Formula Sheet - Qubit Electronics

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1 Qubit Introduction

Fundamental Qubit Representation and State Vectors

- **General Superposition:** $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ where α and β are complex probability amplitudes.
- **Normalization Condition:** $|\alpha|^2 + |\beta|^2 = 1 = \langle\psi|\psi\rangle$
- **Hermitian Conjugate:** $|\psi\rangle^\dagger = \langle\psi| = [\alpha^*, \beta^*]$
- **Multi-Qubit System Scaling:** A system of N qubits scales to 2^N components. For $N = 3$: $|\psi\rangle = c_1|000\rangle + c_2|001\rangle + \cdots + c_8|111\rangle$

The Bloch Sphere Geometry

- **General State Formula:** $|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$
- **Bloch Vector Coordinates:** $|0\rangle$ (North Pole): $(0, 0, 1)$; $|1\rangle$ (South Pole): $(0, 0, -1)$; $|+\rangle$: $(1, 0, 0)$; $|-\rangle$: $(-1, 0, 0)$

Quantum Measurement and the Born Rule

- **State Collapse:** Measurement forces $|\psi\rangle$ to collapse into either $|0\rangle$ or $|1\rangle$.
- **Born Rule:** $P(|a\rangle) = |\langle\psi|a\rangle|^2$

Basis States in Different Representations

- **Computational Basis:** $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- **Hadamard Basis:** $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
- **Circular Basis:** $|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$, $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

Single-Qubit Unitary Operators

- **X (NOT):** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- **Y:** $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
- **Z:** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- **Hadamard (H):** $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

- **Phase (S):** $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
- **T:** $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
- **Rotation Gates:** $R_x(\theta) = e^{-i\theta X/2}, \quad R_y(\theta) = e^{-i\theta Y/2}, \quad R_z(\theta) = e^{-i\theta Z/2}$

Two-Qubit Interactions: CNOT Gate

- **Mapping Rule:** $|A\rangle|B\rangle \to |A\rangle|A \oplus B\rangle$
- **Matrix Representation:** $\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Density Matrix Formalism

- **Definition:** $\rho = |\psi\rangle\langle\psi|$
- **Mixed State Ensemble:** $\widehat{\rho} \equiv \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$ where p_i are probabilities and $\sum_i p_i = 1$
- **Trace Property:** $\text{Tr}(\rho^2) = 1$ if pure state;< 1 if mixed state

Qubit Performance Metrics

- **Coherence Time:** $\frac{1}{T_2} = \frac{1}{T_\phi} + \frac{1}{2T_1}$
- **Gate Capacity:** $N_{\text{gates}} \sim \frac{T_2}{t_{\text{gate}}}$