FAST- National University of Computer and Emerging Sciences, Karachi.

FAST School of Computing

Quiz-I, Fall 2021 -- Solution

07th October 2021

Course Code: CS1005	Course Name: Discrete Structures
Instructors: Mr. Shoaib Raza	
Student Roll No:	Section:

Time Allowed: 50 minutes. Maximum Points: 30 points

NOTE: Each question carries equal points. In order to get maximum marks, step-by-step solutions are required.

Question #1:

Let p, q, r and s be the propositions.

p: Ali works hard. q: Ali is a dull boy. r: Ali will get the job. s: Ali is ambitious.

Write these propositions using *p, q, r and s* and logical connectives (including negations):

a) Ali works hard and he is ambitious. Solution: $p \land s$

b) Ali is a dull boy if he works hard. Solution: $p \to q$

c) Ali is a dull boy only if he does not get the job. Solution: $q \to \neg r$

Question #2:

Using the premises(statements) from Question #1, apply rules of inference to obtain conclusion from those premises.

Solution:

Now we can write the premises as, $(p \land s) \land (p \to q) \land (q \to \neg r)$

$\equiv \underline{(p \land q)} \land (p \to q) \land (q \to \neg r)$ Simplification

$\equiv \underline{p \land (p \to q)} \land (q \to \neg r)$ Modus Ponen

$\equiv \underline{q \land (q \to \neg r)}$ Modus Ponen

$\equiv \neg r$ Hence, the conclusion is "Ali will not get the job."

Question #3:

Prove or disprove the following logical equivalence using the laws of logic: $p \leftrightarrow q \cong (p \land q) \lor (\neg p \land \neg q)$

Solution:

$p \leftrightarrow q$

$\equiv (p \to q) \land (q \to p)$ *Definition of bi-implication*

$\equiv (\neg p \lor q) \land (\neg q \lor p)$ *Definition of implication*

$\equiv [(\neg p \lor q) \land \neg q] \lor [(\neg p \lor q) \land p]$ *Distributive*

$\equiv [(\neg p \land \neg q) \lor (q \land \neg q)] \lor [(\neg p \land p) \lor (q \land p)]$ *Distributive*

$\equiv [(\neg p \land \neg q) \lor F] \lor [F \lor (q \land p)]$ *Negation*

$\equiv (\neg p \land \neg q) \lor (q \land p)$ *Identity*

$\equiv (\neg p \land \neg q) \lor (p \land q)$ *Commutative*

$\equiv (p \land q) \lor (\neg p \land \neg q)$ *Commutative*

Question #4:

Use truth table to prove that the given statement $((p \to q) \land (q \to r)) \to (p \to r)$ is a Tautology OR Contradiction.

Solution: It's a tautology.

p	q	r	$(p \to q)$	$(q \to r)$	$(p \to q) \land (q \to r)$	$(p \to r)$	$((p \to q) \land (q \to r)) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Question #5:

(a) Translate the statement into English, where the domain for each variable consists of all real numbers. $\exists x \forall y\ (xy = y)$

Solution: There exists a real number x such that for every real number y, xy = y.

(b) Use quantifiers to express the statements. "For every real numbers x, y, there exist a real number z such that x = y + z."

Solution: $\forall x \forall y \exists z\ (x = y + z)$

Question #6:

Let $f(p, q)$ means "$p + q = 0$", where p and q are integers. Determine the truth value of the statement.

(a) $\exists q \forall p\ \ f(p, q)$

Solution: False Let q=1 $p + q = -1+1=0$ but $0 + 1=1 \neq 0$ and $1+1 =2 \neq 0$

(b) $\forall q \exists p\ f(p, q)$

Solution: True Let q=1 $p + q = -1+1=0$ or $0 + 0 = 0$ and $2 + -2 = 0$

Question #7:

Use set-builder notation and logical equivalences to establish the given expression. $(X - Y) \cup (X \cap Y) = X$

$\equiv \{x \mid ((x \in X) \wedge (x \notin Y)) \vee ((x \in X) \wedge (x \in Y))\}$

$\equiv \{x \mid (x \in X) \wedge ((x \notin Y) \vee (x \in Y))\}$ Distributive Law

$\equiv \{x \mid (x \in X) \wedge (x \in U)\}$ Complement or Negation Law

$\equiv \{x \mid (x \in X)\} \equiv$ R.H.S

Question #8:

In a class of 100 students, 35 like science and 45 like math. 10 like both. How many like either of them and how many like neither? Also draw Venn diagram.

Solution:

Total number of students, $n(\mu) = 100$

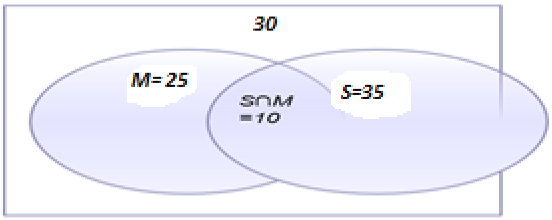
Number of science students, $n(S) = 35$

Number of math students, $n(M) = 45$

Number of students who like both, $n(M \cap S) = 10$

Number of students who like either of them,

$n(M \cup S) = n(M) + n(S) - n(M \cap S) = 45+35-10 = 70$



Question #9:

Determine whether the function from $\mathbf{R}$ to $\mathbf{Z}$ is Injective OR Surjective. $f(n) = \lceil \frac{n}{2} \rceil$

Solution:

It is Surjective (onto function). This can be shown by an example; $f(1) = 1$, and $f(2) = 1$.

Question #10:

Let f be the function from $\{w, x, y, z\}$ to $\{1,2,3,4\}$ such that $f(w) = 2$, $f(x) = 3$, $f(y) = 4$ and $f(z) = 1$. Is f invertible and if so, what is its inverse?

Solution:

The function f is invertible because it is a one-to-one correspondence. The inverse function f^{-1} reverses the correspondence given by f, so $f^{-1}(1) = z$, $f^{-1}(2) = w$, $f^{-1}(4) = y$ and $f^{-1}(3) = x$.

BEST OF LUCK ☺