Formula Sheet - Qubit Electronics

1 Fundamental Qubit Representation and State Vectors

A single qubit exists in a state $|\psi\rangle$, representing a superposition of basis states $|0\rangle$ and $|1\rangle$.

1.1 General Superposition Formula

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

where α and β are complex probability amplitudes.

1.2 Normalization Condition

$$|\alpha|^2 + |\beta|^2 = 1$$

1.3 Hermitian Conjugate

$$|\psi\rangle^\dagger = \langle\psi| = [\alpha^*, \beta^*]$$

1.4 Multi-Qubit System Scaling

A system of N qubits scales to 2^N components. For $N = 3$:

$$|\psi\rangle = c_1|000\rangle + c_2|001\rangle + \cdots + c_8|111\rangle$$

2 The Bloch Sphere Geometry

2.1 General State Formula

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

2.2 Bloch Vector Coordinates

- $|0\rangle$ (North Pole): $(0, 0, 1)$
- $|1\rangle$ (South Pole): $(0, 0, -1)$
- $|+\rangle$: $(1, 0, 0)$
- $|-\rangle$: $(-1, 0, 0)$

3 Quantum Measurement and the Born Rule

3.1 State Collapse

Measurement forces $|\psi\rangle$ to collapse into either $|0\rangle$ or $|1\rangle$.

3.2 Probability

$$P(0) = |\alpha|^2, \quad P(1) = |\beta|^2$$

4 Single-Qubit Unitary Operators

4.1 X-Gate (NOT)

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

4.2 Y-Gate

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

4.3 Z-Gate

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

4.4 Hadamard Gate

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

4.5 Phase Gate

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

4.6 T-Gate

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

5 Rotation Gates

$$R_x(\theta) = e^{-i\theta X/2}, \quad R_y(\theta) = e^{-i\theta Y/2}, \quad R_z(\theta) = e^{-i\theta Z/2}$$

6 Two-Qubit Interactions: CNOT Gate

6.1 Mapping Rule

$$|A\rangle|B\rangle \to |A\rangle|A \oplus B\rangle$$

6.2 Matrix Representation

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

7 Density Matrix Formalism

7.1 Definition

$$\rho = |\psi\rangle\langle\psi|$$

7.2 Trace Property

$$\mathrm{Tr}(\rho) = 1$$

7.3 State Purity

- Pure State: $\mathrm{Tr}(\rho^2) = 1$
- Mixed State: $\mathrm{Tr}(\rho^2) < 1$

8 Qubit Performance Metrics

8.1 Coherence Time

$$\frac{1}{T_2} = \frac{1}{T_\phi} + \frac{1}{2T_1}$$

8.2 Gate Capacity

$$N_{\mathrm{gates}} \sim \frac{t_{\mathrm{gate}}}{T_2}$$

8.3 DiVincenzo Criteria

The seven requirements for a scalable quantum system include: scalable physical qubits, initialization capability, long coherence times, universal gates, qubit-specific measurement, qubit conversion, and faithful transmission.