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A survey of formal methods for determining functional joint axes

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Abstract

Axes of rotation e.g. at the knee, are often generated from clinical gait analysis data to be used in the assessment of kinematic abnormalities, the diagnosis of disease, or the ongoing monitoring of a patient's condition. They are additionally used in musculoskeletal models to aid in the description of joint and segment kinematics for patient specific analyses. Currently available methods to describe joint axes from segment marker positions share the problem that when one segment is transformed into the coordinate system of another, artefacts associated with motion of the markers relative to the bone can become magnified. In an attempt to address this problem, a symmetrical axis of rotation approach (SARA) is presented here to determine a unique axis of rotation that can consider the movement of two dynamic body segments simultaneously, and then compared its performance in a survey against a number of previously proposed techniques.

Using a generated virtual joint, with superimposed marker error conditions to represent skin movement artefacts, fitting methods (geometric axis fit, cylinder axis fit, algebraic axis fit) and transformation techniques (axis transformation technique, mean helical axis, Schwartz approach) were classified and compared with the SARA. Nearly all approaches were able to estimate the axis of rotation to within an RMS error of 0.1 cm at large ranges of motion (90°). Although the geometric axis fit produced the least RMS error of approximately 1.2 cm at lower ranges of motion (5°) with a stationary axis, the SARA and Axis Transformation Technique outperformed all other approaches under the most demanding marker artefact conditions for all ranges of motion. The cylinder and algebraic axis fit approaches were unable to compute competitive AoR estimates. Whilst these initial results using the SARA are promising and are fast enough to be determined "on-line", the technique must now be proven in a clinical environment.

Keywords: Axis of rotation

1. Introduction

Clinical gait analysis is capable of assisting in the assessment and diagnosis of kinematic irregularities that may be unobservable even to a skilled clinician (Andriacchi et al., 1998; Cappozzo et al., 2005). Musculoskeletal analyses (Heller et al., 2001; Stansfield et al., 2003) can determine the internal loading conditions during functional load bearing, using the kinematics delivered from such non-invasive clinical gait analyses (Reinbolt et al., 2005), and can therefore aid in assessing treatment options. The ability to perform patient specific analyses, however, is limited by the accuracy of reconstructing the joint kinematics (Lu and O'Connor, 1999), a process that often requires determining an accurate axis of rotation (AoR). Furthermore, some joints such as e.g. the knee, have a much more complex pattern of motion and require a more detailed description than can be provided by joint centres alone and determining an AoR can provide a time-dependent anatomical reference to rotational motion. Moreover, in the orthopaedic and biomechanical literature, the function of the knee has often been characterised by a flexion axis (Churchill et al., 1998; Li et al., 2002; Most et al., 2004; Piazza et al., 2004; Asano et al., 2005) and hence procedures that are able to identify these axes are required.

The non-invasive determination of body segment motion is usually performed by directly measuring reflective marker positions using infra-red optical measurement

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systems. From this data, there are two main mathematical strategies for the determination of joint axes. The first attempts to fit cylindrical arcs to the orbits of the moving segment markers, where the other segment is assumed to be at rest (Halvorsen et al., 1999; Gamage and Lasenby, 2002). The second general approach considers the distance between markers on each joint segment and the AoR, which enables the definition of local coordinate systems. A transformation of these local systems for all time frames into a common reference system should place the joint axis at a fixed position. Such techniques are here designated as ‘‘transformation techniques’’. Within this general category, helical axes have been widely used (e.g. Kelkar et al., 2001) based on the work of Woltring and co-workers (1985). This approach has often been used for the estimation of time-dependent joint parameters (finite or instantaneous helical axes). Only in recent years have studies considered formal mathematical approaches for the estimation of joint axes from marker position measurements (Halvorsen et al., 1999; Gamage and Lasenby, 2002; Schwartz and Rozumalski, 2005), but very few comparisons between these methods have been performed, and no stringent classification of AoR approaches or comparison of the accuracy of the algorithms under different conditions is yet available.

During measurement of marker positions, the relative motion between the markers and bone (from e.g. local shifting or deformation of the skin) have been shown to be main causes of error, or artefact (Leardini et al., 2005; Stagni et al., 2005; Taylor et al., 2005). Current methods for describing joint axes, however, often share the problem that one segment must first be transformed into the coordinate system of the other, in order to use a common coordinate system. Any artefact of the local markers, and therefore any error in the definition of the local coordinate system, is then transformed into the coordinate system of the second, thereby possibly amplifying any inaccuracies in determining the AoR. Clinically, this could mean that the determination of AoRs during gait analysis is potentially subject to preventable errors.

Improvement in the accuracy of methods to determine this AoR could enhance the assessment and monitoring of kinematic abnormalities, as well as improve joint and limb musculoskeletal modelling for assessing patient specific treatment options. In this study, we compare a number of previously proposed techniques for determining the AoR as well as present an approach for determining a unique AoR, capable of considering two dynamic body segments simultaneously using information from both marker sets. In addition, we propose a consistent system of classification of AoR approaches.

2. Methods

2.1. The virtual hinge joint

In order to provide a direct and fair comparison between different methods and their performance under various conditions, a virtual hinge joint was created, for which the marker positions were generated computationally. The joint consisted of two independent segments, each allowed to rotate around a common axis within a defined angular range of motion (RoM), resulting in short circular planar arcs. The movement of each segment was characterised by a set of four markers, assumed to be rigidly attached to each segment. The joint axis was approximately 10 and 15 cm distant from the two segment marker sets. Marker positions were then randomly distributed around the arc and within the defined RoMs to account for each time frame measurement.

In order to study the influence of the marker errors associated with local shifting or deformation of the skin, two different error types were applied to the generated marker positions. Firstly isotropic, independent, and identically distributed Gaussian noise (standard deviation 0.1 cm) was applied to each of the individual marker positions. For the second error condition, Gaussian noise (also standard deviation 0.1 cm) was applied to the marker set collectively, i.e. the same deviations were applied to all markers, in order to simulate the movement of the skin with respect to the underlying bone (Taylor et al., 2005). The two error conditions were also combined.

Two joint movement scenarios were superimposed on the marker positions. In the first, one of the two segments could rotate randomly around the AoR, but within a specified plane and angular RoM, and was affected by the aforementioned noise conditions. In this case, the other segment was held stationary. In the second scenario, one of the two segments could again rotate randomly around the AoR, but now noise conditions were applied to both marker sets. Additionally, the complete joint configuration (AoR and marker sets) was able to randomly translate in space, enabling the simulation of a moving AoR.

A complete description of the approaches from the literature is available in the Supplementary material, in which each technique is classified into fitting approaches (geometric axis fit e.g. Shakarij, 1998, cylinder axis fit, algebraic axis fit, Halvorsen et al., 1999; Gamage and Lasenby, 2002; Cerveri et al., 2005) or transformation techniques (axis transformation technique (ATT), mean helical axis approach based on Woltring et al., 1985 and used by e.g. Stokdijk et al., 1999; Halvorsen, 2002; Besier et al., 2003; Camomilla et al., 2006, Schwartz approach, Schwartz and Rozumalski, 2005). The AoR was then determined using each approach and compared against a further method, the symmetrical axis of rotation approach (SARA), as presented below.

2.2. Symmetrical axis of rotation approach

The SARA is a natural extension of the ATT that is capable of considering the rotational movement of two segments independently, by including a rotation and transformation term for the second segment. The objective function:

$$f_{\mathrm{SARA}}(c_1, c_2) = \sum_{i=1}^{n} \|R_i c_1 + t_i - (S_i c_2 + d_i)\|^2, \tag{1}$$

must therefore be minimised, in which c_1, c_2 are arbitrary points on the joint axes in the local coordinate systems and (R_i,t_i), (S_i,d_i) are the transformations from an appropriate global system into local segment CSs (Fig. 1). This can then be written as the linear least squares problem:

$$\begin{pmatrix} R_1 & -S_1 \\ \vdots & \vdots \\ R_n & -S_n \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} d_1 - t_1 \\ \vdots \\ d_n - t_n \end{pmatrix}. \tag{2}$$

It is then possible to obtain a unique representation of the joint axes in the local CSs of both segments, using the same SVD techniques as described above. The SARA algorithm has the benefit that no preliminary transformation of the marker coordinates of one segment into the CS of the other is required. An equivalent approach has been proposed by O'Brien and co-workers (2000) for the determination of joint parameters from magnetic motion capture data, but has never been compared to other AoR methods.

All methods except the Schwartz and SARA approaches require that the marker positions of one segment are transformed from global into the local coordinates of the second segment before any analysis in which both segments are in motion is performed. The accuracy of this transformation is therefore influenced by any skin marker artefacts.

2.3. Numerical simulation

During numerical simulations, the exact axis position and orientation were known. For an assessment of the performance of the specific methods, a measure of the distance between the exact and the estimated axes is required. Mathematically, the set of possible axes, or lines in three dimensions, builds a four-dimensional manifold, the Grassmann manifold $GL(2,4)$, which can be represented locally by four parameters (Fig. 2). Here, two planes were constructed perpendicular to the exact axis at a distance of 1 cm from the "centre" of the joint, i.e. the intersection of the two segment long axes. Any line not perpendicular to the exact AoR therefore has an intersection point with each of these planes. The coordinates of these intersections allow a measure of distance, $d(c,a;c_{\mathrm{ex}},a_{\mathrm{ex}})$, for all possible axis estimates to be calculated:

$$d(c,a;c_{\mathrm{ex}},a_{\mathrm{ex}}) = \sqrt{\frac{d_1^2 + d_2^2}{2}} \tag{3}$$

for all lines (c,a) not perpendicular to the exact axis $(c_{\mathrm{ex}},a_{\mathrm{ex}})$. For any axis that is exactly parallel to the exact joint axis, such a measure simply gives the distance between the two axes.

In order to perform a fair comparison between the various approaches, all simulations were repeated $n_t = 1000$ times, each with 200 time frames, with different conditions i.e. distribution of the marker positions within a specified RoM and Gaussian noise attributed to each

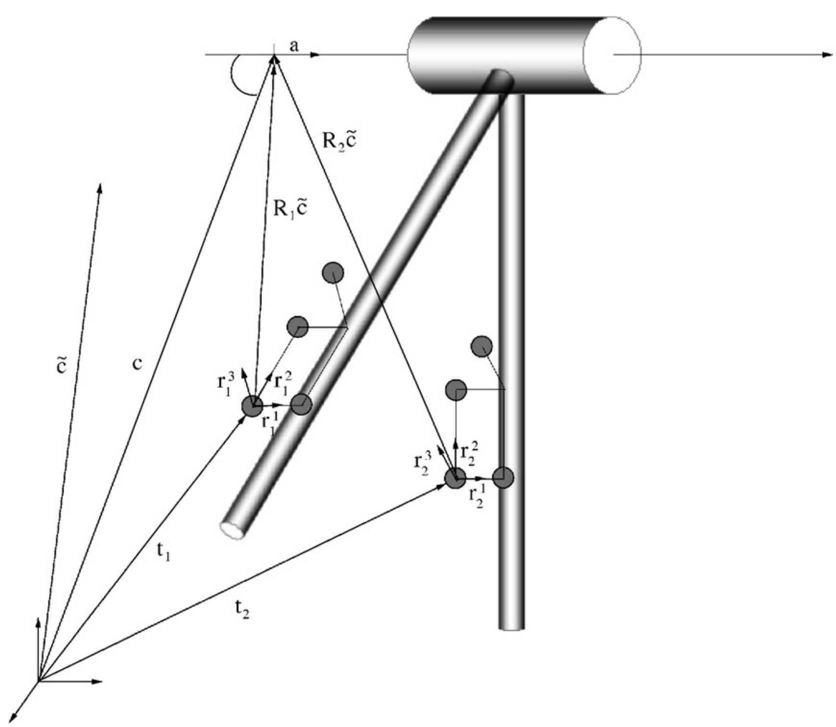

Fig. 1. Representation of the construction of local coordinates on one segment of the virtual joint. The translations t_1 and t_2, together with the rotations $R_i = (r_i^1, r_i^2, r_i^3)$, where the three r_i are the unit basis vectors for constructing the local coordinate systems, transform the point $\tilde{c}$ on the axis from the global coordinate system into the axis point, c, in these local systems.

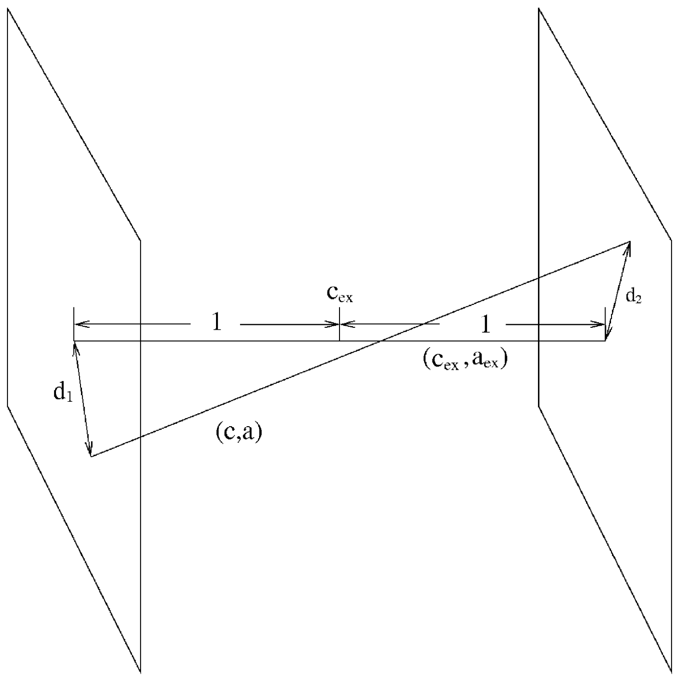

Fig. 2. Illustration of the distance measure between the exact axis defined by the parameters (c_{ex}, a_{ex}) and an approximate axis (c,a). Two planes are set perpendicular to the exact axis at a distance of 1 cm from the "centre" of the joint, i.e. the intersection of the two segment long axes. Any line not exactly perpendicular to the exact axis produces two intersection points on these planes, enabling the definition of the distance measure $d(c, a; c_{ex}, a_{ex})$.

marker. As a measure of the performance of each method, the root mean square (RMS) error

$$\sqrt{\frac{1}{n_t}\sum_{i=1}^{n_t} d(c_i, a_i; c_{ex}, a_{ex})^2} \tag{4}$$

was calculated, where (c_i, a_i) is the AoR estimate of the ith simulation. The AoR was determined for simulated movements of either one or both segments within six different specified RoMs of 5°, 10°, 20°, 30°, 45° and 90°.

3. Results

In general, the RMS error of the calculated axes relative to the known AoR decreased for each approach with increasing RoM under all conditions tested (Figs. 3 and 4). When one segment was fixed, with noise applied independently to each marker of the other (Fig. 3), the geometric axis fit determined the most accurate AoR (maximum RMS error of 1.16 cm), with somewhat better predictions than the remainder of the approaches, particularly at lower ranges of motion. Under these conditions (movement applied only to a single segment), the SARA approach reduces to the same formulation as the ATT and the two methods therefore delivered identical results. Although the SARA, ATT, algebraic fit, mean helical axis and Schwartz approach (using mode averaging) could all determine the axis to within approximately 1 cm at 20° RoM and above, the errors in these approaches rose rapidly below this range. Under larger ranges of motion (at 90°), these approaches were all able to determine the AoR to within an RMS error of 0.1 cm. The cylinder axis fit was far less accurate throughout the tested RoMs.

When noise was applied to the complete marker set of the segment, the SARA, ATT and mean helical axis techniques produced the least errors of 0.2 cm, even for small RoMs (5°). Although they produced smaller errors under these conditions, the geometric axis fit and the Schwartz approach were relatively less accurate compared to these methods when noise was applied to individual markers. The cylinder axis fit and algebraic axis fit techniques produced considerably less accurate AoRs.

When a combination of individual and group marker noise was applied, the effects were almost exactly a summation of the two errors. The accuracy of the AoR under this combination of noise conditions thus tended to be dominated by the errors associated with the independent marker motion. In general, a similar result was therefore observed to that under the application of independent noise alone, but the absolute amount depended upon the relative magnitudes of the different noise errors applied.

When both segments were allowed to move (Fig. 4) and noise applied to each independent marker, the most accurate predictions were produced by the ATT and SARA approaches. Only these and the mean helical axis method produced RMS errors of less than 6 cm at a RoM of 5°. Again the cylinder axis fit and the algebraic fit approaches failed to produce reasonable results. The best approaches here were able to estimate the AoR to within an RMS error of 0.1 cm at larger ranges of motion.

When noise was applied to the marker set as a whole, the SARA again outperformed all other approaches, producing RMS errors of no greater than 0.3 cm for all RoMs. Similar to the application of independent marker noise, the cylinder and algebraic axis fit approaches were unable to compute competitive AoR estimates.

Once again, when a combination of independent and collective set errors were applied to the marker positions, approximately additive results were produced, the errors again dominated by individual marker noise. Under these conditions, only the SARA, ATT and the mean helical axis produced the most accurate AoR with RMS errors under 3.5 cm throughout.

4. Discussion

In this study we have compared a number of different techniques for determining the AoR under a variety of numerically generated error conditions. Until now, methods to determine the AoR of two dynamic bodies have generally only considered the relative positions of one set of markers in the coordinate system of the second. In this study, we have presented an approach for estimating a unique AoR, capable of considering two dynamic body segments simultaneously, using information from both segments. Furthermore, this method, SARA, has been

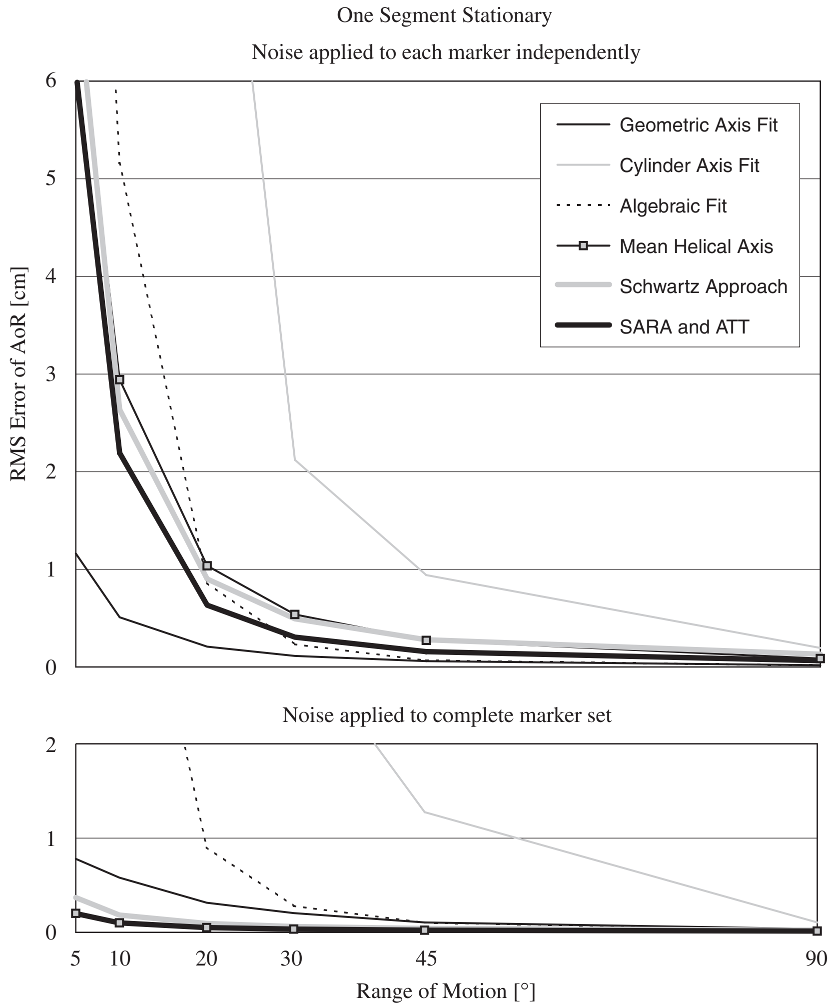

Fig. 3. RMS error of estimated AoR for different approaches over 1000 simulations, assuming one static segment (no movement or Gaussian noise). Top: isotropic, independent and identically distributed Gaussian noise was applied to each marker of the segment set. Bottom: Gaussian noise was applied to all markers on the segment simultaneously. Note that the results for the SARA, ATT and Schwartz median techniques are virtually identical for this case, and their graphs are thus almost indistinguishable from one another.

shown capable of producing among the best AoR estimates throughout the different tests performed in this study.

Axes of rotation are important, particularly during clinical movement analysis for the assessment of e.g. knee (Schache et al., 2006) or finger (Cerveri et al., 2005) kinematics, where abnormalities can be assessed and monitored. Moreover, the determination of joint centres alone is often not capable of describing the complex pattern of motion that is required in joints such as e.g. the knee. In these cases, AoRs can provide a detailed time-dependent description of the motion, as well as an anatomical reference to rotation. In such analyses, kinematic crosstalk, a process whereby the functional axes of rotation of a joint are not aligned with the chosen joint coordinate system (Piazza and Cavanagh, 2000), can play a role in defining the functional AoRs, particularly for the knee, where internal–external rotation of approximately 10° (Lafortune et al., 1992) can cause the AoR to move. This process has not been addressed in the current study, but must certainly be considered when using these AoR approaches during clinical gait analysis. It is therefore important that an assessment of the SARA be performed on clinical data in order to test its reliability.

Within the description of the various approaches examined in this study, a natural system of classification became apparent, in which the methods were divided into either fitting or transformation methods, as well as approaches that consider the movement of one or both segments, according to their mathematical strategy. Although the geometric axis fit, classified as a fitting approach, produced the most accurate axis estimate when one segment was held stationary, the approaches that are classified as transformation approaches demonstrated a clear advantage if the noise was applied to the marker sets on both segments.

In this study, the different AoR approaches have been compared using an RMS error definition that includes not only a measure of the distance of the estimated axis from the exact, known axis, but also considers the angle at which the two are positioned relative to one another. For

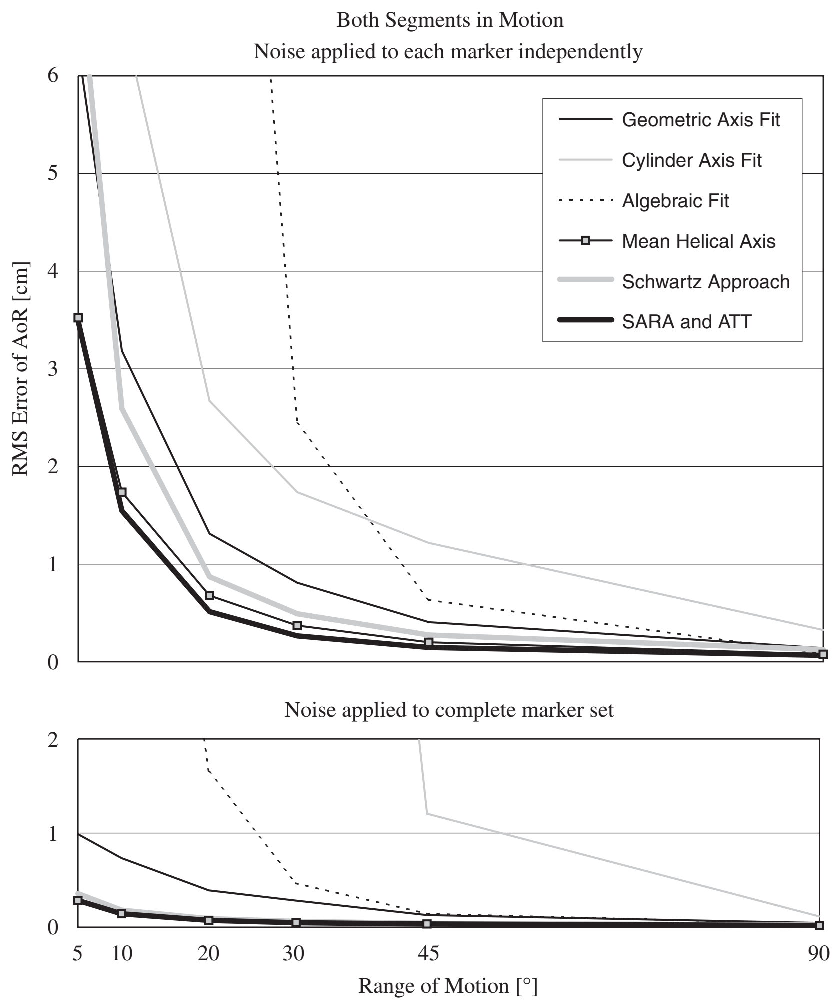

Fig. 4. RMS error of the estimated AoRs for different approaches over 1000 simulations, assuming two segments moving and with Gaussian noise. Top: isotropic, independent and identically distributed Gaussian noise was applied to each marker of the segment set. Bottom: Gaussian noise was applied to all markers on the segment simultaneously.

conditions when noise was applied to each marker of the set independently, a maximum RMS error of 6 cm has been shown (Figs. 3 and 4). In practical terms, this RMS error of 6 cm is equivalent to e.g. a parallel axis at a distance of 6 cm, or to an axis that precisely bisects the exact axis, but at an angle of approximately 37°. The magnitude of this error is directly associated with the marker-to-axis distances and the marker-to-marker distances, as well as the magnitude and form of the marker artefact, here chosen to be representative of typical measurements for human gait, but are not necessarily general to all configurations and marker-to-axis distances. Although the majority of the axis estimates in this study produced an RMS error of considerably less than 6 cm, the results demonstrate that in certain cases, particularly at low ranges of motion, the calculated AoRs may not even come close to the real situation. Under the most demanding conditions, the SARA and ATT methodologies produced no more than 0.7 cm RMS error at 20° RoM.

The ATT produced results that were practically indistinguishable from the SARA. Future studies must address how the two approaches cope with motion conditions that include kinematic crosstalk. Although the ATT approach was originally hinted at by Schwartz and Rozumalski (2005) this method seems never to have been previously implemented. From the results presented in this study, it is certainly an approach that is capable of producing excellent results. The actual Schwartz approach, however, has been shown to be less accurate and require longer time for computation (Ehrig et al., 2005). This was independent of the averaging approach used (mean, median, mode). Although the results for the mode were presented in this study and were indeed the best, the median produced only slightly less accurate results. The mean, however, produced a far less accurate AoR based on a number of outlying results that seemed to skew the axes.

In a companion publication (Ehrig et al., 2005), a number of transformation methods have been considered for the determination of a spherical joint centre, or centre of rotation (CoR). Each of these methods may also be used as an approach to compute the joint axis by solving the corresponding least squares problem by using SVD algorithms or the corresponding system of normal equations with eigenvalue decomposition. From each of the transformation methods for determining a CoR, it is therefore possible to define approaches for the determination of an AoR e.g. for the Holzreiter CoR approach (Holzreiter, 1991). As for the centre determination, this technique yields an algorithm equivalent to the ATT. The SARA is also a derivation of the symmetrical centre of rotation estimation, or SCoRE (Ehrig et al., 2005). A similar approach to Eq. (2), in the form of normal equations, has also been proposed by Biryukova et al. (2000). Here, the authors used rotation matrices in terms of Euler angles, whereas any orthogonal matrix with determinant 1, i.e. any basis of a local coordinate system, may be used (Fig. 3). Furthermore, Cameron and Lasenby (2005) developed a symmetrical algorithm for the determination of joint centres, which may be interpreted as a variant of SCoRE. Here the rotations (R_i, t_i), (S_i, d_i) were applied to describe the motion of each marker instead of only the motion of the local CSs. To the authors' knowledge, this method has, however, never been exploited as a method for determining joint axes. The SARA has the advantage that it is possible to interpret the joint as either a ball or a hinge joint, depending upon the magnitude of the smallest singular value of the matrix in Eq. (2) or (S9), a feature already alluded to by O'Brien and co-workers (2000). This balance may allow the advantage of a combined analysis where the joint may not rotate entirely as either a hinge or a ball joint in e.g. the knee where internal/external rotation acts in addition to flexion/extension, and will be addressed in future studies.

Helical axes, based on the work of Woltring and co-workers (1985), have been widely used in the literature and are useful, especially when an AoR is to be determined between only two time points. Even if the derivations are different, it can be shown that the mean helical axis approach is identical to the axis transformation technique, i.e. Eq. (S12), if the weighting parameter is set to $w_i = \sin^2(\vartheta_i/2)$. Thus both algorithms differ only by the weighting of each pair of time frames. A general drawback of this mean helical axis, as well as the Schwartz approach, is that a large number of AoRs need to be averaged, but no unique method exists to perform this task. The averaging procedure proposed by Woltring (1990) determines the point nearest to all helical axes, a process that delivers progressively worse results the more parallel and accurate the AoRs themselves become. An "optimal" direction vector is then computed through this point, which minimises the distances of this vector to the surrounding helical axes, but problems may arise when this process does not lead to a unique direction. A simpler technique, used in the Schwartz approach (Schwartz and Rozumalski, 2005), averages of the points of each axis nearest to the origin as well as their direction vectors. Normalising the resulting direction and orthogonalising the position vector often seems to yield a successful AoR, but the results are not always unambiguous, since the direction vectors cannot be uniquely defined.

The accuracy of determining an AoR is dependent upon the measurement artefacts from e.g. skin marker motion relative to the underlying bones, here modelled by applying combinations of independent and collective noise to the markers. Although more invasive methods can certainly determine the kinematics of the bones to a greater accuracy (Stagni et al., 2005), for the non-invasive approaches generally used in clinical movement analysis, markers that are generally attached to prominent skeletal landmarks detected by palpation are subject to skin and soft tissue errors. Although a number of methods have attempted to minimise these effects (Andriacchi et al., 1998; Alexander and Andriacchi, 2001; Taylor et al., 2005), they continue to be the largest source of error (Benoit et al., 2006; Filipe et al., 2005) and limit the widespread applicability of gait analysis, especially in more obese patients (Vaughan et al., 1999). While devices to attach rigid marker sets or bandages to bind soft tissues, as well as the amount of the patients' soft tissue coverage, will all effect the relative error and artefact magnitudes, in this study we have attempted to generate errors (Gaussian error of SD 0.1 cm) that may be appropriate to clinical movement analysis.

In conclusion, a complete survey and classification of formal methods for determining the AoR has, for the first time, been performed in this study. The symmetrical axis of rotation approach, or SARA, which requires no assumptions regarding the segment movements relative to the AoR, produced among the smallest errors in the estimation of joint axes for all test scenarios investigated in this study. Whilst these results using the SARA are both promising and fast, full tests must now be performed on clinical data during normal functional movements.

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Appendix A. Supplementary Materials

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.jbiomech.2006.10.026.

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