Copying answers and steps are strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. The calculations and answers should be written neatly on paper which is attached as a single pdf. Box your answers where appropriate. Thanks!

Problem 1

Solve the following `homogeneous linear differential equations with constant coefficient`.

(a) $(D^4 + 6D^3 + 15D^2 + 20D + 12)y = 0$

(b) $(D^3 - 27)y = 0$

Problem 2

Solve the differential equations by method of `Undetermine Coefficient-Superposition Approach`.

(a) $(D^2 - 7D + 12)y = e^{2x}(x^3 - 5x^2)$

(b) $y'' + y' - 2y = x^2 + 2\sin x - e^{3x}$

Problem 3

Solve differential using `Variation of Perimeter`.

(a) $(D^2 + 1)y = \csc x$

(b) $(D^2 - 1)y = \dfrac{2}{1 + e^x}$

Problem 4

Solve the differential equations by `method of Undetermine Coefficient-Annihilator Approach`.

(a) $y'' + y' + \dfrac{1}{4}y = e^x(\sin 3x + \cos 3x)$

(b) $y'' + 2y' + y = x^2e^{-x}$

Problem 5

Solve `Cauchy Euler equation`.

$x^2y'' + xy' - y = x^3e^x$

Bonus Problem 6

(a) Determine whether the the given set of fucntion is `linearly indepent or lineearly dependent` on interval $(-\infty, \infty)$.

i. $y_1 = \cos 2x, y_2 = 1, y_3 = \cos^2 x$

ii. $y_1 = x, y_2 = x^{-2}, y_3 = x^2 \ln x$

(b) Solve differential equation using `reduction of order` (first by substitution then again by formulation $y_2 = y_1 \int \dfrac{e^{-\int P(x)dx}}{y_1^2}dx$).

i. $9y'' - 12y' + 4y = 0, y_1 = e^{2x/3}$

ii. $y'' - 3y' + 2y = 5e^{3x}, y_1 = e^x$