Assignment #1 Date: 27/02/2022

Problem 1 :-

a) $\frac{\partial^3 u}{\partial x^3} + \frac{\partial^2 u}{\partial y^2} + \left(\frac{\partial u}{\partial z}\right)^2 + ux^3 + uy^2 + vz = 0$

Sol:-
This equation is a partial D.E, the order of this D.E is 3rd and it is of 2ND degree. The equation is non-linear.

b) $\left(\frac{dy}{dx}\right)^2 = \left(\frac{d^2y}{dx^2} + y\right)^{3/2}$

Sol:-
This equation is an ordinary D.E, the order of this D.E is 2ND and it is of 4th degree, it is non-linear.

Problem 2 :-

a) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0$; $y = 0 = -2$, $y'(0) = 6$

where $y = C_1 e^{4x} + C_2 e^{-3x}$ is the general sol of given D.E.

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(A)

a) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0$; $y(0) = -2$, $y'(0) = 6$

Sol:-

$$y = c_1e^{4x} + c_2e^{-3x} \quad -(1)$$

$$y' = 4c_1e^{4x} - 3c_2e^{-3x} \quad -(2)$$

$$y'' = 16c_1e^{4x} + 9c_2e^{-3x} \quad -(3)$$

Verify:-

Place values (1) (2) (3) in (A)

$\Rightarrow 16c_1e^{4x} + 9c_2e^{-3x} - (4c_1e^{4x} - 3c_2e^{-3x}) - 12(c_1e^{4x} + c_2e^{-3x})$

$\Rightarrow \cancel{16c_1e^{4x}} + \cancel{9c_2e^{-3x}} + (\cancel{-4c_1e^{4x}} + \cancel{3c_2e^{-3x}}) \cancel{-12c_1e^{4x}} \cancel{-12c_2e^{-3x}}$

$\Rightarrow \boxed{0 = 0}$ Ans

b) $x^3\frac{d^3y}{dx^3} - 3x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} - 6y = 0$;

$y(2) = 0$; $y(2) = 0$; $y'(2) = 2$, $y''(2) = 6$

where $y = c_1x + c_2x^2 + c_3x^3$ is general equation.

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Sol:-

$$\cancel{y = c_1x + c_2x^2 + c_3x^3 \quad -(1)}$$
$$\cancel{y' = c_1 + 2c_2x + 3c_3x^2 \quad -(2)}$$
$$\cancel{y'' = 0 + 2c_2 + 6c_3x \quad -(3)}$$
$$\cancel{y''' = 6c_3}$$

To find c_1 & c_2

$y(0) = -2$

$$y = c_1e^{4x} + c_2e^{-3x}$$
$$-2 = c_1e^{4(0)} + c_2e^{-3(0)}$$
$$-2 = c_1 + c_2$$
$$\boxed{c_1 = -2 - c_2} \rightarrow (a)$$

For c_2

$$y' = 4c_1e^{4x} - 3c_2e^{-3x} \qquad \because y'(0) = 6$$
$$6 = 4c_1e^{0} - 3c_2e^{0}$$
$$\boxed{6 = 4c_1 - 3c_2} \rightarrow (b) \quad \text{put value of } c_1 \text{ in } b$$
$$6 = 4(-2 - c_2) - 3c_2$$
$$6 = -8 - 4c_2 - 3c_2$$
$$6 + 8 = -7c_2$$
$$-\frac{\cancel{14}^{2}}{\cancel{7}_{1}} = c_2 \Rightarrow \boxed{c_2 = -2}$$

$$c_1 = -2 + 2$$
$$c_1 = 0$$

$$y = 0 + (-2)e^{-3x}$$
$$\boxed{y = -2e^{-3x}}$$

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b)

Sol:-

$y = c_1x + c_2x^2 + c_3x^3$ — ①

$y' = c_1 + 2c_2x + 3c_3x^2$ — ②

$y'' = 2c_2 + 6c_3x$ — ③

$y''' = 6c_3 \rightarrow$ ④

To verify:-

$\Rightarrow x^3(6c_3) - 3x^2(2c_2 + 6c_3x) + 6x(c_1 + 2c_2x + 3c_3x^2) - 6(c_1x + c_2x^2 + c_3x^3) = 0$

$\Rightarrow \cancel{6x^3c_3} - \cancel{6x^2c_2} - \cancel{18x^3c_3} + \cancel{6xc_1} + \cancel{12c_2x^2} + \cancel{18c_3x^3} - \cancel{6c_1x} - \cancel{6c_2x^2} - \cancel{6c_3x^3} = 0$

$\boxed{0 = 0 \text{ verified}}$

$y(2) = 0$:-

$0 = c_1 2 + c_2(2)^2 + c_3(2)^3$

$0 = 2c_1 + 4c_2 + 8c_3$

$c_1 = \frac{-4c_2 - 8c_3}{2}$

$c_1 = \frac{-\cancel{4}^{2}c_2}{\cancel{2}} - \frac{\cancel{8}^{4}c_3}{\cancel{2}}$

$\boxed{c_1 = -2c_2 - 4c_3}$ — ⑤

$y'(2) = 2$ –

$2 = c_1 + 2c_2(2) + 3c_3(2)^2$

$2 = c_1 + 4c_2 + 3c_3 4$

$2 = c_1 + 4c_2 + 12c_3$

$2 = (-2c_2 - 4c_3) + 4c_2 + 12c_3$

$2 = -2c_2 - 4c_3 + 4c_2 + 12c_3$

$2 = 2c_2 + 8c_3$

$2c_2 = -2 + 8c_3$

$c_2 = \frac{\cancel{+2}(-1 + 4c_3)}{\cancel{2}}$

$\boxed{c_2 = -1 + 4c_3}$ – b

$y''(2) = 6$ –

$6 = 2c_2 + 6c_3(2)$

$6 = 2c_2 + 12c_3$

$6 = 2(-1 + 4c_3) + 12c_3$

$6 = -2 + 8c_3 + 12c_3$

$6 + 2 = 8c_3 + 12c_3$

${}_2\cancel{8} = \cancel{4}(2c_3 + 3c_3)$

$2 = 5c_3$

$\boxed{c_3 = \frac{5}{2}}$, $c_2 = -1 + 4\left(\frac{5}{2}\right)$, $c_1 = -2(9) - 4\left(\frac{5}{2}\right)$

$\boxed{c_2 = 9}$, $\boxed{c_1 = -28}$

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$$y = -28(x) + 9x^2 + \frac{5}{2}x^3 \quad \text{Ans}$$

$\rightarrow \frac{d^2y}{dx^2} + y = 0$; $y(0) = 1$, $y'\left(\frac{\pi}{2}\right) = -1$

Sol:-

$y = c_1 \sin x + c_2 \cos x$

$y' = c_1 \cos x + (-c_2 \sin x)$

$\cancel{c_1 \cos x - \sin x\, c_2 + c_1 \sin x + c_2 \cos x}$

$y'' = -c_1 \sin x - c_2 \cos x$

$-c_1 \sin x \cancel{- c_2 \cos x} \cancel{+ c_1 \sin x} \cancel{+ c_2 \cos x} = 0$

$\boxed{0 = 0}$ verified

$y(0) = 1$;

$y = c_1 \sin x + c_2 \cos x$

$1 = c_1 \sin 0 + c_2 \cos(0)$

$1 = 0 + c_2$

$c_2 = 1$

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$y'\left(\frac{\pi}{2}\right) = -1$:-

$-1 = c_1 \cos\left(\frac{\pi}{2}\right) - c_2\left(\sin\frac{\pi}{2}\right)$

$-1 =$

Q.3,

a,

for $\frac{x\,dy}{dx} + (1 + x\cot x)y = x$

Domain $= 0, \pm\pi, \pm 2\pi$

Interval $= (0, \pi)$

for $y = -\cot x + \frac{1}{x} + \frac{c_1}{x}\operatorname{cosec} x$

Domain $= 0, \pm\pi, \pm 2\pi.$

interval $= (0, \pi)$

b,

for $(x^2+1)\frac{dy}{dx} + 2xy = 4x^2$

Domain $= -\alpha \;\; +\alpha$

interval $= (-\alpha, +\alpha)$

for $3y = \frac{4x^3}{(x^2+1)} + \frac{3c}{(x^2+1)}$

interval $= (-\alpha, +\alpha)$

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Q. $(xy+2x+y+2)dx+(x^2+2x)dy=0$

Sol:-

$x(y+2)+1(y+2)\,dx+(x^2+2x)dy=0$

$\{(1+x)(y+2)\}dx+x(x+2)dy=0$

divide b.s by $x(x+2)(y+2)$

$$\frac{(x+1)\cancel{(y+2)}}{x(x+2)\cancel{(y+2)}}dx+\frac{\cancel{x(x+2)}}{\cancel{x(x+2)}(y+2)}dy=0$$

$$\frac{(x+1)}{x(x+2)}dx+\frac{1}{(y+2)}dy=0$$

$$\frac{dy}{(y+2)}=-\left(\frac{x-1}{x(x+2)}\right)dx$$

$\int$grate on b.s

$$\frac{dy}{y+2}=\frac{-1/2}{x}-\frac{1/2}{x+2}$$

$$\int\frac{dy}{y+2}=-1/2\int\frac{1}{x}-\frac{1}{2}\int\frac{1}{x+2}$$

$\ln|y+2|=-1/2\ln|x|-\frac{1}{2}\ln|x+2|+c_1$

$\ln|y+2|=\frac{1}{2}\ln(x(x+2))^{2}-\frac{1}{2}\ln c_1$

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$$e^{\ln|y+2|} = e^{\ln(x(x+2))^{-1/2}}$$

$$y+2 = e^{c_1} \cdot (x(x+2))^{-1/2}$$

$$c = e^{c_1}$$

$$\boxed{y+2 = \frac{c}{\sqrt{x(x+2)}}}$$

$$\boxed{y = \frac{c}{\sqrt{x(x+2)}} - 2}$$

$$\frac{x-1}{x(x+2)} = \frac{A}{x} + \frac{B}{(x+2)}$$

$$x-1 = A(x+2) + Bx$$

$$\boxed{A = 1/2} \qquad \boxed{B = 1/2}$$

$$\frac{1/2}{x} + \frac{1/2}{x+2}$$