Mock Exam Advanced Labor Economics

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1 The impact of automation on the labor market

1.1 Environment

Assume the following competitive economy:

- Aggregate production of a single final good, Y, is given by the following Cobb-Douglas production function:
$$Y = \exp\left[\int_0^1 \ln(y(z))dz\right] \tag{1}$$
with $y(z)$ the quantity of task z used in the production of Y. There is a continuum of tasks spread out over the unit-interval such that $z \in [0, 1]$.

- Each task is produced using a quantity of capital, $k(z)$, or of labor, $l(z)$, according to:
$$y(z) = \begin{cases} \gamma^K(z)k(z) \text{ if } z \in [0, I] \\ \gamma^L(z)l(z) \text{ if } z \in (I, 1] \end{cases} \tag{2}$$
with $\gamma^K(z)$ and $\gamma^L(z)$ the task-specific productivity of capital and labor, respectively. Task $z = I \in (0, 1)$ is an exogenous task threshold such that all tasks $z \leqslant I$ can only be produced by capital and all tasks $z > I$ can only be produced by labor.

- In equilibrium, clearing of capital and labor markets requires that:
$$\int_0^1 k(z)dz = K \text{ and } \int_0^1 l(z)dz = L \tag{3}$$
with K and L, respectively, a given aggregate quantity of capital and labor in the economy.

- Given the Cobb-Douglas production function in equation (1), the corresponding expression for the marginal cost of producing Y is given by:
$$MC = \exp\left[\int_0^1 \ln(p(z))dz\right] \tag{4}$$
with $p(z)$ the price or the unit-cost of task z.

1.2 Equilibrium

1. Define R as the price of capital and W as the price of labor. That is, R is the price of one unit of $k(z)$ and W is the price of one unit of $l(z)$. For given values of R and W, use that marginal revenue products of capital and labor must equal their marginal costs in equilibrium to replace the question marks in the following expression for the unit-cost of task z, $p(z)$:
$$p(z) = \begin{cases} \textbf{?} \text{ if } z \in [0, I] \\ \textbf{?} \text{ if } z \in (I, 1] \end{cases} \tag{5}$$

2. Given that equation (1) is a Cobb-Douglas production function using a continuum of tasks, cost shares must be constant and equal across all tasks in equilibrium. In particular, given that tasks are spread out over the unit-interval, we must have that $\forall z : p(z)y(z) = Y$. Using that $y(z) = Y/p(z)$ together with equations (2) and (5), show that:

$$k(z) = \begin{cases} \frac{Y}{R} \text{ if } z \in [0, I] \\ 0 \text{ if } z \in (I, 1] \end{cases} \tag{6}$$

and

$$l(z) = \begin{cases} 0 \text{ if } z \in [0, I] \\ \frac{Y}{W} \text{ if } z \in (I, 1] \end{cases} \tag{7}$$

which gives the demand for capital and labor for each task z, respectively.

3. Use equations (3), (6) and (7) to solve for equilibrium expressions for R and W.

4. In equilibrium, the marginal cost of Y, given by equation (4), must equal the price of Y which we denote by P. Choosing Y as the numeraire, we must therefore have that $P = MC = 1$ in equilibrium. Taking logarithms such that $\ln(P) = \ln(MC) = 0$ and using equations (4) and (5) together with the equilibrium expressions for R and W, re-write Y as an aggregate Cobb-Douglas production function and interprete its parameters.

1.3 The impact of automation

1. Assume that tasks are ranked on the unit-interval such that $\gamma^K(z)/\gamma^L(z)$ is decreasing in z. That is, capital has a comparative advantage in the production of lower-indexed tasks, and labor has a comparative advantage in the production of higher-indexed tasks. Now assume that technological progress consists of the automation by capital of labor tasks. As examples, think of the automation by industrial robots of car assembly lines or of software automating tasks done by office clerks. In our model, assume that automation is captured by an increase in the task threshold I.

   Taking the logarithm of the expression for W and differentiating, show that the impact of automation on (the logarithm of) W consists of two effects. The first is a negative *direct displacement effect* because the range of tasks done by workers decreases. The second is a *productivity effect* that captures a change in (the logarithm of) average labor productivity. A priori, the productivity effect can be positive or negative.

2. Proof that the productivity effect will be strictly positive if and only if:

$$\frac{R}{\gamma^K(I)} < \frac{W}{\gamma^L(I)}$$

   and interpret this inequality in terms of the unit-cost to produce the task threshold I.

3. Show that automation always decreases the aggregate labor share in the economy, defined by $s \equiv WL/Y$.

2 Monopsony and firm effects in wages

2.1 Worker preferences over non-pecuniary amenities

- Assume that workers have different preferences over the same workplace amenities. In particular, assume that utility of worker i from working in firm j is given by:

$$u_{ij} = \epsilon \ln(w_j) + \eta_{ij} \tag{1}$$

  where η_{ij} captures idiosyncratic preferences for working at firm j, arising, for example, from non-pecuniary match factors such as distance to work or interactions with co-workers and supervisors. If $\{\eta_{ij}\}$ are independent draws from a type-I Extreme Value distribution, the total number of workers that will be employed at firm j is given by:

$$l_j = \frac{\exp\left(\epsilon \ln(w_j)\right)}{\sum_{k=1}^{J} \exp\left(\epsilon \ln(w_k)\right)} L \tag{2}$$

  with L the total labor force.

- To simplify the analyses and abstract from strategic interactions in wage setting, assume that the number of firms J is very large such that we can write:

$$\ln(l_j) = \epsilon \ln(w_j) + \ln(\lambda L) \tag{3}$$

  where λ is a constant common across all firms.

- Interpret equation (3). What does it mean if $\epsilon \to 0$?

2.2 Firm optimization

- Assume that firms have the following production functions:

$$q_j = \psi_j \ln(l_j) \tag{4}$$

  where ψ_j is a firm-specific productivity shifter and the marginal product of labor is decreasing in employment. Both turn out to play an important role in our comparative statics later. Firms maximize profits by posting a wage that minimizes labor costs given equation (3):

$$\max_{w_j} \left[\psi_j \ln(l_j) - w_j l_j\right] \quad \text{s.t.} \quad l_j = l_j(w_j) \tag{5}$$

  Note that we assume employers do not observe the idiosyncratic taste for amenities of any given worker given by equation (1). This information asymmetry implies employers cannot price discriminate between workers. Instead, if a firm wants to hire more workers it needs to offer higher wages to all workers.

- Using the first-order conditions, derive the following expression:

$$\ln(w_j) = \ln\left(\psi_j \frac{1}{1+\epsilon}\right) - \ln(l_j) \tag{6}$$

2.3 Equilibrium

- Given equations (3) and (6), derive expressions for firm-level wages and employment in equilibrium by replacing the question marks in the following expressions:

$$\ln(w_j) = \textbf{?} - \frac{1}{1+\epsilon} \ln(\lambda L) \tag{7}$$

  where the right-hand side only depends on the model's parameters. Equilibrium firm-level employment is given by:

$$\ln(l_j) = \textbf{?} + \frac{1}{1+\epsilon} \ln(\lambda L) \tag{8}$$

2.4 Firm effects in wages

- Equations (7) and (8) show that relatively more productive firms (i.e. firms with higher ψ_j) pay higher wages and employ more workers. Briefly explain how you could use matched employer-employee data to estimate the importance of firm effects in wage inequality.
- Could an increased difference in firm-level productivity over time explain the rising importance of firm effects and the increased sorting of workers into high-wage firms?