CHAPTER 12

Skills, Tasks and Technologies: Implications for Employment and Earnings[☆]

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4. The model is also silent on the question of why the allocation of skill groups across occupations has substantially shifted in the last two decades, with a rising share of middle educated workers employed in traditionally low education services, or why the importance of occupations as predictors of earnings may have increased over time.
5. Because it incorporates technical change in a factor-augmenting form, it does not provide a natural framework for the study of how new technologies, including computers and robotics, might substitute for or replace workers in certain occupations or tasks.
6. Because it treats technical change as exogenous, it is also silent on how technology might respond to changes in labor market conditions and in particular to changes in supplies.
7. Finally, the canonical model does not provide a framework for an analysis of how recent trends in offshoring and outsourcing may influence the labor market and the structure of inequality (beyond the standard results on the effect of trade on inequality through its factor content).

Recognizing the virtues of the canonical model, we propose a richer conceptual framework that nests the canonical model while allowing for a richer set of interactions among job tasks, technologies, trading opportunities, and skill supplies in determining the structure of wages.

4. A RICARDIAN MODEL OF THE LABOR MARKET

Many of the shortcomings of the canonical model can, we believe, be addressed by incorporating a clear distinction between workers' skills and job tasks and allowing the assignment of skills to tasks to be determined in equilibrium by labor supplies, technologies, and task demands, as suggested by Autor et al. (2003).[62] In this terminology, a task is a unit of work activity that produces output. A skill is a worker's *endowment* of capabilities for performing various tasks. This endowment is a stock, which may be either exogenously given or acquired through schooling and other investments. Workers apply their skill endowments to tasks in exchange for wages. Thus, the task-based approaches emphasize that skills are applied to tasks to produce output—skills do not directly produce output. Task models provide a natural framework for interpreting patterns related to occupations in the labor market, as documented above, since we can think of occupations

[62] The precedent of this approach is the assignment model, introduced in Tinbergen (1974), and further developed in Rosen (1974, 1981, 1982), Sattinger (1975, 1993), Heckman and Sedlacek (1985), Teulings (1995), Saint-Paul (2001) and Garicano (2000). The task-based approach has been used more recently in several papers studying the impact of technology and international trade on the labor market, including Feenstra and Hanson (1999), Acemoglu and Zilibotti (2001), Spitz-Oener (2006), Goos and Manning (2007), Grossman and Rossi-Hansberg (2008), Autor and Dorn (2009, 2010), Firpo et al. (2009), Acemoglu et al. (2010), Rodriguez-Clare and Ramondo (2010), and Costinot and Vogel (forthcoming).

as bundles of tasks. In this light, the canonical model may be seen as a special case of the general task-based model in which there is a one-to-one mapping between skills and tasks.[63]

The distinction between skills and tasks becomes relevant, in fact central, when workers of a given skill level can potentially perform a variety of tasks and, moreover, can change the set of tasks that they perform in response to changes in supplies or technology. Although a growing literature adopts the task-based approach to study technology and its role in the labor market, this literature has not yet developed a flexible and tractable task-based model for analyzing the interactions among skill supplies, technologies, and trade in sharping the earnings distribution.[64] The absence of such a framework has also meant that the power of this approach for providing a unified explanation for recent trends has not been fully exploited.

We believe that a useful task-based model should incorporate several features that are absent in the canonical model, while at the same time explicitly subsuming the canonical model as a special case. In particular,

1. Such a model should allow an explicit distinction between skills and tasks, and allow for general technologies in which tasks can be performed by different types of skills, by machines, or by workers in other countries ("offshored"). This will enable the model to allow for certain tasks to be become mechanized (as in Autor et al., 2003) or alternatively produced internationally.
2. To understand how different technologies may affect skill demands, earnings, and the assignment (or reassignment) of skills to tasks, it should allow for comparative advantage among workers in performing different tasks.
3. To enable a study of polarization and changes in different parts of the earnings distribution during different periods, it should incorporate at least three different skill groups.
4. As with the canonical model, the task-based approach should give rise to a well-defined set of skill demands, with downward sloping relative demand curves for skills (for a given set of technologies) and conventional substitutability and complementarity properties among skill groups.

The following sections present a succinct framework that enriches the canonical model in these three dimensions without sacrificing the underlying logic of the canonical model. This model is a generalization of Acemoglu and Zilibotti (2001) and is also

63 Alternatively, the canonical model can be interpreted as an approximation whereby this assignment is fixed during the period of study.

64 The assignment models mentioned in footnote 62 provide highly flexible task-based models, but are generally not tractable and do not offer a simple framework in which the interaction between technology and the allocation of tasks across different skills can be readily analyzed.

related to Costinot and Vogel (forthcoming).[65] The relationship between the framework here and these models will be discussed further below. Given the central role that the comparative advantage differences across different types of workers play in our model and the relationship of the model to Dornbusch et al. (1977), we refer to it as a *Ricardian model* of the labor market.[66]

4.1. Environment

We consider a static environment with a unique final good. For now, the economy is closed and there is no trade in tasks (a possibility we allow for later). The unique final good is produced by combining a continuum of tasks represented by the unit interval, [0, 1]. We simplify the analysis by assuming a Cobb-Douglas technology mapping the services of this range of tasks to the final good. In particular,

$$Y = \exp\left[\int_0^1 \ln y(i)\mathrm{d}i\right], \tag{11}$$

or equivalently, $\ln Y = \int_0^1 \ln y(i)\mathrm{d}i$, where Y denotes the output of a unique final good and we will refer to $y(i)$ as the "service" or production level of task i. We will also alternately refer to workers "performing" or producing a task. We assume that all markets are competitive. Throughout, we choose the price of the final good as the numeraire.

There are three factors of production, high, medium and low skilled workers. In addition, we will introduce capital or technology (embedded in machines) below. We first assume that there is a fixed, inelastic supply of the three types of workers, L, M and H. We return to the supply response of different types of skills to changes in technology later in this section.

65 The assignment literature, and in particular the recent important paper by Costinot and Vogel (forthcoming), considers a similar model with a continuum of skills (as well as a continuum of tasks as in our framework). Under a comparative advantage (log supermodularity) assumption, which generalizes our comparative advantage assumption below, Costinot and Vogel (forthcoming) characterize the labor market equilibrium in terms of two ordinary differential equations, one determining the match between skills and tasks and the other determining the wage as a function of assignment. They show that a variety of changes in the patterns of comparative advantage will lead to unambiguous comparative static results. The framework of Costinot and Vogel (forthcoming) can thus also be used to study issues similar to those exposited below. As with other assignment models, one would need to impose additional structure on the pattern of comparative advantage to obtain sharp predictions.
Our framework is also related to growth models in which technical progress expands the range of tasks in which machines can be used instead of labor. See, for example, Champernowne (1963), Zeira (1998, 2006), Hellwig and Irmen (2001) and Acemoglu (2009). Finally, Saint-Paul (2008) provides a rich exposition of both conventional and unconventional models of technological change and considers their nuanced implications for wage levels and wage inequality.

66 In particular, our model is isomorphic to a Ricardian trade model à la Dornbusch et al. (1977), with each skill group representing a country (i.e., a single factor, three-country model with a continuum of goods). Wilson (1980) provides a generalization of the Dornbusch, Fischer and Samuelson model to an arbitrary number of countries and more general preferences. Wilson's approach can be used to extend some of the results here to more than three skill groups and to more general preferences than those in Eq. (11).

Each task has the following production function

$$y(i) = A_L \alpha_L(i)\, l(i) + A_M \alpha_M(i)\, m(i) + A_H \alpha_H(i)\, h(i) + A_K \alpha_K(i)\, k(i), \quad (12)$$

where A terms represent factor-augmenting technology, and $\alpha_L(i)$, $\alpha_M(i)$ and $\alpha_H(i)$ are the task productivity schedules, designating the productivity of low, medium and high skill workers in different tasks. For example, $\alpha_L(i)$ is the productivity of low skill workers in task i, and $l(i)$ is the number of low skill workers allocated to task i. The remaining terms are defined analogously. Given this production function, we can think of A_L as (factor-augmenting) low skill biased technology, of A_M as medium skill biased technology, and of A_H as high skill biased technology. It is critical to observe that this production function for task services implies that each task can be performed by low, medium or high skill workers, but the *comparative advantage* of skill groups differ across tasks, as captured by the α terms. These differences in comparative advantage will play a central role in our model.

We impose the following assumption on the structure of comparative advantage throughout:

Assumption 1. $\alpha_L(i)/\alpha_M(i)$ and $\alpha_M(i)/\alpha_H(i)$ are continuously differentiable and strictly decreasing.

This assumption specifies the structure of comparative advantage in the model. It can be interpreted as stating that higher indices correspond to "more complex" tasks in which high skill workers are better than medium skill workers and medium skill workers are better than low skill workers. Though not very restrictive, this assumption ensures a particularly simple and tight characterization of equilibrium in this economy.

Factor market clearing requires

$$\int_0^1 l(i)\mathrm{d}i \le L, \qquad \int_0^1 m(i)\mathrm{d}i \le M \quad \text{and} \quad \int_0^1 h(i)\mathrm{d}i \le H. \quad (13)$$

When we introduce capital, we will assume that it is available at some constant price r.

4.2. Equilibrium without machines

An equilibrium is defined in the usual manner as an allocation in which (final good) producers maximize profits and labor markets clear. For now there is no labor supply decision on the part of the workers.

Let us first ignore capital (equivalently, $\alpha_K(\cdot) \equiv 0$). This implies that initially there are no machines that can substitute for labor in the production of specific tasks.

Allocation of skills to tasks

We first characterize the allocation of skills to tasks.

The characterization of equilibrium in this economy is simplified by the structure of comparative advantage differences in Assumption 1. In particular, there will exist some I_L and I_H such that all tasks $i < I_L$ will be performed by low skill workers, and all tasks $i > I_H$ will be performed by high skill workers. Intermediate tasks will be performed by medium skilled workers. We can think of these intermediate tasks as the routine tasks performed by workers in many production, clerical, and administrative support occupations. More formally, we have:

Lemma 1. *In any equilibrium there exist* I_L *and* I_H *such that* $0 < I_L < I_H < 1$ *and for any* $i < I_L$, $m(i) = h(i) = 0$, *for any* $i \in (I_L, I_H)$, $l(i) = h(i) = 0$, *and for any* $i > I_H$, $l(i) = m(i) = 0$.

The proof of this lemma follows a similar argument to a lemma presented in Acemoglu and Zilibotti (2001), extended to an environment in which there are three types of workers. Intuitively, if at given prices of three types of labor, w_L, w_M and w_H, the costs of producing a unit of services of task I_L using either low skill or medium skill workers are the same, then in view of the fact that $\alpha_L(i)/\alpha_M(i)$ is strictly decreasing (Assumption 1), it will cost strictly less to perform tasks $i < I_L$ using low skill rather than medium skill workers; and similarly, it will be strictly less costly to perform tasks $i > I_L$ using medium skill rather than low skill workers. The same argument applies to the comparison of medium and high skill workers below or above the threshold I_H. Note also that given Assumption 1, we do not need to compare the cost of producing a given task using low and high skill workers, since if the cost were the same with low and high skill workers, it would necessarily be strictly less with medium skill workers. Furthermore, because there is a positive supply of all three types of labor, the threshold tasks I_L and I_H must be both interior and different (i.e., $0 < I_L < I_H < 1$).

Lemma 1 shows that the set of tasks will be partitioned into three (convex) sets, one performed by low skill workers, one performed by medium skill workers and one performed by high skill workers. Crucially, the boundaries of these sets, I_L and I_H, are endogenous and will respond to changes in skill supplies and technology. This introduces the first type of substitution that will play an important role in our model: *the substitution of skills across tasks*. Given the types of skills supplied in the market, firms (equivalently workers) will optimally choose which tasks will be performed by which skill groups.

The law of one price for skills

Even though workers of the same skill level perform different tasks, in equilibrium they will receive the same wage—a simple "law of one price" that has to hold in any competitive equilibrium. We now derive these prices.

Let $p(i)$ denote the price of services of task i. Since we chose the final good as numeraire (setting its price to 1), we have

$$\exp\left[\int_0^1 \ln p(i)\mathrm{d}i\right] = 1.$$

In any equilibrium, all tasks employing low skill workers must pay them the same wage, w_L, since otherwise, given the competitive market assumption, no worker would supply their labor to tasks paying lower wages. Similarly, all tasks employing medium skill workers must pay a wage w_M, and all tasks employing high skill workers must pay a wage w_H. As a consequence, the value marginal product of all workers in a skill group must be the same in all the tasks that they are performing. In particular, in view of Lemma 1 and the production function (12), this implies:

$$\begin{aligned} w_L &= p(i)A_L\alpha_L(i) \quad \text{for any } i < I_L. \\ w_M &= p(i)A_M\alpha_M(i) \quad \text{for any } I_L < i < I_H. \\ w_H &= p(i)A_H\alpha_H(i) \quad \text{for any } i > I_H. \end{aligned}$$

This observation has a convenient implication. We must have that the price difference between any two tasks produced by the same type of worker must exactly offset the productivity difference of this type of worker in these two tasks. For example, for low skill workers we have

$$p(i)\alpha_L(i) = p(i')\alpha_L(i') \equiv P_L, \tag{14}$$

for any $i, i' < I_L$, where the last equality defines P_L as the price "index" of tasks performed by low skill workers. Note, however, that this price is endogenous not only because of the usual supply–demand reasons, but also because the set of tasks performed by low skill workers is endogenously determined. Similarly, for medium skill workers, i.e., for any $I_H > i, i' > I_L$, we have

$$p(i)\alpha_M(i) = p(i')\alpha_M(i') \equiv P_M, \tag{15}$$

and for high skill workers and any $i, i' > I_H$,

$$p(i)\alpha_H(i) = p(i')\alpha_H(i') \equiv P_H. \tag{16}$$

The Cobb-Douglas technology (the unitary elasticity of substitution between tasks) in (11) implies that "expenditure" across all tasks should be equalized, and given our choice of numeraire, this expenditure should be equal to the value of total output. More specifically, the first-order conditions for cost minimization in the production of the final good imply that $p(i)y(i) = p(i')y(i')$ for any i, i'. Alternatively, using our choice of the final good as the numeraire, we can write

$$p(i)y(i) = Y, \quad \text{for any } i \in [0, 1]. \tag{17}$$

(In particular, note that the ideal price index for the final good, P, is defined such that $y(i)/Y = p(i)/P$, and our choice of numeraire implies that $P = 1$, which gives (17)).

Now consider two tasks $i, i' < I_L$ (performed by low skill workers), then using the definition of the productivity of low skill workers in these tasks, we have

$$p(i)\alpha_L(i)\,l(i) = p(i')\alpha_L(i')l(i').$$

Therefore, for any $i, i' < I_L$, we conclude that $l(i) = l(i')$, and using the market clearing condition for low skilled workers, we must have

$$l(i) = \frac{L}{I_L} \quad \text{for any } i < I_L. \tag{18}$$

This is a very convenient implication of the Cobb-Douglas production structure. With a similar argument, we also have

$$m(i) = \frac{M}{I_H - I_L} \quad \text{for any } I_H > i > I_L. \tag{19}$$

$$h(i) = \frac{H}{1 - I_H} \quad \text{for any } i > I_H. \tag{20}$$

The above expressions are derived by comparing expenditures on tasks performed by the same type of worker. Now comparing two tasks performed by high and medium skill workers ($I_L < i < I_H < i'$), we obtain from Eq. (17) that $p(i)A_M\alpha_M(i)\,m(i) = p(i')A_H\alpha_H(i')h(i')$. Next using (14) and (15), we have

$$\frac{P_M A_M M}{I_H - I_L} = \frac{P_H A_H H}{1 - I_H},$$

or

$$\frac{P_H}{P_M} = \left(\frac{A_H H}{1 - I_H}\right)^{-1}\left(\frac{A_M M}{I_H - I_L}\right). \tag{21}$$

Similarly, comparing two tasks performed by medium and high skill workers, we obtain

$$\frac{P_M}{P_L} = \left(\frac{A_M M}{I_H - I_L}\right)^{-1} \left(\frac{A_L L}{I_L}\right). \tag{22}$$

No arbitrage across skills

The above derivations show that the key equilibrium objects of the model are the threshold tasks I_L and I_H. These will be determined by a type of "no arbitrage" condition equalizing the cost of producing these threshold tasks using different skills. We now derive these no arbitrage conditions and determine the threshold tasks.

Recall, in particular, that the threshold task I_H must be such that it can be profitably produced using either high skilled or medium skilled workers. This is equivalent to task I_H having the same equilibrium supply either when produced only with skilled or unskilled workers.[67] That is, it implies our first no arbitrage condition (between high and medium skills) is:

$$\frac{A_M \alpha_M (I_H) M}{I_H - I_L} = \frac{A_H \alpha_H (I_H) H}{1 - I_H}. \tag{23}$$

With an analogous argument, we obtain our second no arbitrage condition (between low and medium skills) as:

$$\frac{A_L \alpha_L (I_L) L}{I_L} = \frac{A_M \alpha_M (I_L) M}{I_H - I_L}. \tag{24}$$

Equilibrium wages and inequality

Once the threshold tasks, I_L and I_H, are determined, wage levels and earnings differences across skill groups can be found in a straightforward manner. In particular, wages are obtained simply as the values of the marginal products of different types of skills. For example, for low skill workers, this is:

$$w_L = P_L A_L. \tag{25}$$

Equally, or perhaps even more, important than the level of wages are their ratios, which inform us about the wage structure and inequality. For example, comparing high

[67] Alternatively, the unit cost of producing task I_H should be the same with medium and high skill workers, i.e., $A_M \alpha_M (I_H) w_M = A_H \alpha_H (I_H) w_H$. We then obtain (23) using (26). Similarly, (24) can be obtained from $A_M \alpha_M (I_L) w_M = A_L \alpha_L (I_L) w_L$ using (27).

and medium skill wages, we have

$$\frac{w_H}{w_M} = \frac{P_H A_H}{P_M A_M}.$$

A more convenient way of expressing these is to use (21) and write the relative wages simply in terms of relative supplies and the equilibrium allocation of tasks to skill groups, given by I_L and I_H. That is,

$$\frac{w_H}{w_M} = \left(\frac{1 - I_H}{I_H - I_L}\right)\left(\frac{H}{M}\right)^{-1}. \tag{26}$$

Similarly, the wage of medium relative to low skill workers is given by

$$\frac{w_M}{w_L} = \left(\frac{I_H - I_L}{I_L}\right)\left(\frac{M}{L}\right)^{-1}. \tag{27}$$

These expressions highlight the central role that allocation of tasks to skills plays in the model. Relative wages can be expressed simply as a function of relative supplies and equilibrium task assignments (in particular, the threshold tasks, I_L and I_H).

These equations, together with the choice of the numeraire, $\int_0^1 \ln p(i)\mathrm{d}i = 0$, fully characterize the equilibrium. In particular, using (14)–(16), we can write the last equilibrium condition as:

$$\int_0^{I_L} (\ln P_L - \ln \alpha_L(i))\,\mathrm{d}i + \int_{I_L}^{I_H} (\ln P_M - \ln \alpha_M(i))\,\mathrm{d}i + \int_{I_H}^{1} (\ln P_H - \ln \alpha_H(i))\,\mathrm{d}i = 0. \tag{28}$$

Equations (26) and (27) give the relative wages of high to medium and medium to low skill workers. To obtain the wage *level* for any one of these three groups, we need to use the price normalization in (28) together with (21) and (22) to solve out for one of the price indices, for example, P_L, and then (25) will give w_L and the levels of w_M and w_H can be readily obtained from (26) and (27).

4.2.1. Summary of equilibrium

The next proposition summarizes our equilibrium characterization and highlights several important features of the equilibrium.

Proposition 1. *There exists a unique equilibrium summarized by* $(I_L, I_H, P_L, P_M, P_H, w_L, w_M, w_H)$ *given by Eqs (21)–(28).*

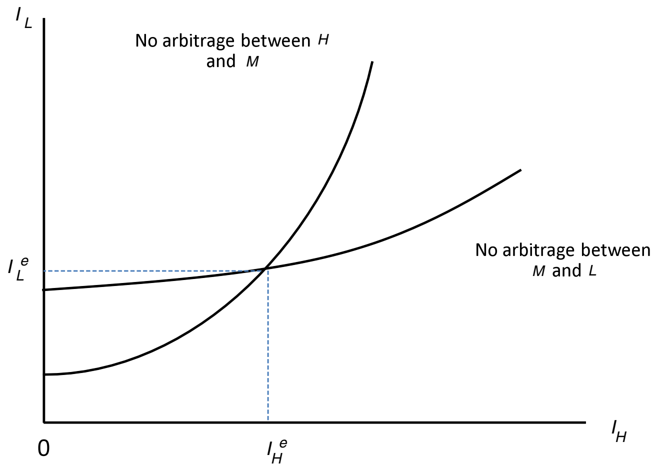

Figure 22 ***Determination of equilibrium threshold tasks.***

The only part of this proposition that requires proof is the claim that equilibrium is unique (the rest of it follows from the explicit construction of the equilibrium preceding the proposition). This can be seen by noting that in fact the equilibrium is considerably easier to characterize than it first appears, because it has a block recursive structure. In particular, we can first use (23) and (24) to determine I_L and I_H. Given these we can then compute relative wages from (26) and (27). Finally, to compute wage and price levels, we can use (21), (22), (25) and (28).

Figure 22 shows a diagrammatic representation of the equilibrium, in which curves corresponding to (23) and (24) determine I_L and I_H. Both curves are upward sloping in the (I_L, I_H) space, but the first one, (23), is steeper than the second one everywhere, (24)—see below for a proof. This establishes the existence of a unique intersection between the two curves in Fig. 22, and thus there exist unique equilibrium values of I_L and I_H. Given these values, P_L, P_M, P_H, w_L, w_M and w_H are uniquely determined from (21), (22) and (25)–(28).

While Fig. 22 depicts the determination of the two thresholds, I_L and I_H, it does not illustrate the allocation of tasks to different types of skills (workers). We do this in Fig. 23, which can also be interpreted as a diagram showing "relative effective demand" and "relative effective supply". In particular, we write (23) as follows:

$$\frac{1 - I_H}{I_H - I_L} \frac{\alpha_M (I_H)}{\alpha_H (I_H)} = \frac{A_H H}{A_M M}. \tag{29}$$

The right-hand side of this equation corresponds to the relative effective supply of high to medium skills (we use the term "effective" since the supplies are multiplied by their

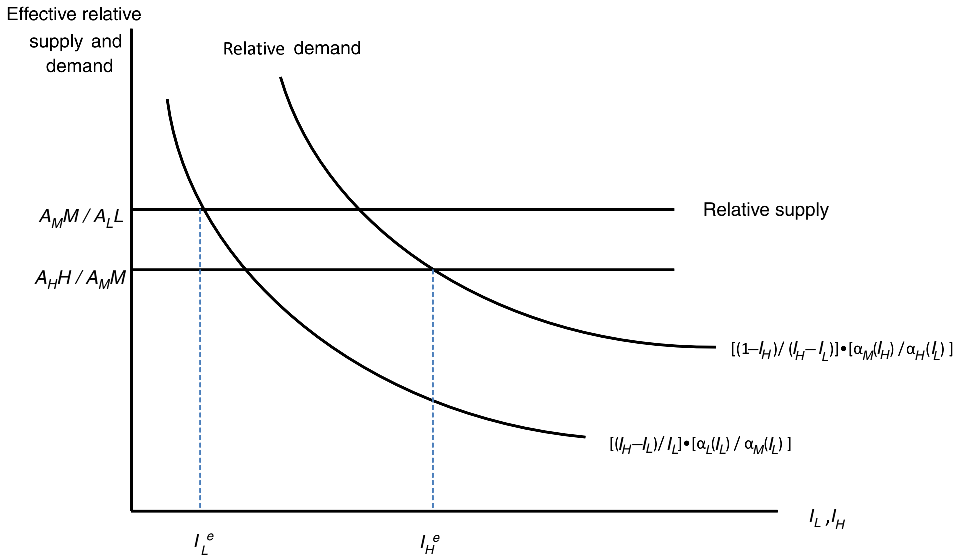

Figure 23 ***Equilibrium allocation of skills to tasks.***

respective factor-augmenting technologies). The left-hand side, on the other hand, can be interpreted as the effective demand for high relative to medium skills. The left-hand side of (29) is shown as the outer curve (on the right) in Fig. 23. It is downward sloping as a function of I_H (for a given level of I_L) since $\alpha_M(I_H)/\alpha_H(I_H)$ is strictly decreasing in view of Assumption 1. Similarly, we rewrite (24) as:

$$\frac{I_H - I_L}{I_L}\frac{\alpha_L(I_H)}{\alpha_M(I_H)} = \frac{A_M M}{A_L L}$$

for given I_H, and this expression has the same relative effective demand and supply interpretation. Since $\alpha_L(I_H)/\alpha_M(I_H)$ is strictly decreasing again from Assumption 1, the left-hand side traces a downward sloping curve as a function of I_L (for given I_H) and is shown as the inner (on the left) curve in Fig. 23. Where the outer curve equals $A_H H/A_M M$, as shown on the vertical axis, gives the threshold task I_H, and where the second curve is equal to $A_M M/A_L L$ gives I_L. This picture does not determine the two thresholds simultaneously as Fig. 22 does, since the dependence of the two curves on the other threshold is left implicit. Nevertheless, Fig. 23 is helpful in visualizing the equilibrium because it shows how equilibrium tasks are partitioned between the three types of skills. We will return to this figure when conducting comparative static exercises.

4.3. Special cases

We now study some special cases that help clarify the workings of the model. Suppose first that there are no medium skill workers. Assumption 1 in this case simply implies that

$\alpha_L(i)/\alpha_H(i)$ is strictly decreasing in i. Then we are back to a two-factor world as in the canonical model.

In addition, we could assume that instead of a continuum of tasks, there are only two tasks, one in which high skill workers have a strong comparative advantage and the other one in which low skill workers have a strong comparative advantage.[68] This would be identical to the canonical model, except with a Cobb-Douglas production function (elasticity of substitution between high and low skill workers equal to one).

Another special case is found in the model studied by Acemoglu and Zilibotti (2001), who also assume that there are only two types of workers, high and low skill. In addition, Acemoglu and Zilibotti impose the following functional form on the schedule of comparative advantage schedules:

$$\alpha_L(i) = 1 - i \quad \text{and} \quad \alpha_H(i) = i. \tag{30}$$

Then an equivalent of (23) implies that all tasks below I will be performed by low skill workers and those above I will be performed by high skill workers. Moreover, exactly the same reasoning that led to the no arbitrage conditions, (23) and (24), now determines the single threshold task, I, separating tasks performed by low and high skill workers. In particular, using (30), the equivalent of (23) and (24) gives I as

$$\frac{1-I}{I} = \left(\frac{A_H H}{A_L L}\right)^{1/2}.$$

In addition, the equivalent of (21) and (22) now gives the relative price of tasks performed by skilled compared to unskilled workers as

$$\frac{P_H}{P_L} = \left(\frac{A_H H}{A_L L}\right)^{-1/2},$$

and the equivalent of (26) and (27) gives the skill premium as

$$\frac{w_H}{w_L} = \left(\frac{A_H}{A_L}\right)^{1/2} \left(\frac{H}{L}\right)^{-1/2}.$$

Therefore, in this case the model is isomorphic to the canonical model with an elasticity of substitution equal to 2. This also shows that by choosing different forms for the comparative advantage schedules in the special case with only two types of skills,

[68] Or in fact, one could replicate a model with two tasks using a continuum of tasks, for example, assuming that $\alpha_L(i) = 1$ if $i \leq I$ and 0 otherwise, and $\alpha_H(i) = 0$ if $i \leq I$ and 1 otherwise (or a smooth approximation to this that would satisfy Assumption 1).

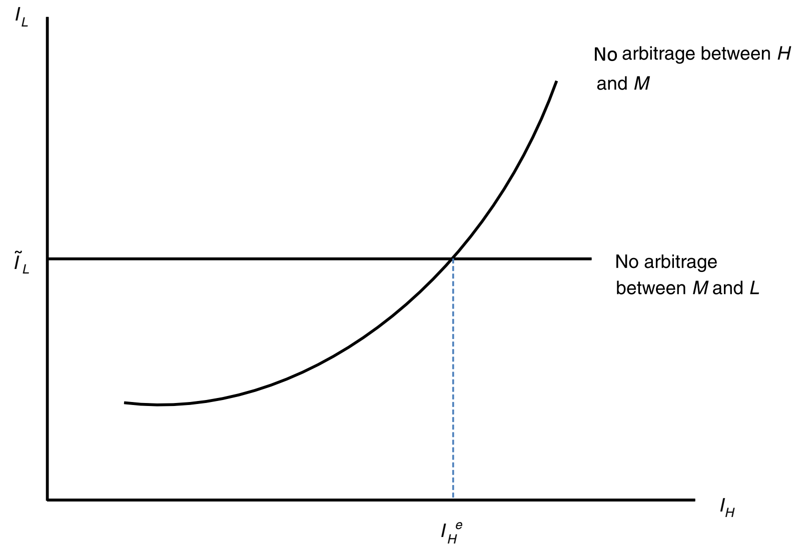

Figure 24 ***Determination of threshold high skill task (I_H) with task assignment for low skilled workers fixed.***

one could obtain any elasticity of substitution, or in fact any constant returns to scale production function (with an elasticity of substitution greater than or equal to 1) as a special case of the model shown here. This is the sense in which the canonical model, and thus all of economic forces emphasized by that model, are already embedded in our more general task-based framework.

Finally, another special case is useful both to show how insights from the two-skill model continue to hold in the three-skill model and also to illustrate how technical change in this task-based model can reduce the wages of some groups. For this, let us return to our general three-skill model introduced above, but suppose that

$$\alpha_L(i) = \begin{cases} \tilde{\alpha}_L & \text{if } i \leq \tilde{I}_L \\ 0 & \text{if } i > \tilde{I}_L \end{cases} \tag{31}$$

where $\tilde{\alpha}_L$ is large and $\tilde{I}_L$ is small. While this task productivity schedule for low skill workers is neither continuous nor strictly decreasing (and thus does not satisfy Assumption 1), we can easily take a strictly decreasing continuous approximation to (31), which will lead to identical results. The implication of this task schedule is that the no arbitrage condition between low and medium skills, (24), can only be satisfied at the threshold task $I_L = \tilde{I}_L$. This fixes one of the equilibrium thresholds, while the other one, I_H, is still determined in the usual fashion from the other no arbitrage condition, (23). Figure 24 adapts Fig. 22 and shows how the determination of equilibrium task thresholds looks in this case.

This case is of interest for two reasons. First, the model is now essentially identical to the two-skill version we have just discussed, since the set of tasks performed by low skill workers is fixed by the task productivity schedule (31) (without reference to other parameters in the model). Thus the mechanics of the equilibrium are simpler. Second, in the three-skill model, as we will see further in the next subsection, a variety of changes that directly affect I_H will have an indirect impact on I_L and these tend to "soften the blow" of some of these changes on the medium skill workers. With I_L fixed at $\tilde{I}_L$, this will not be the case and thus the wage effects of certain types of technical change on medium skilled workers will be exacerbated in this case. We return to this special case again in the next subsection.

4.4. Comparative statics

The usefulness of any framework is related to the insights that it generates, which are most clearly illustrated by its comparative static results. We discuss these here.

To derive these comparative statics, we return to the general model, and take logs in Eq. (23) and (24) to obtain slightly simpler expressions, given by the following two equations:

$$\ln A_M - \ln A_H + \beta_H(I_H) + \ln M - \ln H - \ln(I_H - I_L) + \ln(1 - I_H) = 0, \quad (32)$$

and

$$\ln A_L - \ln A_M + \beta_L(I_L) + \ln L - \ln M + \ln(I_H - I_L) - \ln(I_L) = 0, \quad (33)$$

where we have defined

$$\beta_H(I) \equiv \ln \alpha_M(I) - \ln \alpha_H(I) \quad \text{and} \quad \beta_L(I) \equiv \ln \alpha_L(I) - \ln \alpha_M(I),$$

both of which are strictly decreasing in view of Assumption 1. It can be easily verified that both of these curves are upward sloping in the (I_H, I_L) space, but (32) is everywhere steeper than (33) as claimed above, which also implies that there is indeed a unique intersection between the two curves as shown in Fig. 22.

Basic comparative statics

Basic comparative statics for the allocation of tasks across different skill groups can be obtained from this figure. For example, an increase in A_H, corresponding to high skill biased technical change, shifts (32) inwards, as shown in Fig. 25, so both I_L and I_H decrease (the implications of an increase in H for task allocation, though not for wages, are identical). This is intuitive: if high skill workers become uniformly more productive because of high skill biased technical change—generating an expansion of the set of tasks in which they hold comparative advantage—then they should perform a larger range

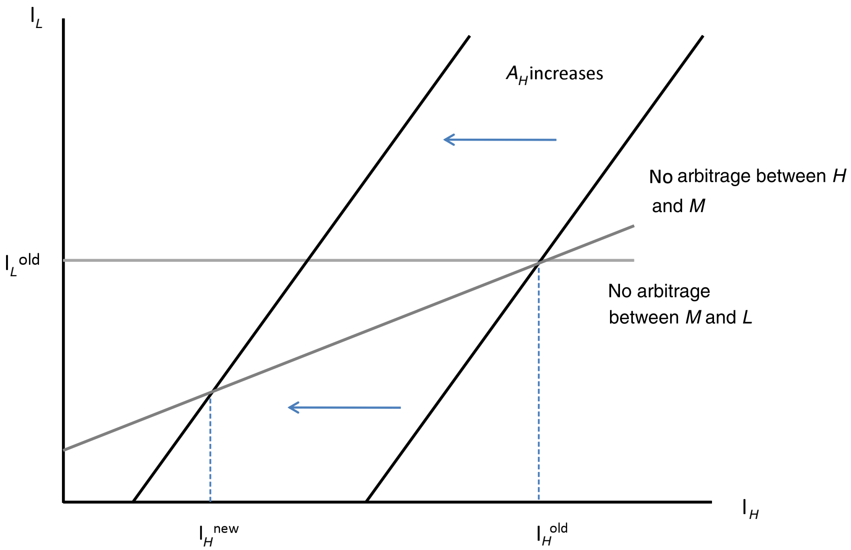

Figure 25 ***Comparative statics.***

of tasks. Thus the allocation of tasks endogenously shifts away from medium to high skill workers (I_H adjusts downward). If I_L remained constant following the downward movement of I_H, this would imply from (19) an "excess" supply of medium skill workers in the remaining tasks. Therefore, the indirect effect of the increase in A_H (or H) is also to reduce I_L, thus shifting some of tasks previously performed by low skill workers to medium skill workers.

Similarly, we can analyze the implications of skill biased technical change directed towards low skill workers, i.e., an increase in A_L, (or a change in the supply of low skill workers, L), which will be to increase I_L and I_H. This has exactly the same logic (there are either more low skill workers or low skill workers are more productive, and thus they will perform more tasks, squeezing medium skill workers, who now have to shift into some of the tasks previously performed by high skill workers). The implications of an increase in A_M, i.e., medium skill biased technical change, or of an increase in M again have a similar logic, and will reduce I_L and increase I_H, thus expanding the set of tasks performed by medium skill workers at the expense of both low and high skill workers. (Formally, in this case, the curve corresponding to (32) shifts up, while that for (33) shifts down). Each of these comparative statics illustrates the substitution of skills across tasks.

It is also useful to return to Fig. 23 to visually represent changes in the task allocation resulting from an increase in A_H, and we do this in Fig. 26. Such a change shifts the outer curve in Fig. 23 downward, as shown in Fig. 26, reducing I_H. This first shift holds I_L constant. However, the inner curve in this figure also shifts, as noted above and as highlighted by Figs 22 and 24. The decline in I_H also shifts this curve down, this time

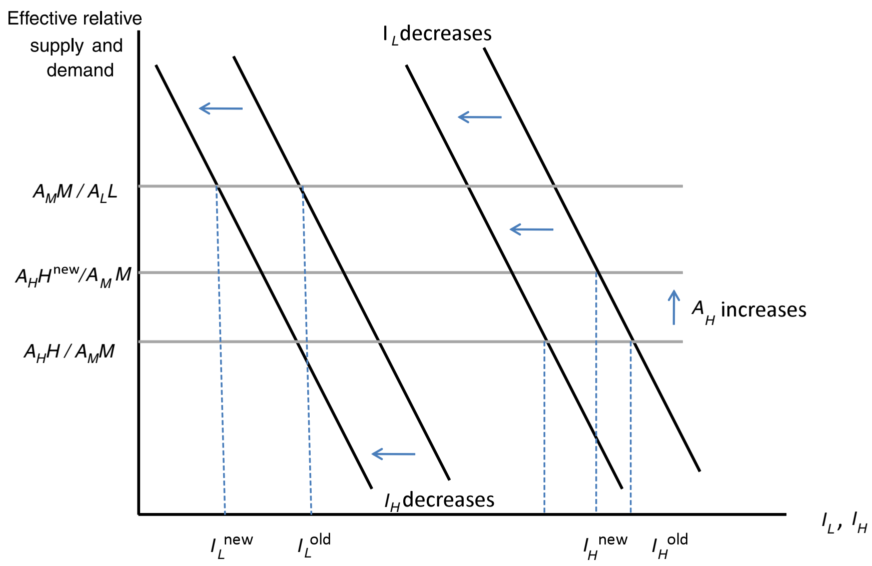

Figure 26 ***Changes in equilibrium allocation.***

reducing I_L. Then there is a second round of adjustment as the decline in I_L shifts the outer curve further down. Ultimately, the economy reaches a new equilibrium, as shown in Fig. 26.

It is a little more difficult to visually represent the changes in the wage structure resulting from changes in technology or supplies, because these depend on how I_L changes relative to I_H. Nevertheless, obtaining these comparative static results is also straightforward. To do this, let us consider a change in A_H and let us totally differentiate (32) and (33). We thus obtain:

$$\begin{pmatrix} \beta_H'(I_H) - \dfrac{1}{I_H - I_L} - \dfrac{1}{1 - I_H} & \dfrac{1}{I_H - I_L} \\ \dfrac{1}{I_H - I_L} & \beta_L'(I_L) - \dfrac{1}{I_H - I_L} - \dfrac{1}{I_L} \end{pmatrix} \begin{pmatrix} \mathrm{d}I_H \\ \mathrm{d}I_L \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mathrm{d}\ln A_H.$$

It can be easily verified that all of the terms in the diagonals of the matrix on the left hand side are negative (again from Assumption 1). Moreover, its determinant is positive, given by

$$\Delta = \left(\beta_H'(I_H) - \frac{1}{1 - I_H}\right)\left(\beta_L'(I_L) - \frac{1}{I_L}\right) + \frac{1}{I_H - I_L}\left(\frac{1}{I_L} + \frac{1}{1 - I_H} - \beta_L'(I_L) - \beta_H'(I_H)\right).$$

Therefore,

$$\frac{\mathrm{d}I_H}{\mathrm{d}\ln A_H} = \frac{\beta_L'(I_L) - \frac{1}{I_H - I_L} - \frac{1}{I_L}}{\Delta} < 0 \quad \text{and} \quad \frac{\mathrm{d}I_L}{\mathrm{d}\ln A_H} = \frac{-\frac{1}{I_H - I_L}}{\Delta} < 0,$$

confirming the insights we obtained from the diagrammatic analysis. But in addition, we can also now see that

$$\frac{\mathrm{d}(I_H - I_L)}{\mathrm{d}\ln A_H} = \frac{\beta_L'(I_L) - \frac{1}{I_L}}{\Delta} < 0.$$

Using these expressions, we can obtain comparative statics for how relative wages by skill group change when there is high skill biased technical change. A similar exercise can be performed for low and medium skill biased technical change. The next proposition summarizes the main results.

Proposition 2. *The following comparative static results apply:*

1. *(The response of task allocation to technology and skill supplies):*

$$\begin{aligned}
&\frac{\mathrm{d}I_H}{\mathrm{d}\ln A_H} = \frac{\mathrm{d}I_H}{\mathrm{d}\ln H} < 0, \qquad \frac{\mathrm{d}I_L}{\mathrm{d}\ln A_H} = \frac{\mathrm{d}I_L}{\mathrm{d}\ln H} < 0 \\
&\textit{and} \quad \frac{\mathrm{d}(I_H - I_L)}{\mathrm{d}\ln A_H} = \frac{\mathrm{d}(I_H - I_L)}{\mathrm{d}\ln H} < 0; \\
&\frac{\mathrm{d}I_H}{\mathrm{d}\ln A_L} = \frac{\mathrm{d}I_H}{\mathrm{d}\ln L} > 0, \qquad \frac{\mathrm{d}I_L}{\mathrm{d}\ln A_L} = \frac{\mathrm{d}I_L}{\mathrm{d}\ln L} > 0 \\
&\textit{and} \quad \frac{\mathrm{d}(I_H - I_L)}{\mathrm{d}\ln A_L} = \frac{\mathrm{d}(I_H - I_L)}{\mathrm{d}\ln L} < 0; \\
&\frac{\mathrm{d}I_H}{\mathrm{d}\ln A_M} = \frac{\mathrm{d}I_H}{\mathrm{d}\ln M} > 0, \qquad \frac{\mathrm{d}I_L}{\mathrm{d}\ln A_M} = \frac{\mathrm{d}I_L}{\mathrm{d}\ln M} < 0 \\
&\textit{and} \quad \frac{\mathrm{d}(I_H - I_L)}{\mathrm{d}\ln A_M} = \frac{\mathrm{d}(I_H - I_L)}{\mathrm{d}\ln M} > 0.
\end{aligned}$$

2. *(The response of relative wages to skill supplies):*

$$\begin{aligned}
&\frac{\mathrm{d}\ln(w_H/w_L)}{\mathrm{d}\ln H} < 0, \qquad \frac{\mathrm{d}\ln(w_H/w_M)}{\mathrm{d}\ln H} < 0, \qquad \frac{\mathrm{d}\ln(w_H/w_L)}{\mathrm{d}\ln L} > 0, \\
&\frac{\mathrm{d}\ln(w_M/w_L)}{\mathrm{d}\ln L} > 0, \qquad \frac{\mathrm{d}\ln(w_H/w_M)}{\mathrm{d}\ln M} > 0, \quad \textit{and} \\
&\frac{\mathrm{d}\ln(w_H/w_L)}{\mathrm{d}\ln M} \lesseqqgtr 0 \quad \textit{if and only if} \quad \left|\beta_L'(I_L)\, I_L\right| \gtreqqless \left|\beta_H'(I_H)(1 - I_H)\right|.
\end{aligned}$$

3. *(The response of wages to factor-augmenting technologies):*

$$\frac{\mathrm{d}\ln(w_H/w_L)}{\mathrm{d}\ln A_H} > 0, \qquad \frac{\mathrm{d}\ln(w_M/w_L)}{\mathrm{d}\ln A_H} < 0, \qquad \frac{\mathrm{d}\ln(w_H/w_M)}{\mathrm{d}\ln A_H} > 0;$$
$$\frac{\mathrm{d}\ln(w_H/w_L)}{\mathrm{d}\ln A_L} < 0, \qquad \frac{\mathrm{d}\ln(w_M/w_L)}{\mathrm{d}\ln A_L} < 0, \qquad \frac{\mathrm{d}\ln(w_H/w_M)}{\mathrm{d}\ln A_L} > 0;$$
$$\frac{\mathrm{d}\ln(w_H/w_M)}{\mathrm{d}\ln A_M} < 0, \qquad \frac{\mathrm{d}\ln(w_M/w_L)}{\mathrm{d}\ln A_M} > 0, \quad \textit{and}$$
$$\frac{\mathrm{d}\ln(w_H/w_L)}{\mathrm{d}\ln A_M} \lesseqqgtr 0 \quad \textit{if and only if} \quad \left|\beta_L'(I_L)\,I_L\right| \gtreqqless \left|\beta_H'(I_H)\,(1-I_H)\right|.$$

Part 1 of this proposition follows by straightforward differentiation and manipulation of the expressions in (32) and (33) for I_L and I_H. Parts 2 and 3 then follow readily from the expressions for relative wages in (26) and (27) using the behavior of these thresholds. Here we simply give the intuition for the main results.

First, the behavior of I_L and I_H in Part 1 is intuitive as already discussed above. In particular, an increase in A_H or H expands the set of tasks performed by high skill workers and contracts the set of tasks performed by low and medium skill workers. This is equivalent to I_L decreasing and I_H increasing. An increase in A_M or M similarly expands the set of tasks performed by medium skill workers and contracts those allocated to low and high skill workers. Mathematically, this corresponds to a decline in I_L and an increase in I_H. The implications of an increase in A_L or L are analogous, and raise both I_L and I_H, expanding the set of tasks performed by low skill workers.

Second, the fact that relative demand curves are downward sloping for all factors, as claimed in Part 2, parallels the results in the canonical model (or in fact the more general results in Acemoglu (2007), for any model with constant or diminishing returns at the aggregate level). The new result here concerns the impact of an increase in M on w_H/w_L. We have seen that such an increase raises I_H and reduces I_L, expanding the set of tasks performed by medium skill workers at the expense of both low and high skill workers. This will put downward pressure on the wages of both low and high skill workers, and the impact on the relative wage, w_H/w_L, is ambiguous for reasons we will encounter again below. In particular, it will depend on the form of the comparative advantage schedules in the neighborhood of I_L and I_H. When the absolute value of $\beta_L'(I_L)$ is high (relative to $\beta_H'(I_H)$), this implies that low skill workers have a strong comparative advantage for tasks below I_L. Consequently, medium skill workers will not be displacing low skill workers much, instead having a relatively greater impact on high skill workers, and in this case w_H/w_L will decline. Conversely, when the absolute value of $\beta_L'(I_L)$ is low relative to the absolute value of $\beta_H'(I_H)$, high skill workers have a strong comparative advantage for tasks right above I_H, and medium skill tasks will expand at the expense of low skill workers relatively more, thus increasing w_H/w_L.

Third, the results summarized in Part 3 of the proposition, linking wages to technologies, are also intuitive. For example, an increase in A_H, corresponding to high skill biased technical change, increases both w_H/w_L and w_H/w_M (i.e., high skill wages rise relative to both medium skill and low skill wages) as we may have expected from the canonical model. Perhaps more interestingly, an increase in A_H also unambiguously reduces w_M/w_L despite the fact that it reduces the set of tasks performed by both medium and low skill workers. Intuitively, the first order (direct) effect of an increase in A_H is to contract the set of tasks performed by medium skill workers. The impact on low skill workers is indirect, resulting from the fact that medium skill workers become cheaper and this makes firms expand the set of tasks that these workers perform. This indirect effect never dominates the direct effect, and thus the wages of medium skill workers decrease relative to those of low skill workers when there is high skill biased technical change.

The implications of medium skill biased technical changes are distinct from the canonical case. Medium skill biased technical changes have a direct effect on both high skill and low skill workers. Consequently, the behavior of w_H/w_L is ambiguous. Similarly to how an increase in M affects w_H/w_L, the impact of a rise in A_M on w_H/w_L depends on the exact form of the comparative advantage schedules. When $\beta_L'(I_L)$ is larger in absolute value than $\beta_H'(I_H)$, w_H/w_L is more likely to decline. Intuitively, this corresponds to the case in which low skill workers have strong comparative advantage for tasks below I_L relative to the comparative advantage of high skill workers for tasks above I_H. In this case, medium skill workers will expand by more into (previously) high skill tasks than (previously) low skill tasks. The levels of I_L and $1 - I_H$ also matter for this result; the higher is I_L, the smaller is the effect on low skill wages of a given size reduction in the set of tasks performed by low skill workers (and vice versa for $1 - I_H$).

Finally, we can further parameterize the task productivity schedules, $\alpha_L(i)$, $\alpha_M(i)$ and $\alpha_H(i)$, and perform comparative statics with respect to changes in these schedules. Naturally in this case unambiguous comparative statics are not always obtainable—though, as discussed below, changes that twist or shift these schedules in specific ways lead to intuitive results.

One attractive feature of the model, highlighted by the characterization results and the comparative statics in Proposition 2, is that all equilibrium objects depend on the set of tasks performed by the three different groups of workers. Depending on which set of tasks expands (contracts) more, wages of the relevant group increase (decrease). This is useful for understanding the workings of the model and also provides a potentially tractable connection between the model and the data.

Wage effects

Given the comparative static results on the relative wages and the numeraire equation, Eq. (28), we can derive predictions on the effects of technical change on wage levels.

Although these are in general more complicated than the effects on *relative* wages, it should be intuitively clear that there is a central contrast between our framework and the canonical model: any improvement in technology in the canonical model raises the wages of all workers, whereas in our task-based framework an increase in A_H (high skill biased technical change), for example, can reduce the wages of medium skilled workers because it erodes their comparative advantage and displaces them from (some of) the tasks that they were previously performing.[69]

To see how high skill biased technical change, i.e., an increase in A_H, can reduce medium skill wages more explicitly, let us work through a simple example. Return to the special case discussed above where the task productivity schedule for the low skill workers is given by (31), implying that $I_L = \tilde{I}_L$. Suppose also that $\beta_H(i) \equiv \ln \alpha_M(i) - \ln \alpha_H(i)$ is constant, so that the no arbitrage condition between high and medium skills in Fig. 25 (or Fig. 22) is flat. Now consider an increase in A_H. This will not change I_L (since $I_L = \tilde{I}_L$ in any equilibrium), but will have a large impact on I_H (in view of the fact that the no arbitrage locus between high and medium skills is flat). Let us next turn to an investigation of the implications of this change in A_H on medium skill wages.

Recall from the same argument leading to (25) that

$$w_M = P_M A_M.$$

Since A_M is constant, the effect on medium skill wages works entirely through the price index for tasks performed by medium skill workers. To compute this price index, let us use (21) and (22) to substitute for P_L and P_H in terms of P_M in (28). This gives

$$\begin{aligned}\ln P_M = {} & I_L \left[\ln \left(\frac{A_L L}{A_M M} \right) + \ln (I_H - I_L) - \ln I_L \right] \\ & + (1 - I_H) \left[\ln \left(\frac{A_H H}{A_M M} \right) + \ln (I_H - I_L) - \ln (1 - I_H) \right] \\ & + \int_0^{I_L} \ln \alpha_L(i)\, \mathrm{d}i + \int_{I_L}^{I_H} \ln \alpha_M(i)\, \mathrm{d}i + \int_{I_H}^{1} \ln \alpha_H(i)\, \mathrm{d}i.\end{aligned}$$

Now differentiating this expression, we obtain

$$\frac{\partial \ln P_M}{\partial \ln A_H} = \frac{1 - I_H}{A_H} + (\ln \alpha_M(I_H) - \ln \alpha_H(I_H)) \frac{\mathrm{d}I_H}{\mathrm{d} \ln A_H}$$

[69] One could, however, draw a parallel between changes in (factor-augmenting) technology in this model and changes in the distribution parameter, γ, in the canonical model (recall footnote 54). Unlike factor-augmenting technologies, shifts in the distribution parameter can reduce the wages of the skill group whose corresponding multiplier is reduced.

$$
+ \left[\left(\frac{I_L}{I_H - I_L} \right) + 1 + \frac{1 - I_H}{I_H - I_L} \right.
$$
$$
\left. - \left(\ln \left(\frac{A_H H}{A_M M} \right) + \ln \left(I_H - I_L \right) - \ln \left(1 - I_H \right) \right) \right] \frac{\mathrm{d} I_H}{\mathrm{d} \ln A_H}.
$$

The first term is positive and results from the indirect effect of the increase in productivity of high skill workers on the wages of medium skill workers operating through q-complementarity (i.e., an increase in productivity increases the wages of all workers because it increases the demand for all types of labor). We know from our comparative static analysis that $\mathrm{d}I_H/\mathrm{d}\ln A_H$ is negative, and moreover given the assumptions we have imposed here, this effect is large (meaning that there will be a large expansion of high skill workers into tasks previously performed by medium skill workers following an increase in A_H). Therefore, if $\alpha_M (I_H) \geq \alpha_H (I_H)$, $A_H H \leq A_M M$, and $1 - I_H \leq I_H - I_L$, the remaining terms in this expression are all negative and can be arbitrarily large (and in fact, some of these inequalities could be reversed and the overall expression could still be negative and arbitrarily large). This implies that an increase in A_H can significantly reduce P_M and thus w_M.

This result illustrates that in our task-based framework, in which changes in technology affect the allocation of tasks across skills, a factor-augmenting increase in productivity for one group of workers can reduce the wages of another group by shrinking the set of tasks that they are performing. This contrasts with the predictions of the canonical model and provides a useful starting point for interpreting the co-occurrence of rising supplies of high skill labor, ongoing skill biased demand shifts (stemming in part from technical change), and falling real earnings among less educated workers.

4.5. Task replacing technologies

A central virtue of our general task-based framework is that it can be used to investigate the implications of capital (embodied in machines) directly displacing workers from tasks that they previously performed. In general, we expect that tasks performed by all three skill groups are subject to machine displacement. Nevertheless, based on the patterns documented in the data above, as well as the general characterization of machine-task substitution offered by Autor et al. (2003), we believe the set of tasks most subject to machine displacement in the current era are those that are routine or codifiable. Such tasks are primarily, though not exclusively, performed by medium skill (semi-skilled) workers. For this reason, let us suppose that there now exists a range of tasks $[I', I''] \subset [I_L, I_H]$ for which $\alpha_K (i)$ increases sufficiently (with fixed cost of capital r) so that they are now more economically preformed by machines than middle skill workers. For all the remaining tasks, i.e., for all $i \notin [I', I'']$, we continue to assume that

$\alpha_K(i) = 0$. What are the implications of this type of technical change for the supply of different types of tasks and for wages?

Our analysis directly applies to this case and implies that there will now be a new equilibrium characterized by thresholds $\hat{I}_L$ and $\hat{I}_H$. Moreover, we have the following proposition generalizing Lemma 1 and Proposition 1 for this case:

Proposition 3. *Suppose we start with an equilibrium characterized by thresholds* $[I_L, I_H]$ *and technical change implies that the tasks in the range* $[I', I''] \subset [I_L, I_H]$ *are now performed by machines. Then after the introduction of machines, there exists new unique equilibrium characterized by new thresholds* $\hat{I}_L$ *and* $\hat{I}_H$ *such that* $0 < \hat{I}_L < I' < I'' < \hat{I}_H < 1$ *and for any* $i < \hat{I}_L$, $m(i) = h(i) = 0$ *and* $l(i) = L/\hat{I}_L$*; for any* $i \in (\hat{I}_L, I') \cup (I'', \hat{I}_H)$, $l(i) = h(i) = 0$ *and* $m(i) = M/(\hat{I}_H - I'' + I' - \hat{I}_L)$*; for any* $i \in (I', I'')$, $l(i) = m(i) = h(i) = 0$*; and for any* $i > \hat{I}_H$, $l(i) = m(i) = 0$ *and* $h(i) = H/(1 - \hat{I}_H)$.

This proposition immediately makes clear that, as a consequence of machines replacing tasks previously performed by medium skill workers, there will be a reallocation of tasks in the economy. In particular, medium skill workers will now start performing some of the tasks previously allocated to low skill workers, thus increasing the supply of these tasks (the same will happen at the top with an expansion of some of the high skill tasks). This proposition therefore gives us a way of thinking about how new technologies replacing intermediate tasks (in practice, most closely corresponding to routine, semi-skilled occupations) will directly lead to the expansion of low skill tasks (corresponding to service occupations).

We next investigate the wage inequality implications of the introduction of these new tasks. For simplicity, we focus on the case where we start with $[I', I''] = \varnothing$, and then the set of tasks expands to an interval of size ε', where ε' is small. This mathematical approach is used only for expositional simplicity because it enables us to apply differential calculus as above. None of the results depend on the set of tasks performed by machines being small.

Under the assumptions outlined here, and using the results in Proposition 3, we can write the equivalents of (32) and (33) as

$$\ln A_M - \ln A_H + \beta_H(I_H) + \ln M - \ln H - \ln(I_H - I_L - \varepsilon) + \ln(1 - I_H) = 0, \quad (34)$$

and

$$\ln A_L - \ln A_M + \beta_L(I_L) + \ln L - \ln M + \ln(I_H - I_L - \varepsilon) - \ln(I_L) = 0. \quad (35)$$

When $\varepsilon = 0$, these equations give the equilibrium before the introduction of machines replacing medium skill tasks, and when $\varepsilon = \varepsilon' > 0$, they describe the new equilibrium. Conveniently, we can obtain the relevant comparative statics by using these two equations. In particular, the implications of the introduction of these new machines on the allocation of tasks is obtained from the following system:

$$\begin{pmatrix} \beta_H'(I_H) - \dfrac{1}{I_H - I_L} - \dfrac{1}{1-I_H} & \dfrac{1}{I_H - I_L} \\ \dfrac{1}{I_H - I_L} & \beta_L'(I_L) - \dfrac{1}{I_H - I_L} - \dfrac{1}{I_L} \end{pmatrix} \begin{pmatrix} \mathrm{d}I_H \\ \mathrm{d}I_L \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{I_H - I_L} \\ \dfrac{1}{I_H - I_L} \end{pmatrix} \mathrm{d}\varepsilon.$$

It is then straightforward to verify that

$$\begin{aligned} \frac{\mathrm{d}I_H}{\mathrm{d}\varepsilon} &= \frac{1}{I_H - I_L} \frac{-\beta_L'(I_L) + \frac{1}{I_L}}{\Delta} > 0, \\ \frac{\mathrm{d}I_L}{\mathrm{d}\varepsilon} &= \frac{1}{I_H - I_L} \frac{\beta_H'(I_H) - \frac{1}{1-I_H}}{\Delta} < 0, \\ \frac{\mathrm{d}(I_H - I_L)}{\mathrm{d}\varepsilon} &= \frac{1}{I_H - I_L} \frac{-\beta_L'(I_L) - \beta_H'(I_H) + \frac{1}{1-I_H} + \frac{1}{I_L}}{\Delta} > 0, \end{aligned}$$

where recall that Δ is the determinant of the matrix on the left hand side. These results confirm the statements in Proposition 3 concerning the set of tasks performed by low and high skill workers expanding.

Given these results on the allocation of tasks, we can also characterize the impact on relative wages. These are stated in the next proposition. Here, we state them for the general case, rather than the case in which the range of tasks performed by machines is infinitesimal, since they can be generalized to this case in a straightforward manner (proof omitted).

Proposition 4. *Suppose we start with an equilibrium characterized by thresholds $[I_L, I_H]$ and technical change implies that the tasks in the range $[I', I''] \subset [I_L, I_H]$ are now performed by machines. Then:*

1. w_H/w_M *increases;*
2. w_M/w_L *decreases;*
3. w_H/w_L *increases if* $\left|\beta_L'(I_L)\, I_L\right| < \left|\beta_H'(I_H)(1 - I_H)\right|$ *and* w_H/w_L *decreases if* $\left|\beta_L'(I_L)\, I_L\right| > \left|\beta_H'(I_H)(1 - I_H)\right|$.

The first two parts of the proposition are intuitive. Because new machines replace the tasks previously performed by medium skill workers, their relative wages, both compared to high and low skill workers, decline. In practice, this corresponds to the wages of workers in the middle of the income distribution, previously performing relatively routine tasks, falling compared to those at the top and the bottom of the wage distribution. Thus the introduction of new machines replacing middle skilled tasks in this framework provides a possible formalization of the "routinization" hypothesis and a possible explanation for job and wage polarization discussed in Section 2.

Note that the impact of this type of technical change on the wage of high skill relative to low skill workers is ambiguous; it depends on whether medium skill workers displaced by machines are better substitutes for low or high skill workers. The condition $\left|\beta_L'(I_L) I_L\right| < \left|\beta_H'(I_H)(1 - I_H)\right|$ is the same as the condition we encountered in Proposition 3, and the intuition is similar. The inequality $\left|\beta_L'(I_L)\right| < \left|\beta_H'(I_H)\right|$ implies that medium skill workers are closer substitutes for low than high skill workers in the sense that, around I_H, there is a stronger comparative advantage of high skill relative to medium skill workers than there is comparative advantage of low relative to medium skill workers around I_L. The terms I_L and $(1 - I_H)$ have a similar intuition. If the set of tasks performed by high skill workers is larger than the set of tasks performed by low skill workers $((1 - I_H) > I_L)$, the reallocation of a small set of tasks from high to medium skill workers will have a smaller effect on high skill wages than will an equivalent reallocation of tasks from low to medium skill workers (in this case, for low skill wages).

It appears plausible that in practice, medium skill workers previously performing routine tasks are a closer substitute for low skill workers employed in manual and service occupations than they are for high skill workers in professional, managerial and technical occupations.[70] Indeed the substantial movement of medium skill high school and some college workers out of clerical and production positions and into service occupations after 1980 (Fig. 14) may be read as *prima facie* evidence that the comparative advantage of middle skill workers (particularly middle skill males) is relatively greater in low rather than high skill tasks. If so, Part 3 of this proposition implies that we should also see an increase in w_H/w_L. Alternatively, if sufficiently many middle skill workers displaced by machines move into high skill occupations, w_H/w_L may also increase. This latter case would correspond to one in which, in relative terms, low skill workers are the main beneficiaries of the introduction of new machines into the production process.

Let us finally return to the basic comparative statics and consider a change in the task productivity schedule of high skill workers, $\alpha_H(i)$. Imagine, in particular, that this schedule is given by

$$\alpha_H(i) = \begin{cases} \theta^{\tilde{I}_H - i}\tilde{\alpha}_H(i) & \text{if } i \leq \tilde{I}_H \\ \tilde{\alpha}_H(i) & \text{if } i > \tilde{I}_H \end{cases} \tag{36}$$

where $\tilde{\alpha}_H(i)$ is a function that satisfies Assumption 1 and $\theta \geq 1$, and suppose that $\tilde{I}_H$ is in the neighborhood of the equilibrium threshold task for high skill workers, I_H. The presence of the term $\theta^{\tilde{I}_H - i}$ in (36) implies that an increase in θ creates a rotation of the task productivity schedule for high skill workers around $\tilde{I}_H$.

70 Juhn (1994) develops a model in which middle skill workers are closer substitutes for low than high skill workers. A decline in demand for middle skill workers consequently places greater downward pressure on low than high skill wages.

Consider next the implications of an increase in θ. This will imply that high skill workers can now successfully perform tasks previously performed by medium skill workers, and hence high skill workers will replace them in tasks close to $\tilde{I}_H$ (or close to the equilibrium threshold I_H). Therefore, even absent machine-substitution for medium skill tasks, the model can generate comparative static results similar to those discussed above. This requires that the task productivity schedule for high skill (or low skill) workers twists so as to give them comparative advantage in the tasks that were previously performed by medium skill workers. The parallel roles that technology (embodied in machinery) and task productivity schedules (represented by $\alpha(\cdot)$) play in the model is also evident if we interpret the task productivity schedule of high skill workers more broadly as including not only their direct productivity when performing a task, but also their productivity when supervising (or operating) machinery used in those tasks. Thus the framework offers a parallel between the analytics of, on the one hand, new machinery that replaces medium skill workers and, on the other hand, changes in the task productivity schedule of high skill workers that enable them to replace medium skill workers in a subset of tasks.

4.6. Endogenous choice of skill supply

We have so far focused on one type of substitution, which we referred to as substitution of skills across tasks. A complementary force is *substitution of workers across different skills*, meaning that in response to changes in technology or factor supplies, workers may change the types of skills they supply to the market. We now briefly discuss this additional type of substitution.

Environment

To allow for substitution of workers across different types of skills, we now assume that each worker j is endowed with some amount of "low skill," "medium skill," and "high skill," respectively l^j, m^j and h^j. Workers have one unit of time, which is subject to a "skill allocation" constraint

$$t_l^j + t_m^j + t_h^j \leq 1.$$

The worker's income is

$$w_L t_l^j l^j + w_M t_m^j m^j + w_H t_h^j h^j,$$

which captures the fact that the worker with skill vector $\left(l^j, m^j, h^j\right)$ will have to allocate his time between jobs requiring different types of skills. Generally, we will see that each worker will prefer to allocate his or her time entirely to one type of skill.

The production side of the economy is identical to the framework developed so far. Our analysis then applies once we know the aggregate amount of skills of different types.

Let us denote these by

$$L=\int_{j\in E_l} l^j \mathrm{d}j, \qquad M=\int_{j\in E_m} m^j \mathrm{d}j, \quad \text{and} \quad H=\int_{j\in E_h} h^j \mathrm{d}j,$$

where E_l, E_m and E_h are the sets of workers choosing to supply their low, medium and high skills respectively.

Clearly, the worker will choose to be in the set E_h only if

$$\frac{l^j}{h^j} \le \frac{w_H}{w_L} \quad \text{and} \quad \frac{m^j}{h^j} \le \frac{w_H}{w_M}.$$

There are similar inequalities determining when a worker will be in the sets E_m and E_l. To keep the model tractable, we now impose a type of *single-crossing* assumption in supplies. We order workers over the interval $(0, 1)$ in such a way that lower indexed workers have a comparative advantage in supplying high relative to medium skills and in medium relative to low skills. More specifically, we impose:

Assumption 2. h^j/m^j and m^j/l^j are both strictly decreasing in j and $\lim_{j\to 0} h^j/m^j = \infty$ and $\lim_{j\to 1} m^j/l^j = 1$.

This assumption implies that lower index workers have a comparative advantage in high skill tasks and higher index workers have a comparative advantage in low skill tasks. Moreover, at the extremes these comparative advantages are strong enough that there will always be some workers choosing to supply high and low skills. An immediate implication is the following lemma:

Lemma 2. *For any ratios of wages w_H/w_M and w_M/w_L, there exist $J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ such that $t_h^j = 1$ for all $j < J^*(w_H/w_M)$, $t_m^j = 1$ for all $j \in (J^*(w_H/w_M), J^{**}(w_M/w_L))$ and $t_l^j = 1$ for all $j > J^{**}(w_M/w_L)$. $J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ are both strictly increasing in their arguments.*

Clearly, $J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ are defined such that

$$\frac{m^{J^*(w_H/w_M)}}{h^{J^*(w_H/w_M)}} = \frac{w_H}{w_M} \quad \text{and} \quad \frac{l^{J^{**}(w_M/w_L)}}{m^{J^{**}(w_M/w_L)}} = \frac{w_M}{w_L}. \tag{37}$$

In light of this lemma, we can write

$$\begin{aligned} H &= \int_0^{J^*(w_H/w_M)} h^j \mathrm{d}j, \qquad M=\int_{J^*(w_H/w_M)}^{J^{**}(w_M/w_L)} m^j \mathrm{d}j \quad \text{and} \\ L &= \int_{J^{**}(w_M/w_L)}^1 l^j \mathrm{d}j. \end{aligned} \tag{38}$$

Note that given Assumption 2, $J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ are both strictly increasing in their arguments. This implies that all else equal, a higher wage premium for high relative to medium skills encourages more workers to supply high rather than medium skills to the market. The same type of comparative static applies when there is a higher premium for medium relative to low skills. In particular, rewriting (38), we have

$$\frac{H}{M} = \frac{\int_0^{J^*(w_H/w_M)} h^j \mathrm{d}j}{\int_{J^*(w_H/w_M)}^{J^{**}(w_M/w_L)}} \quad \text{and} \quad \frac{M}{L} = \frac{\int_{J^*(w_H/w_M)}^{J^{**}(w_M/w_L)}}{\int_{J^{**}(w_M/w_L)}^{1} l^j \mathrm{d}j}. \tag{39}$$

The first expression, together with the fact that $J^*(w_H/w_M)$ is strictly increasing, implies that holding w_M/w_L constant, an increase in w_H/w_M increases H/L. Similarly, holding w_H/w_M constant, an increase in w_M/w_L increases M/L. Consequently, in addition to the comparative advantage of different types of skills across different tasks, we now have comparative advantage of workers in supplying different types of skills, which can be captured by two "upward sloping" relative supply curves.

The next proposition and the associated comparative static results exploit these insights.

Proposition 5. *In the model with endogenous supplies, there exists a unique equilibrium summarized by* $(I_L, I_H, P_L, P_M, P_H, w_L, w_M, w_H, J^*(w_H/w_M), J^{**}(w_M/w_L), L, M, H)$ *given by Eqs* (21)–(28), (37) and (38).

To prove the uniqueness of the equilibrium requires a little more work in this case, and the argument is thus relegated to the Theoretical Appendix.

Comparative statics and interpretation

The major change to the analysis introduced by allowing for the endogenous supply of skills is that when there is factor-augmenting technical change (or the introduction of capital that directly substitutes for workers in various tasks), the induced changes in wages will also affect supplies (even in the short run). Accordingly, there will also be substitution of workers across different types of skills. When, for example, new machines replace medium skill workers in a set of tasks, this will induce some of the workers that were previously supplying medium skills to now supply either low or high skills. If the more elastic margin is the one between medium and low skills, we would expect a significant fraction of the workers previously supplying medium skills and working in intermediate tasks to now supply low skills and perform relatively low-ranked tasks. This type of substitution therefore complements the substitution of skills across tasks. Finally, assuming that effective supplies are distributed unequally across workers, this model also generates a richer distribution of earnings inequality (and richer implications for overall inequality).

We can potentially interpret the changes in the US wage and employment structures over the last several decades through the lens of this framework. Let us take the comparative advantage schedules as given, and consider what combinations of factor-augmenting technical changes, introduction of new machines replacing tasks previously performed by different types of workers, and supply changes would be necessary to explain the patterns we observe. As we have seen, during the 1980s the US labor market experienced declining wages at the bottom of the distribution together with a relative contraction in employment in low wage occupations (though notably, a rise in employment in service occupations as underscored by Autor and Dorn (2010)), and also rising wages and employment in high skill occupations. In terms of our model, this would be a consequence of an increase in A_H/A_M and A_M/A_L, which is the analog of skill biased technical change in this three factor model. We see a different pattern commencing in the 1990s, however, where the behavior of both employment shares and wage percentiles is U-shaped, as documented above. In terms of our model, this would result from rising penetration of information technology that replaces middle skill tasks (i.e., those with a substantial routine component). This will depress both the wages of medium skill workers and reduce employment in tasks that were previously performed by these medium skill workers. In the most recent decade (2000s), employment in low wage service occupations has grown even more rapidly. In terms of our model, this could be an implication of the displacement of medium skill workers under the plausible assumption that the relative comparative advantage of middle skill workers is greater in low than high skill tasks. This would therefore be an example of substitution of skills across tasks. This process is amplified in our model if we also allow for substitution of workers across skills. In that case, some of the workers previously supplying medium skills to routine tasks switch to supplying low skills to manual and service tasks.

We stress that this interpretation of the gross patterns in the data is speculative and somewhat coarse. Our objective here is not to provide a definitive explanation for the rich set of facts offered by the data but rather to offer a set of tools that may be applied towards a more refined set of explanations.[71]

[71] Autor and Dorn (2010), for example, offer a closely related but distinct interpretation of the same patterns. In their model, advancing information technology displaces non-college workers performing routine tasks in production of goods, leading these workers to supply manual labor to service tasks instead. This is equivalent to substitution of skills across tasks in the current model. In Autor and Dorn (2010), this supply effect initially depresses wages in low skill services. But as the price of automating routine tasks becomes ever cheaper, the opportunity for further substitution of skills across tasks is eventually exhausted when essentially all non-college workers have exited goods production. At this point, the imperfect substitutability in consumption between goods and services outputs drives wage setting in services as in Baumol (1967). If the substitution elasticity between goods and services is less than or equal to unity, wage inequality between college workers (who supply abstract tasks to goods production) and non-college workers (who supply manual tasks to service production) either asymptotes to a constant or reverses direction—leading to wage and employment polarization. The Autor and Dorn (2010) hypothesis, as well as the framework developed here, can explain the rapid growth in service occupation employment starting in the 1980s, a period when routine-intensive occupations were in decline (see Fig. 13).

4.7. Offshoring

Alongside technological advances, a major change potentially affecting the US and other advanced market economies over the past two decades has been the change in the structure of international trade, whereby instead of simply trading finished goods and services, there has been a greater tendency to engage in trade in tasks through "outsourcing" and "offshoring" certain tasks to countries where they can now be performed at lower cost. This process particularly applies to information-based tasks, which in recent years have become nearly costless and instantaneous to transport. An advantage of our task-based model is that it provides a unified framework for the analysis of this type of offshoring (or outsourcing) in a way that parallels the impact of machines replacing tasks previously performed by certain types of workers.

To illustrate how offshoring of tasks affects the structure of wages, suppose that a set of tasks $[I', I''] \subset [I_L, I_H]$ can now be offshored to a foreign country, where wages are sufficiently low that such offshoring is cost minimizing for domestic final good producers. This assumption, of course, parallels our analysis of machines replacing tasks. In return, these firms can trade in the final good to ensure trade balance. In this case, it is straightforward to see that the equivalents of Propositions 3 and 4 will hold. In particular, the next proposition contains the relevant results summarizing the implications of offshoring for the allocation of tasks across workers and for wage inequality.

Proposition 6. *Suppose we start with an equilibrium characterized by thresholds* $[I_L, I_H]$ *and changes in technology allow tasks in the range* $[I', I''] \subset [I_L, I_H]$ *to be offshored. Then after offshoring, there exists new unique equilibrium characterized by new thresholds* $\hat{I}_L < I_L$ *and* $\hat{I}_H > I_H$ *such that* $0 < \hat{I}_L < I' < I'' < \hat{I}_H < 1$ *and for any* $i < \hat{I}_L$, $m(i) = h(i) = 0$ *and* $l(i) = L/\hat{I}_L$*; for any* $i \in (\hat{I}_L, I') \cup (I'', \hat{I}_H)$, $l(i) = h(i) = 0$ *and* $m(i) = M/(\hat{I}_H - I'' + I' - \hat{I}_L)$*; for any* $i \in (I', I'')$, $l(i) = m(i) = h(i) = 0$*; and for any* $i > \hat{I}_H$, $l(i) = m(i) = 0$ *and* $h(i) = H/(1 - \hat{I}_H)$*. The implications of offshoring on the structure of wages are as follows:*

1. w_H/w_M *increases;*
2. w_M/w_L *decreases;*
3. w_H/w_L *increases if* $\left|\beta_L'(I_L)\, I_L\right| < \left|\beta_H'(I_H)(1 - I_H)\right|$ *and* w_H/w_L *decreases if* $\left|\beta_L'(I_L)\, I_L\right| > \left|\beta_H'(I_H)(1 - I_H)\right|$.

While the extension of the model to offshoring is immediate, the substantive point is deeper. The task-based model offers an attractive means, in our view, to place labor supply, technological change, and trading opportunities on equal economic footing. In our model, each is viewed as offering a competing supply of tasks that, in equilibrium, are allocated to productive activities in accordance with comparative advantage and cost minimization. This approach is both quite general and, we believe, intuitively appealing.

4.8. Directed technical change

We have so far investigated the implications of extending and, in some senses rewriting, the canonical model by allowing for the endogenous allocation of skill groups across tasks and workers across skill groups, and considering how technology and offshoring interact with this process. A final, potentially significant aspect of the economic environment absent from the canonical model is the endogeneity of technological progress to other changes in the labor market. We now discuss how this endogenous technology aspect can be incorporated to enrich our understanding of the operation of the labor market as well as the task-based model we have so far developed.

General discussion

Acemoglu (1998, 2002a) argues that both long run and medium run changes in US labor markets can be understood, at least partly, as resulting from endogenous changes in technology that responds to changes in supplies. From this perspective, Tinbergen's race between supplies and technology is endogenously generated. Autonomous changes in skill supplies—resulting from demographic trends, evolving preferences, and shifts in public and private education—induce endogenous changes in technology, which increase the demand for skills. These demand shifts in turn lead to endogenous increases in skill supplies and, subsequently, further technological progress. While the impact of technological change on the supply of skills (responding to the skill premium) is standard, the response of technology to (relative) supplies is the more central and novel part of this explanation.

Formally, papers by Acemoglu (1998, 2002b) generalize the canonical model with two types of skills and two types of factor-augmenting technologies so as to endogenize the direction of technical change (and thus the relative levels of the two technologies). This work shows that an increase in the relative supply of skills will endogenously cause technology to become more skill biased. Moreover, this induced skill bias could be strong enough that endogenous technology (or "long-run") relative demand curves can be upward sloping rather than downward sloping. This contrasts with the necessarily downward sloping relative demand for skills in the canonical model and also in the Ricardian model studied here (which, so far, holds technology constant). If the induced response of technology is sufficiently strong to make the endogenous relative demand curves upward sloping, then the increase in the skill premium that the US and many OECD labor markets experienced during the last three decades may be, at least in part, a response to the large increase in the supply of skills that commenced in these economies some decades earlier (around the 1960s).

Acemoglu (2002b) showed that for this strong form of endogenous skill bias (in the context of the canonical model), an elasticity of substitution between high and low skill labor greater than a certain threshold (which is somewhere between one and two) is sufficient. Thus for reasonable values of the elasticity of substitution, the induced response of technology to supplies will be strong enough to make the long-run price of skills increase in response to increases in the supply of skills—a stark contrast to the neoclassical model with constant technology, which always predicts that demand curves for factors are downward sloping.

A shift in focus from the canonical model to a task-based framework significantly enriches the mechanisms by which technology can respond endogenously to changes in (relative) supplies. In particular, in the context of our Ricardian model, we can allow two types of endogenous responses of technologies to changes in supplies. First, we can assume that factor-augmenting technologies respond to skill supplies (namely the terms A_L, A_M, and A_H). This idea is analyzed by Acemoglu and Zilibotti (2001) for the special case of our model discussed in Section 4.3.[72] Second, we can also allow for the comparative advantage schedules (the $\alpha(\cdot)$'s) to respond endogenously to skill supplies. This case is both more novel and more relevant to our discussion of the importance of tasks to understanding major labor market developments, and we pursue it here.

While we would have to impose specific functional forms to derive exact results on how comparative advantage schedules will endogenously respond to skill supplies, we can derive more abstract (though nevertheless quite tight) predictions about the direction of change of technology by using the more general framework introduced in Acemoglu (2007). To do this, let us suppose that technologies are presented by a finite dimensional variable (vector) $\theta \in \Theta$, and all three comparative advantage schedules are functions of this vector of technology, i.e., we have $\alpha_L(i \mid \theta)$, $\alpha_M(i \mid \theta)$ and $\alpha_H(i \mid \theta)$. Since any changes in the factor-augmenting terms, A_L, A_M, and A_H, can be incorporated into these comparative advantage schedules, we hold the factor-augmenting terms constant.

We assume as in Acemoglu (2007) that a set of monopolistically competitive or oligopolistic firms invest in technologies θ, produce intermediate goods (or machines) embedding this technology, and sell them to final good producers. We also assume that the cost of producing technology θ is convex in θ. An equilibrium is given by a set of allocations (prices, employment levels and technology levels) such that taking technology levels as given, final good producers maximize profits, and simultaneously, taking the demands for technologies from the final good sector as given, technology monopolists (oligopolists) maximize profits. Also, following Acemoglu (2007), we will say that a change in technology is *(absolutely) biased* towards factor f (where $f \in \{L, M, H\}$) if the change in technology increases the price of that factor, w_f (where again $f \in \{L, M, H\}$) at the prevailing factor proportions (i.e., when the supplies of the three factors are given

[72] Acemoglu and Zilibotti (2001) showed that the response of factor-augmenting technology to supplies works exactly in the same way in this task-based model as in the canonical model studied in Acemoglu (1998, 2002b). In particular, because the special case studied in Acemoglu and Zilibotti (2001) is equivalent to a version of the canonical model with an elasticity of substitution equal to two, technology adjusts in the long run in that model to make the relative demand for skills entirely flat. It is straightforward to extend this result, again in the model with only high and low skill workers, so that technology adjusts more or less than this amount. Hence, all of the results in Acemoglu (1998, 2002b) generalize for factor-augmenting technical change in this task-based environment.

by L, M, and H).[73] Mathematically, a change in technology is biased towards factor f if $w_f(L, M, H \mid \theta)$, written as a function of the supply levels of the three factors, is nondecreasing in θ. In particular, when θ is a continuous one-dimensional variable (i.e., $\theta \in \mathbb{R}$) and the wage levels are differentiable, this is equivalent to:[74]

$$\frac{\partial w_f(L, M, H \mid \theta)}{\partial \theta} \geq 0.$$

Moreover, we say that an increase in the supply of a factor induces technical change that is *weakly biased* towards that factor (again focusing on the continuous one-dimensional variable representing technology) if

$$\frac{\partial w_f(E_{-f}, E_f \mid \theta)}{\partial \theta} \frac{\mathrm{d}\theta}{\mathrm{d}E_f} \geq 0,$$

where E_f is the supply level of factor f (for $f \in \{L, M, H\}$), $w_f(E_{-f}, E_f \mid \theta) = w_f(L, M, H \mid \theta)$, and $\mathrm{d}\theta/\mathrm{d}E_f$ is the induced change in technology resulting from a change in the supply of this factor. Using the same notation, we also say that an increase in the supply of a factor induces technical change that is *strongly biased* towards that factor if

$$\frac{\mathrm{d}w_f(E_{-f}, E_f \mid \theta)}{\mathrm{d}E_f} = \frac{\partial w_f(E_{-f}, E_f \mid \theta)}{\partial E_f} + \frac{\partial w_f(E_{-f}, E_f \mid \theta)}{\partial \theta} \frac{\mathrm{d}\theta}{\mathrm{d}E_f} > 0,$$

where the notation makes it clear that in contrast to the weak bias case, we are evaluating in this case the change in the price as the supply also changes (and thus we have the first term, which is the direct effect of a change in supply for given technology). Put differently, we are now tracing an "endogenous technology" demand curve. In the case of weak bias, however, factor supplies are held constant (as emphasized by the use of the partial derivative), so weak bias requires only that the technology-constant demand curve shifts in favor of the factor whose increased supply induced the initial change in technology (represented by $\mathrm{d}\theta/\mathrm{d}E_f$).

Without specifying either the shape of the comparative advantage schedules or how specifically they depend upon θ, the results in Acemoglu (2007) enable us to have the following two results. Here we state the results without the full mathematical details.

[73] The qualifier "absolutely" is introduced, since in Acemoglu (1998, 2002b), bias refers to changes in technologies affecting relative prices, whereas in this more general framework, the focus is on the price level of a factor. To obtain sharp results on relative price changes, one needs to restrict the focus to factor-augmenting changes (see Acemoglu (2007)). In what follows, all of the references to biased technical change refer to factor price levels, and thus one could insert the qualifier "absolute," though we will not do so as to simplify terminology.

[74] When θ is a continuous multidimensional variable (a vector), there is a straightforward generalization of this definition (see Acemoglu (2007)). All of the results we discuss here are valid in this general case, but to simplify the exposition, we will not introduce the necessary notation.

More rigorous statements of these propositions follow the formulation in Acemoglu (2007), where proofs for these results can be found.

Proposition 7. *Under regularity conditions (which ensure the existence of a locally isolated equilibrium), an increase in the supply of factor* f *(for* $f \in \{L, M, H\}$*) will induce technical change weakly biased towards that factor.*

This proposition thus shows that even under the richer form of technical change considered in our Ricardian model (in particular shifts in the comparative advantage schedules in response to changes in supplies), the response of the economy to any increase in the supply of a factor will be to undergo an endogenous change in technology that *weakly* increases the demand for that factor. Therefore, even in the context of the richer task-based approach developed here, this result implies that there are strong theoretical reasons to expect the increase in the supply of high skill workers, which the US and OECD economies experienced over the past three decades, to have induced the development of technologies favoring these high skill workers. This result does not, however, state that this induced response will be strong enough to increase the price of the factor that it is becoming more abundant (i.e., it does not state that long-run demand curves incorporating endogenous technological change will be upward sloping). This question is investigated in the next proposition.

Proposition 8. *Under regularity conditions (which ensure the existence of a locally isolated equilibrium), an increase in the supply of factor* f *(for* $f \in \{L, M, H\}$*) will induce technical change strongly biased towards that factor—thus increasing the wage of that factor—if and only if the aggregate production possibilities set of the economy is locally nonconvex in factor* f *and technology* θ.

This local nonconvexity condition implies, loosely, that if we double both the supply of factor f and the quality or quantity of technology θ, output will more than double. This form of nonconvexity is quite common in models of endogenous technical change (e.g., Romer, 1990, and see Acemoglu, 2002b), and it is *not* a very demanding condition for one primary reason: the technology is not chosen by the same set of firms that make the factor demand decisions; if it were, and if these firms were competitive, then the equilibrium could not exhibit such local nonconvexity. In our setting (as in Acemoglu, 2007), however, final good producers make factor demands decisions taking technology as given (while facing constant or diminishing returns), and technology monopolists or oligopolists make technology decisions taking the factor demands of final good producers as given (while again facing convex decision problems). In this formulation, the aggregate production possibilities set of the economy need not be locally convex (in each of the factors and the vector of technologies). For example, the result on upward sloping relative demand curves with endogenous technologies in Acemoglu (1998, 2002b) mentioned above corresponds to this type of nonconvexity, and as explained above, only relies on an elasticity of substitution greater than a certain threshold (between one and two). Therefore, strong bias of technology does not require unduly strong conditions, though of course whether it applies in practice is an empirical question on which there is limited evidence.

An example

We now provide a simple example illustrating how endogenous technology enriches the insights of our task-based model here (and conversely, how the task-based approach enriches the implications of existing models of directed technical change). Let us return to the task productivity schedule for high skill workers in (36) discussed in Section 4.5. Suppose, as we did there, that the equilibrium threshold task for high skill workers, I_H, is close to $\tilde{I}_H$. Assume, however, that θ is now an endogenous variable, taking the value θ_{low} or $\theta_{\mathrm{high}} > \theta_{\mathrm{low}}$. As in the general directed technical change framework described so far in this section, we continue to assume that θ is chosen by profit maximizing technology firms, which then sell machines (intermediate goods) embodying this technology to final good producers.

When will technology firms choose θ_{high} instead of θ_{low}? Recall that, as a starting point, the equilibrium threshold I_H is close to $\tilde{I}_H$. This implies that high skill workers are not performing many tasks below $\tilde{I}_H$ (or in fact, if $I_H > \tilde{I}_H$, they are not performing any tasks below $\tilde{I}_H$). As a result, the return from increasing their productivity in tasks lower than $\tilde{I}_H$ would be limited. Therefore, we can presume that to start with, $\theta = \theta_{\mathrm{low}}$.

Now imagine that the supply of high skill workers, H, increases. The general results we have discussed so far imply that technology will adjust (if technology is indeed endogenous) in a way that is biased towards high skill workers. However, these results are silent on what the impact of this induced change in technology will be on medium skill (or low skill) workers. With the specific structure outlined here, however, this endogenous technology response will create effects that are predictable. In particular, as H increases, the equilibrium threshold task for high skill workers, I_H, will decline given the existing technology (θ_{low}). Suppose that after the change, I_H lies significantly below $\tilde{I}_H$. This generates a potentially large economic return to increasing the productivity of high skill workers in the tasks on the interval I_H to $\tilde{I}_H$. This is accomplished by raising θ from θ_{low} to θ_{high}. From our discussion in Section 4.5, however, we know that this corresponds to a change in technology that will induce high skill workers to become more productive in tasks previously performed by medium skill workers, which potentially further contracts the set of tasks performed by medium skill workers. As per our interpretation in Section 4.5, this process is analytically similar to the case in which new machines replace medium skill workers in the tasks that they were previously performing.

Hence, the endogenous technology response to an expansion in the supply of high skill workers (in this case from θ_{low} to θ_{high}) may not only bias technology in their favor (i.e., raising their productivity), but may also induce them to perform some of the tasks previously performed by medium skill workers (either directly, or by supervising the operation of new machinery). With an analysis similar to that in Section 4.4, this process of endogenous technological change can lead to a decline in the wages of medium skill workers.

Overall, this example illustrates how the endogenous response of technology to changes in relative supplies—or, similarly, to changes in trade or offshoring possibilities—may lead to a rich set of changes in both task productivities and the allocation of skills to tasks. Whether this endogenous technology response is in fact a central determinant of the changes in task allocations that have taken place over the past 30 years is an area for further research.

5. COMPARATIVE ADVANTAGE AND WAGES: AN EMPIRICAL APPROACH

We finally take a step back from the theoretical framework to consider how the broad implications of the model might be brought to the data. A key implication of the theory is that holding the schedule of comparative advantage (that is, the $\alpha(\cdot)'s$) constant, changes in the market value of tasks should affect the evolution of wages by skill group. In particular, our model makes a relatively sharp prediction: if the relative market price of the tasks in which a skill group holds comparative advantage declines, the relative wage of that skill group should also decline—*even if* the group reallocates its labor to a different set of tasks (i.e., due to the change in its comparative advantage).

Critical to this prediction is the distinction made between the wages paid to a skill group and the wages paid to a given task—a distinction that is meaningful because the assignment of skills to tasks is endogenous. To see the implications of this distinction, consider a technological change that raises the productivity of high skill workers in all tasks (e.g., an increase in A_H). The model implies that this would expand the set of tasks performed by high skill workers (i.e., lower I_H), so that some tasks formerly performed by medium skilled workers would now be performed by high skill workers instead. Thus, relative wages paid to workers performing these (formerly) "middle skill" tasks would actually increase (since they are now performed by the more productive high skill workers). But crucially, our analysis also shows that the relative wage of medium skill workers, who were formerly performing these tasks, would fall.[75]

This discussion underscores that because of the endogenous assignment of skills to tasks, it is possible for the relative wage paid to a task to move in the *opposite* direction from the relative wage paid to the skill group that initially performed the task.[76] By contrast,

75 Recall in particular from Proposition 2 that $dI_H/d\ln A_H < 0$ and $d\ln(w_H/w_M)/d\ln A_H > 0$, and thus w_M/w_H will fall.

76 Nor is this notion far-fetched. Skill levels in production and clerical occupations, as measured by the college employment or wage-bill share, have risen as employment in these occupations has declined (Autor et al., 1998). A plausible interpretation of this pattern is that educated workers have comparative advantage in the set of non-routine tasks in these occupations that remain.

the relative wage paid to a given skill group always moves in the same direction as its comparative advantage—that is, a technological change that increases the productivity of a skill group necessarily raises its relative wage. Simultaneously, it alters the set of tasks to which that skill is applied.

As a stylized example of how this insight might be brought to the data, we study the evolution of wages by skill groups, where skill groups are defined according to their initial task specialization across abstract-intensive, routine-intensive, and manual-intensive occupations. We take these patterns of occupational specialization as a rough proxy for comparative advantage. Consider the full set of demographic groups available in the data, indexed by gender, education, age, and region. At the start of the sample in 1959, we assume that these groups have self-selected into task specialities according to comparative advantage, taking as given overall skill supplies and task demands (reflecting also available technologies and trade opportunities). Specifically, let γ^A_{sejk}, γ^R_{sejk} and γ^S_{sejk} be the employment shares of a demographic group in abstract, routine and manual/service occupations in 1959, where s denotes gender, e denotes education group, j denotes age group, and k denotes region of the country.[77] By construction, we have that $\gamma^A_{sejk} + \gamma^R_{sejk} + \gamma^S_{sejk} = 1$.

Let w_{sejkt} be the mean log wage of a demographic group in year t and $\Delta w_{sejk\tau}$ be the change in w during decade τ. We then estimate the following regression model:

$$\Delta w_{sejk\tau} = \sum_t \beta^A_t \cdot \gamma^A_{sejk} \cdot 1\left[\tau = t\right] + \sum_t \beta^S_t \cdot \gamma^S_{sejk} \cdot 1\left[\tau = t\right] + \delta_\tau + \phi_e + \lambda_j + \pi_k + e_{sejk\tau}, \tag{40}$$

where δ, ϕ, λ, and π are vectors of time, education, age and region dummies. The β^S_t and β^A_t coefficients in this model estimate the decade specific slopes on the initial occupation shares in predicting wage changes by demographic group. The routine task category (γ^R_{sejk}) serves as the omitted reference group. Thus we are conceiving of demographic groups as skill groups, and the γ parameters as reflecting their patterns of comparative advantage in 1959.

Our working hypothesis is that the labor market price of routine tasks has declined steeply over the last three decades due to rising competition from information technology. Conversely, we conjecture that the labor market prices of abstract and manual tasks will have increased since these tasks are relatively complementary to the routine tasks (now produced at lower cost and in greater quantity by capital). This hypothesis implies that we should expect the wages of workers with comparative advantage in either abstract or manual/service tasks to rise over time while the opposite should occur for skill

[77] Here, abstract occupations are professional, managerial and technical occupations; routine occupations are sales, clerical, administrative support, production, and operative occupations; and service occupations include protective service, food preparation, cleaning, buildings and grounds, and personal care and personal services.

groups with comparative advantage in routine tasks. Formally, we anticipate that β_t^A and β_t^S will rise while the intercepts measuring the omitted routine task category (δ_τ) will decline. These expected effects reflect the rising earnings power of skill groups that hold comparative advantage in abstract and manual/service tasks relative to skill groups that hold comparative advantage in routine tasks.

Table 10 presents initial descriptive OLS regressions of Eq. (40) using Census wage and occupation data from years 1959 through 2008. Although this empirical exercise is highly preliminary—indeed, it is intended as an example of an empirical approach rather than a test of the theory—the pattern of results appears roughly consistent with expectations. Starting with the estimate for males in column 1, we find a rise in relative wages from the 1980s forward for male skill groups that were initially specialized in abstract tasks. Similarly, starting in the 1980s, we see a substantial increase in the relative wage of male demographic subgroups that had an initial specialization in manual/service tasks. In fact, this task specialty moved from being a strongly negative predictor of wages in the 1960s and 1970s, to a positive predictor from the 1980s forward.

Since the interactions between time dummies and each demographic group's initial routine occupation share (γ_{sejk}^R) serves as the omitted reference category in the regression model, these time intercepts estimate wage trends for demographic groups that hold comparative advantage in routine tasks. Consistent with a decline in the wages of workers with comparative advantage in routine tasks, the routine occupation intercepts fall from strongly positive in the 1960s to weakly positive in the 1970s, and then become negative from the 1980s forward.

The second column repeats the initial estimate, now adding main effects for education, age group, and region. Here, the model is identified by differences in initial comparative advantage among workers within education-age-region cells. The inclusion of these demographic group main effects does not appreciably alter the results.

Columns 3 and 4 repeat these estimates for females. As with males, the estimates indicate rising relative wages from 1980 forward for female demographic subgroups that were initially specialized in abstract tasks. The pattern for the service tasks is less clear cut for females, however. Service task specialization is surprisingly associated with strong wage gains during the 1960s and 1970s. This association becomes negative in the 1980s, which is not consistent with the hypothesis above. It then becomes positive (as predicted) in the final two decades of the sample (column 4).

Finally, the routine task specialty intercepts for females go from weakly positive in the 1960s to strongly negative in the 1970s forward. Thus, the decline in the routine task intercepts starts a decade earlier for females than males. Inclusion of main effects for education, age group and region generally strengthens these results and brings them closer in line with our hypotheses.

We stress that this initial cut of the data is intended as an example of how linking the comparative advantage of skill groups to changes over time in the demands for

Table 10 OLS stacked first-difference estimates of the relationship between demographic group occupational distributions in 1959 and subsequent changes in demographic groups' mean log wages by decade, 1959-2007.

	A. Males		B. Females	
	(1)	(2)	(1)	(2)
Abstract occupation share				
1959 share × 1959–1969 time dummy	0.021	0.033	0.146	0.159
	(0.044)	(0.104)	(0.041)	(0.081)
1959 share × 1969–1979 time dummy	−0.129	−0.123	−0.054	−0.032
	(0.044)	(0.105)	(0.036)	(0.079)
1959 share × 1979–1989 time dummy	0.409	0.407	0.143	0.174
	(0.046)	(0.106)	(0.033)	(0.079)
1959 share × 1989–1999 time dummy	0.065	0.060	0.070	0.107
	(0.049)	(0.109)	(0.033)	(0.079)
1959 share × 1999–2007 time dummy	0.198	0.194	0.075	0.113
	(0.051)	(0.11)	(0.033)	(0.08)
Service occupation share				
1959 share × 1959–1969 time dummy	−0.836	−1.014	0.359	0.404
	(0.278)	(0.303)	(0.064)	(0.09)
1959 share × 1969–1979 time dummy	−0.879	−0.991	0.304	0.363
	(0.295)	(0.316)	(0.065)	(0.091)
1959 share × 1979–1989 time dummy	1.007	0.917	−0.143	−0.060
	(0.332)	(0.349)	(0.074)	(0.096)
1959 share × 1989–1999 time dummy	0.202	0.143	0.117	0.221
	(0.378)	(0.39)	(0.086)	(0.104)
1959 share × 1999–2007 time dummy	0.229	0.212	−0.056	0.058
	(0.398)	(0.408)	(0.094)	(0.109)
Decade dummies				
1959–1969	0.274	0.274	0.120	0.046
	(0.031)	(0.037)	(0.021)	(0.032)
1969–1979	0.084	0.085	−0.083	−0.163
	(0.033)	(0.038)	(0.020)	(0.033)
1979–1989	−0.287	−0.283	−0.011	−0.099
	(0.036)	(0.041)	(0.021)	(0.034)
1989–1999	−0.002	0.002	0.061	−0.035
	(0.039)	(0.045)	(0.022)	(0.035)

(*continued on next page*)

their task specialties could be used to explore and interpret the evolution of wages by skill group. The evidence in Table 10 is therefore only suggestive. But we believe the premise on which this exercise is based is a sound one and has the virtue of exploring a theoretically-grounded set of empirical implications. This exercise and our discussion

Table 10 (continued)

	A. Males		B. Females	
	(1)	**(2)**	**(1)**	**(2)**
1999–2007	−0.157	−0.157	−0.073	−0.171
	(0.041)	(0.046)	(0.024)	(0.036)
Education, age group, and region main effects?	No	Yes	No	Yes
R-squared	0.789	0.821	0.793	0.844
N	400	400	400	400

Source: Census IPUMS 1960, 1970, 1980, 1990 and 2000, and American Community Survey 2008. Each column presents a separate OLS regression of stacked changes in mean log real hourly wages by demographic group and year, where demographic groups are defined by sex, education group (high school dropout, high school graduate, some college, college degree, post-college degree), age group (25-34, 35-44, 45-54, 55-64), and region of residence (Northeast, South, Midwest, West). Models are weighted by the mean start and end-year share of employment of each demographic group for each decadal change. Occupation shares are calculated for each demographic group in 1959 (using the 1960 Census) and interacted with decade dummies. Occupations are grouped into three exhaustive and mutually exclusive categories: (1) abstract—professional, managerial and technical occupations; (2) service—protective service, food service and cleaning, and personal services occupations; (3) routine—clerical, sales, administrative support, production, operative and laborer occupations. The routine group is the omitted category in the regression models.

at the beginning of this section, also emphasize that an alternative, and at first appealing, approach of regressing wages on measures of current tasks performed by workers could generate potentially misleading results.[78] In contrast, the approach here exploits the fact that task specialization in the cross section is informative about the comparative advantage of various skill groups, and it marries this source of information to a well-specified hypothesis about how the wages of skill groups that differ in their comparative advantage should respond to changes in technology, shifts in trade and offshoring opportunities, and fluctuations in skill supplies.[79]

78 As above, because the allocation of workers to tasks is endogenous, the wages paid to a set of workers previously performing a given task can fall even as the wages paid to the workers now performing that task rise. Our framework therefore suggests that a regression of wages on tasks currently performed, or their change over time, would be difficult to interpret.

79 A recent working paper by Firpo et al. (2009) also develops an innovative method for measuring the impact of changing task prices on wage structure. Using a simple statistical model of occupational wage setting, they predict that occupations that are specialized in tasks that have declining market value should see a reduction in both mean occupational wages and the *variance* of occupational wages (and vice versa for tasks with rising prices). This latter (variance) effect stems from the interaction between a falling task price and a fixed distribution of task efficiencies within an occupation; as the market value of a given task falls, the variances of wages paid to workers with differing productivities in that task compresses along with it. An issue that needs further study in their approach is that changes in task prices will presumably lead to changes in self-selection into occupations, as implied by our model (and more generally by the assumption that workers are making maximizing choices). This should also affect occupational wage means and variances. Firpo, Fortin and Lemieux's exploratory analysis finds a significant role for both routine-task displacement and, to a lesser extent, offshoring in contributing to US wage polarization between 1984 and 2001. In addition, their analysis emphasizes the contribution of declining labor union penetration and shifts in demographic composition to wage polarization.

6. CONCLUDING REMARKS

In this paper, we argue that to account for recent changes in the earnings and employment distribution in the United States and other advanced economies, and also to develop a better understanding of the impact of technology on labor market outcomes, it is necessary to substantially enrich the canonical model. Specifically, we propose relaxing the assumptions implicit in this model that: (i) the assignment of skills to tasks is fixed (or, more precisely, that skills and tasks are equivalent); and (ii) technical change takes a purely factor-augmenting form. These strictures, we believe, prevent the model from shedding light on key phenomena presented by the data and documented above. These include: (1) substantial declines in real wages of low skill workers over the last three decades; (2) marked, non-monotone changes in earnings levels in different parts of the earnings distribution during different decades; (3) the polarization in the earnings distribution, particularly associated with a "convexification" in the returns to schooling (and perhaps in the returns to other skills); (4) systematic, non-monotone changes in the distribution of employment across occupations of various skill levels; (5) the introduction of new technologies—as well as offshoring possibilities in part enabled by those technologies—that appear to directly substitute machines (capital) for a range of tasks previously performed by (moderately-skilled) workers.

Having documented these patterns and highlighted why they are particularly challenging for the canonical model, we argue that a task-based framework, in which tasks are the basic unit of production and the allocation of skills to tasks is endogenously determined, provides a fruitful alternative framework.

In the task-based framework proposed in this chapter, a unique final good is produced combining services of a continuum of tasks. Each worker has one of three types of skills, low, medium and high. We assume a pattern of comparative advantage such that tasks are ranked in order of complexity, and medium skill workers are more productive than low skill workers, and less productive than high skill workers in more complex tasks. We show that the equilibrium allocation of skills to tasks is determined by two thresholds, I_L and I_H, such that all tasks below the lower threshold (I_L) are performed by low skill workers, all tasks above the higher threshold (I_H) are performed by high skill workers, and all intermediate tasks are performed by medium skill workers. In terms of mapping this allocation to reality, we think of the lowest range of tasks as corresponding to service occupations and other manual occupations that require physical flexibility and adaptability but little training. These tasks are straightforward for the large majority of workers, but require a degree of coordination, sightedness, and physical flexibility that are not yet easily automated. The intermediate range corresponds to moderately skilled blue-collar production and white-collar administrative, clerical, accounting and sales positions that require execution of well-defined procedures (such as calculating or monitoring) that can increasingly be codified in software and performed by inexpensive machinery. Finally, the highest range corresponds to the abstract reasoning, creative, and problem-solving tasks performed by professionals, managers and some technical occupations. These tasks require a skill set that is currently challenging to automate because the procedures used to perform these tasks are poorly understood.

We show that despite the endogenous allocation of skills to tasks, the model is tractable, and that relative wages among skill groups depend only on relative supplies and the equilibrium threshold tasks. Comparative statics of relative wages then depend on how these thresholds change. For example, whenever I_L increases (for fixed supplies of low, medium and high skills in the market), there is a larger range of tasks performed by low skill workers and their relative wages increase. Similarly, when I_H decreases, the wages of high skill workers increase and when $I_H - I_L$ increases, the relative wages of medium skill workers increase. We also show that an increase in the supply of high skills, or alternatively, technical change that makes high skill workers uniformly more productive, reduces I_H (intuitively, because there is greater "effective supply" of high skills). In addition to this direct effect, such a change also has an indirect effect on I_L, because the decrease in I_H, at given I_L, creates an "excess supply" of medium skill workers in intermediate tasks and thus induces firms to substitute these workers for tasks previously performed by low skill workers.

A noteworthy implication of this framework is that technical change favoring one type of worker can reduce the real wages of another group. Therefore, the richer substitution possibilities between skill groups afforded by the endogenous allocation of skills to tasks highlights that, distinct from canonical model, technical change need not raise the wages of all workers. As importantly, this framework enables us to model the introduction of new technologies that directly substitute for tasks previously performed by workers of various skill levels. In particular, we can readily model how new machinery (for example, software that corrects spelling and identifies grammatical errors) can directly substitute for job tasks performed by various skill groups. This type of technical change provides a richer perspective for interpreting the impact of new technologies on labor market outcomes. It also makes negative effects on the real wages of the group that is being directly replaced by the machinery more likely. These same ideas can also be easily applied to the process of outsourcing and offshoring. Since some tasks are far more suitable to offshoring than others (e.g., developing web sites versus cutting hair), it is natural to model offshoring as a technology (like computers) that potentially displaces domestic workers of various skill levels performing certain tasks, thereby altering their wages by increasing their effective supply and causing a shift in the mapping between skills and tasks (represented by I_L and I_H).

We also show how the model can be extended to incorporate choices on the side of workers to allocate their labor hours between different types of activities and how technical change can be endogenized in this framework. When the direction of technical change and the types of technologies being adopted are endogenous, not only do we obtain the same types of insights that the existing literature on directed technical change generates, but we can also see how the development and the adoption of technologies substituting machines for tasks previously performed by (middle skill) workers can emerge as a response to changes in relative supplies.

We view our task-based framework and the interpretation of the salient labor market facts through the lenses of this framework as first steps towards developing a richer and more nuanced approach to the study of interactions between technology, tasks and skills in modern labor markets. Indeed, it will be a successful first step if this framework provides a foundation for researchers to generate new theoretical ideas and test them empirically. In the spirit of a first step, we suggest one means of parsing changes in real wages over time by demographic groups that is motivated by this theoretical model. Clearly, more needs to be done to derive tighter predictions from this framework and from other complementary task-based approaches for the evolution of earnings and employment distribution both in the United States and other countries. We view this as a promising area for future research.

We also believe that the study of a number of closely related topics in labor economics may be enriched when viewed through this task perspective, though we must only mention them cursorily here:

Organizational change: Acemoglu (1999), Bresnahan (1999), Bresnahan et al. (1999), Caroli and van Reenen (1999), Kremer and Maskin (1996), Garicano (2000), Autor et al. (2002), Dessein and Santos (2006), and Garicano and Rossi-Hansberg (2006) among others, have emphasized the importance of organizational changes as an autonomous factor shaping the demand for skills or, alternatively, as a phenomenon accompanying other equilibrium changes impacting earnings inequality. A task-based approach is implicit in several of these studies, and a systematic framework, like the one proposed here, may enrich the study of the interactions between organizational changes and the evolution of the distribution of earnings and employment. We also note that substitution of machines for tasks previously performed by semi-skilled workers, or outsourcing and offshoring of their tasks, may necessitate significant organizational changes. One might reinterpret the changes in equilibrium threshold tasks in our model as corresponding to a form of organizational change. One might alternatively take the perspective that organizational change will take place in a more discontinuous manner and will involve changes in several dimensions of the organization of production (managerial and job practices, the allocation of authority within the organization, the form of communication, and the nature of responsibility systems). In addition, organizational change might also create tasks, demanding both low and high skill labor inputs, that were not previously present, exerting another force towards polarization. These considerations suggest that the two-way interaction between these organizational changes and the allocation of tasks to different skill groups and technologies is an important area for theoretical and empirical study.

Labor market imperfections: The framework proposed here crucially depends on competitive labor markets, where each worker is paid the value of his or her marginal product. In reality, many frictions—some related to information and search and others resulting from collective bargaining, social norms, firing costs and minimum wage legislation—create a wedge between wages and marginal products. The allocation of skills to tasks is more complex in the presence of such labor market imperfections. Moreover, some of these imperfections might directly affect the choice of threshold tasks. The implications of different types of technical change are potentially quite different in the presence of labor market imperfections, and may in particular depend on the exact form of these frictions. Further work tractably integrating various forms of labor market imperfections within a framework that incorporates the endogenous allocation of skills to tasks appears to be another fruitful area for research.

The role of labor market institutions: Closely related to labor market imperfections, a perspective that emphasizes the importance of tasks also calls for additional study of the role of labor market institutions in the changes in employment and inequality in recent decades. Certain work practices, such as collective bargaining and unionized workplace arrangements, might have greater impact on the earnings distribution because of the way they impact the assignment of tasks to labor or capital. These institutions may restrict the substitution of machines for certain tasks previously performed by workers, particularly in the case of labor unions. Additionally, even if the substitution of machines for labor is not fully impeded, it may occur more slowly than otherwise due to the influence of these institutions. If this force raises the opportunity cost of union membership for some subset of workers (for example, by depressing the return to skill), it may undermine union coalitions, leading to an amplified impact on employment and wages (e.g., Acemoglu et al., 2001). Richer and empirically more important forms of two-way interactions between technology and unions and other workplace arrangements are another fruitful area for future research.

Cross-country trends: We have shown that changes in the occupation of distribution are surprisingly comparable across a sizable set of advanced economies. This fact not withstanding, changes in the earnings distribution have been quite different in different countries (e.g., Davis, 1992; Blau and Kahn, 1996; Card et al., 1996; Katz and Autor, 1999; Card and Lemieux, 2001a,b; Atkinson, 2008; Dustmann et al., 2009; Atkinson et al., 2010; Boudarbat et al., 2010). One interpretation of these facts is that while many advanced countries have experienced similar technological forces that have altered occupational structures, the manner in which their labor markets (in particular their wage schedules) has responded to them have been far from identical. As of yet, there is no satisfactory understanding of the root causes of these differences. One possibility is that the adoption of new technologies either replacing or complementing workers in certain tasks requires up-front fixed investments, and the incentives for adopting these technologies are not only affected by labor supply and demand, but also by existing regulations. It is then possible that firms select different technologies in different countries in accordance with these constraints, and this may affect the evolution of real wages for various skill groups. For example, Acemoglu (2003) suggests a model in which institutionally-imposed wage compression encourages the adoption of technologies that increase the productivity of low skill workers and thus slows demand shifts against these skill groups.

Changes in male-female and white-nonwhite wage differentials: Our empirical analysis highlighted the substantial differences in the evolution of employment and earnings between men and women. The framework and data both suggest that the comparatively poor labor market performance of males may in part be due to the fact that men were more heavily represented in middle skill production occupations that were undercut by automation and offshoring.[80] A similar contrast might exist between white and nonwhite workers. Juhn et al. (1991) provided an early attempt to explain the differential evolution of earnings and employment by race and gender as a result of skill biased demand shifts. A similar comprehensive exercise, with a richer conception of technology potentially rooted in a task-based approach, is a logical next step to obtain a more complete understanding of the recent changes in the distribution of employment and earnings among minority and non-minority groups.

The importance of service occupations: Our framework highlights why recent technical change might have increased employment in service occupations. The idea here is related to Baumol's classic argument, where the demand for labor from sectors experiencing slower technical advances might be greater if there is sufficient complementarity between the goods and services that they and more rapidly growing sectors produce (Baumol, 1967; see also, Acemoglu and Guerrieri, 2007; Pissarides and Ngai, 2007; Autor and Dorn, 2009, 2010). Our framework captures this phenomenon to some degree, but because of the unit elasticity of substitution across all tasks, the extent of this effect is limited. A somewhat different variant of our framework may be necessary to better capture the evolution of the demand for services during the past several decades.

DATA APPENDIX

May/Outgoing Rotation Groups Current Population Survey

Wages are calculated using May/ORG CPS data for earnings years 1973-2009 for all workers aged 16-64 who are not in the military, institutionalized or self-employed. Wages are weighted by CPS sample weights. Hourly wages are equal to the logarithm of reported hourly earnings for those paid by the hour and the logarithm of usual weekly earnings divided by hours worked last week for non-hourly workers. Top-coded earnings observations are multiplied by 1.5. Hourly earners of below \$1.675/hour in 1982 dollars (\$3.41/hour in 2008 dollars) are dropped, as are hourly wages exceeding

[80] We should caveat, however, that female workers have also been substantially displaced over the last two decades from a different set of middle skill tasks (in particular, administrative support and clerical jobs), without seemingly experiencing the adverse wage and employment consequences observed among men.

1/35th the top-coded value of weekly earnings. All earnings are deflated by the chain-weighted (implicit) price deflator for personal consumption expenditures (PCE). Allocated earnings observations are excluded in all years, except where allocation flags are unavailable (January 1994 to August 1995).

March Current Population Survey

Wages are calculated using March CPS data for earnings years 1963-2008 for full-time, full-year workers aged 16-64, excluding those who are in the military or self-employed. Full-time, full-year workers are those who usually worked 35 or more hours per week and worked forty or more weeks in the previous year. Weekly earnings are calculated as the logarithm of annual earnings divided by weeks worked. Calculations are weighted by CPS sampling weights and are deflated using the personal consumption expenditure (PCE) deflator. Earnings of below $67/week in 1982 dollars ($136/week in 2008 dollars) are dropped. Allocated earnings observations are excluded in earnings years 1967 forward using either family earnings allocation flags (1967-1974) or individual earnings allocation flags (1975 earnings year forward).

Census/American Community Survey

Census Integrated Public Use Micro Samples for years 1960, 1970, 1980, 1990, and 2000, and American Community Survey for 2008 are used in this paper. All Census samples include 5% of the population, except 1970, which includes 1% of the population. Wages are calculated for full-time, full-year workers aged 16-64, excluding those who are in the military, institutionalized or self-employed. Weekly earnings are calculated as the logarithm of annual earnings divided by weeks worked. Calculations are weighted by Census sampling weights and are deflated using the personal consumption expenditure (PCE) deflator.

Education categories used for the May/ORG and March CPS files and Census/ACS files are equivalent to those employed by Autor et al. (2003), based on the consistent classification system proposed by Jaeger (1997).

Dictionary of Occupational Titles

The US Labor Department's Dictionary of Occupational Titles (DOT) task measures used in this paper follow the construction of Autor et al. (2006), who collapse Autor et al.'s (2003) original five task measures into three categories: routine, manual and abstract. Routine corresponds to a simple average of two DOT measures: "set limits, tolerances and standards," and "finger dexterity." Manual corresponds to the DOT measure "eye-hand-foot coordination". Abstract is the simple average of two DOT measures: "direction, control and planning" and "GED math." DOT task measures are converted from their original 14,000 detailed occupations to 326 consistent occupations, which allow for merging with CPS and Census data files.

O*NET

O⋆NET task measures used in this paper are composite measures of O⋆NET Work Activities and Work Context Importance scales:
Non-routine cognitive: Analytical

4.A.2.a.4 Analyzing data/information
4.A.2.b.2 Thinking creatively
4.A.4.a.1 Interpreting information for others

Non-routine cognitive: Interpersonal

4.A.4.a.4 Establishing and maintaining personal relationships
4.A.4.b.4 Guiding, directing and motivating subordinates
4.A.4.b.5 Coaching/developing others

Routine cognitive

4.C.3.b.7 Importance of repeating the same tasks
4.C.3.b.4 Importance of being exact or accurate
4.C.3.b.8 Structured v. Unstructured work (reverse)

Routine manual

4.C.3.d.3 Pace determined by speed of equipment
4.A.3.a.3 Controlling machines and processes
4.C.2.d.1.i Spend time making repetitive motions

Non-routine manual physical

4.A.3.a.4 Operating vehicles, mechanized devices, or equipment
4.C.2.d.1.g Spend time using hands to handle, control or feel objects, tools or controls
1.A.2.a.2 Manual dexterity
1.A.1.f.1 Spatial orientation

Offshorability

4.C.1.a.2.l Face to face discussions (reverse)
4.A.4.a.5 Assisting and Caring for Others (reverse)
4.A.4.a.8 Performing for or Working Directly with the Public (reverse)
4.A.1.b.2 Inspecting Equipment, Structures, or Material (reverse)
4.A.3.a.2 Handling and Moving Objects (reverse)
4.A.3.b.4 0.5⋆Repairing and Maintaining Mechanical Equipment (reverse)
4.A.3.b.5 0.5⋆Repairing and Maintaining Electronic Equipment (reverse)

O⋆NET scales are created using the O⋆NET-SOC occupational classification scheme, which we collapse into SOC occupations. Each scale is then standardized to have mean zero and standard deviation one, using labor supply weights from the pooled 2005/6/7 Occupational Employment Statistics (OES) Survey, one of the few large surveys that uses the SOC occupational classification system. The composite task measures listed above are equal to the summation of their respective constituent scales, then standardized to mean zero and standard deviation one. In order to merge the composite task measures with the Census data, the task measures are collapsed to the Census 2000 occupational code level using the OES Survey labor supply weights and then collapsed to the 326 consistent occupations as detailed in Autor and Dorn (2010), using Census 2000 labor supply weights.

THEORETICAL APPENDIX: UNIQUENESS OF EQUILIBRIUM IN PROPOSITION 5

Let us proceed in steps. First, rewrite (23) and (24) as

$$\ln\left(\frac{w_H}{w_M}\right) = \ln\left(\frac{A_H}{A_M}\right) - \beta_H\left(I_H\right), \tag{41}$$

and

$$\ln\left(\frac{w_M}{w_L}\right) = \ln\left(\frac{A_M}{A_L}\right) - \beta_L\left(I_L\right), \tag{42}$$

where recall that $\beta_H(I) \equiv \ln\alpha_M(I) - \ln\alpha_H(I)$ and $\beta_L(I) \equiv \ln\alpha_L(I) - \ln\alpha_M(I)$ are both strictly decreasing in view of Assumption 1. Now substituting these two equations into (38), we have

$$\begin{aligned}
H &= \Gamma_H\left(\ln\left(\frac{A_H}{A_M}\right) - \beta_H\left(I_H\right)\right)\\
M &= \Gamma_M\left(\ln\left(\frac{A_H}{A_M}\right) - \beta_H\left(I_H\right), \ln\left(\frac{A_M}{A_L}\right) - \beta_L\left(I_L\right)\right)\\
L &= \Gamma_L\left(\ln\left(\frac{A_M}{A_L}\right) - \beta_L\left(I_L\right)\right),
\end{aligned}$$

where we denote derivatives of these functions by Γ'_H, Γ'_L, and Γ^1_M and Γ^2_M for the first and second derivatives of Γ_M. The arguments so far immediately imply that $\Gamma'_H > 0$, $\Gamma'_L < 0$ and $\Gamma^1_M < 0$ and $\Gamma^2_M > 0$. Now rewriting (32) and (33) substituting for these, we again have a two-equation system in I_H and I_L characterizing the equilibrium. It is given by

$$\ln A_M - \ln A_H + \beta_H(I_H) + \ln \Gamma_M\left(\ln\left(\frac{A_H}{A_M}\right) - \beta_H(I_H), \ln\left(\frac{A_M}{A_L}\right) - \beta_L(I_L)\right)$$
$$- \ln \Gamma_H\left(\ln\left(\frac{A_H}{A_M}\right) - \beta_H(I_H)\right) - \ln(I_H - I_L) + \ln(1 - I_H) = 0, \tag{43}$$

and

$$\ln A_L - \ln A_M + \beta_L(I_L) + \ln \Gamma_L\left(\ln\left(\frac{A_M}{A_L}\right) - \beta_L(I_L)\right)$$
$$- \ln \Gamma_M\left(\ln\left(\frac{A_H}{A_M}\right) - \beta_H(I_H), \ln\left(\frac{A_M}{A_L}\right) - \beta_L(I_L)\right)$$
$$+ \ln(I_H - I_L) - \ln(I_L) = 0. \tag{44}$$

Let us evaluate the Jacobian of this system at an equilibrium. Following similar steps to those we used in the comparative static analysis before, this can be written as

$$\begin{pmatrix} \beta'_H(I_H)\left[1 + \frac{\Gamma'_H}{\Gamma_H} - \frac{\Gamma^1_M}{\Gamma_M}\right] - \frac{1}{I_H - I_L} - \frac{1}{1 - I_H} & \frac{1}{I_H - I_L} - \frac{\Gamma^2_M}{\Gamma_M}\beta'_L(I_L) \\ \frac{1}{I_H - I_L} + \frac{\Gamma^1_M}{\Gamma_M}\beta'_H(I_H) & \beta'_L(I_L)\left[1 - \frac{\Gamma'_L}{\Gamma_L} + \frac{\Gamma^2_M}{\Gamma_M}\right] - \frac{1}{I_H - I_L} - \frac{1}{I_L} \end{pmatrix}.$$

Since $\Gamma'_H > 0$, $\Gamma'_L > 0$, $\Gamma^1_M > 0$ and $\Gamma^2_M < 0$, the diagonal elements of this matrix are always negative. In addition, we verify that the determinant of this matrix is also always positive. In particular, denoting the determinant by Δ, we have

$$\begin{aligned} \Delta = &\left(\beta'_H(I_H)\left[1 + \frac{\Gamma'_H}{\Gamma_H} - \frac{\Gamma^1_M}{\Gamma_M}\right] - \frac{1}{I_H - I_L} - \frac{1}{1 - I_H}\right) \\ &\times \left(\beta'_L(I_L)\left[1 - \frac{\Gamma'_L}{\Gamma_L} + \frac{\Gamma^2_M}{\Gamma_M}\right] - \frac{1}{I_H - I_L} - \frac{1}{I_L}\right) \\ &- \left(\frac{1}{I_H - I_L} - \frac{\Gamma^2_M}{\Gamma_M}\beta'_L(I_L)\right) \times \left(\frac{1}{I_H - I_L} + \frac{\Gamma^1_M}{\Gamma_M}\beta'_H(I_H)\right) \\ = &\left(\beta'_H(I_H)\left[1 + \frac{\Gamma'_H}{\Gamma_H}\right] - \frac{1}{1 - I_H}\right) \times \left(\beta'_L(I_L)\left[1 - \frac{\Gamma'_L}{\Gamma_L}\right] - \frac{1}{I_L}\right) \\ &- \frac{1}{I_H - I_L} \times \left(\beta'_H(I_H)\left[1 + \frac{\Gamma'_H}{\Gamma_H} - \frac{\Gamma^1_M}{\Gamma_M}\right]\right. \end{aligned}$$

$$
\begin{aligned}
& - \frac{1}{1-I_H} + \beta_L'(I_L)\left[1 - \frac{\Gamma_L'}{\Gamma_L} + \frac{\Gamma_M^2}{\Gamma_M}\right] - \frac{1}{I_L}\Bigg) \\
& - \frac{\Gamma_M^1}{\Gamma_M} \times \left(\beta_L'(I_L)\left[1 - \frac{\Gamma_L'}{\Gamma_L}s\right] - \frac{1}{I_L}\right) \\
& + \frac{\Gamma_M^2}{\Gamma_M} \times \left(\beta_H'(I_H)\left[1 + \frac{\Gamma_H'}{\Gamma_H}\right] - \frac{1}{1-I_H}\right).
\end{aligned}
$$

All five lines of the last expression are positive, and thus so is Δ. This implies that the Jacobian is everywhere a P-matrix, and from Simsek et al. (2007), it follows that there exists a unique equilibrium.

Moreover, given that the determinant is everywhere positive, comparative static results are similar to those of the equilibrium with fixed supplies. For example, an increase in A_H will reduce I_H and increase w_H/w_M and w_M/w_L as before, but also it will increase H/L. Similarly, if new machines replace tasks previously performed by middle skills, this will increase w_H/w_M and reduce w_M/w_L, as workers previously performing middle skill tasks are reallocated to low and high skills. In addition, there will now be a supply response, and workers previously supplying their middle skills will shift to supplying either low or high skills. In particular, if the relevant margin of substitution in the supply side is between middle and low, many of these workers will start supplying low skills to the market, leading to an expansion of low skill tasks.

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