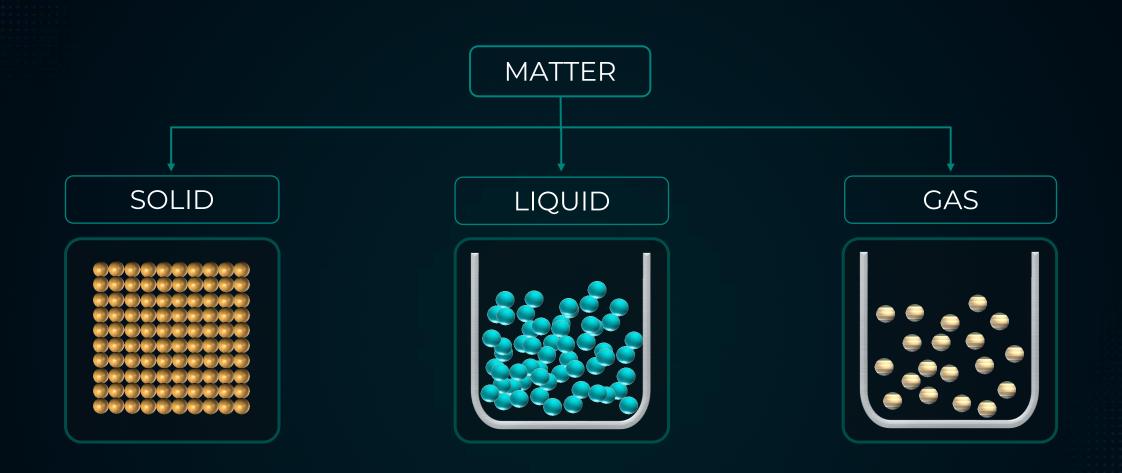




States of Matter

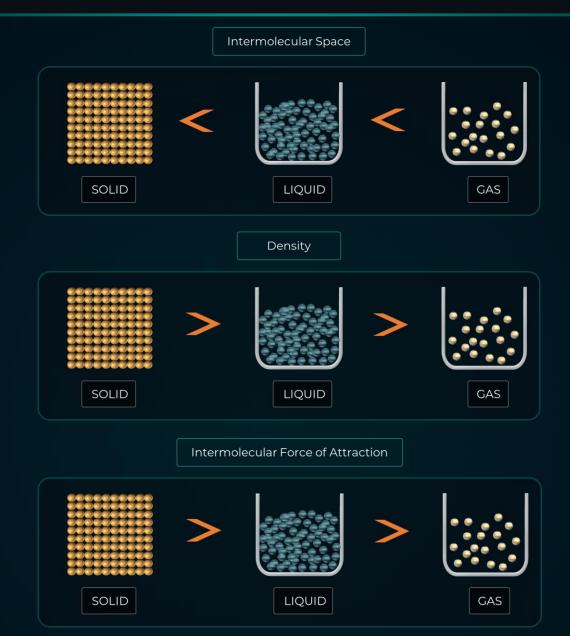














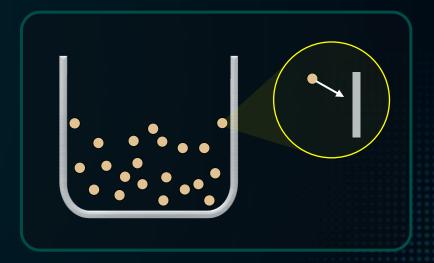
Characteristics of Gas



- It has no shape and size and it can expands indefinitely and uniformly to fill the available space.
- The intermolecular force of gases are very weak, molecules may fly apart.
- It exerts pressure on its surroundings (collision with wall of container).











Volume

Macroscopic Properties of Gas

Pressure

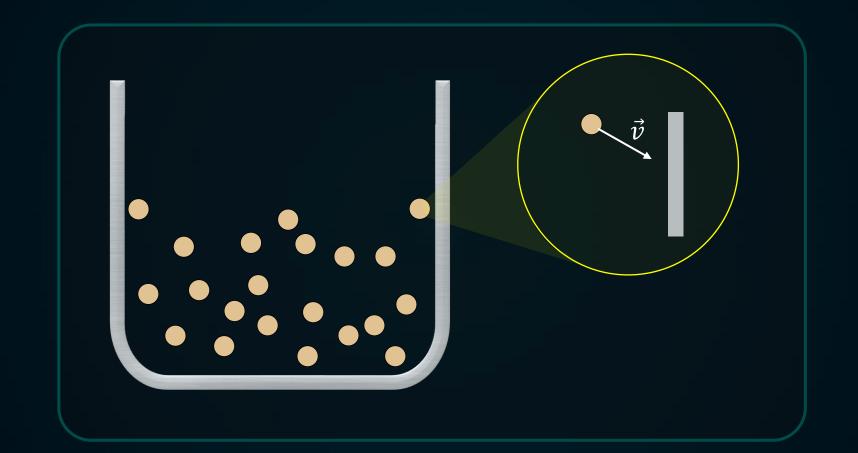
Temperature



Pressure



- Pressure is the force exerted per unit area on the walls of the container.
- It is a result of the collisions of the atoms with the container's walls.





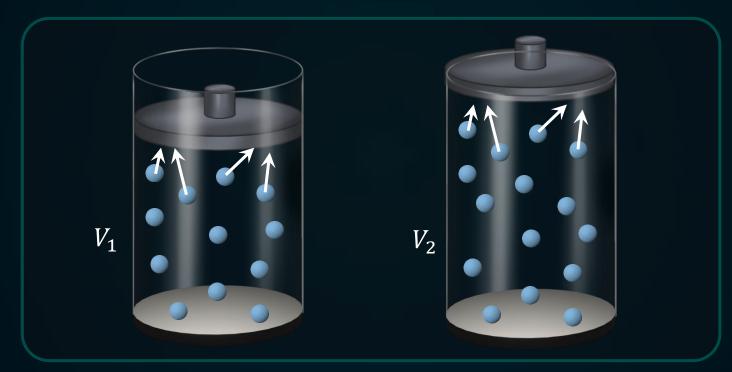
Volume



Volume is the space occupied by the gas.

Volume of the gas = Volume of the container

• It is a result of freedom of the atoms to spread throughout the container..

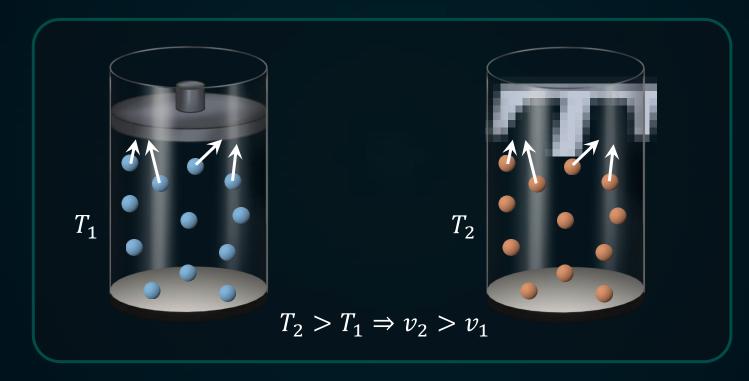




Temperature



- Temperature is the measure of degree of hotness.
- It is a result of motion of the constituent particles rapidly within the container.
- Faster the particles move, higher is the temperature of gas





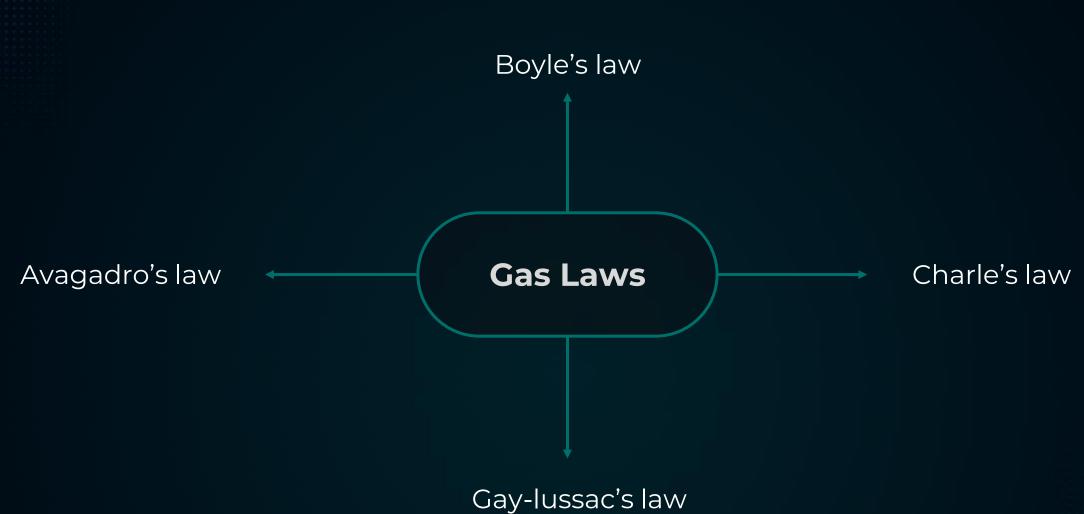
Assumption of Ideal gas



- The gas consists of a large number of molecules, which are in random motion.
- The volume of the molecules is negligibly small compared to the volume occupied by the gas.
- There are no (or entirely negligible) intermolecular forces between the gas molecules.
- All collisions, both between the molecules themselves, and between the molecules and the walls of the container, are perfectly elastic.







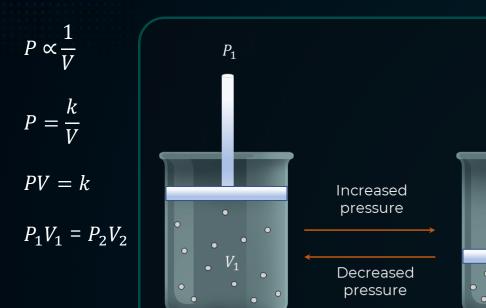


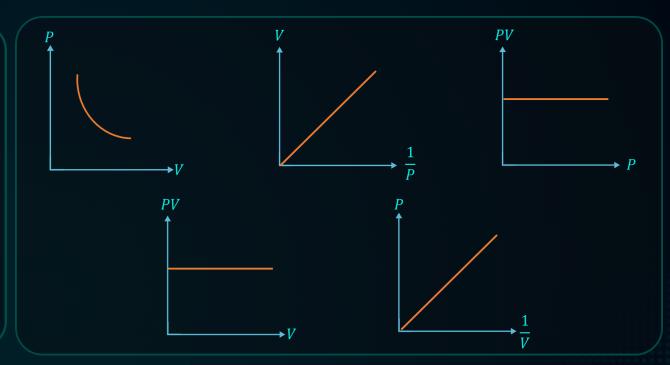
Boyle's Law



For a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure.

If n and T are constant







Charle's Law



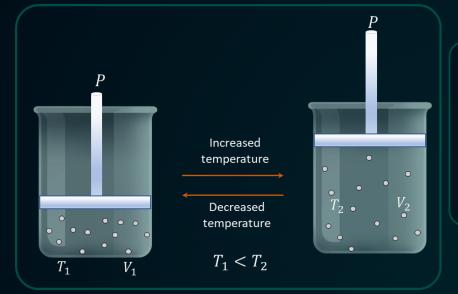
For constant pressure, the volume of the given mass of a gas is directly proportional to its absolute temperature. If n and P are constant

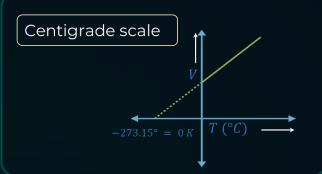
$$V \propto T$$

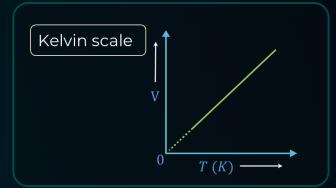
$$V = kT$$

$$\frac{V}{T} = k$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$



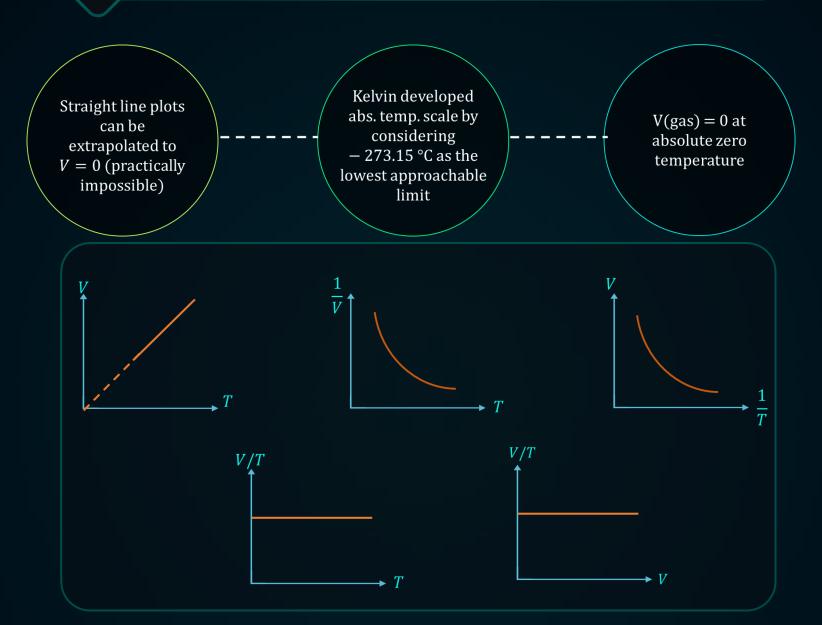






Charle's Law







Gay-Lussac's Law



For constant volume, the pressure of a given mass of a gas is directly proportional to its absolute temperature.

If n and V are constant

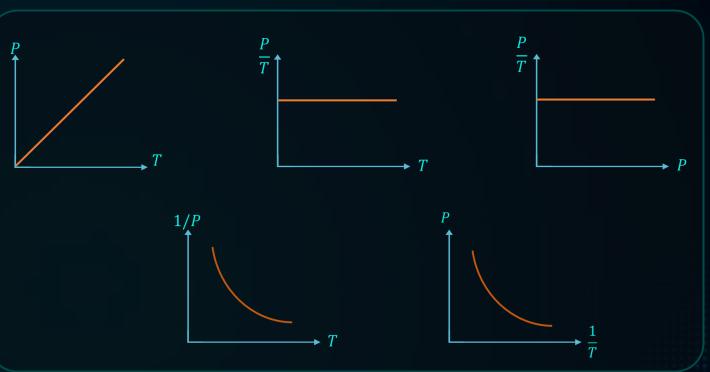
$$P \propto T$$

$$P = kT$$

$$\frac{P}{T} = k$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$







Avogadro's Law



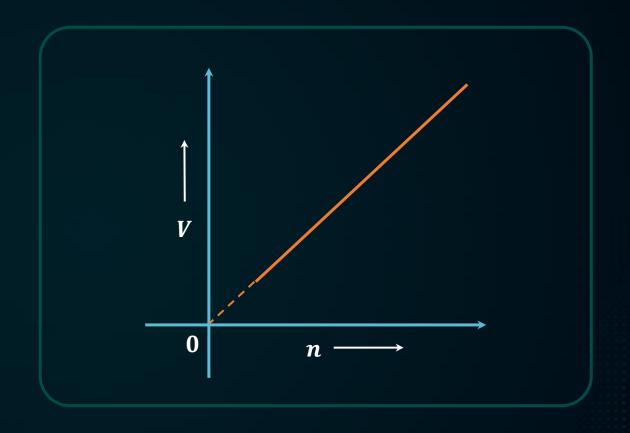
 For same condition of temperature and pressure, equal volumes of different gases contain an equal number of molecules.

At constant P and T

$$V \propto n$$

$$V = kn$$

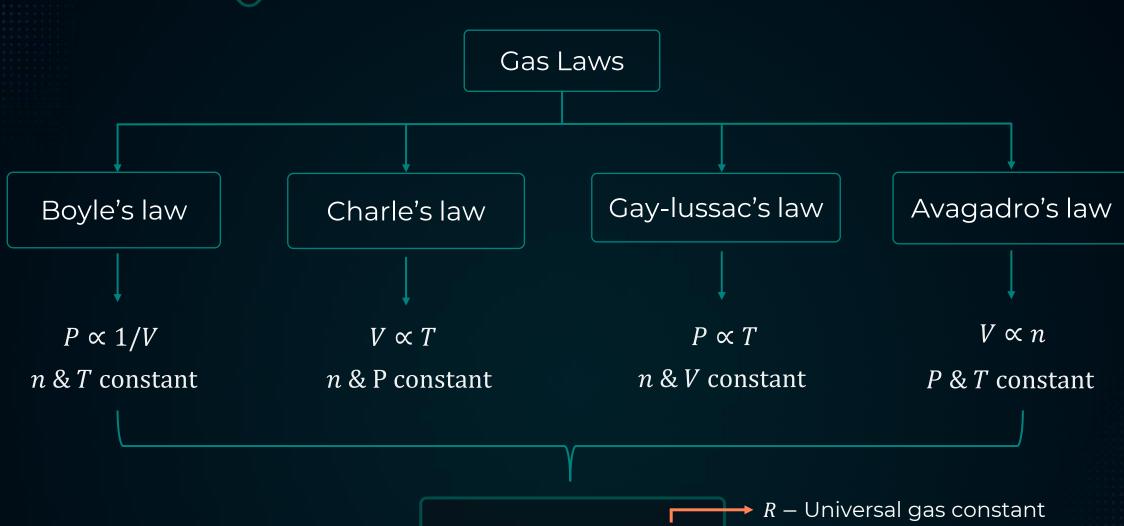
$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$





Gas Laws





Ideal Gas equation⇒

$$PV \propto nT \Rightarrow PV = nRT$$



Universal Gas Constant



$$PV = nRT$$

$$R = \frac{PV}{nT}$$

At STP conditions-

$$P = 1 \text{ atm} = 101325 \text{ N/m}^2$$
 $n = 1 \text{ mole}$ $V = 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3$

$$n = 1 mole$$

$$V = 22.4 L = 22.4 \times 10^{-3} m^{2}$$

$$R = \frac{101325 \frac{N}{m^2} \times 22.4 \times 10^{-3} m^3}{1 \ mol \times 273 \ K} = 8.314 \frac{N.m}{mol. K} = 8.314 \frac{J}{mol - K}$$

$$1 \ cal = 4.18 \ J$$

$$R = 8.314 \frac{\frac{1}{4.18} \, cal}{mol. \, K} = 1.98 \, \frac{cal}{mol. \, K} \approx 2 \, \frac{cal}{mol. \, K}$$



Ideal Gas equation



• The equation which relates the pressure (P) volume (V) and temperature (T) of the given state of an ideal gas.

$$PV = nRT$$

$$n = \frac{mass\ of\ the\ gas}{Molar\ mass\ of\ the\ gas} = \frac{m}{M} \Rightarrow PV = m\frac{R}{M}T$$

$$r-Specific\ gas\ constant$$

$$n = \frac{Total\ no.of\ molecules}{Avagadro\ no.} = \frac{N}{N_A} \Rightarrow PV = N \frac{R}{N_A} T$$

 $\rightarrow k$ -Boltzmann's constant = $1.38 \times 10^{-23} J/K$





A gas occupies a volume of $400 cm^3$ at $27^{\circ}C$ and 780 mm of Hg. What volume (in liters) will it occupy at $87^{\circ}C$ and 780 mm of Hg?

Given:
$$T_1 = 27^{\circ}C = 300 \text{ K}$$
, $P_1 = 780 \text{ mm}$ of Hg, $V_1 = 400 \text{ cm}^3$, $T_2 = 87^{\circ}C = 360 \text{ K}$, $P_2 = 780 \text{ mm}$ of Hg

To find: $V_2 = ?$

Solution: Here the pressure is constant and only the temperature is changed. Using Charles Law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{400}{300} = \frac{V_2}{360} \implies V_2 = \frac{400 \times 360}{300} = 480 \ cm^3 \implies V_2 = 0.48 \ L$$





0.24

0.48





A fixed mass of gas at a constant temperature has a pressure of $2000 \, Pa$ and a volume of $0.02 \, m^3$. It is compressed until the volume is $0.005 \, m^3$. What is its new pressure (Pa)?

Given:
$$P_1 = 2000pa$$
, $V_1 = 0.02m^3$, $V_2 = 0.005m^3$

To find:
$$V_2 = ?$$

Solution: Using Boyle's Law- $P_1V_1 = P_2V_2$

$$\Rightarrow P_2 = \frac{P_1 V_1}{V_2}$$

 $P_2 = 8 \, Pa$

$$P_2 = \frac{(2000Pa)(0.02 \, m^3)}{0.005m^3}$$

8

16

d





In the given (V-T) diagram, what is the relation between pressures P_1 and P_2 ?

Solution: Assuming the graph for a gas of given mass,

a

we have,
$$PV = nRT$$

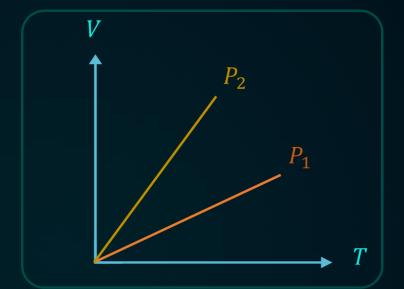
$$\therefore \quad \frac{V}{T} \propto \frac{1}{P}$$

From the graph,
$$\frac{V}{T} = \tan \theta$$

$$\therefore \quad \frac{1}{P} \propto \tan \theta$$

As angle θ increases, $\tan \theta$ increases and pressure decreases.

$$\therefore P_1 > P_2$$



 $P_2 = P_1$

$$C$$
 $P_2 > P_1$







What is the mass of gas at a pressure of 350 k Pa and a temperature of $35 \,^{\circ}\text{C}$ if it occupies volume of $0.030 \, m^3$. The gas has a characteristic gas constant of $290 \, Jkg^{-1}K^{-1}$?

Given:
$$P = 350 \times 10^3 Pa$$
, $T = 273 + 35^{\circ}C = 308 K$, $V = 0.03m^3$ and $R = 290J kg^{-1}K^{-1}$

To find: m = ?

Solution:
$$PV = mRT \Rightarrow m = \frac{PV}{RT}$$

$$m = \frac{(350 \times 10^3)(0.03)}{(290)(308)}$$

$$m = 0.12 kg$$

 $\left(\begin{array}{c} \mathsf{a} \end{array}\right) \quad 0.12\,\mathit{Kg}$

C 12 Kg

b 1.2 *Kg*

d 0.012 Kg





The P-V diagram of two different mass m_1 and m_2 are drawn at constant temperature T, What is the relation between m_1 and m_2 ?

Given: P - v diagram

To find: Relation between m_1 and m_2

Solution:

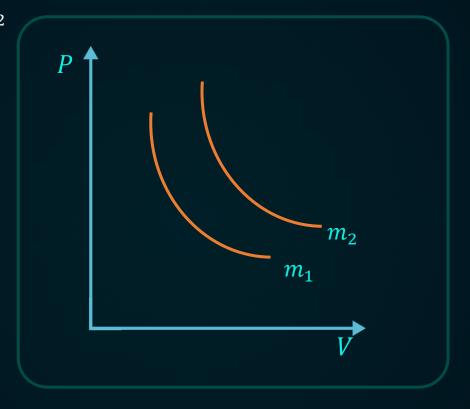
$$PV = nRT$$

$$PV = \frac{m}{M}RT$$

 $PV \propto m$

Keeping P constant, $V_1 < V_2$

 $m_1 < m_2$



$$b \qquad m_1 = m_2$$

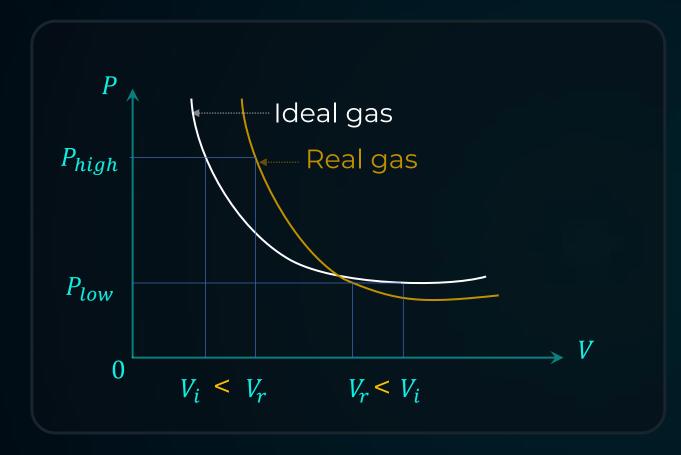
$$C$$
 $m_1 < m_2$



Real gases



Gases which do not obey ideal gas laws exactly under all conditions of temperature and pressure.



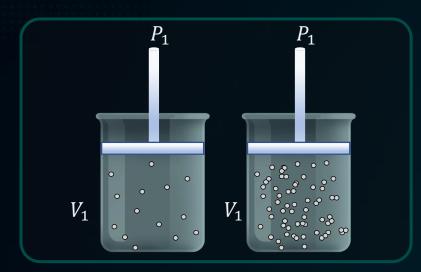
At higher pressure, $V_{real} > V_{ideal}$

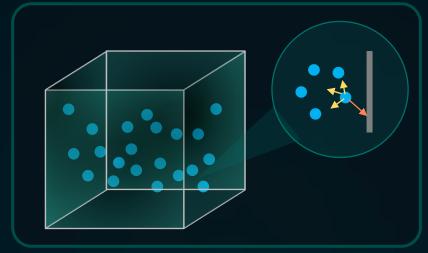
At lower pressure, $V_{ideal} > V_{real}$

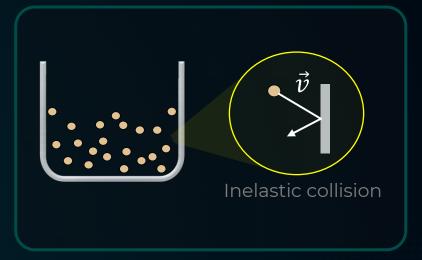


Real Gases Vs Ideal Gas









Volume of gas particles is not negligible with respect to container. Interparticle forces of attraction and repulsion are present.

Collisions are not perfectly elastic.



Pressure correction



Due to the interparticle attractions

- Speed during collisions will be reduced
- Momentum will be less
- Force applied will be less
- Pressure will be less. $(P_{ideal} > P_{Real})$

$$\Rightarrow P_{ideal} = P_{Real} + dP$$

$$dP \propto \frac{n^2}{V^2} \Rightarrow dP = a \frac{n^2}{V^2}$$

$$dP \propto \frac{n^2}{V^2} \Rightarrow dP = a \frac{n^2}{V^2}$$
 $a = \text{Van der Waal's constant \& } a \propto \text{Attractive force}$

$$P_{Ideal} = P_{Real} + \frac{an^2}{V^2}$$



Volume corrections



 V_{ideal} = Volume available for free motion of a gas = V_{free}

$$V_{free} = V_{container} - V_{excluded}$$

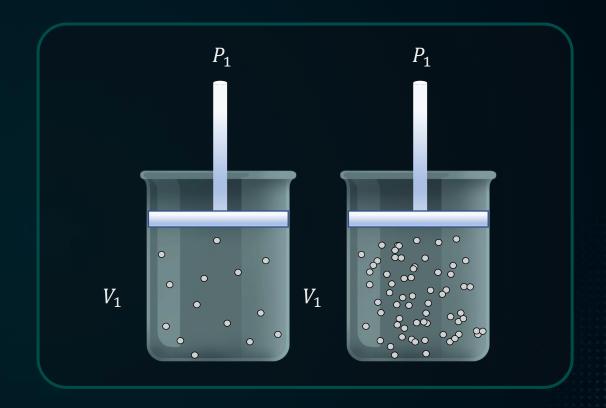
 $\overline{V_{\text{container}}} = V$

 $V_{\text{excluded}} = nb$

b = volume of 1 mole of molecule

 $b \propto \text{size of molecules}$

$$V_{free} = V - nb$$





Van Der Waals Equation







$$P_{Ideal}V_{Ideal} = nRT$$

Pressure Corrections

$$P_{Ideal} = P_{Real} + \frac{an^2}{V^2}$$

Van Der Waals Equation

$$\left(P_{Real} + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

Volume Corrections

$$V_{ideal} = V_{real} - nb$$

Compressibility Factor(*Z*)

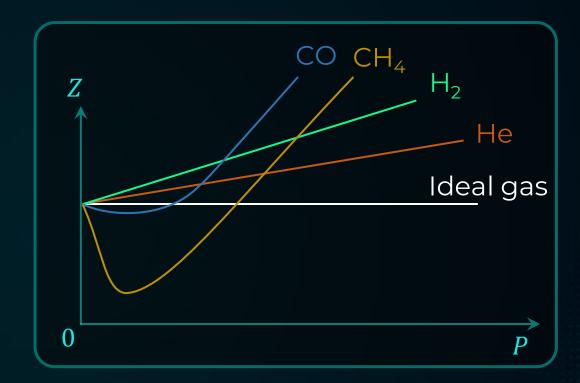


Ratio of the observed volume of the gas to the calculated volume at the same n, T and P.

$$Z = \frac{V_{real}}{V_{ideal}}$$

For Ideal gas: Z = 1

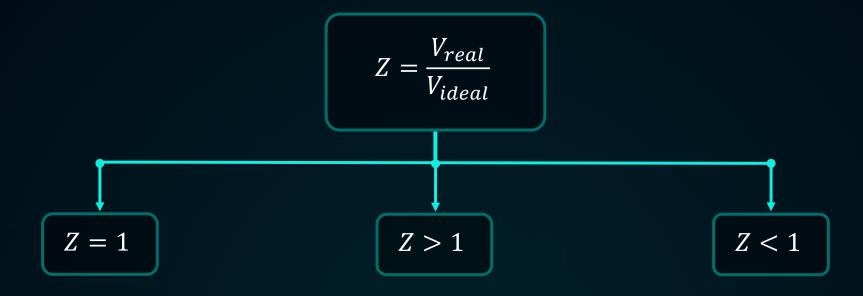
For Real gas: $Z \neq 1$





Compressibility Factor(*Z*)





$$V_{real} = V_{ideal}$$

$$F_{att} = F_{rep} = 0$$

$$V_{real} > V_{ideal}$$

$$F_{att} < F_{rep}$$

$$V_{real} < V_{ideal}$$

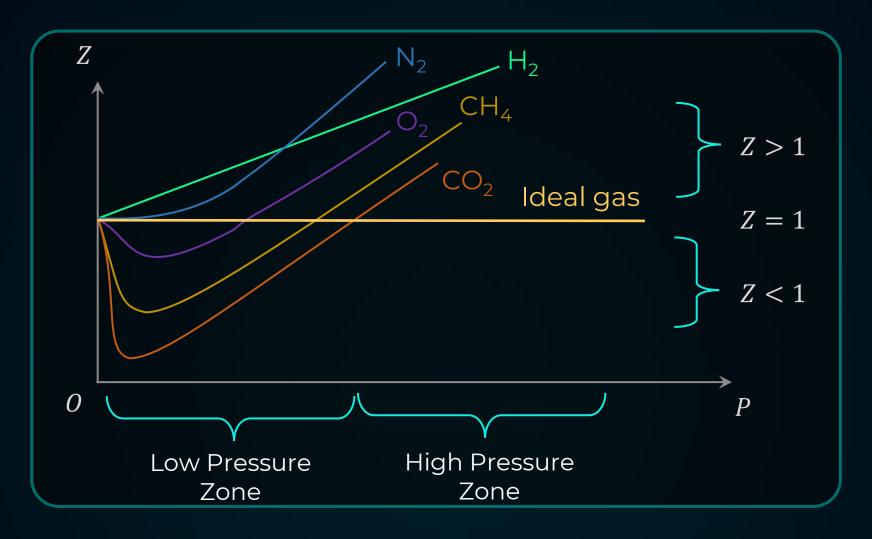
$$F_{att} > F_{rep}$$

Easy to liquify



Graph of some Real Gases





For fixed amount of real gas Z depends upon P, T of gas.



Real gases at Low Pressure



Van Der Waals Equation-

$$\left(P_{Real} + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

For
$$n = 1$$
 $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

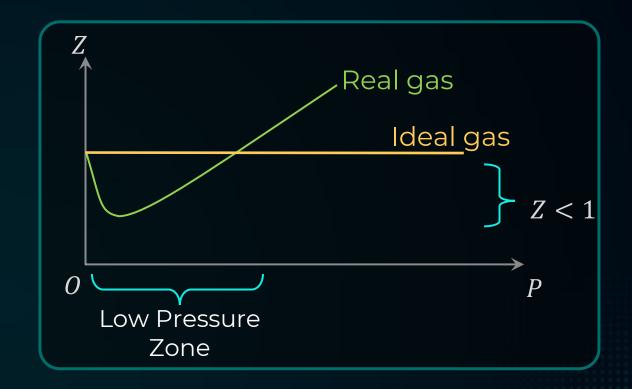
At low pressure $\Rightarrow V \uparrow \Rightarrow b$ can be neglected

$$\left(P + \frac{a}{V^2}\right)V = RT \Rightarrow PV + \frac{a}{V} = RT \Rightarrow \frac{PV}{RT} + \frac{a}{VRT} = 1$$

$$\Rightarrow 7 - 1 - \frac{a}{V}$$

$$\Rightarrow Z = 1 - \frac{a}{VRT}$$

$$\therefore Z < 1$$





Real gases at High Pressure



Van Der Waals Equation-

$$\left(P_{Real} + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

For
$$n = 1$$
 $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

At high pressure $\rightarrow \frac{a}{V^2}$ can be neglected

$$P(V - b) = RT \Rightarrow PV - Pb = RT \Rightarrow \frac{PV}{RT} = 1 + \frac{Pb}{RT}$$
$$\Rightarrow Z = 1 + \frac{Pb}{RT}$$





Real gases as Ideal gas



Van Der Waals Equation-
$$\left(P_{Real} + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

For
$$n = 1$$
 $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

$$PV = RT$$

Ideal gas Equation

At low pressure

$$P \downarrow \Rightarrow V \uparrow$$

∴ b can be neglected

At high temperature

$$T \uparrow \Rightarrow F_{attractions} \approx 0$$

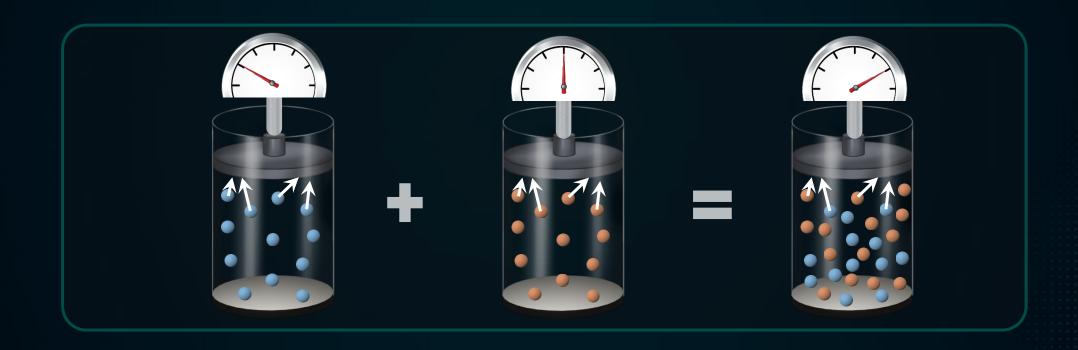
$$\therefore \frac{a}{V^2}$$
 can be neglected

Dalton's Law of Partial Pressures



 Total pressure exerted by a mixture of gases is equal to the sum of the partial pressures exerted by each individual gases in the mixture.

$$P_{total} = P_1 + P_2 + \dots$$





Diffusion & Effusion



Diffusion-

- Spontaneous intermixing of gas molecules under concentration gradient.
- Molecules move from region of higher concentration to region of lower concentration to reduce the concentration gradient.

Effusion-

 Escape of molecules from region of high pressure to region of lower pressure through pin hole without molecular collision.







Graham's Law



 Graham's law states that the rate of diffusion or of effusion of a gas is inversely proportional to the square root of its molecular weight.

$$r \propto \frac{1}{\sqrt{M}}$$

In the same conditions of temperature and pressure, the molar mass is
proportional to the mass density. Therefore, the rates of diffusion of different gases
are inversely proportional to the square roots of their mass densities.

$$r \propto \frac{1}{\sqrt{\rho}}$$
 P and T are constant

For two gases A and B diffusing into each other-

$$\frac{r_A}{r_B} = \sqrt{\frac{M_B}{M_A}} = \sqrt{\frac{
ho_B}{
ho_A}}$$





Macroscopic propetries

Pressure

Volume

Temperature



Microscopic propetries

Speed

Momentum

Kinetic energy



Maxwell's distribution



Number of molecules possessing very high and very low speed is very small. The maximum in the curve represents speed possessed by maximum number of molecules. This speed is called most probable speed, v_{mp} .

$$v_{mp} = \sqrt{\frac{2RT}{M_{molar}}}$$

We know
$$PM_{molar} = \rho RT \implies v_{mp} = \sqrt{\frac{2P}{\rho}}$$

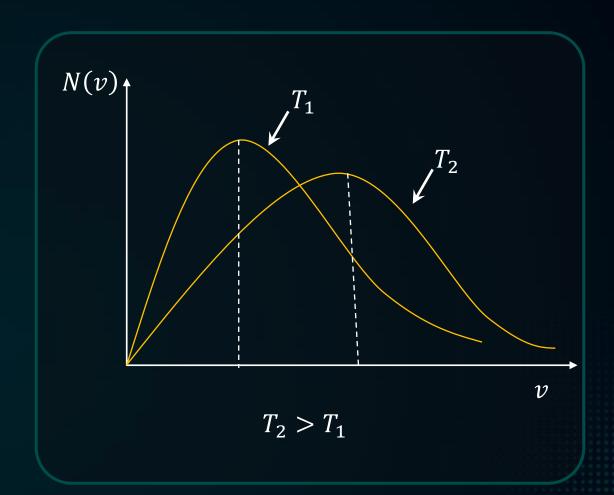




Maxwell's distribution



- On increasing the temperature most probable speed increases.
- Distribution curve broadens at higher temperature. Implies that, number of molecules moving at higher speed increases.
- Speed distribution also depends upon mass of molecules. At the same temperature, gas molecules with heavier mass have slower speed than lighter gas molecules.





Average Speed



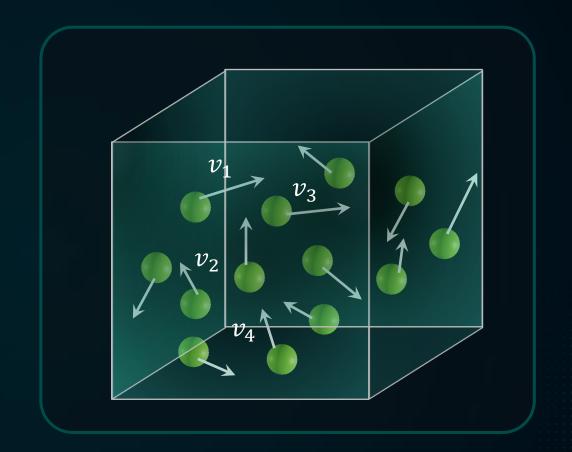
Arithmetic mean of the speed of all the molecules in a gas.

$$v_{avg} = \frac{(v_1 + v_2 + v_3 + \dots + v_n)}{n}$$

$$v_{avg} = \sqrt{\frac{8RT}{\pi M_{molar}}}$$

We know $PM_{molar} = \rho RT$,

$$v_{avg} = \sqrt{\frac{8P}{\pi \rho}}$$







KINETIC ENERGY,
$$KE = \frac{1}{2}mv^2$$

To find average kinetic energy

$$< KE > = \frac{1}{2}m < v >^2$$

or

$$\langle \mathit{KE} \rangle = \frac{1}{2}m \langle v^2 \rangle$$



Root Mean Square Speed



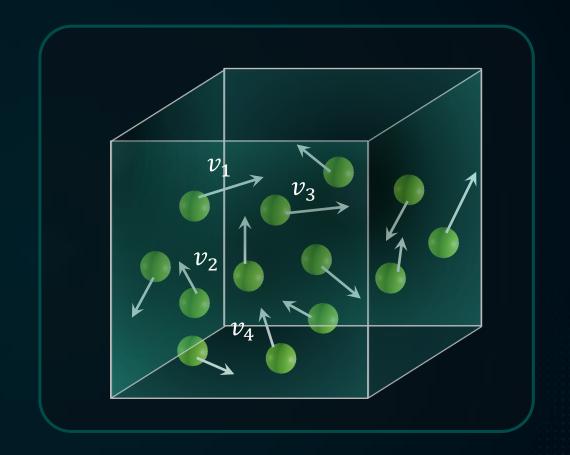
Root of the mean of the square's of the speeds of all molecules in a gas

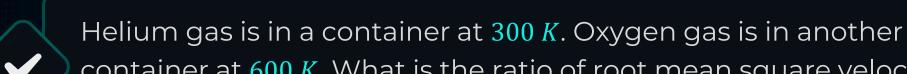
$$v_{rms} = \sqrt{\frac{(v_1^2 + v_1^2 + v_1^2 + \dots + v_n^2)}{n}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M_{molar}}}$$

$$v_{rms} \propto \sqrt{T}$$

$$v_{rms} \propto \sqrt{\frac{1}{M}}$$







container at 600 K. What is the ratio of root mean square velocity of Helium gas to oxygen gas?

Given:
$$T_{O_2} = 600 \, K$$
 and $T_{He} = 300 \, K$

To find:
$$v_{rms}(He): v_{rms}(O_2)$$

Solution:
$$v_{rms} = \sqrt{\frac{3RT}{M_{molar}}}$$

$$\frac{v_{He}}{v_{O_2}} = \sqrt{\frac{T_{He} \cdot M_{O_2}}{T_{O_2} \cdot M_{He}}} = \sqrt{\frac{300 \times 32}{600 \times 4}}$$

$$\frac{v_{He}}{v_{O_2}} = \frac{2}{1}$$





The r.m.s speed of hydrogen molecules at a certain temperature is v. If the temperature is doubled and the hydrogen gas dissociates into atomic hydrogen, the r.m.s speed will become-

Given: T is doubled and M is halved

To find: v_{rms} after dissociation

$$v_{rms} = \sqrt{\frac{3RT}{M}} = v$$

$$T_2 = 2T_1 \text{ and } M_2 = \frac{M_1}{2} \Rightarrow \qquad v_{rms} = \sqrt{\frac{3R(2T)}{\left(\frac{M}{2}\right)}} = 2\sqrt{\frac{3RT}{M}}$$

16

$$v_{rms} = 2v$$







Ratio of v_{mp} , v_{avg} and v_{rms}

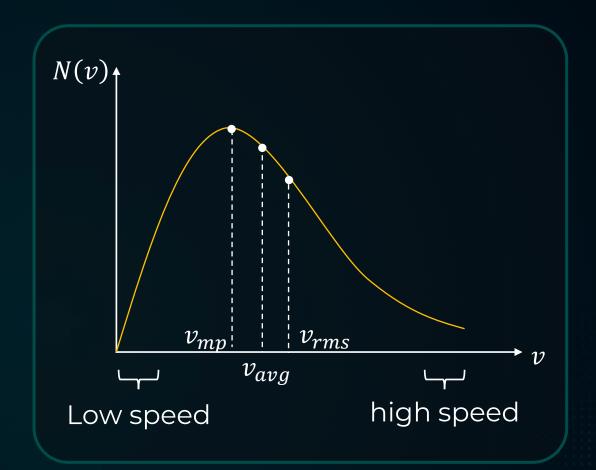


$$v_{mp}: v_{avg}: v_{rms}$$

$$\sqrt{\frac{2RT}{M_{molar}}}: \sqrt{\frac{8RT}{\pi M_{molar}}}: \sqrt{\frac{3RT}{M_{molar}}}$$

$$\sqrt{2}: \qquad \sqrt{\frac{8}{\pi}} \qquad : \sqrt{3}$$

$$v_{mp} < v_{avg} < v_{rms}$$

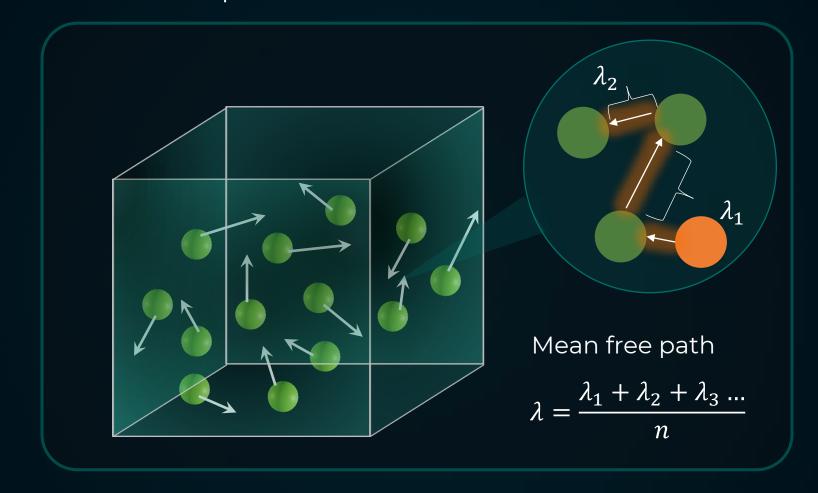




Free Path



 The distance that a moving particle travels between successive collisions or impacts is called the free path.



Mean Free Path



 Mean free path refers to the average distance that a moving particle travels between successive collisions or impacts

$$\lambda \propto \frac{1}{\rho} \Rightarrow \lambda \propto \frac{1}{\frac{n}{V}}$$

$$\lambda \propto \frac{1}{\text{size of the molecule}} \Rightarrow \lambda \propto \frac{1}{d^2}$$

$$\rho$$
 – density of the gas and

$$n/V$$
 – number of molecules per unit volume

$$d$$
 – diameter of the gas molecule

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 \left(\frac{n}{V}\right)}$$



At what temperature will rms speed of oxygen molecules become just sufficient for escaping from the Earth's atmosphere?

$$(m_{O_2} = 2.76 \times 10^{-26} \, kg, k_B = 1.38 \times 10^{-23} \, J \, K^{-1}, \text{ escape velocity} = 11.2 \, km/s)$$

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Given:
$$m_{O_2} = 2.76 \times 10^{-26} \ kg$$
, $k_B = 1.38 \times 10^{-23} \ J \ K^{-1}$, escape velocity = 11.2 km/s

To find: 7

Solution:
$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

$$11.2 \times 10^{3} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times T}{2.76 \times 10^{-26}}} \Rightarrow \qquad T = 8.363 \times 10^{4} \, K$$





Kinetic Energy



Average Kinetic energy of the gas molecules -

$$< KE > = \frac{1}{2}m < v^2 >$$
 where m -mass of each molecule

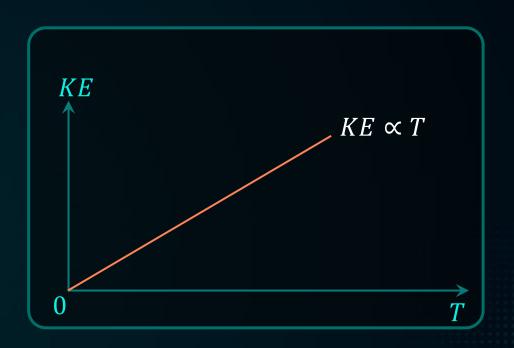
$$\langle KE \rangle = \frac{1}{2} m v_{rms}^2$$

$$= \frac{1}{2}m\frac{3RT}{M_{molar}}$$
 $\left(v_{rms} = \sqrt{\frac{3RT}{M_{molar}}}\right)$

$$\langle KE \rangle = \frac{3}{2}nRT$$
 n -number of moles $=\frac{m}{M_{molar}}$

Average Kinetic energy of the gas molecules is directly proportional to the temperature

From Ideal Gas Equation-
$$\langle KE \rangle = \frac{3}{2}PV$$







Estimate the average thermal energy of one mole of helium atoms at room temperature (27 $^{\circ}$ C).

Given:
$$n = 1, T = 27 \, ^{\circ}\text{C} = 300 \, K$$

To find: KE

Solution:
$$KE = \frac{3}{2}nRT$$

$$KE = \frac{3}{2} \times 1 \times 8.3 \times (300)$$

$$KE = 3.735 \, kJ$$



Pressure of an Ideal Gas

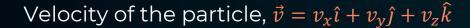


Assumptions:

- 1. All the particles are moving with different velocities.
- 2. Number of particles moving in a direction remains same with time.

$$P = \frac{F}{A}$$
 and $F = \frac{dp}{dt}$

For
$$t = 1 s$$
, $F_{average} = \Delta p$



The particles have velocity components in all three directions,

$$\Rightarrow \sum v_i^2 = \sum v_{x_i}^2 + \sum v_{y_i}^2 + \sum v_{z_i}^2$$

For a gas consisting of large number of particles,

$$v_z$$
 v_y
 v_y

$$\sum v_{xi}^2 = \sum v_{y_i}^2 = \sum v_{z_i}^2 = \frac{\sum v_i^2}{3}$$



Pressure of an Ideal Gas



Change in momentum of one particle in one collision with wall:

$$\Delta P = 2mv_{\chi}$$

Time between two successive collisions with the wall:

$$t = \frac{2l}{v_x}$$

Change in momentum of one particle in 1 s:

$$\Delta P = \frac{mv_x^2}{l}$$

Total change in momentum in 1 s:

$$(\Delta P)_{1S} = \frac{m}{l} \left(v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 + \dots + v_{x_N}^2 \right) = \frac{m}{l} \sum v_{x_i}^2$$

$$(\Delta P)_{1s} = \frac{m}{l} \times \frac{\sum v_i^2}{3} = F_{average}$$

Pressure of gas,
$$P = \frac{F_{average}}{l^2}$$

$$\Rightarrow P = \frac{m}{3l} \frac{\sum v_i^2}{l^2} \Rightarrow P = \frac{mN}{3V} \times v_{rms}^2$$

$$\Rightarrow P = \frac{1}{3}\rho v_{rms}^2$$

$$v_{z}$$
 v_{y}
 v_{y}
 v_{z}
 v_{z}



Relation between P and $\overline{v_{rms}}$



$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_0}}$$

 \Rightarrow If pressure increases isothermally then ρ also increases,

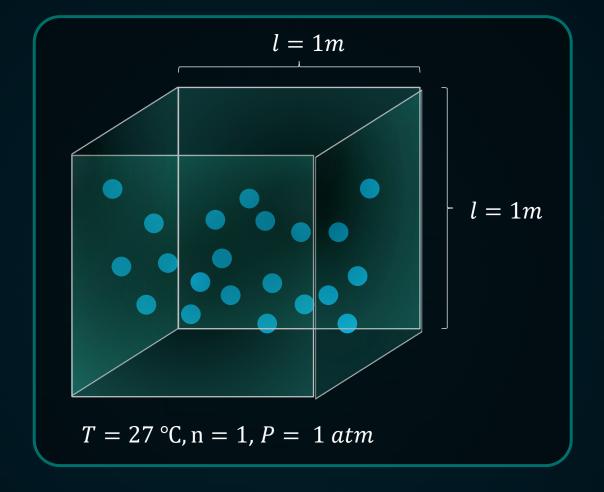
so v_{rms} remains constant. (: $\frac{P}{\rho}$ =constant)





1 mole of oxygen at 27 °C and 1 atm is enclosed in a vessel. Assuming the molecules to be moving with v_{rms} , find the number of collisions per second which the molecules make with 1 m^2 area of the vessel wall.

$$[k = 1.38 \times 10^{-23} \, J \, K^{-1} \text{ and } N_A = 6.02 \times 10^{23} \, mol^{-1}].$$









Given: T = 27 °C, n = 1, P = 1 atm, $k = 1.38 \times 10^{-23}$ J K^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1} .

To Find: Number of collisions per second with $1 m^2$ area

Solution: Mass of one oxygen molecule, $m = \frac{M}{N_A} = \frac{32}{6.02 \times 10^{23}}$ $g = 5.316 \times 10^{-26}$ kg

So,
$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{5.316 \times 10^{-26}}} \approx 483.36 \ m \ s^{-1}$$

Change in momentum per collision, $\Delta p = 2mv_{rms} \Rightarrow \Delta p = 2 \times 5.316 \times 10^{-26} \times 483.35$ $\Rightarrow \Delta p = 5.14 \times 10^{-23} \ kg \ m \ s^{-1}$

For
$$n$$
 particles striking per second $\Rightarrow F = \frac{dp}{dt} = n\Delta p \Rightarrow 1.01 \times 10^5 \times 1 = n \times 5.14 \times 10^{-23}$
 $\Rightarrow n = 1.965 \times 10^{27} \text{ collisions/s}$



Degree of Freedom



Degree of freedom of a gas molecule refers to the number of ways in which the molecule in the gas phase can keep energy. It is denoted by f.

$$f = 3A - B$$

A – number of molecules and B – number of restrictions



• Monatomic gases are composed of single atoms. Example: He, Ne, Ar, Kr



• Diatomic gases are made of molecules that are composed of two atoms. Example: H_2 , N_2 , F_2 , O_2

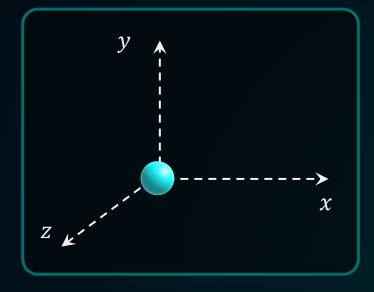


Triatomic gases are made of molecules that are composed of three atoms. It may be linear or Non-linear in structure. Example: \mathcal{CO}_2 , $\mathcal{H}_2\mathcal{O}$, \mathcal{O}_3 , \mathcal{NO}_2





Monatomic Gas

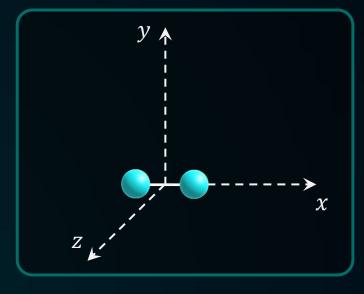


$$f = 3A - B = 3(1) - 0$$

$$f = 3$$

Three translational kinetic energy along three cartesian axes, namely x, y, and z.

Diatomic Gas



$$f = 3A - B = 3(2) - 1$$

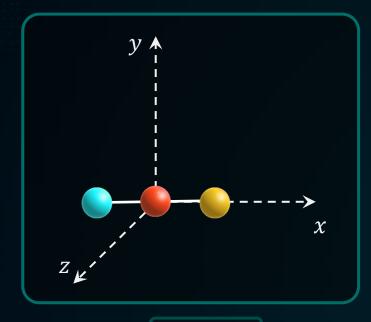
$$f = 5$$

Three translational degrees of freedom and two rotational degrees of freedom.

Degree of Freedom for Triatomic Gases



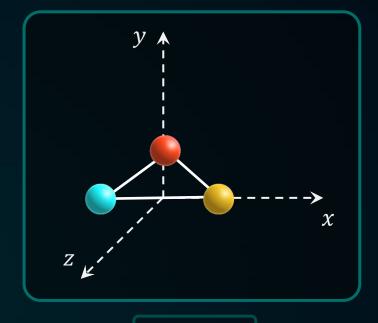
Linear Triatomic Gas



$$f = 5$$

It will also have three translational and two rotational degrees of freedom.

Non Linear Triatomic Gas

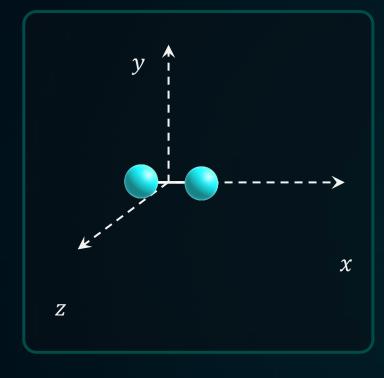


$$f = 6$$

It will have three translational and three rotational degrees of freedom.

Maxwell's Law for Equipartition of Energy

The total energy of a gas is distributed uniformly along all the degrees of freedom. If one mole of atom contain E energy, then one atom have $\frac{E}{N_A}$ energy



Total Energy E

$$K.E_{x} = \frac{E}{5} = \frac{1}{2}mv_{x}^{2}$$

$$K.E_y = \frac{E}{5} = \frac{1}{2}mv_y^2$$

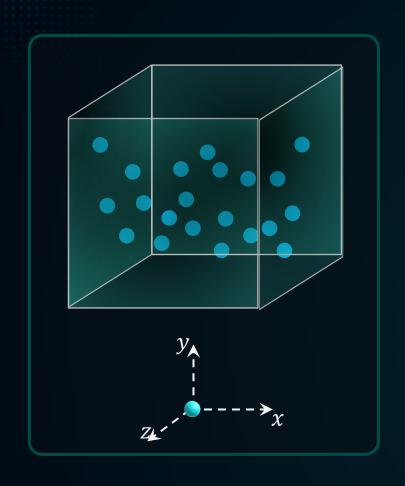
$$K.E_z = \frac{E}{5} = \frac{1}{2}mv_z^2$$

$$R.K.E_y = \frac{E}{5} = \frac{1}{2}I\omega_y^2$$

$$R.K.E_z = \frac{E}{5} = \frac{1}{2}I\omega_z^2$$

Maxwell's Law for Equipartition of Energy





For monatomic gas,

$$K.E = \frac{3}{2}nRT$$

$$T.E = K.E + P.E \Rightarrow T.E = K.E = \frac{3}{2}nRT \qquad (P.E = 0)$$

Energy of one particle
$$=\frac{3}{2}\frac{R}{N_A}T=\frac{3}{2}kT$$

$$f = 3 \Rightarrow \frac{1}{2}kT$$
 in along all axis x, y and z .

Vibrational Degree of Freedom



- Vibrational degree of freedom (D.O.F) is only observed at very high temperature.
- Each vibrational frequency has two modes of energy (kinetic and potential) with corresponding energy equal to, $2 \times \frac{1}{2}kT = kT$.

Types of Gas

Monatomic

Diatomic

Triatomic (Linear)

Triatomic (Non-linear)

D. O. F
$$(T + R + V)$$
.

$$3 + 0 + 0 = 3$$

$$3 + 2 + 2 = 7$$

$$3 + 2 + 2 = 7$$

$$3 + 3 + 2 = 8$$



Specific Heat Capacity



	Monoatomic ($f = 3$)	Diaatomic ($f = 5$) *Rigid	Diatomic (f = 5 + 1) *not rigid, considering vibrational mode
Total Energy of one particle	$U = \frac{3}{2}kT$	$U = \frac{5}{2}kT$	$U = \frac{5}{2}kT + kT = \frac{7}{2}kT$
<i>U</i> for one mole	$U = \frac{3}{2}kT \times N_A = \frac{3}{2}RT$	$U = \frac{5}{2}RT$	$U = \frac{7}{2}RT$
Molar Specific heat at constant volume	$C_V = \frac{dU}{dT} = \frac{3}{2}R$	$C_V = \frac{5}{2}R$	$C_V = \frac{7}{2}R$
Molar Specific heat at constant pressure	$C_P - C_V = R \Rightarrow C_P = \frac{5}{2}R$	$C_P = \frac{7}{2}R$	$C_P = \frac{9}{2}R$
Ratio of \mathcal{C}_P and \mathcal{C}_V	$\gamma = \frac{C_P}{C_V} = \frac{5}{3}$	$\gamma = \frac{C_P}{C_V} = \frac{7}{5}$	$\gamma = \frac{C_P}{C_V} = \frac{9}{7}$



A gas mixture consists of 2 moles of O_2 and 4 moles of Ar at temperature T. Neglecting all the vibrational modes, what is the total internal energy of the system?



NEET 2017

Given: $n_{O_2} = 2$ and $n_{Ar} = 4$

To find: *U*

Solution: For a mole of monoatomic gas $U = \frac{3}{2}RT$

For a mole of diatomic gas
$$U = \frac{5}{2}RT$$

Total internal energy,
$$U_{total} = \left(2 \times \frac{5}{2}RT\right) + \left(4 \times \frac{3}{2}RT\right)$$

$$U_{total} = 11RT$$





One mole of an ideal diatomic gas undergoes a transition from A to B along a path AB as shown in the figure. The change in internal energy of the gas during the transition is-

Given:
$$P_i = 5kPa$$
, $P_f = 2kPa$, $V_i = 4m^3$ and $V_f = 6m^3$

To find: ΔU

Solution: For a mole of diatomic gas

$$U = \frac{5}{2}RT = \frac{5}{2}PV$$



$$U_f = \frac{5}{2} \times 2 \times 10^3 \times 6 = 30kJ$$

$$\Delta U = U_f - U_i = (30 - 50)kJ \quad \Rightarrow \Delta U = -20kJ$$



-20kJ



 $\begin{bmatrix} \mathsf{C} \end{bmatrix}$ -12kJ





Specific Heat Capacity



In general a polyatomic molecule has 3 translational, 3 rotational degrees of freedom and a certain number z of vibrational modes.

Total Energy of one particle,
$$U = \frac{3}{2}kT + \frac{3}{2}kT + zkT$$

For one mole, U = (3 + z)RT

Molar Specific heat at constant volume, $C_V = (3 + z)R$

Molar Specific heat at constant pressure, $C_P = (4 + z)R$

$$\gamma = \frac{C_P}{C_V} = \frac{4+z}{3+z}$$





If for a gas at , $\frac{R}{c_V} = 0.67$, what is the nature of the gas molecules? Assume that the vibrational modes can be ignored.

NEET 1992

Given:
$$\frac{R}{C_V} = 0.67$$

To find: nature of the gas molecules

Solution:
$$\frac{R}{C_V} = 0.67 \Rightarrow C_V = \frac{R}{0.67} = 1.49R$$

Monoatomic- Diatomic- Triatomic-

Polyatomic-

$$C_V = \frac{3}{2}R = 1.5R$$

$$C_V = \frac{3}{2}R = 1.5R$$
 $C_V = \frac{5}{2}R = 2.5R$ $C_V = \frac{7}{2}R = 3.5R$ $C_V = 3R$

$$C_V = \frac{7}{2}R = 3.5R$$

$$C_V = 3R$$

: the gas is monoatomic