrix :

A matrix with only one row and plural columns.

$$\begin{bmatrix} a_{11} & a_{12} \ldots\ldots\ldots & a_{1n} \end{bmatrix}_{1 \times n}$$

e.g. Let A =[10 -3 4 6]

2) Vertical matrix :

A matrix with only one column and many rows.

$$\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ \vdots \\ a_{m1} \end{bmatrix}_{m \times 1}$$

e.g. Let A = $\begin{bmatrix} -8 \\ 9 \\ 12 \end{bmatrix}$.

Types of Matrices

3) Zero/Null matrix :

Matrix in which all the elements are Zero.

e.g. i) Let $A_{2x2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

ii) Let $A_{2x3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 & \dots\dots\dots\dots & 0 \\ 0 & 0 & \dots\dots\dots\dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots\dots\dots\dots & 0 \end{bmatrix}_{m \times n}$$

i.e. $a_{ij} = 0, \forall \; i, j$

Types of Matrices

4) Square matrix :

Heera

A matrix that has equal number of rows and columns.

a_{ii} : diag. elts

$$\begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \vdots & & a_{ii} & \vdots \\ a_{n1} & a_{n2} & \cdots\cdots & a_{nn} \end{bmatrix}_{m\times n}$$

e.g. Let $A = \begin{bmatrix} 2 & 3 & -7 \\ 6 & 8 & 9 \\ 10 & 5 & 4 \end{bmatrix}$

For Square matrices: $m = n$

4) Square matrix :

$$\begin{pmatrix} -1 & 2 & 3 \\ 4 & 0 & 5 \\ -10 & 6 & 7 \end{pmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \vdots & & a_{ii} & \vdots \\ a_{n1} & a_{n2} & \cdots\cdots & a_{nn} \end{bmatrix}$$

Note:

- $a_{ii} \rightarrow$ diagonal elements $\rightarrow a_{11}, a_{22}, \ldots a_{nn}$.
- a_{ij} & $a_{ji} \rightarrow$ conjugate elements $\rightarrow a_{12}$ & a_{21}, a_{23} & a_{32}, …. a_{1n} & a_{n1}
- Trace $\rightarrow$ Trace is the sum of diagonal elements.

$$\sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + a_{33} + \cdots \text{ and so on}$$

SQUARE MATRIX

- Diagonal
- Upper Triangular
- Lower Triangular

$a_{ij} = 0 \ \forall \ i \neq j$ (non-diag)

$$\begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \vdots & & a_{ii} & \vdots \\ a_{n1} & a_{n2} & \cdots\cdots & a_{nn} \end{bmatrix}$$

Diagonal

$$\begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \vdots & & a_{ii} & \vdots \\ a_{n1} & a_{n2} & \cdots\cdots & a_{nn} \end{bmatrix}$$

Neeche = 0

$$\begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \vdots & & a_{ii} & \vdots \\ a_{n1} & a_{n2} & \cdots\cdots & a_{nn} \end{bmatrix}$$

Upar = 0

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Triangular Matrix

Upper Triangular

Neeche 0

$$\begin{bmatrix} a_{11} & a_{12} & \dots\dots\dots & a_{1n} \\ 0 & a_{22} & \dots\dots\dots & a_{2n} \\ 0 & 0 & a_{33} & a_{3n} \\ \vdots & & & \vdots \\ 0 & 0 & 0\dots\dots\dots & a_{nn} \end{bmatrix}$$

$\boxed{a_{ij} = 0;\ \forall\, i > j}$

Lower Triangular

Upar 0

$$\begin{bmatrix} a_{11} & 0 & 0 & \dots\dots\dots & 0 \\ a_{21} & a_{22} & 0 & \dots\dots\dots & 0 \\ a_{31} & a_{32} & a_{33} & & 0 \\ \vdots & & & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots\dots\dots & a_{nn} \end{bmatrix}$$

$\boxed{a_{ij} = 0;\ \forall\, i < j}$

Types of Matrices

5) Diagonal Matrix:

Matrix with, all non-diagonal entries: Zero

i.e. $a_{ij} = 0$, if '$i \neq j$'

$$\begin{bmatrix} a_{11} & 0 & 0 & \cdots\cdots & 0 \\ 0 & a_{22} & 0 & \cdots\cdots & 0 \\ 0 & 0 & a_{33} & \cdots\cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots\cdots & a_{nn} \end{bmatrix}$$

$a_{ii} = k \neq 0$

Sq matrix

$\downarrow$

Diag matrix

$\downarrow$

Scalar

$\downarrow$

Identity/Unit

Types of Matrices

6) Scalar Matrix:

Matrix with, all diagonal entries same

i.e. $a_{ij} = 0$, if '$i = j$'

$$\begin{bmatrix} k & 0 & 0 & \cdots\cdots & 0 \\ 0 & k & 0 & \cdots\cdots & 0 \\ 0 & 0 & k & \cdots\cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots\cdots & k \end{bmatrix}$$

Types of Matrices

7) Identity/Unit Matrix:

Matrix with, all diagonal entries are equal to 1

i.e. $a_{ij} = 0$, if '$i = j = 1$'

$$\begin{bmatrix} 1 & 0 & 0 & \cdots\cdots & 0 \\ 0 & 1 & 0 & \cdots\cdots & 0 \\ 0 & 0 & 1 & \cdots\cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots\cdots & 1 \end{bmatrix}$$

e.g. Let $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Algebra of Matrices

Equality of Matrices:

Two matrices are said to be equal if:

1. Their orders are equal ONLY TRUE. Equivalent matrices
2. Corresponding elements are equal.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$$

$$\begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}_{2\times3}$$

Q. If $\begin{bmatrix} 2\alpha+1 & 3\beta \\ 0 & \beta^2-5\beta \end{bmatrix} = \begin{bmatrix} \alpha+3 & \beta^2+2 \\ 0 & -6 \end{bmatrix}$

Find the equation whose roots are α and β.

$\boxed{x^2-4x+4=0}$

$\beta^2-5\beta+6=0$

$(\beta-2)(\beta-3)=0$

$\Rightarrow \beta=2,3$

$2\alpha+1=\alpha+3$

$\alpha=2$

$\beta=2$

$\beta^2+2-3\beta=0$

$(\beta-2)(\beta-1)=0$

$\beta=1,2$

Solution: The given matrices will be equal, iff

$$2\alpha + 1 = \alpha + 3 \Rightarrow \alpha = 2$$

$$3\beta = \beta^2 + 2 \Rightarrow \beta^2 - 3\beta + 2 = 0$$

$$\therefore \quad \beta = 1, 2 \quad \text{and} \quad \beta^2 - 5\beta = -6 \qquad \text{...(i)}$$

$$\Rightarrow \quad \beta^2 - 5\beta + 6 = 0$$

$$\therefore \quad \beta = 2, 3 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $\beta = 2$

$$\Rightarrow \quad \alpha = 2, \beta = 2$$

$\therefore$ Required equation is $x^2 - (2 + 2)x + 2 \cdot 2 = 0$

$$\Rightarrow \quad x^2 - 4x + 4 = 0$$

Algebra of Matrices

Addition of matrices

Two matrices can be added only if :

1. order of the matrices are equal
2. are to be added only term by term

E.g. $\begin{bmatrix} 4 & 8 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 8 & 9 \end{bmatrix}$

Algebra of Matrices

Addition of matrices

Two matrices can be added only if :

1. order of the matrices are equal

2. are to be added only term by term

E.g. $\begin{bmatrix} 4 & 8 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 4+1 & 8+0 \\ 3+5 & 7+2 \end{bmatrix}$

Algebra of Matrices

Subtraction of matrices

Two matrices can be subtracted only if :

1. order of the matrices are equal

2. are to be subtracted only term by term

E.g. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ 1 & 1 \end{bmatrix}$

Algebra of Matrices

Subtraction of matrices

Two matrices can be subtracted only if :

1. order of the matrices are equal

2. are to be subtracted only term by term

E.g. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1-5 & 2-6 \\ 3-2 & 4-3 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ 1 & 1 \end{bmatrix}$

Algebra of Matrices

Multiplication by a constant

$$k \times \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{bmatrix}$$

$$3 \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} \checkmark$$

100 mins DAILY DOSE 2.0

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Properties of Algebra of Matrices

Operation	Input	Output	Properties
Sum	$A_{m\times n}+B_{m\times n}$	$(A+B)_{m\times n}$	1. $A+B=B+A$ (Commutative) 2. $(A+B)+C=A+(B+C)$ (Associative)
Difference	$A_{m\times n}-B_{m\times n}$	$(A-B)_{m\times n}$	Negative of a matrix $A_{m\times n}$ is $-A_{m\times n}$
Scalar Multiplication	$K(A)_{m\times n}$ $K\neq 0$	$(KA)_{m\times n}$	K gets multiplied with every element.
Additive identity	$\underset{m\times n}{A}+\underset{m\times n}{O}$	$\underset{m\times n}{O}+\underset{m\times n}{A}$	$A+O=O+A=A$
Additive inverse	$\underset{m\times n}{A}+\underset{m\times n}{(-A)}$	$\underset{m\times n}{(-A)}+\underset{m\times n}{A}$	$A+(-A)=(-A)+A=O$

Algebra of Matrices

Multiplication of Two Matrices

Imp.

$$A_{m\times n} \times B_{n\times p} = C_{m\times p}$$

Same

Pre multiplier

Post multiplier

$$\boxed{AB \neq BA \text{ in genl.}}$$

$$(AB)C = A(BC)$$

Q. The order of $\underbrace{[x\ y\ z]}_{1\times 3}\underbrace{\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}}_{3\times 3}\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{3\times 1}$ *is*

$(1\times 3)\ (3\times 1)$

A. 3×1

B. 1×1

C. 1×3

D. 3×3

Solution:

Order will be $(1\times 3)(3\times 3)(3\times 1)=(1\times 1)$

Q. The order of $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is

A 3×1

B 1×1

C 1×3

D 3×3

TAK TAK

Q. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$

find AB and BA. → not psbl

$$AB = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}_{3\times3} \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}_{3\times2} = \begin{bmatrix} 0-1+4 & 0+0-2 \\ +1-2+6 & -2+0-3 \\ +2-3+8 & -4+0-4 \end{bmatrix}_{3\times2}$$

$$= \begin{bmatrix} 3 & -2 \\ 5 & -5 \\ 7 & -8 \end{bmatrix}$$

$$BA = [\quad]_{3\times2} [\quad]_{3\times3}$$

$2 \neq 3$

Solution:

$$AB = \begin{array}{c} \\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \end{array} \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array} \times \begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} 0\times1+1\times(-1)+2\times2 & 0\times(-2)+1\times0+2\times(-1) \\ 1\times1+2\times(-1)+3\times2 & 1\times(-2)+2\times0+3\times(-1) \\ 2\times1+3\times(-1)+4\times2 & 2\times(-2)+3\times0+4\times(-1) \end{bmatrix}_{3\times2}$$

$$= \begin{bmatrix} 0-1+4 & 0+0-2 \\ 1-2+6 & -2+0-3 \\ 2-3+8 & -4+0-4 \end{bmatrix}_{3\times2} = \begin{bmatrix} 3 & -2 \\ 5 & -5 \\ 7 & -8 \end{bmatrix}_{3\times2}$$

Since, the number of columns of B is 2 and the number of rows of A is 3, BA is not defined $(\because 2 \neq 3)$.

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

Properties of Matrix Multiplication

✱ Matrix mult^n is def^d.

$A_{m\times n}.B_{p\times q}$ $n=p$	$(AB)_{m\times q}$ A sq. matrix	1. $AB\neq BA$ 2. $A(B+C)=AB+AC$ 3. $A(BC)=(AB)C$ 4. $\boxed{A_{m\times m}.I_{m\times m}=A_{m\times m} = I_{m\times m}.A_{m\times m}}$

I multiplicative Identity

Result:

Q. If $AB = A$ and $BA = B$ then $B^2 =$

A A^2

B B

C A

D -B

$A^2 = ?$

$AB = A$

$(AB)A = A^2$

$\Rightarrow A(BA) = A^2$

$\Rightarrow AB = A^2$

$\Rightarrow A = A^2$

$(BA)B = BB$ (Post multiply with B)

$= B(AB) = B^2$

$\Rightarrow BA = B^2$

$\Rightarrow B = B^2$

Solution:

Given, $AB = A$ and $BA = B$

multiply both sides by B

$BA \times B = B \times B$

$\Rightarrow B(AB) = B^2$

$\Rightarrow B(A) = B^2 \; [\because BA = B]$

$\Rightarrow B = B^2$

$\therefore \boldsymbol{B^2 = B}$

Q. If $AB = A \, and \, BA = B \, then \; B^2 =$

A A^2

B B

C A

D -B

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

NTA PAPER 86

Q. If A and B are two matrices of order 3×3 satisfying $AB = A$ and $BA = B$, then $(A + B)^5$ is equal to

$\Rightarrow A^2 = A$ and $\Rightarrow B^2 = B$

A $5(A + B)$

B $5I$

C $16(A + B)$

D $32I$

Iska matlab

$(A+B)^4 = 2^3(A+B)$

$(A+B)^5 = 2^4(A+B)$

$(A+B)^2 = A^2 + B^2 + AB + BA$

$= A + B + A + B$

$(A+B)^2 = 2^1(A+B)$

$(A+B)^3 = (A+B)^2(A+B)$

$= 2(A+B)(A+B)$

$(A+B)^3 = 2(2(A+B)) = 4(A+B)$

Solution:

$AB = A,\ BA = B$ (Given)

$\therefore A^2 = A \cdot A = A(BA) = ABA = AB$
$= A$

Similarly,

$B^2 = B \cdot B = (BA) \cdot B = B(AB) = BA$
$= B$

So, $(A+B)^2 = A^2 + B^2 + AB + BA$
$= 2(A+B)$

$(A+B)^3 = (A+B)^2 (A+B)$
$= 2(A+B)^2 = 4(A+B)$

Similarly,

$(A+B)^4 = 8(A+B)$

$(A+B)^5 = 16(A+B)$

NTA PAPER 86

Q. If A and B are two matrices of order 3×3 satisfying $AB = A$ and $BA = B$, then $(A + B)^5$ is equal to

A. $5(A + B)$

B. $5I$

C. $16(A + B)$

D. $32I$

$\cancel{(A+B)^2 = A^2 + B^2 + 2AB}$

Q. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ $and\,(A+B)^2 = A^2 + B^2$

Then value of a and b are

A. a=4,b=1

B. a=1,b=4

C. a=0,b=4

D. a=2,b=4

We know

$$\boxed{\begin{aligned}(A+B)^2 &= A^2 + B^2 + AB + BA \\ &= A^2 + B^2\end{aligned}}$$

$$AB + BA = O$$

$$\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Solution:

We have $(A+B)^2 = A^2 + B^2 + A.B + BA = A^2 + B^2$

$\because \; AB + BA = 0$ $\qquad \therefore \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} + \begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix} = 0$

$$\begin{bmatrix} 2a+2-b & -a+1 \\ 2a-2 & 4-b \end{bmatrix} = 0$$

On comparing, we get, $-a+1=0$

$\Rightarrow a=1$ and $4-b=0 \Rightarrow b=4$

Q. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix} B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ $and\,(A+B)^2=A^2+B^2$

Then value of a and b are

A a=4,b=1

B a=1,b=4

C a=0,b=4

D a=2,b=4

Positive Integral Powers of a Square Matrix

(A^n Type Matrices)

The positive integral powers of a matrix A are defined only when A is a square matrix.

Also then, $A^2 = A \cdot A, A^3 = A \cdot A \cdot A = A^2 \cdot A$. Also, for any positive integers m, n

a) $A^m \cdot A^n = A^{m+n}$ ✓

b) $(A^m)^n = A^{mn} = (A^n)^m$

c) $I^n = I, I^m = I$ $\quad m, n \in N$ or Z^+

d) $A^0 = I_n$

Q. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$ is equal to

can be anything

JEE-Main 2019 (9th Jan-1st shift)

A $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

B $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

C $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

D $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

$$A^{-50} = \begin{bmatrix} \cos 50\theta & +\sin 50\theta \\ -\sin 50\theta & \cos 50\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\cos\left(50 \times \frac{\pi}{12}\right)$$

$$\cos\left(4\pi + \frac{2\pi}{12}\right)$$

$$= \cos\frac{\pi}{6} = \sqrt{3}/2$$

$$A^{-50} = \begin{bmatrix} \cos(-50\theta) & -\sin(-50\theta) \\ \sin(-50\theta) & \cos(-50\theta) \end{bmatrix}$$

$$A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

$$A^2=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

$$A^2=\begin{bmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$$

By using symmetry

$$A^{-50}=\begin{bmatrix}\cos(-50\theta) & -\sin(-50\theta)\\ \sin(-50\theta) & \cos(-50\theta)\end{bmatrix}$$

At $\theta=\frac{\pi}{12}$

$$A^{-50}=\begin{bmatrix}\cos\frac{25\pi}{6} & \sin\frac{25\pi}{6}\\ -\sin\frac{25\pi}{6} & \cos\frac{25\pi}{6}\end{bmatrix}=\begin{bmatrix}\cos\frac{\pi}{6} & \sin\frac{\pi}{6}\\ -\sin\frac{\pi}{6} & \cos\frac{\pi}{6}\end{bmatrix}=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

Q. If $A=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta=\frac{\pi}{12}$ is equal to

JEE-Main 2019 (9th Jan-1st shift)

A $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

B $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

C $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

D $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

Q. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then the value of α for which $A^2 = B$ is

kar lena !

(A) 1

(B) -1

(C) 4

(D) No real values

Solution:

$$A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} \because A^2 = B \text{ (given)}$$

Then $\begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \Rightarrow \boxed{\alpha^2 = 1}$ and $\boxed{\alpha + 1 = 5}$. Clearly no real value of α

no soln.

$\boxed{\alpha = \pm 1}$ $\boxed{\alpha = 4}$

No common value

Q. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix},$ then the value of α *for which* $A^2 = B$ *is*

A 1

B -1

C 4

D No real values

JEE(Main): 6th April 2023 -I

Q. Let $A = [a_{ij}]_{2\times 2}$, where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and $b = |A|$. Then $3a^2 + 4b^2$ is equal to :

A 14

B 4

C 3

D 7

$|A^2| = |I|$

$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$

$\left(\because |A^n| = |A|^n \right)$ V imp.

$b = \pm 1$

$a = 0$

$a = Tr(A)$

$b = |A|$

$A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \alpha\beta + \beta\delta = 0$

$\Rightarrow \beta(\alpha + \delta) = 0 \quad \beta \neq 0$

$\alpha + \delta = 0 \leftarrow a$

Solution:

$$A^2 = I \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1 = b$$

$$\text{Let } A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = I$$

$$\begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta + \beta\delta \\ \alpha\gamma + \gamma\delta & \gamma\beta + \delta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Rightarrow \alpha^2 + \beta\gamma = 1$$

$$(\alpha + \delta)\beta = 0 \Rightarrow \alpha + \delta = 0 = a$$ (sum of all diagonal elements of A)

$$(\alpha + \delta)\gamma = 0$$

$$\beta\gamma + \delta^2 = 0$$

$$\text{Now } 3a^2 + 4b^2 = 3(0)^2 + 4(1) = 4$$

JEE(Main): 6th April 2023 -I

Q. Let $A = [a_{ij}]_{2\times2}$, where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and $b = |A|$. Then $3a^2 + 4b^2$ is equal to :

A 14

B 4

C 3

D 7

JEE(Main): 29 Jan 2023 -I

Q. Let α and β real numbers. Consider a 3×3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

(A) $\alpha = 1$

(B) $\alpha = 4$

(C) $\beta = 8$

(D) $\beta = -8$

Solution:

$$A^2 = 3A + \alpha I$$

$$A^3 = 3A^2 + \alpha A$$

$$A^3 = 3(3A + \alpha I) + \alpha A$$

$$A^3 = 9A + \alpha A + 3\alpha I$$

$$A^4 = (9 + \alpha)A^2 + 3\alpha A$$

$$= (9 + \alpha)(3A + \alpha I) + 3\alpha A$$

$$= A(27 + 6\alpha) + \alpha(9 + \alpha)$$

$$\Rightarrow 27 + 6\alpha = 21 \Rightarrow \alpha = -1$$

$$\Rightarrow \beta = \alpha(9 + \alpha) = -8$$

JEE(Main): 29 Jan 2023 -I

Q. Let α and β real numbers. Consider a 3×3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

A. $\alpha = 1$

B. $\alpha = 4$

C. $\beta = 8$

D. $\beta = -8$

JEE(Main): 31 Jan 2023 -I

Q. Let A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$ .Then the sum of the diagonal elements of the matrix $(A+I)^{11}$ is equal to

HACK

A 6144

B 4094

C 4097

D 2050

A^n ques.

Dhyan se Samjho!

$^{11}C_0 + {}^{11}C_1 + \cdots + {}^{11}C_{10} + {}^{11}C_{11} - {}^{11}C_{11}$

$2^{11} - 1$

$(A+I)^{11}$

$= {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} I + {}^{11}C_2 A^9 I^2 + \cdots$

$= A({}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \cdots + {}^{11}C_{10}) + {}^{11}C_{11} I$

$\mathrm{Tr}(A+I)^{11}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$

$\Rightarrow A^3 = A^4 = \cdots = A^{11} = A$

$A(2^{11} - 1) + I$

Solution:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = \mathrm{A}$$

$$\Rightarrow \mathrm{A}^3 = \mathrm{A}^4 = \ldots\ldots = \mathrm{A}$$

$$(\mathrm{A}+\mathrm{I})^{11} = {}^{11}\mathrm{C}_0\,\mathrm{A}^{11} + {}^{11}\mathrm{C}_1\,\mathrm{A}^{10} + \ldots .\,{}^{11}\mathrm{C}_{10}\,\mathrm{A} + {}^{11}\mathrm{C}_{11}\mathrm{I}$$

$$= \left({}^{11}\mathrm{C}_0 + {}^{11}\mathrm{C}_1 + \ldots .\,{}^{11}\mathrm{C}_{10}\right)\mathrm{A} + \mathrm{I}$$

$$= \left(2^{11}-1\right)\mathrm{A} + \mathrm{I} = 2047\,\mathrm{A} + \mathrm{I}$$

$\therefore$ Sum of diagonal elements $= 2047(1+4-3)+3$

$$= 4094 + 3 = 4097$$

$$2047\begin{bmatrix} 1 & & \\ & 4 & \\ & & -3 \end{bmatrix} + \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$= 2047(1+4-3)+3$$

$$=$$

JEE(Main): 31 Jan 2023 -I

Q. Let $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{bmatrix}$. Then the sum of the diagonal elements of the matrix $(A+I)^{11}$ is equal to

A 6144

B 4094

C 4097

D 2050

JEE(Main): 20 July 2021 - I

Q. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ and $B=7A^{20}-20A^{7}+2I$, where I is an identity matrix of order 3×3. If $B=[b_{ij}]$, then b_{13} is equal to

HACK

** A^{20}

**

$A = I + \underline{\underline{C}}$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^{20} = (I+C)^{20} = \boxed{I + {}^{20}C_1 IC + {}^{20}C_2 C^2} + 0 \cdots + 0$$

$$C^2 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C^3 = O = C^4 = C^5 = \cdots$$

Solution:

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = I + C$

where $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

Then,

$C^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$C^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = C^4 = C^5 = \ldots\ldots\ldots$

Now,

$B = 7A^{20} - 20A^7 + 2I$

$= 7(I + C)^{20} - 20(I + C)^7 + 2I$

$= 7(I + 20C + {}^{20}C_2 C^2 + \ldots) - 20(I + 7C + {}^{7}C_2 C^2 + \ldots) + 2I$

$= -11I + 910C^2$

$= \begin{bmatrix} -11 & 0 & 910 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix}$

$-11\begin{bmatrix} & & 0 \\ & & \\ & & \end{bmatrix} + \begin{bmatrix} - & - & 910 \\ - & - & - \\ - & - & \end{bmatrix}$

So,

$b_{13} = 910$

JEE Advanced 2022 : Paper 2

HACK

Q. If $M = \begin{pmatrix} \frac{5}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{1}{2} \end{pmatrix}$, then which of the following matrices is equal to M^{2022}?

$= I + (2022)\frac{3}{2}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$

A $\begin{pmatrix} 3034 & 3033 \\ -3033 & -3032 \end{pmatrix}$

B $\begin{pmatrix} 3034 & -3033 \\ 3033 & -3032 \end{pmatrix}$

C $\begin{pmatrix} 3033 & 3032 \\ -3032 & -3031 \end{pmatrix}$

D $\begin{pmatrix} 3032 & 3031 \\ -3031 & -3030 \end{pmatrix}$

$M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 3/2 & 3/2 \\ -3/2 & -3/2 \end{pmatrix}$

$M = I + A$

$\boxed{M = I + 3/2 \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}}$

$A^2 = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow A^3 = A^4 = \ldots = O$

$M^{2022} = (I + 3/2\, A)^{2022} = \boxed{I + {}^{2022}C_1 I (3/2) A} + 0 + 0 + \ldots$

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

Solution:

$$M = \begin{bmatrix} 1+\frac{3}{2} & \frac{3}{2} \\ -\frac{3}{2} & 1-\frac{3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{3}{2} \end{bmatrix} = I + \frac{3}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$M = I + \frac{3}{2}A$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^3 = A^4 = \dots\dots\dots = O$$

$$M^{2022} = \left(I + \frac{3}{2}A\right)^{2022} = I + 2022\,\frac{3}{2}A + O + O \dots = I + 1011 \times 3A = I + 3033A$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3033 & 3033 \\ -3033 & -3033 \end{bmatrix} = \begin{bmatrix} 3034 & 3033 \\ -3033 & -3032 \end{bmatrix}$$

JEE Advanced 2022 : Paper 2

Q. If $M = \begin{pmatrix} \frac{5}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{1}{2} \end{pmatrix}$, then which of the following matrices is equal to M^{2022}?

A. $\begin{pmatrix} 3034 & 3033 \\ -3033 & -3032 \end{pmatrix}$

B. $\begin{pmatrix} 3034 & -3033 \\ 3033 & -3032 \end{pmatrix}$

C. $\begin{pmatrix} 3033 & 3032 \\ -3032 & -3031 \end{pmatrix}$

D. $\begin{pmatrix} 3032 & 3031 \\ -3031 & -3030 \end{pmatrix}$

Transpose of a Matrix

Matrix obtained by Interchanging Rows and Columns is called Transpose of a Matrix.

Symbol : A^T or A'

$$\text{If } A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}_{3\times 2} \Rightarrow A^T = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}_{2\times 3}$$

Transpose of a Matrix

Matrix obtained by Interchanging Rows and Columns is called Transpose of a Matrix.

Symbol : A^T or A'

$$\text{If} \quad A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}_{3 \times 2} \Rightarrow A^T = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}_{2 \times 3}$$

Transpose of a Matrix

Interchanging Rows and Columns

A^T or A'

$\boxed{\text{Sleep} \equiv \text{Wake Up}}$

$|A^m| = |A|^m$

PROPERTIES

1) $(A^T)^T = A$

2) $(A \pm B)^T = A^T \pm B^T$

3) $(KA)^T = K(A^T)$

4) $\boxed{(AB)^T = B^T A^T}$ (Reversal law holds)

5) $(A^T)^m = (A^m)^T$

6) $\text{adj}A^T = (\text{adj}A)^T$

7) $(A^T)^{-1} = (A^{-1})^T$

8) $\boxed{(ABC)^T = C^T B^T A^T}$

Q. Let $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ such that $A^TA = I$,

NTA Paper 63

then the value of $x^2 + y^2 + z^2$ is

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{3}$$

$$\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & & \\ & 6y^2 & \\ & & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

Solution:

$$A^T = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$

$$= \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^T A = I$$

$$2x^2 = 1 \Rightarrow x^2 = \tfrac{1}{2}$$

$$3z^2 = 1 \Rightarrow z^2 = \tfrac{1}{3}$$

$$6y^2 = 1 \Rightarrow y^2 = \tfrac{1}{6}$$

$$x^2 + y^2 + z^2 = \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{6}$$

$$= \tfrac{1}{2} + \tfrac{1}{2}$$

$$= 1$$

Some Special Types of Matrices

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

Symmetric Matrix

A SQUARE Matrix A is said to be a **Symmetric Matrix** if:

$\boxed{A^T = A}$ $\boxed{\text{i.e. } a_{ij} = a_{ji}}$

→ conjugate elements

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

Skew – Symmetric Matrix:

A SQUARE Matrix A is said to be **Skew–Symmetric** if:

$\boxed{A^T = -A}$ $\boxed{\text{i.e. } a_{ij} = -a_{ji}}$

conjugate elts -ve

Important Points: Transpose of a Matrix

1. If A is a symmetric matrix, then $-A, kA, A^T, A^n, A^{-1}, B^{\top}AB$ are also symmetric matrices, where $n \in N$, $k \in R$ and B is a square matrx of order that A.

2. If A is a skew symmetric matrix, then
(a) A^{2n} is a symmetric matrix for n $\in$ **N**,
(b) A^{2n+1} is a skew-symmetric matrix for **n** $\in$ **N**,
(c) kA is also skew-symmetric matrix, where **k** $\in$ **N**,
(d) $B^{\top}AB$ is also skew-symmetric matrix where B is a square matrix of order that of A.

** Imp.

Imp.

A

Symm $\Rightarrow A^n$ is symm

Skew-Symm: A^{2n} S, A^{2n+1} SS

Important Points: Transpose of a Matrix

3. If A, B are two symmetric matrix, then
(a) $A \pm B$, $AB + BA$ are also symmetric matrix,
(b) $AB - BA$ is a skew-symmetric matrix,
(c) AB is a symmetric matrix, when $AB = BA$.

4. If A, B are two skew symmetric matrix, then
(a) $A \pm B$, $AB - BA$ are skew-symmetric matrices,
(b) $AB + BA$ is a symmetric matrix.

JEE(Main): 25 Jan 2023 -II

Q. Let A,B,C,3×3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements

(S_1): $A^{13}B^{26}-B^{26}A^{13}$ is symmetric ✗

(S_2): $A^{26}C^{13}-C^{13}A^{26}$ is symmetric Then,

A symm

B SS

A Only S_2 is true

B Only S_1 is true

C Both S_1 and S_2 are false

D Both S_1 and S_2 are true

$$(A^{13}B^{26}-B^{26}A^{13})^T$$

$$=(A^{13}B^{26})^T-(B^{26}A^{13})^T$$

$$=(B^{26})^T(A^{13})^T-(A^{13})^T(B^{26})^T$$

$$=(B^T)^{26}(A^T)^{13}-(A^T)^{13}(B^T)^{26}$$

$$=B^{26}A^{13}-A^{13}B^{26}$$

$$=-SS$$

Solution:

Given, $A^T = A$, $B^T = -B$, $C^T = -C$

Let $M = A^{13} B^{26} - B^{26} A^{13}$

Then, $M^T = (A^{13} B^{26} - B^{26} A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$

$= B^{26}A^{13} - A^{13} B^{26} = -M$

Hence, M is skew symmetric

Let, $N = A^{26}C^{13} - C^{13}A^{26}$

then, $N^T = (A^{26} C^{13})^T - (C^{13} A^{26})^T$

$= -(C)^{13}(A)^{26} + A^{26}C^{13} = N$

Hence, N is symmetric.

$\therefore$ Only S2 is true.

JEE(Main): 25 Jan 2023 -II

Q. Let A,B,C,3×3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements

(S_1): $A^{13}B^{26}-B^{26}A^{13}$ is symmetric

(S_2): $A^{26}C^{13}-C^{13}A^{26}$ is symmetric

Then,

A. Only S_2 is true

B. Only S_1 is true

C. Both S_1 and S_2 are false

D. Both S_1 and S_2 are true

Trace of Matrix

Trace is the sum of diagonal elements.

$$\sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + a_{33} + \cdots \text{ and so on}$$

NTA Paper 26

Q. If A is a 2×2 matrix such that $A\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$ and $A^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$, then trace of A is (where , the trace of the ,matrix is the sum of all principal diagonal elements of the matrix)

A 1

B 0

C 2

D 5

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}_{2\times2}\begin{bmatrix}1\\-1\end{bmatrix}_{2\times1}=\begin{bmatrix}-1\\2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}a-b\\c-d\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$$

let $\begin{bmatrix}a & b\\c & d\end{bmatrix}=A$

$a+d=?$

$$A^2\begin{bmatrix}1\\-1\end{bmatrix}=A\left\{A\begin{bmatrix}1\\-1\end{bmatrix}\right\}=A\begin{bmatrix}-1\\2\end{bmatrix}$$

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$$

Solution:

Let, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

On solving, we get,

$a - b = -1$, $c - d = 2$

Also,

$$A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = A\left(A\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = A\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

This gives, $-a + 2b = 1$, $-c + 2d = 0$

$\Rightarrow b = 0,\ a = -1,\ d = 2,\ c = 4$

$\therefore$ Trace of A $= a + d = 1$

NTA Paper 26

Q. If A is a 2×2 matrix such that $A\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$ and $A^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$, then trace of A is (where , the trace of the ,matrix is the sum of all principal diagonal elements of the matrix)

A 1

B 0

C 2

D 5

Important Concept

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then

$T_r(AA^T) = T_r(A^TA) = a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{33}^2$

Important Concept

Q. If $A = \begin{pmatrix} 0 & 1 & -2 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{pmatrix}$ then find $\mathrm{Tr}(AA^T)$.

$0^2+1^2+(-2)^2+$

$1^2+0^2+3^2+$

$2^2+(-3)^2+0^2$

$=28$

Solution:

$a_{11}= 0, a_{12} = 1, a_{13} = -2, a_{21}= 1, a_{22} = 0, a_{23} = 3, a_{31}= 2, a_{32}= -3, a_{33} = 0,$

$T_r(AA^T)=a_{11}^2+a_{12}^2+a_{13}^2+a_{21}^2+a_{22}^2+a_{23}^2+a_{31}^2+a_{32}^2+a_{33}^2$

$T_r(AA^T)= 0^2 + 1^2 + (-2)^2 + 0^2 + 3^2 + 2^2 + 2^2 + (-3)^2 + 0$

$= \boxed{\cancel{31}}$ $\underline{28}$

Q. The number of 3×3 matrices M with entries from { 0, 1, 2}, such that the sum of the diagonal elements of M^TM is 5, are

NTA Paper 63

A 198

B 126

C 135

D 162

$\mathrm{Tr}(M^TM) = 5$ ✱✱

let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\Rightarrow a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2+i^2 = 5$

CASE 1 5 1's & 4 0's

${}^9C_5 \times 1$

CASE 2 ② 1ce & 1 1ce Rest 0's

${}^9C_1 \times {}^8C_1$

$= 72$

Solution:

Let, $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\therefore$ Sum of the diagonal elements,

$Tr\left(M^T M\right) = a^2 + b^2 + c^2 + d^2 + e^2$

$+f^2 + g^2 + h^2 + i^2 = 5,$

where entries are $\{0, 1, 2\}$

Only two cases are possible.

(I) Five entries are 1 and other fourare 0

$\therefore\ {}^9C_5 \times 1$

(II) One entry is $2,$ one entry is 1 and others are $0.$

$\therefore\ {}^9C_2 \times 2$

Total cases $= 126 + 72 = 198.$

Q. The number of 3×3 matrices M with entries from $\{0, 1, 2\}$, such that the sum of the diagonal elements of M^TM is 5, are

NTA Paper 63

A 198

B 126

C 135

D 162

ONLY for SQUARE MATRICES

Nilpotent	$A^m = 0$ and $A^{m-1} \neq 0$	$\|A\| = 0$
Idempotent	$A^2 = A$	$\|A^2\| = 0$ or 1
Involutory	$A^2 = I$	$\|A\| = \pm 1$
Symmetric	$A^T = A$	$\|A^T\| = \|A\|$
Skew - Symmetric	$A^T = -A$	$\|A^T\| = \|-A\|$
Orthogonal ★	$A \times A^T = I$	$\|A\| = \pm 1$

- $a_{ii} = 0 \ \forall i$
- Det. of SS matrix of odd order $= 0$

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

ORTHOGONAL TRICK

C_1, C_2, C_3 mag = 1

Dot prod of all 2 cols. = 0

$C_1C_2 = 1\times 2 + 2\times 1 + (-2)\times 2$
$= 0$

Q. The matrix $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is

C_1 $\sqrt{1^2+2^2+(-2)^2}$
$= \frac{3}{3} = 1$

C_2 1

C_3 1

$AA^T = I$ (A) Orthogonal

$A^2 = I$ (B) Involutory

$A^2 = A$ (C) Idempotent

$A^m = 0$, $A^{m-1} \neq 0$ (D) Nilpotent

Solution:

Since for given $A = \frac{1}{3}\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{vmatrix}$. For orthogonal matrix $AA^T = A^T A = I_{(3\times 3)}$

$$\Rightarrow AA^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = 3I.$$

Similarly $A^T A = 3I$. Hence A is orthogonal

Q. The matrix $A=\frac{1}{3}\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ -2 & 2 & -1\end{bmatrix} is$

A. Orthogonal

B. Involutary

C. Idempotent

D. Nilpotent

Some Special types of Matrices

Orthogonal matrices examples:

Mag C_1 & $C_2 = 1$

$C_1 \cdot C_2 = 0$

1. $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

2. $A = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$

3. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Note: Columns /rows are perpendicular unit vector.

JEE Main 2019

Q. Let $A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$. $If\ \underbrace{AA^T = I_3}\ then\ |p|\ is$

A is orthogonal

mag $C_1 = 1$

$\sqrt{0^2 + p^2 + p^2} = 1$

$\Rightarrow \sqrt{2}\, p = 1$

A $\frac{1}{\sqrt{2}}$

B $\frac{1}{\sqrt{5}}$

C $\frac{1}{\sqrt{6}}$

D $\frac{1}{\sqrt{3}}$

Solution:

$$\sqrt{0^2+p^2+p^2}=1$$

$$\sqrt{2}|p|=1$$

$$|p|=\frac{1}{\sqrt{2}}$$

JEE Main 2019

Q. Let A $= \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$. $If\ AA^T = I_3\ then\ |p|\ is$

A $\frac{1}{\sqrt{2}}$

B $\frac{1}{\sqrt{5}}$

C $\frac{1}{\sqrt{6}}$

D $\frac{1}{\sqrt{3}}$

Q. If $\sum_{i=1}^{3} p_i q_i = \sum_{i=1}^{3} q_i r_i = \sum_{i=1}^{3} p_i r_i = 0$ and $\sum_{i=1}^{3} p_i^2 = \sum_{i=1}^{3} q_i^2 = \sum_{i=1}^{3} r_i^2 = 1$ then the value of

$$A = \begin{vmatrix} p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \\ p_3 & q_3 & r_3 \end{vmatrix}$$

ORTHOGONAL mein VECTORS ka Swaad

$\Rightarrow$ A is orthogonal

$\therefore |A| = \pm 1$

Solution:

Given matrix represent an orthogonal matrix.

As we know determinant of orthogonal matrix $= \pm 1$.

Hence, $\begin{bmatrix} p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \\ p_3 & q_3 & r_3 \end{bmatrix} = \pm 1$

JEE(Main): 26 Aug 2021 -I

Q. Let $A = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix}$, $i=\sqrt{-1}$, and $Q = A^TBA$, then the inverse of the matrix $AQ^{2021}A^T$ is

A is orthog. $\Rightarrow AA^T = I$

$A\,Q^{2021}A^T$

$= A(A^TBA)(A^TBA)\;\;(A^TBA)A^T$

2021 times

B^{2021}

$B^2 = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2i & 1 \end{pmatrix}$

$\vdots$

$B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix} \Rightarrow (B^{2021})^{-1} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

A $\begin{bmatrix} 1 & -2021 \\ 0 & 1 \end{bmatrix}$

B $\begin{bmatrix} 1 & 0 \\ -2021i & 1 \end{bmatrix}$

C $\begin{bmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{bmatrix}$

D $\begin{bmatrix} 1 & 0 \\ 2021i & 1 \end{bmatrix}$

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Solution:

$$AA^T = \begin{bmatrix} \frac{1}{\sqrt5} & \frac{2}{\sqrt5} \\ \frac{-2}{\sqrt5} & \frac{1}{\sqrt5} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt5} & \frac{-2}{\sqrt5} \\ \frac{2}{\sqrt5} & \frac{1}{\sqrt5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$Q = A^T BA$$

$$\Rightarrow Q^2 = (A^T BA)(A^T BA)$$

$$= A^T B(AA^T)BA$$

$$= A^T B(I)BA$$

$$= A^T B^2 A$$

$$\Rightarrow Q^3 = A^T B^3 A$$

$$\Rightarrow Q^{2021} = A^T B^{2021} A$$

Now let $P = AQ^{2021} A^T$

$P = A(A^T B^{2021} A)A^T$

given $AA^T = I$

$P = B^{2021}$

$$B^2 = \begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2i & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3i & 1 \end{bmatrix}$$

$$B^{2021} = \begin{bmatrix} 1 & 0 \\ 2021i & 1 \end{bmatrix}$$

inverse of $P = (P^{-1}) = (B^{2021})^{-1} = \begin{bmatrix} 1 & 0 \\ -2021i & 1 \end{bmatrix}$

JEE(Main): 26 Aug 2021 -I

Q. Let $A = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix}$, $i=\sqrt{-1}$, and $Q=A^TBA$, then the inverse of the matrix $AQ^{2021}A^T$ is

A $\begin{bmatrix} 1 & -2021 \\ 0 & 1 \end{bmatrix}$

B $\begin{bmatrix} 1 & 0 \\ -2021i & 1 \end{bmatrix}$

C $\begin{bmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{bmatrix}$

D $\begin{bmatrix} 1 & 0 \\ 2021i & 1 \end{bmatrix}$

DETERMINANT

DETERMINANT

A Determinant is a representation of numbers that has a particular value

$$\begin{array}{c|ccc|}
 & \text{Column 1} & \text{Column 2} & \text{Column 3} \\
\text{Row 1} & a_{11} & a_{12} & a_{13} \\
\text{Row 2} & a_{21} & a_{22} & a_{23} \\
\text{Row 3} & a_{31} & a_{32} & a_{33}
\end{array}$$

The vertical arrangement of elements are called Columns

The horizontal arrangement of elements are called Rows

The numbers in the arrangement are called Elements

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Order & Expansion of Determinant

Determinant	Order	Expansion
$\|A\| = \|a_{11}\|$	1	a_{11}
$\|A\| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$	2	$\|A\| = a_{11}a_{22} - a_{21}a_{12}$
$\|A\| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$	$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$ 3	$\|A\| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Minors and Cofactors

- **Minor** of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

- **Cofactor** of an element a_{ij}, denoted by A_{ij} is defined by $A_{ij} = (-1)^{i+j}.M_{ij}$, where M_{ij} is minor of a_{ij}.

$C_{ij} = (-1)^{i+j} M_{ij}$

$\boxed{\begin{array}{l} \text{even } + \\ \text{odd } - \end{array}}$

Properties:

- The sum of product of elements of any row (column) with their corresponding cofactors is value of determinant.

$$\Delta = \begin{vmatrix} 9 & 2 & 1 \\ 5 & -1 & 6 \\ 4 & 0 & -2 \end{vmatrix} = a_{12}C_{12} + a_{22}C_{22} + a_{32}C_{32}$$

Remark:

$5C_{11} + (-1)C_{12} + 6C_{13} = 0$

If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero.

Note:

For easy calculation in a determinant of order 3 the signs used to find the cofactors are

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Q. If $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$, then the minor of a_{22} is

M_{22}

A. -56

B. 51

C. -43

D. 41

$M_{22} = \begin{vmatrix} 1 & 9 \\ 9 & 25 \end{vmatrix}$

$= 25 - 81$

even

$C_{22} = -56$

Solution:

Given, $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$

So, By property of minor

minor of $a_{22} = M_{22}(-1)^{2+2} = M_{22}$

$= \begin{vmatrix} 1^2 & 3^2 \\ 3^2 & 5^2 \end{vmatrix}$

$= 25 - 9 \times 9$

$= -56$

Q. If $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$, then the minor of a_{22} is

A -56

B 51

C -43

D 41

Q. The cofactors of elements in second row of the determinant $\begin{vmatrix} 2 & -1 & 4 \\ 4 & 2 & -3 \\ 1 & 1 & 2 \end{vmatrix}$ are

$-2-4=-6$

$C_{21} = -(-6) = 6$

$C_{22} = (0)$

$C_{23} = -(3)$

$2-(-1)=3$

(A) 5,6,4

(B) 6,0,-3

(C) 5,1,8

(D) 6,0,3

Solution:

$$\begin{vmatrix} 2 & -1 & 4 \\ 4 & 2 & -3 \\ 1 & 1 & 2 \end{vmatrix}$$

$$C_{21} = -\begin{vmatrix} -1 & 4 \\ 1 & 2 \end{vmatrix} = 6$$

$$C_{22} = \begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 0$$

$$C_{23} = -\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = -3$$

Q. The cofactors of elements in second row of the determinant $\begin{vmatrix} 2 & -1 & 4 \\ 4 & 2 & -3 \\ 1 & 1 & 2 \end{vmatrix}$ are

A. 5,6,4

B. 6,0,-3

C. 5,1,8

D. 6,0,3

Basic Properties of Determinant

A. n×n matrix

- $|kA| = k^n|A|$
- $|AB| = |A||B|$
- $|A^T| = |A|$
- $|A^n| = |A|^n$

$|A^n| = |A|^n$

Q. If the matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then the value of $\dfrac{|A^{100}+A^{98}|}{|A^{20}+A^{18}|}$ is equal to

$|A| = 6-5 = 1$

$$\frac{|A^{98}(A^2+I)|}{|A^{18}(A^2+I)|} = \frac{|A^{98}|\,\cancel{|A^2+I|}}{|A^{18}|\,\cancel{|A^2+I|}}$$

$$= \frac{|A|^{98}}{|A|^{18}} = |A|^{80}$$

A) 0

B) 1

C) 2

D) 3

Solution

$$|A| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1$$

$$\frac{\left|A^{100} + A^{98}\right|}{\left|A^{20} + A^{18}\right|} = \frac{\left|A^{98}\right|\left|A^2 + I\right|}{\left|A^{18}\right|\left|A^2 + I\right|} = |A|^{80} = 1$$

JEE MAIN : MATRICES & DETERMINANTS (NEW Syllabus)

Q. If the matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then the value of $\dfrac{|A^{100}+A^{98}|}{|A^{20}+A^{18}|}$ is equal to

A. 0

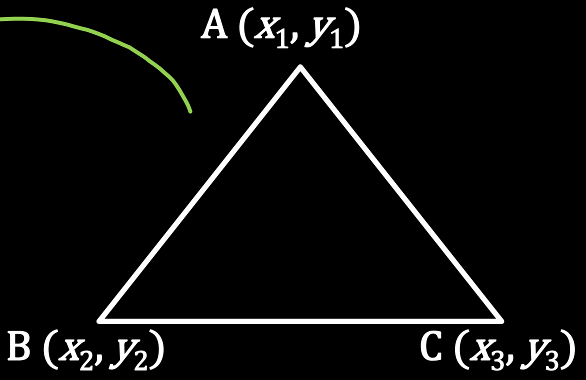
B. 1

C. 2

D. 3

Area of triangle(Using Determinant Method)

$$\text{Area of } \Delta = \left| \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right|$$



We must have to apply modulus so that area is positive

Q. If the area of a triangle with vertices (-3,0),(3,0) and (0,k) is 9 sq.units, then what is the value of k ?

A 3

B 6

C 9

D 12

$$\frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix} = \pm 9$$

$$\Rightarrow -3(-k) - 0 + 1(3k) = \pm 18$$

$$\Rightarrow 6k = \pm 18$$

$$\Rightarrow k = \pm 3$$

Solution:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Rightarrow \text{Area} = \frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}$$

$$\Rightarrow \text{Area} = \frac{1}{2}[-3(0 - k) - 0 + 1(3k)]$$

$\Rightarrow$ Area = 3k

The area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 square units.

$\Rightarrow$ 3k = 9

$\Rightarrow$ k = 3

Q. If the area of a triangle with vertices (-3,0),(3,0) and (0,k) is 9 sq.units, then what is the value of k ?

A 3

B 6

C 9

D 12

NTA PAPER 86

Atrangi ques

Q. Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{bmatrix}$ and $\boxed{N = \frac{M^2}{2}}$. If $(a-b)^2 + (d-e)^2 = 36$, $(b-c)^2 + (e-f)^2 = 64$, $(a-c)^2 + (d-f)^2 = 100$, then value of $|N|$ is equal to

(A) 1152

(B) 48

(C) 144

(D) 288

$|kA| = k^n|A|$

$|N| = \left|\frac{1}{2}M^2\right|$

$= \left(\frac{1}{2}\right)^3 |M^2|$

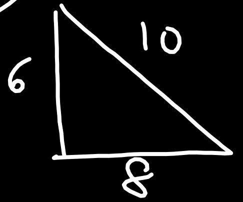
$N = \frac{1}{8}|M|^2$

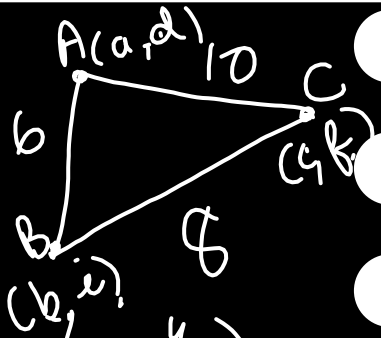
Neha ma'am

$|M| = |M^T| = \begin{vmatrix} a & d & 1 \\ b & e & 1 \\ c & f & 1 \end{vmatrix}$

$= 2\,Ar\,\Delta(ABC)$

$= 2(\text{r}t\,\Delta) = 2\left(\frac{1}{2} \times 6 \times 8\right)$





Solution:

$|M| = 2\times$Area of the triangle with vertices $(a, d), (b, e)$ & (c, f) with the length of sides as $6, 8, 10$ respectively.

$\because$ Triangle is a right angle triangle, hence,

$$\text{Area} = \frac{1}{2} \times 6 \times 8 = 24$$

$$|M| = 48$$

$$|N| = \left|\frac{M^2}{2}\right| = \frac{1}{8}|M|^2 = \frac{(48)^2}{8} = 288$$

NTA PAPER 86

Q. Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ 1 & 1 & 1 \end{bmatrix}$ and $N = \frac{M^2}{2}$. If

$(a - b)^2 + (d - e)^2 = 36$, $(b - c)^2 + (e - f)^2 = 64$, $(a - c)^2 + (d - f)^2 = 100$, then value of $|N|$ is equal to

A 1152

B 48

C 144

D 288

Adjoint of a Matrix

Adjoint of a Matrix

For any Square Matrix A, Adjoint of 'A' is defined as 'Transpose' of the Matrix obtained by replacing all elements of Matrix A by their "Co–Factors".

A

$\downarrow$

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \text{Co factor Matrix of A}$$

$\downarrow$

$$\text{Adj}(A) = (\text{Co–factor Matrix }(A))^{T}$$

$$\text{Adj }(A) = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

Properties of Adjoint of a Matrix

For **Non–Singular Square** Matrices, A and B we have the following results:

Property 1	$A\,\mathrm{adj}(A) = \lvert A\rvert I_n = \mathrm{adj}(A)A$ → kill adj	
Property 2	$\mathrm{adj}(AB) = \mathrm{adj}(B)\times\mathrm{adj}(A)$	(Reversal law holds)
Property 3	$\mathrm{adj}(kA_n) = k^{n-1}\,\mathrm{adj}(A_n)$	($k \Rightarrow$ constant)
Property 4	$\mathrm{adj}(A^T) = (\mathrm{adj}\,A)^T$	($A^T \Rightarrow$ Transpose)
Property 5	$\lvert\mathrm{adj}(A)\rvert = \lvert A\rvert^{n-1}$ ☆☆	($\lvert A\rvert \Rightarrow$ Determinant)
Property 6	$\mathrm{adj}(\mathrm{adj}(A)) = \lvert A\rvert^{n-2}\times A$ ✓	
Property 7	$\lvert\mathrm{adj}(\mathrm{adj}(A))\rvert = \lvert A\rvert^{(n-1)^2}$	

Properties of Adjoint of a Matrix

Property 8 $\quad \text{adj}(A^{-1}) = (\text{adj } A)^{-1}$

Property 9 $\quad \text{adj}(A^{m}) = (\text{adj } A)^{m}$

$**\ |adj\,A| = |A|^{n-1}$

$|kA| = k^n|A|$

JEE Main 2022: 26 June - I

Q. Let A be a matrix of order 3×3 and if $|adj\,(24A)| = |adj\,(3\,adj\,2A)|$, then $|A|^2$ is equal to:

$|(24A)|^2 = |(3\,adj\,2A)|^2$

$(24)^3|A| = (3)^3\,|adj(2A)|$

$\Rightarrow 8^3|A| = |2A|^2$

$8^3|A| = (2^3)^2|A|^2$

$\Rightarrow |A| = \frac{8^3}{2^6} = \frac{2^9}{2^6} = 2^3$

A) 1

B) 2^6

C) 2^{12}

D) 6^6

Solution:

$$|24A|^2 = |3adj2A|^2$$

$$\Rightarrow 24^6\left|A^2\right| = 3^6|2A|^4$$

$$\Rightarrow 8^6|A|^2 = 2^{12}|A|^4$$

$$\Rightarrow |A|^2 = 2^6$$

JEE Main 2022: 26 June - I

Q. Let A be a matrix of order 3×3 and if $|\text{adj }(24A)|=|\text{adj }(3\text{ adj }2A)|$, then $|A|^2$ is equal to:

A. 1

B. 2^6

C. 2^{12}

D. 6^6

JEE MAIN 2022 24th JUNE - I

Q. If $A = \begin{bmatrix} 1 & 0 & a \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ where $a \in N$ from 1 to 50 and $\sum_{a=1}^{50} |adj A| = 100k$ then value of k is:

(A) 1723/2

(B) 1717/2

(C) 1719/2

(D) 1821/4

$$\sum_{k=1}^{n} k^2 = \frac{n(2n+1)}{6}$$

$$|A| = 1(1) - 0 + a(1)$$
$$= a+1$$

$$|adj A| = |A|^2$$

$$1^2 +$$

$$\sum_{a=1}^{50} |A|^2 = \sum_{a=1}^{50} (a+1)^2 = 2^2 + 3^2 + 4^2 + \cdots + 51^2 - 1^2$$

$$= \frac{(51)(52)(103)}{6} - 1$$

$$= 100k$$

Solution:

$$\left|adj\ A\right| = |A|^{n-1} = |A|^{3-1} = |A|^2$$

$$\sum_{a=1}^{50}(a+1)^2 = 2^2 + 3^2 + 4^2 + \ldots + 51^2$$

$$= 1^2 + 2^2 + 3^2 + 4^2 + \ldots + 51^2 - 1^2$$

$$= \frac{51\times52\times103}{6} - 1$$

$$= 45526 - 1$$

$$= 45525 = 100k$$

$$\therefore k = \frac{1821}{4}$$

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

JEE MAIN 2022 24th JUNE - I

Q. If $A = \begin{bmatrix} 1 & 0 & a \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ where a∈N from 1 to 50 and

$$\sum_{a=1}^{50} \left| adjA \right| = 100k$$

then value of k is:

A. 1723/2

B. 1717/2

C. 1719/2

D. 1821/4

JEE(Main): 8th April 2023 -I

Q. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. if $|adj(adj(adj2A))| = (16)^n$, then n is equal to:

$|A| = 4$

$n = 3$

$|2A|^{(3-1)^3} = (16)^n$

$\Rightarrow |2A|^8 = (16)^n$

$\Rightarrow (2^3)^8 |A|^8 = (16)^n$

$\Rightarrow (2^3)^8 \cdot 4^8 = (16)^n$

$\Rightarrow 2^{24+16} = 2^{4n}$

A. 8

B. 9

C. 12

D. 10

Solution:

$|A| = 2[3] - 1[2] = 4$

$\therefore \left|\operatorname{adj}\left(\operatorname{adj}\left(\operatorname{adj} 2A\right)\right)\right|$

$= |2A|^{(n-1)^3} \Rightarrow |2A|^8 = 16^n$

$\Rightarrow \left(2^3 |A|\right)^8 = 16^n$

$\Rightarrow \left(2^3 \times 2^2\right)^8 = 16^n$

$\Rightarrow 2^{40} = 16^n$

$\Rightarrow 16^{10} = 16^n \Rightarrow n = 10$

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

JEE(Main): 8th April 2023 -I

Q. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. $if\ |adj(adj(adj2A))| = (16)^n$,

Then n is equal to:

A 8

B 9

C 12

D 10

JEE MAIN : MATRICES & DETERMINANTS (NEW Syllabus)

JEE(Main): 15th April 2023 -I

Q. Let the determinant of a square matrix A of order m be m – n, where m and n satisfy $4m + n = 22$ and $17m + 4n = 93$. If det $(n.adj(adj(mA))) = 3^a5^b6^c$, then $a + b + c$ is equal to

A 101

B 84

C 109

D 96

Solution:

$|A| = m - n$

$4m + n = 22$ ..... (i)

$17m + 4n = 93$ ..... (ii)

On solving (i) & (ii)⇒ $m = 5, n = 2$

$|A| = 3$

$$
\begin{aligned}
|2\,\text{adj}\,(\text{adj}\,5A)| &= 2^5\,|5A|^{16} \\
&= 2^5 . 5^{80}\,|A|^{16} \\
&= 2^5 . 5^{80} . 3^{16} \\
&= 3^{11} . 5^{80} . 6^5
\end{aligned}
$$

$a + b + c = 96$

JEE(Main): 15th April 2023 -I

Q. Let the determinant of a square matrix A of order m be m – n, where m and n satisfy $4m + n = 22$ and $17m + 4n = 93$. If $\det(n.\mathrm{adj}(\mathrm{adj}(mA))) = 3^a 5^b 6^c$, then a + b + c is equal to

A 101

B 84

C 109

D 96

Singular Matrix

A square matrix is said to be a singular matrix if its determinant is zero, i.e., $\det A = 0$

Non-singular Matrix

A square matrix is said to be a non-singular matrix if its determinant is zero, i.e., $\det A \neq 0$.

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

Q. The matrix $A = \begin{bmatrix} 2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is non-singular if

$|A| \neq 0$

$\Rightarrow 2(-9+8) - \lambda(-1)$

$- 4(-1) \neq 0$

(A) $\lambda \neq -2$

(B) $\lambda \neq 2$

(C) $\lambda \neq 3$

(D) $\lambda \neq -3$

Solution:

The given matrix $A = \begin{bmatrix} 2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is non singular If $|A| \neq 0$

Directly expand

$$\Rightarrow |A| = \begin{vmatrix} 2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{vmatrix} \neq 0 \Rightarrow \begin{vmatrix} 1 & \lambda+3 & 0 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{vmatrix} \neq 0 \; [R_1 \to R_1 + R_2]$$

$$\Rightarrow |A| = \begin{bmatrix} 1 & \lambda+3 & 0 \\ 0 & 1 & 1 \\ 0 & -\lambda-5 & -3 \end{bmatrix} \neq 0 \quad , \begin{bmatrix} R_2 \to R_2 + R_3 \\ R_3 \to R_3 - R_1 \end{bmatrix}$$

$$\Rightarrow 1(-3+\lambda+5) \neq 0 \Rightarrow \lambda + 2 \neq 0 \Rightarrow \lambda \neq -2$$

Q. The matrix $\begin{bmatrix} 2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is non-singular if

A $\lambda \neq -2$

B $\lambda \neq 2$

C $\lambda \neq 3$

D $\lambda \neq -3$

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

Inverse of Matrix

A square matrix of order n is invertible if there exists a square matrix B of the same order such that, $AB = BA = I$

In such a case, we say that the **inverse** of **A** is **B** and we write, $\mathbf{A^{-1} = B}$. or $B^{-1} = A$

Clearly if A is non-singular (i.e. $|A| \neq 0$) then A^{-1} is defined, and is given by

$$\boxed{A^{-1} = \frac{1}{|A|} \times (\operatorname{adj}(A))}$$

Note: $A^{-1} \times A = I$

Properties of Inverse of a Matrix

Property 1 $(A^{-1})^{-1} = A$

Property 2 $(AB)^{-1} = B^{-1}A^{-1}$ (Reversal law)

Property 3 $(A^{T})^{-1} = (A^{-1})^{T}$

Property 4 $(kA)^{-1} = \dfrac{1}{k} A^{-1},\ k \neq 0$

Property 5 $|A^{-1}| = |A|^{-1}$ i.e. $\boxed{|A^{-1}| = \dfrac{1}{|A|}}$ **

Q. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10\, B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of matrix A, then α is

A 5

B -1

C 2

D -2

$M_{11} = 4,\ M_{12} = 5,\ M_{13} = 1$

$C_{11} = 4,\ C_{12} = -5,\ C_{13} = 1$

$B = A^{-1}$

$|A| = 1(4) + 1(5) + 1(1)$

$\text{adj} A = (C_{ij})^T$

$= \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{10}\begin{bmatrix} & & \\ & & \end{bmatrix}$

$\Rightarrow 10A^{-1} = 10B$

Solution:

We have, $A=\begin{bmatrix}1 & -1 & 1\\ -2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, $\therefore\ |A|=1(4)+1(5)+1(1)=10$ and $adj(A)=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$

Then $A^{-1}=\dfrac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$

According to question, B is the inverse of matrix A. Hence $\alpha=5$

Q. Let $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$ and $10.B=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{bmatrix}$. If B is the inverse of matrix A, then α is

A 5

B -1

C 2

D -2

JEE Main 2021: 27 July - I

Q. Let $A=\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $\boxed{A^{-1} = \alpha I + \beta A}$, $\alpha, \beta \in R$, I is a 2×2 identity matrix, then $\underline{4(\alpha - \beta)}$ is equal to :

Trick

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -\beta & 4\beta \end{bmatrix}$$

$$\Rightarrow \quad \frac{2}{3} = \alpha + \beta \quad \Bigg| \quad \frac{-2}{6} = 2\beta \Rightarrow \beta = -1/6$$

A 5

B 4

C 2

D 8/3

Solution:

$$A=\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}, |A|=6$$

$$A^{-1}=\frac{\text{adjA}}{|A|}=\frac{1}{6}\begin{bmatrix}4 & -2\\ 1 & 1\end{bmatrix}=\begin{bmatrix}\frac{2}{3} & -\frac{1}{3}\\ \frac{1}{6} & \frac{1}{6}\end{bmatrix}$$

$$\begin{bmatrix}\frac{2}{3} & -\frac{1}{3}\\ \frac{1}{6} & \frac{1}{6}\end{bmatrix}=\begin{bmatrix}\alpha & 0\\ 0 & \alpha\end{bmatrix}+\begin{bmatrix}\beta & 2\beta\\ -\beta & 4\beta\end{bmatrix}$$

$$\left.\begin{aligned}\alpha+\beta&=\frac{2}{3}\\ \beta&=-\frac{1}{6}\end{aligned}\right\}\Rightarrow \alpha=\frac{2}{3}+\frac{1}{6}=\frac{5}{6}$$

$$4(\alpha-\beta)=4(1)=4$$

JEE Main 2021: 27 July - I

Q. Let $A\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = aI + \beta A, \alpha, \beta \in R,$ I is a 2× 2 identity matrix, then $4(a-\beta)$ is equal to :

A 5

B 4

C 2

D 8/3

JEE Main 2021: 27 Aug-I

Q. If the matrix $A = \begin{pmatrix} 0 & 2 \\ k & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$,

Then the value of k is $|A| \neq 0 \Rightarrow A^{-1}$ exists

A $-\frac{1}{2}$

B -1

C 1

D $\frac{1}{2}$

$$\underbrace{A^{-1}A}(A^3+3I) = 2A^{-1}I$$

$$\Rightarrow A^3 + 3I = 2A^{-1}$$

$$\Rightarrow A^3 + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \frac{2}{-2k}\begin{bmatrix} -1 & -2 \\ -k & 0 \end{bmatrix}$$

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

Solution:

$$A=\begin{bmatrix}0 & 2\\ k & -1\end{bmatrix} \text{ then } A^2=\begin{bmatrix}0 & 2\\ k & -1\end{bmatrix}\begin{bmatrix}0 & 2\\ k & -1\end{bmatrix}=\begin{bmatrix}2k & -2\\ -k & 2k+1\end{bmatrix}$$

$$A^3=\begin{bmatrix}2k & -2\\ -k & 2k+1\end{bmatrix}\begin{bmatrix}0 & 2\\ k & -1\end{bmatrix}=\begin{bmatrix}-2k & 4k+2\\ 2k^2+k & -4k-1\end{bmatrix}$$

Now, $A\left(A^3+3I\right)=2I$ gives $A^3+3I=2A^{-1}$

$$\begin{bmatrix}-2k & 4k+2\\ 2k^2+k & -4k-1\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}=2\left(\frac{-1}{2k}\right)\begin{bmatrix}-1 & -2\\ -k & 0\end{bmatrix}$$

$$\begin{bmatrix}-2k+3 & 4k+2\\ 2k^2+k & -4k+2\end{bmatrix}=\begin{bmatrix}1/k & 2/k\\ 1 & 0\end{bmatrix}$$

$$\Rightarrow -2k+3=\frac{1}{k}\Rightarrow 2k^2-3k+1=0$$

$$2k^2-2k-k+1=0$$

$$(2k-1)(k-1)=0$$

$$k=\frac{1}{2},1$$

$$\& \ 4k+2=\frac{2}{k} \text{ or } 2k+1-\frac{1}{k} \text{ so or } 2k^2+k-1=0$$

$$2k^2+2k-k-1=0$$

$$(2k-1)(k+1)=0$$

$$\Rightarrow k=\frac{1}{2},-1$$

$$\Rightarrow k=\frac{1}{2}$$

JEE Main 2021: 27 Aug-I

Q. If the matrix $A = \begin{pmatrix} 0 & 2 \\ k & -1 \end{pmatrix}$ satisfied $A(A^3 + 3I) = 2I$,

Then the value of k is

A $-\frac{1}{2}$

B -1

C 1

D $\frac{1}{2}$

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

Homogeneous Equations

$Ax = 0$

$|A| \neq 0$
Trivial Solutions
$x=y=z=0$

$|A| = 0$

Infinite Solutions
(trivial) & nontrivial

No sdn nhi hota

Important Note:

1. Homogeneous system is always consistent (as (0,0,0) always satisfies it)
2. (0,0,0) is also called trivial solution.
3. Homogeneous system has non-trivial (i.e., non-zero) solution if $|A|=0$.

CRAMER'S RULE

$AX = B$

$D = |A|$

$D \neq 0$

- At least One $D_1, D_2, D_3 \neq 0$: Consistent Unique non-zero trivial solution

  $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$, $z = D_3/D$

- $D_1 = D_2 = D_3 = 0$: Consistent trivial solution

$D = 0$

- $D_1 = D_2 = D_3 = 0$: Consistent Infinitely Solutions (except || lines) OR no solution
- One of $D_1, D_2, D_3 \neq 0$: Inconsistent solution

  No soln.

Exceptional

Q. The system of equations
$x + 2y + 3z = 6$
$2x + 4y + 6z = 12$
$3x + 6y + 9z = 22$ has

$D = D_1 = D_2 = D_3 = 0$
No soln
Lines are //

(A) Unique solution

(B) No solution

(C) Infinitely many solutions

(D) None of these

$D_2 = 0$

$D_3 = 0$

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 6 & 2 & 3 \\ 12 & 4 & 6 \\ 22 & 6 & 9 \end{vmatrix} = 0$$

Solution:

The given system of equations are

$x + 2y + 3z = 6$

$2x + 4y + 6z = 12$

$3x + 6y + 9z = 22$

Here, $D = 0$

Also, $D_1 = D_2 = D_3 = 0$

This is the case of || lines, hence **no solution**

Q. The system of equations

$x + 2y + 3z = 6$

$2x + 4y + 6z = 12$

$3x + 6y + 9z = 22$ has

A. Unique solution

B. No solution

C. Infinitely many solutions

D. None of these

JEE(Main): 6th April 2023 -I

Q. If the system of equations

$x + y + az = b$

$2x + 5y + 2z = 6$

$x + 2y + 3z = 3$ has infinitely many solutions, then $2a + 3b$ is equal to :

$D = D_1 = D_2 = D_3 = 0$

$D = 0 \Rightarrow \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \boxed{a =}$

$D_3 = 0 \Rightarrow \begin{vmatrix} 1 & 1 & b \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \boxed{b =}$

A 28

B 20

C 25

D 23

Solution:

$$x + y + az = b$$
$$2x + 5y + 2z = 6$$
$$x + 2y + 3z = 3$$

For ∞ solution

$$\Delta = 0, \Delta_x = 0, \Delta_y = 0, \Delta_z = 0$$

$$\Delta = \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 11 - 4 - a = 0 \Rightarrow a = 7$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & b \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 3 - 0 - b = 0 \Rightarrow b = 3$$

Hence $2a + 3b = 23$

JEE(Main): 6th April 2023 -I

Q. If the system of equations

$x + y + az = b$

$2x + 5y + 2z = 6$

$x + 2y + 3z = 3$ has infinitely many solutions, then $2a + 3b$ is equal to :

A 28

B 20

C 25

D 23

JEE Main 2022: 25 July - II

Q The number of real values of λ such that the system of linear equations

$2x - 3y + 5z = 9, x + 3y - z = -18$ and

$3x - y + (\lambda^2 - |\lambda|)z = 16$ has no solution is

D=0 but atleast one $D_i \neq 0$

$$D = 0 \Rightarrow \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2 - |\lambda| \end{vmatrix} = 0$$

(A) 0

(B) 1

(C) 2

(D) 4

Solution:

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2 - |\lambda| \end{vmatrix} = 2\left(3\lambda^2 - 3|\lambda| - 1\right) + 3\left(\lambda^2 - |\lambda| + 3\right) + 5(-1-9)$$

$$= \boxed{9\lambda^2 - 9|\lambda| - 43}$$

$$= 9|\lambda|^2 - 9|\lambda| - 43$$

Prod $= -\frac{43}{9}$

$\Delta = 0$ for 2 values of $|\lambda|$ out of which one is –ve and other is +ve

$|\lambda| \to$ +ve, –ve ✗

So, 2 values of λ satisfy the system of equations to obtain no solution

$|\lambda| = +ve$
$\Rightarrow \lambda = \pm$

JEE Main 2022: 25 July - II

Q The number of real values of λ such that the system of linear equations

$2x - 3y + 5z = 9, x + 3y - z = -18$ and

$3x - y + (\lambda^2 - |\lambda|)z = 16$ has no solution is

A 0

B 1

C 2

D 4

JEE MAIN : MATRICES & DETERMINANTS (NEW Syllabus)

JEE Main 2023: 01 Feb - I

Q. Let S denote the set of all real values of λ such that the system of equations

$\lambda x+y+z=1$

$x+\lambda y+z=1$

$x+y+\lambda z=1$ is inconsistent, then $\sum_{\lambda \in S}(|\lambda|^2+|\lambda|)$ is equal to

no soln

A 2

B 12

C 4

D 6

Solution:

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda+2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda+2)\left[1\left(\lambda^2-1\right)-1(\lambda-1)+(1-\lambda)\right]=0$$

$$(\lambda+2)\left[\left(\lambda^2-2\lambda+1\right)=0\right.$$

$$(\lambda+2)(\lambda-1)^2=0 \Rightarrow \lambda=-2, \lambda=1$$

at $\lambda = 1$ system has infinite solution,

for inconsistent $\lambda = -2$

so $\sum\left(|-2|^2+|-2|\right)=6$

JEE Main 2023: 01 Feb - I

Q. Let S denote the set of all real values of λ such that the system of equations

$\lambda x+y+z=1$

$x+\lambda y+z=1$

$x+y+\lambda z=1$ is inconsistent, then $\sum_{\lambda \in S}\left(|\lambda|^2+|\lambda|\right)$ is equal to

A) 2

B) 12

C) 4

D) 6

JEE Main 2022: 26 June - I

Q. The ordered pair(a,b) , for which the system of linear equations
$3x-2y+z=b$, $5x-8y+9z=3$, $2x+y+az=-1$ has no solution , is

A (3,⅓)

B (-3,⅓)

C (-3,-⅓)

D (3,-⅓)

$\Delta = 0$

$$\Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0$$

$\Rightarrow a = -3$

No soln atleast one $D_i \neq 0$

$$D_1 = \begin{vmatrix} b & -2 & 1 \\ 3 & -8 & 9 \\ -1 & 1 & -3 \end{vmatrix} \neq 0$$

$= b(15) + 2(0) + 1(-5)$

$15b - 5 \neq 0$

$b \neq \frac{1}{3}$

Solution:

Given system of linear equations are

$3x - 2y + z = b$

$5x - 8y + 9z = 3$

$2x + y + az = -1$

For system to have no solution,

$$\Delta = \begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0$$

$\Rightarrow 3(-8a - 9) + 2(5a - 18) + 1(21) = 0$

$\Rightarrow a = -3$

$$\text{Also, } \Delta_2 = \begin{vmatrix} 3 & -2 & b \\ 5 & -8 & 3 \\ 2 & 1 & -1 \end{vmatrix} = 3(5) + 2(-11) + b(21) = 21b - 7$$

If $b = \frac{1}{3}$, $\Delta_2 = 0$

So b must be equal to $-\frac{1}{3}$

JEE Main 2022: 26 June - I

Q. The ordered pair(a,b) , for which the system of linear equations
$3x-2y+z=b$, $5x-8y+9z=3$, $2x+y+az=-1$ has no solution , is

A $(3,\frac{1}{3})$

B $(-3,\frac{1}{3})$

C $(-3,-\frac{1}{3})$

D $(3,-\frac{1}{3})$

JEE(Main): 8th April 2023 -II

Q. Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$$x + y + \sqrt{3}z = 0$$
$$-x + (\tan\theta)y + \sqrt{7}z = 0$$
$$x + y + (\tan\theta)z = 0$$

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta \in S}\theta$ is equal to:

PYQ Series

(A) 20

(B) 40

(C) 30

(D) 10

$AX = 0$

$|A| \neq 0$ Unique $(0,0,0)$

$|A| = 0$ Inf many (Non-trivial)

$$\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix} = 0$$

Solution:

For non trivial solutions

D = 0

$$\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix} = 0$$

$$\Rightarrow \left(\tan^2\theta - \sqrt{7}\right) - \left(-\tan\theta - \sqrt{7}\right) + \sqrt{3}(-1 - \tan\theta) = 0$$

$$\Rightarrow \tan^2\theta + \left(1 - \sqrt{3}\right)\tan\theta - \sqrt{3} = 0$$

$$\Rightarrow (\tan\theta + 1)\left(\tan\theta - \sqrt{3}\right) = 0$$

$$\Rightarrow \tan\theta = \sqrt{3}, -1$$

$$\Rightarrow \theta = \frac{\pi}{3}, \frac{-2\pi}{3}, \frac{-\pi}{4}, \frac{3\pi}{4}$$

Hence,

$$\frac{120}{\pi}\sum\theta = 120\left(\frac{1}{3} - \frac{2}{3} - \frac{1}{4} + \frac{3}{4}\right) = 20$$

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

JEE(Main): 8th April 2023 -II

Q. Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$$x + y + \sqrt{3}z = 0$$
$$-x + (\tan\theta)y + \sqrt{7}z = 0$$
$$x + y + (\tan\theta)z = 0$$

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta \in S} \theta$ is equal to:

A 20

B 40

C 30

D 10

Characteristic equation

The equation $|A-\lambda I|=0$ is a polynomial equation in the variable λ for given A. It is called the **characteristic equation** of the matrix A.

Cayley Hamilton theorem:

Every matrix satisfies its characteristic equation $|A-\lambda I|=0$

Q. Characteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

(A) $A^3 - 20A + 8I = 0$

(B) $A^3 + 20A + 8I = 0$

(C) $A^3 - 80A + 20I = 0$

(D) None of these

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 3-\lambda & -3 \\ -2 & -4 & -4-\lambda \end{vmatrix} = 0$$

$\Rightarrow \cancel{(1-\lambda)}$

$\lambda^3 - 20\lambda + 8 = 0$

Solution:

The characteristic equation is $|A-\lambda I|=0$.

So, $\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 3-\lambda & -3 \\ -2 & -4 & -4-\lambda \end{vmatrix}=0$ i.e. $\lambda^3-20\lambda+8=0$

By cayley-Hamilton theorem , $A^3-20A+8I=0$

Q. Characteristic equation of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

A $A^3 - 20A + 8I = 0$

B $A^3 + 20A + 8I = 0$

C $A^3 - 80A + 20I = 0$

D None of these

JEE Main 2022: 27 July - II

Q. Let $A = \begin{bmatrix} 4 & -2 \\ \alpha & \beta \end{bmatrix}$. If $A^2 + \gamma A + 18I = 0$, then $|A|$ is equal to 18 ____. $4\beta + 2\alpha$

(A) -18

(B) 18

(C) -50

(D) 50

Cayley

$A^2 - A(4+\beta) + (4\beta+2\alpha)I = 0$

$A^2 + A\gamma + 18I = 0$

$4\beta + 2\alpha = 18$

$|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 4-\lambda & -2 \\ \alpha & \beta-\lambda \end{vmatrix} = 0$

$\Rightarrow (4-\lambda)(\beta-\lambda) + 2\alpha = 0$

$\Rightarrow 4\beta - 4\lambda - \lambda\beta + \lambda^2 + 2\alpha = 0$

$\Rightarrow \lambda^2 - \lambda(4+\beta) + (4\beta+2\alpha) = 0$

Solution:

Charcteristic equation of A is given by $|A-\lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 4-\lambda & -2 \\ \alpha & \beta-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - (4+\beta)\lambda + (4\beta + 2\alpha) = 0$$

$$\Rightarrow A^2 - (4+\beta)A + (4\beta + 2\alpha)I = 0$$

$$|A| = 4\beta + 2\alpha = 18$$

JEE Main 2022: 27 July - II

Q. Let $A = \begin{bmatrix} 4 & -2 \\ \alpha & \beta \end{bmatrix}$. If $A^2 + \gamma A + 18I = 0$, then $|A|$ is equal to ______.

(A) -18

(B) 18

(C) -50

(D) 50

Differentiation of Determinant

If only one row (or column) consists of functions of x and other rows (or columns) are constant,

Let $\Delta(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$, *then* $\Delta'(x) = \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

and in general $\Delta^n(x) = \begin{vmatrix} f_1^n(x) & f_2^n(x) & f_3^n(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$, where n is any positive interger and $f^n(x)$ denotes the n^{th} derivative of $f(x)$.

Q. Let $f(x) = \begin{vmatrix} x^3 & sinx & cosx \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant.

Then $\frac{d^3}{dx^3}(f(x))$ at $x = 0$ is

$g(x) = x^3 \quad \sin x \quad \cos x$

$g'(x) = 3x^2 \quad \cos x \quad -\sin x$

$g''(x) = 6x \quad -\sin x \quad -\cos x$

$g'''(x) = 6 \quad -\cos x \quad \sin x$

$$f'''(x) = \begin{vmatrix} 6 & -\cos x & \sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

A. p

B. p - p^3

C. p + p^3

D. Independent of p

Solution:

Given $f(x)=\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where p is constant

$$\Rightarrow f'''(x)=\begin{vmatrix} 6 & -\cos x & \sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow f'''(x)]_{x=0}=\boxed{\begin{vmatrix} 6 & -1 & 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}}=0$$

= Independent of p

Q. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant.

Then $\frac{d^3}{dx^3}(f(x))$ at $x = 0$ is

A) p

B) p - p^3

C) p + p^3

D) Independent of p

Integration of Determinant

$$\Rightarrow \int_a^b \Delta(x)dx = \begin{vmatrix} \int_a^b f(x)dx & \int_a^b g(x)dx & \int_a^b h(x)dx \\ a & b & c \\ l & m & n \end{vmatrix}$$

Note:

If the elements of more than one column or rows are functions of x then the integration can be done only after evaluation/expansion of the determinant.

100 mins DAILY DOSE 2.0

NEHA MA'AM'S ARMY

Q. If $\Delta(x)=\begin{vmatrix}\sin^2 x & \log\cos x & \log\tan x\\ n^2 & 2n-1 & 2n+1\\ 1 & -2\log 2 & 0\end{vmatrix}$, then evaluate $\int_0^{\frac{\pi}{2}}\Delta(x)dx$?

Solution:

$$\Delta(x) = \begin{vmatrix} \sin^2 x & \log\cos x & \log\tan x \\ n^2 & 2n-1 & 2n+1 \\ 1 & -2\log 2 & 0 \end{vmatrix}$$

$$\int_0^{\frac{\pi}{2}} \Delta(x)dx = \begin{vmatrix} \int_0^{\frac{\pi}{2}} \sin^2 x dx & \int_0^{\pi/2} \log\cos x dx & \int_0^{\pi/2} \log\tan x dx \\ n^2 & 2n-1 & 2n+1 \\ 1 & -2\log 2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\pi}{4} & -\frac{\pi}{2}\log 2 & 0 \\ n^2 & 2n-1 & 2n+1 \\ 1 & -2\log 2 & 0 \end{vmatrix}$$

$$= (\frac{\pi}{2})2n\log 2 + (\frac{\pi}{2})\log 2 - (\frac{\pi}{2})2n\log 2 - (\frac{\pi}{2})\log 2$$

$$= 0$$

Product of Two Determinant

Let the two determinants of third order be,

$$D_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and } D_2 = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}.$$ Let D be their product.

Hence,

Same as matrix mult^n.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} = \begin{vmatrix} a_1\alpha_1 + b_1\beta_1 + c_1\gamma_1 & a_1\alpha_2 + b_1\beta_2 + c_1\gamma_2 & a_1\alpha_3 + b_1\beta_3 + c_1\gamma_3 \\ a_2\alpha_1 + b_2\beta_1 + c_2\gamma_1 & a_2\alpha_2 + b_2\beta_2 + c_2\gamma_2 & a_2\alpha_3 + b_2\beta_3 + c_2\gamma_3 \\ a_3\alpha_1 + b_3\beta_1 + c_3\gamma_1 & a_3\alpha_2 + b_3\beta_2 + c_3\gamma_2 & a_3\alpha_3 + b_3\beta_3 + c_3\gamma_3 \end{vmatrix}$$

Note: We can also multiply rows by columns or columns by rows or columns by columns.

$2^6 = 64$

$2^7 = 128$

$2^8 =$

Q. $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix} =$

10

(A) 7

(B) 10

(C) 13

(D) 17

$= \left| (\log_3 512)(\log_2 3) + \log_4(3)\log_3(4) \right.$

$\dfrac{\log 9}{\log(2)}$

$\log 2$

$\dfrac{\log 512}{\cancel{\log 3}} \cdot \dfrac{\cancel{\log 3}}{\log 2} + 1$

Solution:

$$\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix} = \left(\frac{\log 512}{\log 3} \times \frac{\log 9}{\log 4} - \frac{\log 3}{\log 4} \times \frac{\log 8}{\log 3}\right) \times \left(\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} - \frac{\log 3}{\log 8} \times \frac{\log 4}{\log 3}\right)$$

$$= \left(\frac{\log 2^9}{\log 3} \times \frac{\log 3^2}{\log 2^2} - \frac{\log 2^3}{\log 2^2}\right) \times \left(\frac{\log 2^2}{\log 2} - \frac{\log 2^2}{\log 2^3}\right) = \left(\frac{9 \times 2}{2} - \frac{3}{2}\right)\left(2 - \frac{2}{3}\right) = 10$$

Q. $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix} =$

A) 7

B) 10

C) 13

D) 17

Transposed Conjugate

- Transpose of conjugate of a matrix is called its transposed conjugate.
- It is denoted by $(\bar{A})^T$ or A^θ

Example:

If $A = \begin{bmatrix} i & 2+i & 1 \\ 0 & 1-i & 1 \\ 2 & 1+2i & 0 \end{bmatrix}$ then

$$\bar{A} = \begin{bmatrix} -i & 2-i & 1 \\ 0 & 1+i & 1 \\ 2 & 1-2i & 0 \end{bmatrix} \text{ and } (\bar{A})' = \begin{bmatrix} -i & 0 & 2 \\ 2-i & 1+i & 1-2i \\ 1 & 1 & 0 \end{bmatrix} = A^\theta$$

Unitary Matrix

A SQUARE Matrix A is said to be a **Unitary Matrix** if:

$$\boxed{A \times (\bar{A})^T = I}$$

$$A A^\theta = I$$

Hermitian Matrix

A SQUARE Matrix A is said to be a **Hermitian Matrix** if:

$$\boxed{(\bar{A})^T = A}$$

$A^\theta = A$

Example:

$$A = \begin{bmatrix} 1 & i & 3i \\ -i & 0 & 2-i \\ -3i & 2+i & -1 \end{bmatrix}$$

Skew Hermitian Matrix

A SQUARE Matrix A is said to be a **Skew Hermitian Matrix** if:

$$\boxed{(\bar{A})^T = -A}$$

$A^{\theta} = -A$

Example:

$$\begin{bmatrix} i & 1+i & 1+2i \\ -(1-i) & 2i & 2 \\ -(1-2i) & 2 & 0 \end{bmatrix}$$

QUESTION SOLVING

JEE Main 2018

Q. Let A be a matrix such that $A\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ is a scalar matrix and $|3A| = 108.\ then\ A^2$ equals:

A $\begin{bmatrix}4 & 0\\-32 & 36\end{bmatrix}$

B $\begin{bmatrix}36 & -32\\0 & 4\end{bmatrix}$

C $\begin{bmatrix}36 & 0\\-32 & 4\end{bmatrix}$

D $\begin{bmatrix}4 & -32\\0 & 36\end{bmatrix}$

$|3A| = 108$

$\Rightarrow 3^2|A| = 108$

$\Rightarrow |A| = \frac{\cancel{108}^{12}}{\cancel{9}}$

$ad - bc = 12$

$\Rightarrow ad = 12$

let $A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$

$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix} = \begin{bmatrix}k & 0\\0 & k\end{bmatrix}$

$\Rightarrow \begin{bmatrix}a & 2a+3b\\c & 2c+3d\end{bmatrix} = \begin{bmatrix}k & 0\\0 & k\end{bmatrix}$

$\boxed{2a+3b=0}$ $\Rightarrow \boxed{k=a}$ $\Rightarrow c=0$

$2c+3d=k$

$\boxed{3d=k}$ $\Rightarrow \boxed{3d=a}$

Solution:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Now, $A.\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} a & 2a+3b \\ c & 2c+3d \end{bmatrix}$ is a scalar.

$\therefore\ c = 0,\ 2a + 3b = 0,\ a = 2c + 3d \Rightarrow a = 3d$

Given, $|3A| = 108$

$\Rightarrow |A| = \frac{108}{3^2} = 12 \Rightarrow ad - bc = ad = 12$

$3dd = 12$

$d^2 = 4 \Rightarrow d = \pm 2$

$a = \pm 6$

$\Rightarrow (3d)d = 12 \Rightarrow d^2 = 4\ [\because a = 3d]$

$\therefore\ a^2 = (3d)^2 = 9d^2 == 9 \times 4 = 36$ and

$\because\ 2a + 3b = 0 \Rightarrow b = \frac{-2a}{3} = \frac{-2\times 6}{3} \Rightarrow b = -4$

Now, $A^2 = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} = \begin{bmatrix} a^2 & ab+bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

JEE Main 2018

Q. Let A be a matrix such that $A\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ is a scalar matrix and $|3A| = 108. \, then \; A^2$ equals:

A $\begin{bmatrix}4 & 0\\ -32 & 36\end{bmatrix}$

B $\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$

C $\begin{bmatrix}36 & 0\\ -32 & 4\end{bmatrix}$

D $\begin{bmatrix}4 & -32\\ 0 & 36\end{bmatrix}$

JEE Main 2022: 26 July - II

Q. Let $A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$, then the value of A'BA is

(A) 1224

(B) 1042

(C) 540

(D) 539

$9^2+12^2-15^2-10^2+13^2+16^2+11^2-14^2+17^2$

$=$

$\underset{1\times 3}{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 9^2+12^2-15^2 & -10^2+13^2+16^2 & 11^2-14^2+17^2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Solution:

Given $A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$

then $A' = [1 \quad 1 \quad 1]$

So $A'B = [1 \quad 1 \quad 1]\begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$

$= [\, 9^2 + 12^2 - 15^2 \quad -10^2 + 13^2 + 16^2 \quad 11^2 - 14^2 + 17^2 \,]$

$= [0 \quad 325 \quad 214]$

Now $A'BA = [0 \quad 325 \quad 214]\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= [325 + 214]$

$= [539]$

JEE Main 2022: 26 July - II

Q. Let $A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$, then

the value of A'BA is

A. 1224

B. 1042

C. 540

D. 539

Q. If

$$f(x)=\begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix} \text{ then } \int_0^{\pi/2}\left[f(x)+f^1(x)\right]dx=$$

$= 2\cos^2 x(\cos^2 x) + \sin 2x(\sin x \cos x)$

$-\sin x(-\cos x \sin 2x - 2\sin^3 x)$

A) 0

B) π

C) $\frac{\pi}{2}$

D) 2π

Solution:

$$f(x)=\begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$$

$$=2\cos^2 x\left(\cos^2 x\right)-\sin 2x\left(\sin x\cos x\right)-\sin x\left(-\sin 2x\cos x-2\sin^3 x\right)$$

$$=2\left(\cos^4 x+\sin^2 x\right)-\sin^2 2x=2$$

$$=2\left(\left(\cos^2 x\right)^2+\sin^2 x\right)-4\sin^2 x\cos^2 x$$

Therefore, $f'(x)=0$

Hence $\int_0^{\pi/2}\left(f(x)+f'(x)\right)dx=\int_0^{\pi/2}2dx=[2x]_0^{\pi/2}=2\times\frac{\pi}{2}-0=\pi$

Q. If

$$f(x) = \begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix} then \int_0^{\pi/2} [f(x) + f^1(x)]\,dx =$$

A 0

B π

C $\frac{\pi}{2}$

D 2π

Q. If the system of equations

$x + y + z = 6,$

$x + 2y + \lambda z = 10$ and

$x + 2y + 3z = \mu$

has infinite solutions, then the value of $\lambda + 2\mu$ is equal to

NTA Paper 55

$D = 0$

$D_1 = 0$

A 20

B 22

C 23

D 25

For infinite solutions, $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda \\ 1 & 2 & 3 \end{vmatrix} = 1(6-2\lambda) - 1(3-\lambda) + 0 = 0 \Rightarrow \lambda = 3$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & \lambda \\ \mu & 2 & 3 \end{vmatrix} = 0 \Rightarrow 6(6-2\lambda) - 1(30-4\lambda) + 1(20-2\mu) = 0$$

$?\lambda = 3 \Rightarrow \mu = 10$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & 10 & 3 \end{vmatrix} = 0, \Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & 10 \end{vmatrix} = 0$$

$\lambda + 2\mu = 23$

Q. If the system of equations

$x + y + z = 6,$

$x + 2y + \lambda z = 10$ and

$x + 2y + 3z = \mu$

has infinite solutions, then the value of $\lambda + 2\mu$ is equal to

NTA Paper 55

A. 20

B. 22

C. 23

D. 25

JEE(Main): 25 Jan 2023 -II

Q. Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i=\sqrt{-1}$. If $M=A^TBA$, then the inverse of the matrix $AM^{2023}A^T$ is

$\rightarrow B^{2023}$

Trick A is orthogonal

$AA^T = I$

A $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

B $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

C $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$

D $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

JEE(Main): 25 Jan 2023 -II

Q. Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i=\sqrt{-1}$. If $M=A^TBA$, then the inverse of the matrix $AM^{2023}A^T$ is

Solution:

$$AA^{T} = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^{2} = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$$

$$B^{3} = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

$$\vdots$$

$$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

Solution:

$$M = A^T B A$$

$$M^2 = M \cdot M = A^T B A A^T B A = A^T B^2 A$$

$$M^3 = M^2 \cdot M = A^T B^2 A^T B A = A^T B^3 A$$

$$\vdots$$

$$M^{2023} = \ldots\ldots\ldots\ldots\ldots\ldots A^T B^{2023} A$$

$$AM^{2023} A^T = \underline{AA^T} B^{2023} \underline{AA^T} = B^{2023}$$

$$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

Inverse of $\left(AM^{2023} A^T\right)$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

JEE(Main): 25 Jan 2023 -II

Q. Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i=\sqrt{-1}$. If $M=A^TBA$, then the inverse of the matrix $AM^{2023}A^T$ is

A $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

B $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

C $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$

D $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Q. If $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 5 & 2 \\ 1 & 6 & 1 \end{bmatrix}$,then tr(A.adj(adj A)) is equal to

A 7

B 18

C -58

D -1624

$|A| = -28$

$A^2 = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 5 & 2 \\ 1 & 6 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 & -1 \\ 3 & 5 & 2 \\ 1 & 6 & 1 \end{pmatrix}$

$= \left(\quad \right)$

$adj(adj A) = |A|^{n-2} A$

$adj(adj A) = |A| A$

$tr(A \cdot |A| A)$

$tr(|A| A^2) = tr(-28(\quad))$

Solution

$\therefore \text{adj}(\text{adjA}) = |A|A$ (by property) $(\because \text{adj}(\text{adjA}) = |A|^{n-2}A)$

$\text{Aadj}(\text{adjA}) = |A|A^2 \ldots\ldots (\text{i})$

$$|A| = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 5 & 2 \\ 1 & 6 & 1 \end{vmatrix} = 2(5-12) - 1(3-2)$$

$$-1(18-5)$$

$$= -14 - 1 - 13 = -28$$

$$A^2 = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 5 & 2 \\ 1 & 6 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 3 & 5 & 2 \\ 1 & 6 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & -1 \\ 23 & 40 & 9 \\ 21 & 37 & 12 \end{bmatrix}$$

$$\text{tr}(A(\text{adj}(adjA))) = \text{tr}(|A|A^2) =$$

$$-28[6+40+12] = -28 \times 58 = -1624$$

Q. If $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 5 & 2 \\ 1 & 6 & 1 \end{bmatrix}$,then tr(A.adj(adj A)) is equal to

A 7

B 18

C -58

D -1624

JEE(Main): 2019

Q. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$, where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is:

A $2\sqrt{3}$

B $-2\sqrt{3}$

C $\sqrt{3}$

D $-\sqrt{3}$

$|A| = 2(2b^2+2-b^2) - b(2b-b) + 1(b-b^2-1)$

$|A| = b^2+3$

$\frac{b^2+3}{b}$

$= b + \frac{3}{b}$

AM $\geq$ GM

$\geq 2\sqrt{b \times \frac{3}{b}}$

Solution:

$$|A| = \begin{vmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{vmatrix}$$

$$= 2(2b^2+2-b^2) - b(2b-b) + 1(b^2-b^2-1)$$

$$= 2(b^2+2) - b^2 - 1$$

$$= 2b^2 + 4 - b^2 - 1$$

$$= b^2 + 3$$

$$\frac{|A|}{b} = b + \frac{3}{b} = \left(\sqrt{b} - \sqrt{\frac{3}{b}}\right)^2 + 2\sqrt{3}$$

Hence, the minimum value of $\frac{|A|}{b}$ is $2\sqrt{3}$. $\left(\because \left(\sqrt{b} - \sqrt{\frac{3}{b}}\right)^2 \geq 0\right)$

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Q. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$, where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is:

A. $2\sqrt{3}$

B. $-2\sqrt{3}$

C. $\sqrt{3}$

D. $-\sqrt{3}$

D.I.Y

Q. If the system of equations

$x + y + z = 6,$

$x + 2y + \lambda z = 10$ and

$x + 2y + 3z = \mu$

has infinite solutions, then the value of $\lambda + 2\mu$ is equal to

A. 20

B. 22

C. 23

D. 25

Solution:

For infinite solutions, $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda \\ 1 & 2 & 3 \end{vmatrix} = 1(6-2\lambda) - 1(3-\lambda) + 0 = 0 \Rightarrow \lambda = 3$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & \lambda \\ \mu & 2 & 3 \end{vmatrix} = 0 \Rightarrow 6(6-2\lambda) - 1(30-4\lambda) + 1(20-2\mu) = 0$$

$?\lambda = 3 \Rightarrow \mu = 10$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & 10 & 3 \end{vmatrix} = 0, \Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & 10 \end{vmatrix} = 0$$

$\lambda + 2\mu = 23$