More Robust Estimators for Panel Bartik Designs, With An Application to the Effect of Imports from China on US Employment.*

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First version: March 11, 2021
This version: July 13, 2023

Abstract

We show that panel Bartik regressions identify non-convex combinations of location-and-period-specific treatment effects. Thus, those regressions could be biased if effects are heterogeneous. We propose an alternative instrumental-variable correlated-random-coefficient (IV-CRC) estimator, that is more robust to heterogeneous effects. We revisit Autor et al. (2013), who use a panel Bartik regression to estimate the effect of imports from China on US manufacturing employment. Their regression estimates a highly non-convex combination of effects, and our IV-CRC estimator is small and insignificant: without assuming constant treatment effects, one cannot conclude from their data that imports from China negatively affected US employment.

Keywords: Bartik instrument, correlated random coefficients, heterogeneous treatment effects, panel data, China shock.

JEL Codes: C21, C23, F16

*We are particularly grateful to Xavier D'Haultfœuille, Peter Hull, Michal Kolesár, and Isabelle Méjean for their feedback on this paper. We also thank Teresa Fort, Lucie Gadenne, François Gerard, Paul Goldsmith-Pinkham, Shawn Klimek, Ismael Mourifié, Félix Pasquier, Aureo de Paula, Isaac Sorkin, Martha Stinson, and seminar participants at CREST, McMaster University, PSE, Queen Mary, Tilburg University, the Tinbergen Institute, and the Sao Paulo School of Economics for their helpful comments. Clément de Chaisemartin was funded by the European Union (ERC, REALLYCREDIBLE,GA N°101043899). Views and opinions expressed are those of the authors and do not reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

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1 Introduction

The "Bartik" or "shift-share" instrument is a popular tool to estimate the effect of a treatment on an outcome. For instance, Autor et al. (2013), herafter ADH, use a panel data set of US commuting zones (CZs) to estimate the effect of $D_{g,t}$, the imports from China in CZ g at t,[1] on $Y_{g,t}$, the manufacturing employment in g at t. Some of their regressions leverage two time periods per CZ, while others leverage three periods: to simplify the exposition without great loss of generality, we assume the data has two periods in this introduction. Then, one may estimate an OLS regression of ΔY_g on ΔD_g, where Δ denote the first-difference operator. However, ΔD_g may be endogenous: the evolution of imports from China may be correlated with US demand shocks. Therefore, ADH estimate a 2SLS regression. Assume manufacturing is divided into S sectors indexed by s. Let $Z_{s,t}$ denote imports from China in sector s at t, in a group of high-income countries similar to the US. Let

$$Z_{g,t} = \sum_{s=1}^{S} Q_{s,g} Z_{s,t},$$

where $Q_{s,g}$ is the share sector s accounts for in CZ g's manufacturing employment. $Z_{g,t}$ is the imports CZ g would have experienced at t if all its sectors had experienced the same imports levels as non-US high-income countries. Importantly, $Z_{g,t}$ is not directly determined by US demand. The estimator used by ADH is $\hat{\theta}^b$, the coefficient of ΔD_g in a 2SLS regression of ΔY_g on ΔD_g using ΔZ_g as the instrument. One can show that

$$\hat{\theta}^b = \frac{\sum_{g=1}^{G} \Delta Y_g \left(\Delta Z_g - \Delta Z_.\right)}{\sum_{g=1}^{G} \Delta D_g \left(\Delta Z_g - \Delta Z_.\right)}, \tag{1.1}$$

where $\Delta Z_.$ denotes the average instrument across CZs. With two periods, $\hat{\theta}^b$ is numerically equivalent to the coefficient of $D_{g,t}$ in a 2SLS two-way fixed effects regression (TWFE) of $Y_{g,t}$ on $D_{g,t}$ with location and period fixed effects, using $Z_{g,t}$ as the instrument. $\hat{\theta}^b$ has been used by several other influential papers, including Autor et al. (2020) and Acemoglu & Restrepo (2020).

In this paper, we start by showing that $\hat{\theta}^b$ does not estimate a convex combination of location-and-period-specific treatment effects. Our two first results are simple enough to state finite-sample versions of them in this introduction. Let $Y_{g,t}(d)$ denote the potential outcome of location g at period t if $D_{g,t}$ is equal to d. For instance, in ADH $Y_{g,t}(0)$ is CZ g's potential manufacturing employment at t without any imports from China. We assume that

$$Y_{g,t}(d) = Y_{g,t}(0) + \alpha_{g,t} d, \tag{1.2}$$

meaning that g's potential outcome at t is a linear function of its treatment level, with a location-and-period-specific slope $\alpha_{g,t}$. Then, the observed outcome satisfies

$$Y_{g,t} = Y_{g,t}(0) + \alpha_{g,t} D_{g,t}.$$

[1] ADH's treatment is actually a proxy for g's imports from China at t. The simplified description of their treatment we give in this introduction is not of essence to our main conclusions, and we give their exact treatment definition in Section 4.

First-differencing the previous display yields

$$\Delta Y_g = \Delta Y_g(0) + \alpha_{g,2} D_{g,2} - \alpha_{g,1} D_{g,1}. \tag{1.3}$$

(1.3) highlights a simple but important fact: positing a causal model between a first-differenced outcome and a first-differenced treatment implicitly assumes that the effect of the *level* of the treatment is constant over time: it is only if $\alpha_{g,2} = \alpha_{g,1} = \alpha_g$ that (1.3) simplifies to

$$\Delta Y_g = \Delta Y_g(0) + \alpha_g \Delta D_g. \tag{1.4}$$

Now, plugging (1.3) into (1.1) yields

$$\begin{aligned}
\hat{\theta}^b &= \frac{\sum_{g=1}^{G} \Delta Y_g(0)\left(\Delta Z_g - \Delta Z_.\right)}{\sum_{g=1}^{G} \Delta D_g\left(\Delta Z_g - \Delta Z_.\right)} + \frac{\sum_{g=1}^{G} \left(\alpha_{g,2} D_{g,2} - \alpha_{g,1} D_{g,1}\right)\left(\Delta Z_g - \Delta Z_.\right)}{\sum_{g=1}^{G} \Delta D_g\left(\Delta Z_g - \Delta Z_.\right)} \\
&= \frac{\sum_{g=1}^{G} \Delta Y_g(0)\left(\Delta Z_g - \Delta Z_.\right)}{\sum_{g=1}^{G} \Delta D_g\left(\Delta Z_g - \Delta Z_.\right)} \\
&\quad + \sum_{g=1}^{G}\sum_{t=1}^{2} \frac{(1\{t=2\} - 1\{t=1\}) D_{g,t}(\Delta Z_g - \Delta Z_.)}{\sum_{g'=1}^{G}\sum_{t'=1}^{2}(1\{t'=2\} - 1\{t'=1\}) D_{g',t'}(\Delta Z_{g'} - \Delta Z_.)} \alpha_{g,t}.
\end{aligned} \tag{1.5}$$

Thus, $\hat{\theta}^b$ can be decomposed into the sum of two terms. The first is the coefficient one would get from a 2SLS regression of $\Delta Y_g(0)$, locations' outcome evolution without treatment, on ΔD_g, using ΔZ_g as the instrument. If the instrument is exogeneous, meaning that locations' outcome evolutions without treatment are uncorrelated with ΔZ_g, this term converges to zero. The second term is a weighted sum of the location-and-period-specific slopes $\alpha_{g,t}$, where weights sum to one, but where every location is such that either its period-one or its period-two slope is weighted negatively: $\alpha_{g,1}$ is weighted negatively if $\Delta Z_g > \Delta Z_.$, and $\alpha_{g,2}$ is weighted negatively if $\Delta Z_g < \Delta Z_.$. Negative weights are problematic. Because of them, one could have, say, $\alpha_{g,t} \geq 0$ for all (g, t) but $\hat{\theta}^b < 0$, even asymptotically, and even when the instrument is exogeneous. Assuming constant treatment effects over time ($\alpha_{g,2} = \alpha_{g,1} = \alpha_g$), (1.5) simplifies to

$$\hat{\theta}^b = \frac{\sum_{g=1}^{G} \Delta Y_g(0)\left(\Delta Z_g - \Delta Z_.\right)}{\sum_{g=1}^{G} \Delta D_g\left(\Delta Z_g - \Delta Z_.\right)} + \sum_{g=1}^{G} \frac{\Delta D_g(\Delta Z_g - \Delta Z_.)}{\sum_{g'=1}^{G} \Delta D_{g'}(\Delta Z_{g'} - \Delta Z_.)} \alpha_g. \tag{1.6}$$

Even if the instrument is exogenous, $\hat{\theta}^b$ still does not estimate a convex combination of effects: α_g is weighted negatively for locations such that ΔD_g and $\Delta Z_g - \Delta Z_.$ are of a different sign.

The intuition for (1.5) and (1.6) goes as follows. (1.1) shows that locations such that $\Delta Z_g - \Delta Z_. > 0$ are used as "treatment-group" locations by $\hat{\theta}^b$: their outcome and treatment evolutions are weighted positively. On the other hand, locations such that $\Delta Z_g - \Delta Z_. < 0$ are used as "control-group" locations: their outcome and treatment evolutions are weighted negatively. $\alpha_{g,1}$ enters with a negative sign in the ΔY_g of treatment-group locations (see (1.3)), so it gets weighted negatively by $\hat{\theta}^b$. Similarly, $\alpha_{g,2}$ enters with a positive sign in the ΔY_g of control-group locations (see (1.3)), so it gets weighted negatively by $\hat{\theta}^b$. Assuming constant effects over time, now locations' outcome evolutions are only affected by their treatment evolutions, not by their treatment levels. But if there are treatment-group locations that experienced a negative treatment evolution, the effect of this evolution enters with a negative sign in their ΔY_g (see

(1.4)), and it gets weighted negatively by $\hat{\theta}^b$. Similarly, if there are control-group locations that experienced a positive treatment evolution, the effect of this evolution enters with a positive sign in their ΔY_g (see (1.4)), and it gets weighted negatively.

(1.5) and (1.6) are generic decompositions that apply to any first-difference 2SLS regression. We derive two further results that exploit the specificity of the Bartik instrument, but in both cases we still find that $\hat{\theta}^b$ does not estimate a convex combination of effects. First, if we further assume a linear first-stage model tailored to the structure of the Bartik instrument, $\hat{\theta}^b$ may still not estimate a convex combination of effects, even if the treatment effect is constant over time ($\alpha_{g,2} = \alpha_{g,1} = \alpha_g$) and the first-stage effect of the instrument on the treatment is fully homogeneous, across sectors, locations, and time periods. Second, even if we further assume that the shocks $Z_{s,t}$ are as-good-as randomly assigned, we show that $\hat{\theta}^b$ may still not estimate a convex combination of effects if treatment effects vary over time. This result contrasts with results in Borusyak et al. (2022) and Adão et al. (2019), who have shown that with randomly-assigned-shocks, cross-sectional Bartik regressions estimate a convex combination of effects. Applying directly their result to $\hat{\theta}^b$ requires positing a causal model in first-difference, as in (1.4), which implicitly assumes constant treatment effects over time.

We then propose alternative estimators. The alternatives we recommend depend on whether the randomly-assigned shocks assumption holds. Therefore, we start by proposing two novel tests of this assumption.[2] Our tests may be more powerful than that previously proposed by Borusyak et al. (2022): when we revisit ADH, our tests are rejected while theirs is not. With randomly-assigned shocks, we show that a slightly modified panel Bartik estimator, where shocks are standardized by their period-specific standard deviation when constructing the instrument, estimates a convex combination of effects, even if treatment effects vary over time and across locations. Alternatively, combining the results in Borusyak et al. (2022) and Adão et al. (2019) to those in Angrist (1998), it follows that with randomly-assigned shocks, researchers can also use a "pooled-cross-section" 2SLS Bartik regression of $Y_{g,t}$ on $D_{g,t}$ using $Z_{g,t}$ as the instrument, with period fixed effects but no location fixed effects. When the random-shocks assumption is implausible or rejected, we propose an instrumental-variable correlated-random-coefficient (IV-CRC) estimator, inspired from Chamberlain (1992), that can be used if there are at least three time periods in the data. Our IV-CRC estimator is much more robust to heterogeneous effects than $\hat{\theta}^b$: it estimates the average treatment effect, a very natural target parameter, even if the treatment effect varies across locations and over time. It still imposes some restrictions on treatment effects, as it requires that they follow the same evolution over time in every location. Moreover, it relies on a stronger exogeneity assumption than $\hat{\theta}^b$, and it also relies on the assumption that locations' treatment effects are mean-independent of their treatments conditional on their instruments. We propose suggestive tests of those assumptions. Finally, note that our IV-CRC estimator may be applicable to other instruments than Bartik ones. Due

[2] Pre-testing whether shocks are randomly assigned could lead to some bias when pre-tests lack power, a concern analogous to that highlighted by Roth (2022) in difference-in-differences (DID) studies. The benefit of pre-testing may outweight the cost. When we revisit ADH, our tests are very strongly rejected, so pre-tests do not always lack power in Bartik designs. Moreover à priori assessments of the plausibility of identifying assumptions may sometimes be misleading in Bartik designs. For instance, Borusyak et al. (2022) compellingly argue that the random-shocks assumption is à priori plausible in ADH, while our tests strongly reject that assumption.

to their specific structure, Bartik instruments may be good candidates to satisfy the assumptions underlying our estimator, but there may be other instruments that can satisfy those assumptions.

We use our results to revisit the main Bartik regression in ADH, where they estimate the effect of imports from China on US manufacturing employment, and find a large negative effect. We start by using a gravity-based decomposition of trade flows (see, e.g., Arkolakis et al. 2012) to give economically interpretable sufficient conditions of our main econometric assumptions. Then, we test the randomly-assigned shocks assumption, and find that it is strongly rejected. Under this assumption, sectoral shocks should be uncorrelated with sectors' characteristics, and in particular with sectors' average share across locations. In practice, shocks are strongly correlated with sectoral shares, even conditional on other sectors' characteristics: this is evidence that shocks are not as-good-as randomly assigned, even conditionally. Then, we decompose the Bartik regression in this application. Our first decomposition, following (1.5), indicates that it estimates a highly non-convex combination of CZ-and-period specific effects $\alpha_{g,t}$: nearly 50% of effects are weighted negatively, and negative weights sum to -0.734. Weights are correlated with the year variable, and with several CZ characteristics. This could bias the Bartik regression if the effects $\alpha_{g,t}$ change over time and/or are correlated with those characteristics. Our second decomposition, following (1.6), shows that even if one assumes constant effects over time, the Bartik regression still estimates a highly non-convex combination of effects, where negative weights sum to -0.314. Finally, our IV-CRC estimator is small, insignificant, and significantly different from the Bartik estimator. Given its large standard error, our estimator is compatible with a relatively large range of effects. Still, it shows that without assuming that the effect of imports from China is constant over time and across CZs, one cannot conclude from the data used by ADH that those imports negatively affected US manufacturing employment.

The paper is organized as follows. Section 2 presents our decompositions of Bartik regressions. Section 3 presents our alternative IV-CRC estimator. Section 4 presents our re-analysis of ADH. Section 5 concludes with some recommendations for practitioners. Those may come handy for readers primarily interested in the main take-aways of our long and dense paper.

Related literature

On the applied side, our paper builds upon the groundbreaking work of ADH, who tackle a fundamental economic question for which it is intrinsically hard to find a credible empirical design, in view of the macro-nature of the treatment. We leverage their design, show that the standard estimator they have used is not robust to heterogeneous treatment effects, and propose a more robust estimator. Our results show that without assuming constant effects, one cannot conclude from their data that imports from China had a significantly negative effect on US manufacturing employment, though we find suggestive evidence of heterogeneous effects across commuting zones, depending on their employment share in routine occupations. Our re-analysis differs from and complements recent re-analyses of ADH that have used more granular firm-level data (see Bloom et al. 2019, Ding et al. 2022).

On the methods' side, our paper builds upon the pioneering work of Goldsmith-Pinkham et al. (2020), Borusyak et al. (2022), and Adão et al. (2019). A brief overview of their main contributions may go as follows. Goldsmith-Pinkham et al. (2020) and Borusyak et al. (2022)

have proposed two distinct ways of rationalizing instrument-exogeneity in Bartik designs, the so-called shares and shocks approaches, respectively. Following Borusyak et al. (2022), Adão et al. (2019) have shown that in the shocks approach, conventional standard errors may be misleading, and have proposed alternative standard errors. Our paper is not concerned with rationalizing instrument-exogeneity in Bartik designs: our first two decompositions of $\hat{\theta}^b$ hold even if the instrument is not exogenous, as (1.5) and (1.6) show. Our paper is also not concerned with inference. Another difference is that those papers consider both cross-sectional and panel regressions, while we consider only the panel case. Instead, our paper is concerned with the robustness of panel Bartik regressions to heterogeneous effects. Heterogeneous effects is a less central issue in those papers, though Goldsmith-Pinkham et al. (2020) discuss it in an extension, Borusyak et al. (2022) in their online appendix, and Adão et al. (2019) in the main sections of their paper. To preserve space, we defer a detailed discussion of the connections between our and their results for panel Bartik regressions with heterogeneous effects to Section 2.

Our paper is also related to de Chaisemartin & D'Haultfœuille (2020), who derive decompositions of OLS TWFE regressions under a parallel trends assumption. Our first decomposition of $\hat{\theta}^b$ in Theorem 1 below is related to their Theorem 1: replacing the instrument by the treatment in our Theorem 1 yields the same weights as in that result with two time periods. Thus, our Theorem 1 may be seen as an extension of that result to 2SLS regressions, an extension that was not derived by de Chaisemartin & D'Haultfœuille (2020). de Chaisemartin & D'Haultfœuille (2020) had also not specifically derived a decomposition of OLS TWFE regressions in the special case with two time periods. Our Theorem 1 can be used to derive a closed-form expression of the weights in that special case. That closed-form might be of independent interest. For instance, it shows that with two periods and $D_{g,t} > 0$ for all (g, t), exactly 50% of the weights attached to OLS TWFE regressions are negative, a fact not noted in de Chaisemartin & D'Haultfœuille (2020). More generally, our paper is the first to note that the panel Bartik regression used by ADH, Autor et al. (2020), or Acemoglu & Restrepo (2020), among others, is a 2SLS TWFE regression, and suffers from similar issues as OLS TWFE regressions in the presence of heterogeneous effects and under a parallel-trends-like assumption. Finally, our IV-CRC estimator is inspired from Chamberlain (1992).

2 Panel Bartik regressions with heterogeneous effects

2.1 Setup and notation

Location-level panel data. We consider a panel with G locations, indexed by $g \in \{1, ..., G\}$, and T periods indexed by $t \in \{1, ..., T\}$. We want to use this data set to estimate the effect of a treatment $D_{g,t}$ on an outcome $Y_{g,t}$. To simplify exposition, for now we assume that $T = 2$. In Web Appendix B, we extend some of our decomposition results to Bartik regressions with multiple time periods. Locations are typically geographical regions, e.g. commuting zones (CZs).

Bartik instrument (Bartik 1991). Assume there are S sectors indexed by $s \in \{1, ..., S\}$. Sectors could for instance be industries. Let $Z_{s,t}$ denote a shock affecting sector s at period t.

Definition 1 *For all (g,t), the Bartik instrument $Z_{g,t}$ is:*

$$Z_{g,t} = \sum_{s=1}^{S} Q_{s,g} Z_{s,t}.$$

For all (g,t), $Q_{s,g}$ are positive weights summing to 1 or less, reflecting the importance of sector s in location g at period t. For instance, $Q_{s,g}$ could be the share that sector s accounts for in g's employment at $t=1$. Definition 1 assumes time-invariant shares: all our results can readily be extended to allow for time-varying shares.

Definition 2 (2SLS Panel Bartik estimator) *Let $\Delta Z_. = \frac{1}{G}\sum_{g=1}^{G} \Delta Z_g$, and let*

$$\hat{\theta}^b = \frac{\sum_{g=1}^{G} \Delta Y_g \left(\Delta Z_g - \Delta Z_.\right)}{\sum_{g=1}^{G} \Delta D_g \left(\Delta Z_g - \Delta Z_.\right)}.$$

$\hat{\theta}^b$ is the sample coefficient from a 2SLS regression of ΔY_g on an intercept and ΔD_g, using ΔZ_g as the instrument. Throughout the paper, we consider Bartik regressions that are not weighted, say, by locations' population. In Web Appendix B, we extend some of our decompositions to weighted Bartik regressions. We do not extend our decompositions to Bartik regressions with covariates, but doing so would be a mechanical extension.

Potential outcomes. Let $Y_{g,t}(d)$ denote the potential outcome that location g will experience at period t if $D_{g,t} = d$. $Y_{g,t}(0)$ is g's potential outcome at t without any treatment.

Assumption 1 *Linear Treatment Effect: for all $(g,t) \in \{1,...,G\} \times \{1,...,T\}$, there exists $\alpha_{g,t}$ such that for any d:*

$$Y_{g,t}(d) = Y_{g,t}(0) + \alpha_{g,t} d.$$

Assumption 1 is a linear treatment effect assumption. Goldsmith-Pinkham et al. (2020) and Adão et al. (2019) also make linear treatment effect assumptions.

Sampling and sources of uncertainty. We seek to accommodate two sampling schemes. In the first one, hereafter referred to as the shares approach, the shocks $Z_{s,t}$ are fixed (or conditioned upon), and locations are an independent and identically distributed (iid) sample drawn from a super population of locations. Then, the vectors $(\Delta Z_g, \Delta D_g, \Delta Y_g)$ are iid. This approach is similar to that in Goldsmith-Pinkham et al. (2020). In the second one, hereafter referred to as the shocks approach, the locations are fixed (or conditioned upon), and the vectors of shocks $(Z_{s,1}, Z_{s,2})$ are drawn independently across sectors. This approach is similar to that in Borusyak et al. (2022) and Adão et al. (2019). The exogeneity assumptions we make in Sections 2.2 and 2.3 below are compatible with both approaches.

Definition 3 (2SLS Panel Bartik estimand) *Let*

$$\theta^b = \frac{\sum_{g=1}^{G} E\left(\Delta Y_g \left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right)}{\sum_{g=1}^{G} E\left(\Delta D_g \left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right)}. \tag{2.1}$$

In the approach of Goldsmith-Pinkham et al. (2020), locations are iid, so $\theta^b = \text{cov}(\Delta Y, \Delta Z)/\text{cov}(\Delta D, \Delta Z)$, the probability limit of $\hat{\theta}^b$ when $G \to +\infty$. Similarly, in the shocks approach of Borusyak et al. (2022) and Adão et al. (2019), $\hat{\theta}^b - \theta^b$ converges to zero when $S \to +\infty$.

Instrument relevance. Throughout the paper, we assume that the instrument is relevant: $\sum_{g=1}^{G} E\left(\Delta D_g\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right) \neq 0$. Without loss of generality we can further assume that $\sum_{g=1}^{G} E\left(\Delta D_g\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right) > 0$: the population first-stage is strictly positive.

Definition of robustness to heterogeneous effects We say that θ^b is robust to heterogeneous effects if and only if $\theta^b = E\left(\sum_{g=1}^{G}\sum_{t=1}^{2} w_{g,t}\alpha_{g,t}\right)$, with $E\left(\sum_{g=1}^{G}\sum_{t=1}^{2} w_{g,t}\right) = 1$ and $w_{g,t} \geq 0$ almost surely. One may find this definition too weak, and argue that θ^b is only robust to heterogeneous effects if $\theta^b = E\left(\frac{1}{2G}\sum_{g=1}^{G}\sum_{t=1}^{2} \alpha_{g,t}\right)$, meaning that θ^b identifies the average treatment effect (ATE). All our results below show that θ^b is not robust under our weaker criterion, so θ^b is also not robust under any stricter criterion. Requiring that the weights are almost surely positive is important. As we will see later, $w_{g,t}$ and $\alpha_{g,t}$ are often correlated. Then, having $E\left(w_{g,t}\right) \geq 0$ is not enough to prevent a so-called sign reversal, where, say, $\alpha_{g,t} \geq 0$ almost surely for all (g,t), but $\theta^b < 0$.[3] Under our definition of robustness, such sign reversals cannot happen, even if $w_{g,t}$ and $\alpha_{g,t}$ are correlated.

Shares summing to one? We do not assume that shares sum to one, except when we analyze the robustness of θ^b under the randomly-assigned shocks assumption proposed by Borusyak et al. (2022) and Adão et al. (2019). Indeed, when shares do not sum to one, Borusyak et al. (2022) show that under their assumption, one should not estimate θ^b. Instead, one should estimate a 2SLS regression where the intercept in the regression is replaced by locations' sum of shares (this reduces to estimating θ^b when shares sum to one). We conjecture that results similar to those we derive for θ^b under the random-shocks assumption when shares sum to one also apply to that estimand when shares do not sum to one.

2.2 Panel Bartik is not robust under a linear treatment-effect model

Decomposition of θ^b under Assumption 1.

Theorem 1 *Suppose Assumption 1 holds.*

1. *Then,*

$$\theta^b = \frac{\sum_{g=1}^{G} E\left(\Delta Y_g(0)\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right)}{\sum_{g=1}^{G} E\left(\Delta D_g\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right)} + E\left(\sum_{g=1}^{G}\sum_{t=1}^{2} \frac{(1\{t=2\}-1\{t=1\})D_{g,t}(\Delta Z_g - E\left(\Delta Z_.\right))}{E\left(\sum_{g'=1}^{G}\sum_{t'=1}^{2}(1\{t'=2\}-1\{t'=1\})D_{g',t'}(\Delta Z_{g'} - E\left(\Delta Z_.\right))\right)}\alpha_{g,t}\right).$$

2. *If one further assumes that for all g, there exists α_g such that $\alpha_{g,1} = \alpha_{g,2} = \alpha_g$,*

$$\theta^b = \frac{\sum_{g=1}^{G} E\left(\Delta Y_g(0)\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right)}{\sum_{g=1}^{G} E\left(\Delta D_g\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right)} + E\left(\sum_{g=1}^{G} \frac{\Delta D_g(\Delta Z_g - E\left(\Delta Z_.\right))}{E\left(\sum_{g'=1}^{G}\Delta D_{g'}(\Delta Z_{g'} - E\left(\Delta Z_.\right))\right)}\alpha_g\right).$$

[3] For instance, if $G=1$, $w_{1,1} = -1+3X$, $w_{1,2} = 1-w_1$, $\alpha_{1,1} = 1-X$, and $\alpha_{1,2} = X$, where X follows a Bernoulli distribution with parameter $2/3$, then $E\left(\sum_{g=1}^{G}\sum_{t=1}^{2} w_{g,t}\alpha_{g,t}\right) = -1$.

Consequences of Theorem 1. Point 1 of Theorem 1 shows that under Assumption 1, θ^b can be decomposed into the sum of two terms. The first is the population coefficient one would get from a 2SLS regression of $\Delta Y_g(0)$, locations' outcome evolution without treatment, on ΔD_g, using ΔZ_g as the instrument. The second is the expectation of a weighted sum of the treatment effects $\alpha_{g,t}$, with weights

$$\frac{(1\{t=2\}-1\{t=1\})D_{g,t}(\Delta Z_g - E(\Delta Z_.))}{E\left(\sum_{g'=1}^{G}\sum_{t'=1}^{2}(1\{t'=2\}-1\{t'=1\})D_{g',t'}(\Delta Z_{g'} - E(\Delta Z_.))\right)}. \tag{2.2}$$

If $D_{g,t} > 0$ for all (g,t), as is for instance the case in ADH, then every location whose effects do not receive a weight equal to zero is such that either $\alpha_{g,1}$ or $\alpha_{g,2}$ is weighted negatively. Thus, exactly a half of the effects $\alpha_{g,t}$ are weighted negatively, so θ^b is not robust to heterogeneous effects according to our definition. Point 2 of Theorem 1 shows that even if one assumes homogeneous effects over time, θ^b may still not be robust.

Decomposition of θ^b under Assumption 1 and an exogeneity assumption.

Assumption 2 *(Exogenous instrument)*

1. *For all $g \in \{1,...,G\}$, $cov(\Delta Z_g, \Delta Y_g(0)) = 0$.*
2. *$E(\Delta Z_g)$ does not depend on g.*

Assumption 2 ensures that the first term in the decompositions of θ^b in Theorem 1 is equal to zero. Then, it directly follows from, say, Point 1 of Theorem 1 that under Assumptions 1 and Assumption 2, θ^b is equal to the weighted sum of treatment effects therein. The first point of Assumption 2 requires that location g's potential outcome evolution without any treatment be uncorrelated with its first-differenced Bartik instrument. This condition may be interpreted as a parallel trends assumption. The second point of Assumption 2 requires that $E(\Delta Z_g)$ does not vary across locations. Assumption 2 is a "high-level" exogeneity condition, that nests both the "shares" and "shocks" rationalizations of Bartik exogeneity proposed by Goldsmith-Pinkham et al. (2020) and Borusyak et al. (2022) and Adão et al. (2019). Goldsmith-Pinkham et al. (2020) consider shocks as non-stochastic, and their Assumption 2 requires that $\text{cov}(Q_{s,g}, \Delta Y_g(0)) = 0$. This implies Point 1 of Assumption 2. Point 2 trivially holds in their setting, because they assume iid locations. In our panel data setting, with period fixed effects and no other control variables, Assumption 4.ii) in Adão et al. (2019) requires that for all (s,t),

$$E\left(Z_{s,t}|\left(Y_{g,t'}(0),(Q_{s',g,t'})_{s'\in\{1,...,S\}}\right)_{(g,t')\in\{1,...,G\}\times\{1,2\}}\right) = m_t$$

for some real number m_t. When shares sum to one, this implies that $E(\Delta Z_g) = m_2 - m_1$. Then,

$$\begin{aligned}
&\text{cov}(\Delta Z_g, \Delta Y_g(0))\\
=&E\left(\Delta Y_g(0)\left(\sum_{s=1}^{S}Q_{s,g}E\left(Z_{s,2}|\Delta Y_g(0),(Q_{s,g})_{s\in\{1,...,S\}}\right)\right.\right.\\
-&\left.\left.\sum_{s=1}^{S}Q_{s,g}E\left(Z_{s,1}|\Delta Y_g(0),(Q_{s,g})_{s\in\{1,...,S\}}\right)\right)\right) - (m_2-m_1)E(\Delta Y_g(0))\\
=&0,
\end{aligned}$$

so Assumption 2 holds. In their Appendix A.1, Borusyak et al. (2022) allow for heterogeneous effects and also make an assumption that implies Assumption 2.

Placebo test of Assumption 2. Assumption 2 is "placebo testable", when the data contains prior periods where all locations are untreated, as is sometimes the case (see, e.g., ADH). Then, locations' outcome evolutions without any treatment are observed at those periods, and one can assess if those evolutions are correlated with locations' first-differenced Bartik instrument. If they are not, that lends credibility to Point 1 of Assumption 4: if locations' outcome evolutions without any treatment prior to the Bartik "shocks" are uncorrelated to the first-differenced Bartik instrument, it may be plausible that this untreated outcome evolution, though unobserved when the shock takes place, is also uncorrelated with the first-differenced Bartik instrument.

Practical use of Theorem 1. The weights in Point 1 of Theorem 1 can be estimated, replacing $E(\Delta Z_.)$ by $\Delta Z_.$. Estimating the weights, and assessing if many are negative, can be used to assess the robustness of θ^b. Moreover, when one observes some characteristics of location $\times$ period (g,t)s that are likely to be correlated with their treatment effects $\alpha_{g,t}$, one can test if those characteristics are correlated with the weights. If they are not, it may be plausible to assume that the $\alpha_{g,t}$s and the weights are uncorrelated, in which case one can show that θ^b identifies the ATE. On the other hand, if the weights are correlated with those characteristics, it may not be plausible to assume that the $\alpha_{g,t}$s and the weights are uncorrelated, and θ^b may not identify a meaningful parameter. The weights in Point 2 of Theorem 1 can also be estimated. When it seems plausible that the treatment effect does not vary over time, estimating those weights can be useful to assess if θ^b is robust to heterogeneous effects across locations.

Connection with previous literature. When $\Delta Z_g = \Delta D_g$, meaning that $\hat{\theta}^b$ is actually an OLS regression coefficient, the weights in Point 1 of Theorem 1 reduce to those in the decomposition of OLS TWFE regressions under a parallel trends assumption in Theorem 1 of de Chaisemartin & D'Haultfœuille (2020), in the special case where $T = 2$. Thus, Point 1 of Theorem 1 may be seen as a generalization of that result to 2SLS regressions, in the special case where $T = 2$. de Chaisemartin & D'Haultfœuille (2020) do not give the closed-form expression of the weights in their decomposition in the special case where $T = 2$. That closed-form expression can readily be obtained from Point 1 of Theorem 1, replacing ΔZ_g by ΔD_g, and it might be of independent interest. For instance, it follows from Point 1 of Theorem 1 that when $T = 2$ and $D_{g,t} > 0$ for all (g,t), exactly 50% of the non-zero weights attached to OLS TWFE regressions are negative, a fact that was not noted in de Chaisemartin & D'Haultfœuille (2020). In the shares approach with iid locations, Point 2 of Theorem 1 reduces to

$$\theta^b = E\left(\frac{\Delta D(\Delta Z - E(\Delta Z))}{E\left(\Delta D(\Delta Z - E(\Delta Z))\right)}\alpha\right),$$

a first-difference version of a known result for cross-sectional IV regressions under a linear treatment effect model (see e.g. Equation (3) in Benson et al. 2022). Point 2 of Theorem 1 shows that a similar result holds in first-difference if the treatment effect is constant over time, as then one has a linear treatment effect model in first-difference, as shown in Equation (1.4). In the cross-sectional case, the numerator of the weights is $D(Z - E(Z))$. As D is positive, weights are strictly negative if and only if $D > 0$ and $Z < E(Z)$. In the panel case, ΔD may be negative, so weights are strictly negative if and only if ΔD and $\Delta Z - E(\Delta Z)$ are different from zero and of a different sign, thus leading to a different characterization of observations that receive a negative weight. Theorem 1 applies to any first-difference 2SLS regression with two time periods: at this stage, we have not leveraged the particular structure of the Bartik instrument.

2.3 Panel Bartik is still not robust if one further assumes a linear first-stage

Linear first-stage model. For any $(z_1, ..., z_S) \in \mathbb{R}^S$, let $D_{g,t}(z_1, ..., z_S)$ denote the potential treatment of location g at period t if $(Z_{1,t}, ..., Z_{S,t}) = (z_1, ..., z_S)$. And let $D_{g,t}(\mathbf{0}) = D_{g,t}(0, ..., 0)$ denote the potential treatment of g at t without any shocks. The actual treatment of g at t is $D_{g,t} = D_{g,t}(Z_{1,t}, ..., Z_{S,t})$. We make the following assumption:

Assumption 3 *Linear First-Stage Model: for all $(g, t) \in \{1, ..., G\} \times \{1, ..., T\}$, there exists $(\beta_{s,g,t})_{s\in\{1,...,S\}}$ such that for any $(z_1, ..., z_S)$:*

$$D_{g,t}(z_1, ..., z_s) = D_{g,t}(\mathbf{0}) + \sum_{s=1}^{S} Q_{s,g}\beta_{s,g,t}z_s.$$

Assumption 3 requires that the effect of the shocks on the treatment be linear: increasing $Z_{s,t}$ by 1 unit, holding all other shocks constant, increases the treatment of location g at period t by $Q_{s,g}\beta_{s,g,t}$ units. Similar assumptions are also made by Adão et al. (2019) (see their Equation (11)) and Goldsmith-Pinkham et al. (2020) (see their Equation (8), which we discuss in more details later). Under Assumption 3,

$$D_{g,t} = D_{g,t}(\mathbf{0}) + \sum_{s=1}^{S} Q_{s,g}\beta_{s,g,t}Z_{s,t}. \tag{2.3}$$

If the first-stage effects are constant over time ($\beta_{s,g,2} = \beta_{s,g,1}$ for all g), (2.3) implies

$$\Delta D_g = \Delta D_g(\mathbf{0}) + \sum_{s=1}^{S} Q_{s,g}\beta_{s,g}\Delta Z_s, \tag{2.4}$$

a linear first-stage model relating the first-differenced treatment and shocks. With a slight abuse of notation, let

$$\Delta Y_g(D_g(\mathbf{0})) = Y_{g,2}(0) + \alpha_{g,2}D_{g,2}(\mathbf{0}) - (Y_{g,1}(0) + \alpha_{g,1}D_{g,1}(\mathbf{0}))$$

denote the outcome evolution that location g would have experienced from period one to two without any shocks. Plugging (2.3) into (1.3) yields the following first-differenced reduced-form equation:

$$\Delta Y_g = \Delta Y_g(D_g(\mathbf{0})) + \alpha_{g,2}\sum_{s=1}^{S} Q_{s,g}\beta_{s,g,2}Z_{s,2} - \alpha_{g,1}\sum_{s=1}^{S} Q_{s,g}\beta_{s,g,1}Z_{s,1}. \tag{2.5}$$

If the first-stage and treatment effects are constant over time, (2.5) implies

$$\Delta Y_g = \Delta Y_g(D_g(\mathbf{0})) + \alpha_g \sum_{s=1}^{S} Q_{s,g}\beta_{s,g}\Delta Z_s. \tag{2.6}$$

Identifying assumption with a first-stage model. With our first-stage model in hand, the identifying assumption we consider requires that the instrument be uncorrelated with the reduced-form and first-stage residuals $\Delta Y_g(D_g(\mathbf{0}))$ and $\Delta D_g(\mathbf{0})$, rather than with the second-stage residual $\Delta Y_g(0)$.

Assumption 4 *(Exogenous instrument, v2)*

1. *For all $g \in \{1, ..., G\}$, $cov(\Delta Z_g, \Delta Y_g(D_g(\mathbf{0}))) = 0$.*
2. *For all $g \in \{1, ..., G\}$, $cov(\Delta Z_g, \Delta D_g(\mathbf{0})) = 0$.*
3. *$E(\Delta Z_g)$ does not depend on g.*

The random-shocks assumption in Borusyak et al. (2022) and Adão et al. (2019) implies Assumption 4. Assuming $\text{cov}(Q_{s,g}, \Delta Y_g(D_g(\mathbf{0}))) = 0$, $\text{cov}(Q_{s,g}, \Delta D_g(\mathbf{0})) = 0$, non-stochastic shocks, and iid locations, in the spirit of Goldsmith-Pinkham et al. (2020), also implies Assumption 4.

Comparing Assumptions 2 and 4. If $E(\Delta D_g(\mathbf{0})) = 0$ and $\alpha_{g,1} = \alpha_{g,2} = \alpha_g$, Assumptions 2 and 4 can jointly hold under no restrictions on the joint distribution of α_g and ΔZ_g. For instance, if Point 1 of Assumption 2 holds and $E(\Delta D_g(\mathbf{0})|\alpha_g, \Delta Z_g) = 0$, then Points 1 and 2 of Assumption 4 hold. On the other hand, if $E(\Delta D_g(\mathbf{0})) \neq 0$ or $\alpha_{g,1} \neq \alpha_{g,2}$, imposing jointly Assumptions 2 and 4 is essentially equivalent to assuming that $\text{cov}(\Delta Z_g, \alpha_{g,1}) = \text{cov}(\Delta Z_g, \alpha_{g,2}) = 0$, a strong requirement, unless one is ready to assume that the Bartik instrument is as-good-as randomly assigned to locations. Our decompositions of θ^b under Assumption 4 in Theorem 2 below are similar to those under Assumption 2 that follow from Theorem 1. Imposing Assumption 2 or 4 does not change much our assessment of θ^b's robustness to heterogenous effects.

Decompositions of θ^b under Assumptions 1 and 3-4.

Theorem 2 *Suppose Assumptions 1 and 3-4 hold.*

1. *Then,*

$$\theta^b = E\left(\sum_{g=1}^{G}\sum_{t=1}^{2}\frac{(1\{t=2\}-1\{t=1\})\sum_{s=1}^{S}Q_{s,g}\beta_{s,g,t}Z_{s,t}(\Delta Z_g - E(\Delta Z_.))}{E\left(\sum_{g'=1}^{G}\sum_{t'=1}^{2}(1\{t'=2\}-1\{t'=1\})\sum_{s=1}^{S}Q_{s,g'}\beta_{s,g',t'}Z_{s,t'}(\Delta Z_{g'} - E(\Delta Z_.))\right)}\alpha_{g,t}\right).$$

2. *If one further assumes that for all g, there exist α_g and $(\beta_{s,g})_{s\in\{1,...,S\}}$ such that $\alpha_{g,1} = \alpha_{g,2} = \alpha_g$ and $\beta_{s,g,1} = \beta_{s,g,2} = \beta_{s,g}$, then*

$$\theta^b = E\left(\sum_{g=1}^{G}\frac{\sum_{s=1}^{S}Q_{s,g}\beta_{s,g}\Delta Z_s(\Delta Z_g - E(\Delta Z_.))}{E\left(\sum_{g'=1}^{G}\sum_{s=1}^{S}Q_{s,g'}\beta_{s,g'}\Delta Z_s(\Delta Z_{g'} - E(\Delta Z_.))\right)}\alpha_g\right).$$

3. *If on top of the assumptions in Point 2, one further assumes that $\beta_{s,g} = \beta$,*

$$\theta^b = E\left(\sum_{g=1}^{G}\frac{\Delta Z_g(\Delta Z_g - E(\Delta Z_.))}{E\left(\sum_{g'=1}^{G}\Delta Z_{g'}(\Delta Z_{g'} - E(\Delta Z_.))\right)}\alpha_g\right).$$

Consequences of Theorem 2 Point 1 of Theorem 2 shows that under Assumptions 1 and 3-4, θ^b identifies a weighted sum of the treatment effects $\alpha_{g,t}$, with weights

$$\frac{(1\{t=2\}-1\{t=1\})\sum_{s=1}^S Q_{s,g}\beta_{s,g,t}Z_{s,t}(\Delta Z_g - E(\Delta Z_.))}{E\left(\sum_{g'=1}^G\sum_{t'=1}^2(1\{t'=2\}-1\{t'=1\})\sum_{s=1}^S Q_{s,g'}\beta_{s,g',t'}Z_{s,t'}(\Delta Z_{g'} - E(\Delta Z_.))\right)}. \tag{2.7}$$

Those weights are identical to those in (2.2), replacing $D_{g,t}$ by $\sum_{s=1}^S Q_{s,g}\beta_{s,g,t}Z_{s,t}$, the effect of the shocks on $D_{g,t}$. Therefore, unlike the weights in (2.2), those in (2.7) cannot be estimated, as they depend on the first stage effects $\beta_{s,g,t}$. Let us assume that $Z_{s,t}>0$ for all (s,t), as is for instance the case in ADH. If one further assumes that the first-stage effects $\beta_{s,g,t}$ are all positive, an assumption similar to the monotonicity condition in Imbens & Angrist (1994), then every location is such that either $\alpha_{g,1}$ or $\alpha_{g,2}$ is weighted negatively. Therefore, adding a linear first-stage model with a monotonicity condition is not enough to make θ^b robust to heterogeneous effects. Point 2 of Theorem 2 shows that even assuming that the first-stage and treatment effects are homogeneous over time, θ^b may still not be robust to heterogeneous effects across locations. Finally, Point 3 shows that even if one further assumes a fully homogeneous first-stage effect, θ^b may still not be robust. The weights in that last decomposition can be estimated.

Comparing Point 2 of Theorem 2 to Equation (10) in Goldsmith-Pinkham et al. (2020). In their Equation (10), Goldsmith-Pinkham et al. (2020) analyze a Bartik regression with one time period, in a model with location-specific treatment effects (see their Equation (7)). The regression they consider nests that in our Definition 2, if the treatment and outcome in their regression are first-differenced. Then, their Equation (7) is a linear model in first-difference, which assumes constant effects over time, so their Equation (10) should be compared to Point 2 of our Theorem 2. Like Point 2 of our Theorem 2, their Equation (10) shows that θ^b identifies a weighted sum of treatment effects, potentially with some negative weights. However, the weights in their and our decomposition differ. Expressed in our notation, the weight assigned to α_g in their decomposition is

$$\frac{\sum_{s=1}^S\left(\Delta Z_s\left(\sum_{g'=1}^G Q_{s,g'}(\Delta D_{g'}-\Delta D_.)\right)(Q_{s,g}-Q_{s,.})^2\Delta Z_s\beta_{s,g}\right)}{\left(\sum_{s=1}^S\Delta Z_s\left(\sum_{g'=1}^G Q_{s,g'}(\Delta D_{g'}-\Delta D_.)\right)\right)\times\left(\sum_{g=1}^G(Q_{s,g}-Q_{s,.})^2\Delta Z_s\beta_{s,g}\right)},$$

where $Q_{s,.}=\frac{1}{G}\sum_{g=1}^G Q_{s,g}$ is the average share of sector s across locations, and where

$$\frac{\Delta Z_s\left(\sum_{g'=1}^G Q_{s,g'}(\Delta D_{g'}-\Delta D_.)\right)}{\sum_{s=1}^S\Delta Z_s\left(\sum_{g'=1}^G Q_{s,g'}(\Delta D_{g'}-\Delta D_.)\right)}$$

is the so-called Rotemberg weight (see Rotemberg 1983). The weights in our decomposition do not depend on the Rotemberg weights.

Why do Point 2 of Theorem 2 and Equation (10) in Goldsmith-Pinkham et al. (2020) differ? The difference between our decompositions stems from the fact our first-stage assumptions are different and almost incompatible. In what follows, we assume that shocks are non-stochastic, as in Goldsmith-Pinkham et al. (2020). Then, using our notation, and assuming the regression has no control variables, the first-stage assumptions in Goldsmith-Pinkham et al. (2020) (see Equation (8) and Assumption 3 therein) require that for all (s, g),

$$\Delta D_g = \mu^D + Q_{s,g}\Delta Z_s \beta_{s,g} + u_{s,g}, \tag{2.8}$$

$$\text{with } E(Q_{s,g}u_{s,g}\alpha_g) = 0. \tag{2.9}$$

(2.8) is a first-differenced first-stage model similar to (2.4), where the effect of only one sector appears explicitly. (2.4) and (2.8) imply that $u_{s,g} = \Delta D_g(\mathbf{0}) - \mu^D + \sum_{s' \neq s} Q_{s',g}\Delta Z_{s'}\beta_{s',g}$, so

$$E(Q_{s,g}u_{s,g}\alpha_g) = E(Q_{s,g}(\Delta D_g(\mathbf{0}) - \mu^D)\alpha_g) + \sum_{s' \neq s} E(Q_{s,g}Q_{s',g}\alpha_g\beta_{s',g})\Delta Z_{s'}.$$

Then, (2.9) is hard to rationalize. For instance, if for all (s, g) $\Delta Z_s > 0$, $\beta_{s,g} > 0$, $\alpha_g > 0$, and $Q_{s,g} > 0$, $\sum_{s' \neq s} E(Q_{s,g}Q_{s',g}\alpha_g\beta_{s',g})\Delta Z_{s'} > 0$, so (2.9) can only hold if the first and second terms in the right-hand-side of the previous display cancel each other out. Overall, whenever the linear first-stage model with time-invariant effects in (2.4) seems plausible, the first-stage assumptions in Goldsmith-Pinkham et al. (2020) are unlikely to hold, and the decomposition of θ^b in their Equation (10) is also unlikely to hold. Heterogeneous effects is not a central issue in Goldsmith-Pinkham et al. (2020). Except for their Equation (10), all their other results assume homogeneous effects and do not rest on their Equation (8) and Assumption 3.

2.4 Panel Bartik may still not be robust with randomly-assigned shocks

The random-shocks assumption. Let $\mathcal{F} = \left(Y_{g,t}(0), D_{g,t}(\mathbf{0}), \alpha_{g,t}, (Q_{s,g}, \beta_{s,g,t})_{s \in \{1,\ldots,S\}}\right)_{(g,t) \in \{1,\ldots,G\} \times \{1,2\}}$.

Assumption 5 *(Random shocks)*

1. *For all (s, t), $E(Z_{s,t}|\mathcal{F}) = E(Z_{s,t})$.*
2. *For all t, there exists a real number m_t such that $E(Z_{s,t}) = m_t$ for all s.*
3. *The vectors $(Z_{s,1}, Z_{s,2})$ are mutually independent across s, conditional on $\mathcal{F}$.*

Point 1 of Assumption 5 requires that shocks be mean independent of locations' potential outcomes without treatment, potential treatments without shocks, shares, and first-stage and treatment effects. Point 2 requires that at every period, all sector-level shocks have the same expectation. Point 3 requires that the vector of period-one and period-two shocks be independent across sectors, but it allows for serial correlation within sectors. Points 1 and 2 of Assumption 5 are equivalent to Assumption 4.ii) in Adão et al. (2019) with panel data, period fixed effects, and no other control variables. Point 3 is identical to the independence assumption that Adão et al. (2019) make in their Section V.A, with panel data and clusters defined as sectors.

Testability of the randomly-assigned-shocks assumption. We highlight two testable implications of the randomly-assigned-shocks assumption, which to our knowledge had not been acknowledged so far. First, Point 1 of Assumption 5 implies that

$$E\left(Z_{s,t}\middle|\frac{1}{G}\sum_{s=1}^{S} Q_{s,g}\right) = E(Z_{s,t}), \tag{2.10}$$

meaning that at every period, shocks should be mean independent of the average share of sector s across locations. This can be tested, for instance by regressing shocks on the average share of the corresponding sector. Second, Point 2 of Assumption 5 implies that shocks' expectation should not vary with sector-level characteristics, which can for instance be tested by regressing the shocks on such characteristics. This test is similar to but different from that proposed by Borusyak et al. (2022), who propose to regress each sector-level characteristic on the shocks.

Decomposition of θ^b under Assumptions 1, 3, and 5.

Theorem 3 *Suppose Assumptions 1, 3, and 5 hold, and $\sum_{s=1}^{S} Q_{s,g} = 1$ for all (g,t). If one also assumes that for all s and $(t,t') \in \{1,2\}^2$, $E\left(Z_{s,t}Z_{s,t'}|\mathcal{F}\right) = E\left(Z_{s,t}Z_{s,t'}\right)$, then*

$$\theta^b = E\left(\sum_{g=1}^{G}\sum_{t=1}^{2} \frac{\sum_{s=1}^{S} \beta_{s,g,t} Q_{s,g}^2 \left(V\left(Z_{s,t}\right) - cov\left(Z_{s,1}, Z_{s,2}\right)\right)}{E\left(\sum_{g'=1}^{G}\sum_{t'=1}^{2}\sum_{s=1}^{S} \beta_{s,g',t'} Q_{s,g'}^2 \left(V\left(Z_{s,t'}\right) - cov\left(Z_{s,1}, Z_{s,2}\right)\right)\right)} \alpha_{g,t}\right). \tag{2.11}$$

Remarks on the assumptions underlying Theorem 3. On top of Assumption 5, Theorem 3 further assumes that shares sum to one. Remember that if that is not the case, Borusyak et al. (2022) show that one should not estimate θ^b under their random-shocks assumption. Instead, one should replace the intercept by locations' sum of shares in the 2SLS Bartik regression. We conjecture that when shares do not sum to one, a result similar to that in Theorem 3 can be shown for that estimand. Theorem 3 also further assumes that $E\left(Z_{s,t}Z_{s,t'}|\mathcal{F}\right) = E\left(Z_{s,t}Z_{s,t'}\right)$, a mild strengthening of Point 1 of Assumption 5.

Consequences of Theorem 3. (2.11) shows that θ^b may not be robust to heterogeneous effects, even with randomly-assigned shocks. Weights are all positive if $\text{cov}\left(Z_{s,1}, Z_{s,2}\right) = 0$ for all s, or if $V\left(Z_{s,1}\right) = V\left(Z_{s,2}\right)$ for all s (by Cauchy-Schwarz inequality). However, there are applications where those two conditions are very strongly violated. For instance, when we revisit ADH, we find that the sample variance of $Z_{s,2}$ is more than 3 times larger than the sample variance of $Z_{s,1}$ (imports from China are strongly increasing over the study period), while the sample correlation of $Z_{s,1}$ and $Z_{s,2}$ is equal to 0.70.[4] Let us further assume that shocks' second moments do not depend on s: $V\left(Z_{s,t}\right) = \sigma_t^2$ and $\text{cov}\left(Z_{s,1}, Z_{s,2}\right) = \rho\sigma_1\sigma_2$, an assumption in the spirit of Point 2 of Assumption 5. Then, the weights in Theorem 3 simplify to

$$\frac{\left(\sigma_t^2 - \rho\sigma_1\sigma_2\right)\sum_{s=1}^{S} \beta_{s,g,t} Q_{s,g}^2}{E\left(\sum_{g'=1}^{G}\sum_{t'=1}^{2}\left(\sigma_{t'}^2 - \rho\sigma_1\sigma_2\right)\sum_{s=1}^{S} \beta_{s,g',t'} Q_{s,g'}^2\right)}.$$

If $\beta_{s,g,t} \geq 0$ for all (s,g,t), the weights are of the same sign as $\sigma_t^2 - \rho\sigma_1\sigma_2$, which can be estimated (ρ is just the correlation between the period-one and period-two shock of the same sector). In ADH, $\hat{\sigma}_1^2 - \hat{\rho}\hat{\sigma}_1\hat{\sigma}_2 < 0$, so the estimated weight on $\alpha_{g,1}$ is negative for all g.

[4] There are three periods in ADH. The numbers in the text are computed for the first two periods in their data. Results are similar if one instead uses the last two periods.

Standardizing the shocks within each period can eliminate the negative weights. Assume again that $V(Z_{s,t}) = \sigma_t^2$ and $\text{cov}(Z_{s,1}, Z_{s,2}) = \rho\sigma_1\sigma_2$. Let $Z_{g,t}^{\text{sd}} = Z_{g,t}/\sigma_t$ denote the standardized-within-period Bartik instrument, and let

$$\theta^{b,\text{sd}} = \frac{\sum_{g=1}^{G} E\left(\Delta Y_g \left(\Delta Z_g^{\text{sd}} - E\left(\Delta Z_.^{\text{sd}}\right)\right)\right)}{\sum_{g=1}^{G} E\left(\Delta D_g \left(\Delta Z_g^{\text{sd}} - E\left(\Delta Z_.^{\text{sd}}\right)\right)\right)}$$

denote the estimand attached to a 2SLS regression of ΔY_g on ΔD_g using ΔZ_g^{sd} as the instrument. Under the assumptions of Theorem 3, one can show that

$$\theta^{b,\text{sd}} = E\left(\sum_{g=1}^{G}\sum_{t=1}^{2} \frac{\sigma_t \sum_{s=1}^{S} \beta_{s,g,t} Q_{s,g}^2}{E\left(\sum_{g'=1}^{G}\sum_{t'=1}^{2} \sigma_{t'} \sum_{s=1}^{S} \beta_{s,g',t'} Q_{s,g'}^2\right)} \alpha_{g,t}\right),$$

so unlike θ^b, $\theta^{b,\text{sd}}$ is robust to heterogeneous treatment effects. Therefore, with randomly-assigned but serially correlated and heteroscedastic shocks, it may be preferable to standardize shocks within periods when constructing the Bartik instrument. Alternatively, combining the results in Borusyak et al. (2022) and Adão et al. (2019) to those in Angrist (1998), it follows that with randomly-assigned shocks, a 2SLS Bartik regression of $Y_{g,t}$ on $D_{g,t}$ using $Z_{g,t}$ as the instrument, with period fixed effects but no location fixed effects, estimates a weighted average of treatment effects, even if treatment effects vary across locations and over time.

Reconciling Theorem 3 with Proposition 3 in Adão et al. (2019) and Proposition A.1 in Borusyak et al. (2022). Proposition 3 in Adão et al. (2019) implies that with randomly assigned shocks, cross-sectional 2SLS Bartik regressions are robust to heterogeneous effects. To apply this result to the panel data case we consider here, a first possibility is to assume that the shocks are independent across both s and t. But then, Theorem 3 reduces to

$$\theta^b = E\left(\sum_{g=1}^{G}\sum_{t=1}^{2} \frac{\sum_{s=1}^{S} \beta_{s,g,t} Q_{s,g,t}^2 V(Z_{s,t})}{E\left(\sum_{g'=1}^{G}\sum_{t'=1}^{2}\sum_{s=1}^{S} \beta_{s,g',t'} Q_{s,g',t'}^2 V(Z_{s,t'})\right)} \alpha_{g,t}\right),$$

the same result as in Adão et al. (2019). Sections V.A and V.B in Adão et al. (2019) consider the case with panel data and serially correlated shocks within sectors, but the results therein assume constant treatment effects. A second possibility to apply Proposition 3 in Adão et al. (2019) to the panel data case we consider is to apply their assumptions directly to the first-differenced variables ΔY_g, ΔD_g, and $(\Delta Z_s)_{s\in\{1,\ldots,S\}}$. However, their result then relies on a linear second-stage model relating ΔY_g and ΔD_g, and on a linear first-stage model relating ΔD_g and $(\Delta Z_s)_{s\in\{1,\ldots,S\}}$. As explained earlier, such models implicitly assume constant first-stage and treatment effects over time (see (1.4) and (2.4)). With time-invariant first-stage and treatment effects, Theorem 3 reduces to

$$\theta^b = E\left(\sum_{g=1}^{G} \frac{\sum_{s=1}^{S} \beta_{s,g} Q_{s,g}^2 V(Z_{s,2} - Z_{s,1})}{E\left(\sum_{g'=1}^{G}\sum_{s=1}^{S} \beta_{s,g'} Q_{s,g'}^2 V(Z_{s,2} - Z_{s,1})\right)} \alpha_g\right), \tag{2.12}$$

again the same result one obtains by applying the result in Adão et al. (2019) directly to the first-differenced variables ΔY_g, ΔD_g, and $(\Delta Z_s)_{s\in\{1,\ldots,S\}}$. Proposition A.1 in Borusyak et al.

(2022) shows a result similar to that in Adão et al. (2019), without imposing linear first- and second-stage models. Overall, Theorem 3 is not incompatible with Proposition 3 in Adão et al. (2019) and Proposition A.1 Borusyak et al. (2022). Our result fills a gap, by showing that with shocks randomly assigned across sectors but serially correlated and heteroscedastic, panel Bartik regressions are not robust to time-varying first-stage and treatment effects.

Randomly-assigned shocks, or randomly-assigned first-differenced shocks? With constant first-stage and treatment effects over time, the decomposition of θ^b in (2.12) can be obtained by applying Proposition 3 in Adão et al. (2019) to the first-differenced variables ΔY_g, ΔD_g, and $(\Delta Z_s)_{s\in\{1,...,S\}}$, or by applying Theorem 3. The latter route relies on Assumption 5, while the former route relies on Assumption 6 below, which requires that first-differenced shocks be as good as randomly assigned. Let $\mathcal{F}^{\text{fd}} = \left(\Delta Y_g(0), \Delta D_g(\mathbf{0}), \alpha_g, (Q_{s,g}, \beta_{s,g})_{s\in\{1,...,S\}}\right)_{g\in\{1,...,G\}}$.

Assumption 6 *(Randomly-assigned first-differenced shocks)*

1. *For all s, $E\left(\Delta Z_s|\mathcal{F}^{fd}\right) = E\left(\Delta Z_s\right)$.*
2. *There exists a real number $\Delta\mu$ such that $E\left(\Delta Z_s\right) = \Delta\mu$ for all s.*
3. *The variables ΔZ_s are mutually independent across s, conditional on $\mathcal{F}^{fd}$.*

Assumption 6 may be more plausible than Assumption 5. Its testable implications are similar to, but weaker than those of Assumption 5: it implies that shocks' first-differences should be mean independent of the average share of sector s across locations, and of other sectoral characteristics. If one rejects those testable implications, one can reject both Assumptions 5 and 6. When they revisit ADH, Borusyak et al. (2022) test Assumption 6 rather than 5, and we will do the same.

3 IV-CRC estimator with non-random shocks

Group-level panel data set with at least three time periods. In this section, we propose alternative estimators to Bartik regressions, that may be used when the random-shocks assumption is implausible or rejected. They build upon the correlated-random-coefficients (CRC) estimator proposed by Chamberlain (1992). They can be used when the data has at least three periods.[5] As will become clear below, our IV-CRC estimator is not applicable only to Bartik instruments: it is applicable whenever one has an instrument satisfying Assumptions 8 and 9 below. Due to their specific structure, Bartik instruments may sometimes satisfy those assumptions, but there may be other instruments that can satisfy those assumptions. For all $t \geq 2$ and any variable $R_{g,t}$, let $\Delta R_{g,t} = R_{g,t} - R_{g,t-1}$, and let $\boldsymbol{R}_g = (R_{g,1}, ..., R_{g,T})$ be a vector stacking the full time series of $R_{g,t}$. Our decompositions of Bartik regressions extend to the multi-period case: those extensions are given in Appendix B.

Assumptions underlying our IV-CRC estimator.

Assumption 7 *For all $g \in \{1, ..., G\}$ and $t \in \{1, ..., T\}$, there exists real numbers λ_t and random variables α_g such that $\alpha_{g,t} = \alpha_g + \lambda_t$.*

[5] With two periods, one may be able to follow a similar estimation strategy as that proposed in Graham & Powell (2012) and de Chaisemartin et al. (2022).

Assumption 7 allows for location-specific and time-varying effects, provided the treatment effects follow the same evolution over time in every location. Without loss of generality, we normalize λ_1 to 0. Under Assumption 7,

$$\alpha_{ate} = E\left(\frac{1}{G}\sum_{g=1}^{G}\alpha_g\right) + \frac{1}{T}\sum_{t=1}^{T}\lambda_t$$

so identifying $\alpha_{ate,1} \equiv E\left(\frac{1}{G}\sum_{g=1}^{G}\alpha_g\right)$ and $(\lambda_2, ..., \lambda_T)$ is sufficient to identify α_{ate}.

Assumption 8 *For all $t \in \{2, ..., T\}$, there are real numbers μ_t such that $\forall g \in \{1, ..., G\}$, $E(\Delta Y_{g,t}(0)|\boldsymbol{Z}_g) = \mu_t$.*

Assumption 8 requires that locations' outcome evolutions without treatment be mean-independent of the full sequence of their Bartik instruments. This condition is stronger than the first point of Assumption 2: it requires that $\Delta Y_{g,t}(0)$ be mean independent from $(Z_{g,1}, ..., Z_{g,T})$ rather than uncorrelated with $\Delta Z_{g,t}$. If the data contains a period $t_0 \in \{2, ..., T\}$ such that $D_{g,t_0} = D_{g,t_0-1} = 0$ for all g, then $\Delta Y_{g,t_0}(0)$ is observed, and Assumption 8 has the following testable implication:

$$E(\Delta Y_{g,t_0}|\boldsymbol{Z}_g) = E(\Delta Y_{g,t_0}). \tag{3.1}$$

To test (3.1), one can for instance regress $\Delta Y_{g,t_0}$ on $Z_{g,t'}$ for any $t' \neq t_0$. One could also regress $\Delta Y_{g,t_0}$ on a polynomial in $(Z_{g,1}, ..., Z_{g,t_0-2}, Z_{g,t_0+1}, ..., Z_{g,T})$.

Assumption 9 *For all t and g, $E(\alpha_g|D_{g,t}, \boldsymbol{Z}_g) = E(\alpha_g|\boldsymbol{Z}_g)$.*

Assumption 9 requires that locations' treatment effects be independent of $D_{g,t}$, conditional on $\boldsymbol{Z}_g$: locations with the same vector of instruments but different values of $D_{g,t}$ should not have systematically different treatment effects. α_g is unobserved, so Assumption 9 is untestable. Still, if one observes covariates X_g that are likely to be correlated with locations' treatment effects, one can suggestively test Assumption 9, by regressing X_g on $(D_{g,t}, \boldsymbol{Z}_g)$.

Identification result. Let $\tilde{D}_{g,t} = E(D_{g,t}|\boldsymbol{Z}_g)$. For all $t \geq 2$, let $\mu_{1:t} = \sum_{k=2}^{t}\mu_k$. Let $\boldsymbol{\theta} = (\mu_{1:2}, \lambda_2, \mu_{1:3}, \lambda_3, ..., \mu_{1:T}, \lambda_T)'$, let $\mathbf{0}_k$ denote a vector of k zeros, let

$$\mathcal{P}_g = \begin{pmatrix} \mathbf{0}_{2T-2} \\ 1, \tilde{D}_{g,2}, \mathbf{0}_{2T-4} \\ \mathbf{0}_2, 1, \tilde{D}_{g,3}, \mathbf{0}_{2T-6} \\ \vdots \\ \mathbf{0}_{2T-4}, 1, \tilde{D}_{g,T} \end{pmatrix} \text{ and } \mathcal{X}_g = \begin{pmatrix} 1, \tilde{D}_{g,1} \\ 1, \tilde{D}_{g,2} \\ \vdots \\ 1, \tilde{D}_{g,T} \end{pmatrix}.$$

For any $T \times K$ matrix A, let A^+ be its Moore-Penrose inverse, and let $M(A) = \boldsymbol{I}_T - AA^+$ be the orthogonal projector on the kernel of A. For any $K \times 1$ vector x, $(x)_k$ is its kth coordinate.

Theorem 4 *Suppose that Assumptions 1 and 7-9 hold, $E\left(\frac{1}{G}\sum_{g=1}^{G}\mathcal{P}_g'M(\mathcal{X}_g)\mathcal{P}_g\right)$ is invertible, and with probability 1 $\mathcal{X}_g'\mathcal{X}_g$ is invertible for every $g \in \{1, ..., G\}$. Then:*

$$\boldsymbol{\theta} = E\left(\frac{1}{G}\sum_{g=1}^{G}\mathcal{P}_g'M(\mathcal{X}_g)\mathcal{P}_g\right)^{-1} E\left(\frac{1}{G}\sum_{g=1}^{G}\mathcal{P}_g'M(\mathcal{X}_g)\mathbf{Y}_g\right), \tag{3.2}$$

$$\alpha_{ate,1} = \left(E\left(\frac{1}{G}\sum_{g=1}^{G}\left(\mathcal{X}_g'\mathcal{X}_g\right)^{-1}\mathcal{X}_g'\left(\mathbf{Y}_g - \mathcal{P}_g\boldsymbol{\theta}\right)\right)\right)_2. \tag{3.3}$$

Estimation. We propose a simple method to estimate α_{ate}, under a functional-form assumption on $E(D_{g,t}|\boldsymbol{Z}_g)$.

Assumption 10 *There exists an integer K such that for all $t \geq 2$, there is a polynomial of T variables $P_{K,t}$ such that for all g, $E(D_{g,t}|\boldsymbol{Z}_g) = P_{K,t}(\boldsymbol{Z}_g)$.*

Polynomials are well suited to a large class of applications, but when they are not one can of course assume a different functional form. Under Assumption 10, one may estimate α_{ate} as follows. First, one regresses $D_{g,t}$ on a polynomial of order K in $\boldsymbol{Z}_g$, separately for every $t \geq 2$. Then, letting $\widehat{\tilde{D}}_{g,t}$ denote the prediction from that estimation, one lets

$$\widehat{\mathcal{P}}_g = \begin{pmatrix} \mathbf{0}_{2T-2} \\ 1, \widehat{\tilde{D}}_{g,2}, \mathbf{0}_{2T-4} \\ \mathbf{0}_2, 1, \widehat{\tilde{D}}_{g,3}, \mathbf{0}_{2T-6} \\ \vdots \\ \mathbf{0}_{2T-4}, 1, \widehat{\tilde{D}}_{g,T} \end{pmatrix} \text{ and } \widehat{\mathcal{X}}_g = \begin{pmatrix} 1, \widehat{\tilde{D}}_{g,1} \\ 1, \widehat{\tilde{D}}_{g,2} \\ \vdots \\ 1, \widehat{\tilde{D}}_{g,T} \end{pmatrix},$$

and

$$\begin{aligned}
\widehat{\boldsymbol{\theta}} &= \left(\frac{1}{G}\sum_{g=1}^{G} \widehat{\mathcal{P}}_g' M(\widehat{\mathcal{X}}_g)\widehat{\mathcal{P}}_g\right)^{-1} \left(\frac{1}{G}\sum_{g=1}^{G} \widehat{\mathcal{P}}_g' M(\widehat{\mathcal{X}}_g)\mathbf{Y}_g\right), \\
\widehat{\alpha}_{ate,1} &= \left(\frac{1}{G}\sum_{g=1}^{G} \left(\widehat{\mathcal{X}}_g'\widehat{\mathcal{X}}_g\right)^{-1} \widehat{\mathcal{X}}_g' \left(\mathbf{Y}_g - \widehat{\mathcal{P}}_g\widehat{\boldsymbol{\theta}}\right)\right)_2, \\
\widehat{\alpha}_{ate} &= \widehat{\alpha}_{ate,1} + \frac{1}{T}\sum_{t=1}^{T} \widehat{\lambda}_t.
\end{aligned}$$

Estimating α_{ate} without a functional-form assumption on $E(D_{g,t}|\boldsymbol{Z}_g)$ is feasible, using a non-parametric estimator of $E(D_{g,t}|\boldsymbol{Z}_g)$. We leave this extension for future work.

Intuition. Our estimator may be seen as an IV-version of Chamberlain's CRC estimator. In a first step, one uses the vector of instruments $\boldsymbol{Z}_g$ to predict the treatment $D_{g,t}$. Then, one computes the CRC estimator with the predicted treatment in lieu of the endogenous treatment. Readers familiar with cross-sectional IV models may be surprised by the fact we can allow for heterogeneous treatment effects without imposing a first-stage model with monotonicity restrictions. To simplify the presentation of the identification argument, we momentarily assume that $T = 3$, and that treatment effects are location-specific but time invariant: $\alpha_{g,t} = \alpha_g$. Then,

$$\begin{aligned}
E(\Delta Y_{g,t}|\boldsymbol{Z}_g) &= E(\Delta Y_{g,t}(0)|\boldsymbol{Z}_g) + E(\alpha_g \Delta D_{g,t}|\boldsymbol{Z}_g) \\
&= \mu_t + E(\alpha_g|\boldsymbol{Z}_g)\Delta\tilde{D}_{g,t},
\end{aligned} \tag{3.4}$$

where the second equality follows from Assumptions 8 and 9. Then, subtracting (3.4) at $t = 3$ multiplied by $\Delta\tilde{D}_{g,2}\Delta\tilde{D}_{g,3}$ from (3.4) at $t = 2$ multiplied by $\Delta\tilde{D}_{g,3}^2$ yields

$$\Delta\tilde{D}_{g,3}^2 E(\Delta Y_{g,2}|\Delta\boldsymbol{Z}_g) - \Delta\tilde{D}_{g,2}\Delta\tilde{D}_{g,3}E(\Delta Y_{g,3}|\Delta\boldsymbol{Z}_g) = \Delta\tilde{D}_{g,3}^2\mu_2 - \Delta\tilde{D}_{g,2}\Delta\tilde{D}_{g,3}\mu_3, \tag{3.5}$$

an equation that does not depend on the treatment effect. Similarly, subtracting (3.4) at $t = 2$ multiplied by $\Delta\tilde{D}_{g,2}\Delta\tilde{D}_{g,3}$ from (3.4) at $t = 3$ multiplied by $\Delta\tilde{D}^2_{g,2}$ yields

$$\Delta\tilde{D}^2_{g,2}E(\Delta Y_{g,3}|\Delta\boldsymbol{Z}_g) - \Delta\tilde{D}_{g,2}\Delta\tilde{D}_{g,3}E(\Delta Y_{g,2}|\Delta\boldsymbol{Z}_g) = \Delta\tilde{D}^2_{g,2}\mu_3 - \Delta\tilde{D}_{g,2}\Delta\tilde{D}_{g,3}\mu_2, \tag{3.6}$$

an equation that also does not depend on the treatment effect. (3.5) and (3.6) give a system of conditional moment equalities with two unknowns, μ_2 and μ_3, so μ_2 and μ_3 are identified. Then, it follows from (3.4) that $E(\alpha_g|\Delta\boldsymbol{Z}_g)$ is identified.[6]

Inference. We suggest a method to draw inference on the ATE under Assumption 11.

Assumption 11 *Conditional on* $(\Delta Z_{s,t})_{(s,t)\in\{1,...,S\}\times\{2,...,T\}}$, $(\boldsymbol{Z}_g, \boldsymbol{D}_g, \mathbf{Y}_g)_g$ *is iid.*

Assumption 11 requires that conditional on shocks' first-differences, the full time series of their instruments, treatments, and outcomes be iid across locations. It is compatible with the shares approach, not with the shocks approach. If the treatment and outcome are also influenced by unobserved sector-level shocks, as hypothesized in Borusyak et al. (2022) and Adão et al. (2019), such unobserved shocks also need to be conditioned upon for Assumption 11 to be plausible. Under Assumption 11, to estimate the standard error of $\widehat{\alpha}_{ate}$, we propose to bootstrap the whole estimation procedure, clustering the bootstrap at the location level. This bootstrapped standard error does not account for the variance arising from the shocks. Accounting for it would require extending the approach in Adão et al. (2019) to the estimators in Theorem 4, without making the randomly-assigned-shocks assumption. This important extension is left for future work. Similarly, in our applications, to draw inference on Bartik regression coefficients we use standard errors clustered at the location level: those do not account for the variance arising from the shocks, but they give valid estimators of the coefficients' standard errors conditional on the shocks under Assumption 11. The heuristic identification argument above shows that the estimand identifying the ATE involves some of the third moments of $(\Delta Y_{g,2}, \Delta Y_{g,3}, \Delta\tilde{D}_{g,2}, \Delta\tilde{D}_{g,3})$, while the Bartik estimand only involves the first and second moments of that vector. This may explain why when we revisit ADH, the variance of the IV-CRC estimator is substantially larger than that of the Bartik estimator. Noteworthy, using the IV-CRC estimator instead of the Bartik one does not always lead to precision losses as large as those we find in ADH: when we revisit the canonical design in Section A of the Web Appendix, the variance of the IV-CRC estimator is slightly larger than that of the Bartik estimator, but the difference is much lower than in ADH (see Web Appendix Tables A.1 and A.3).

Bartik regressions are still not robust to heterogeneous treatment effects under the assumptions underlying our IV-CRC estimator. Under Assumptions 1 and 7-9, if one further assumes constant effects over time ($\lambda_t = 0$), it follows from Point 2 of Theorem 1 that when $T = 2$, θ^b identifies a weighted sum of the conditional effects $E(\alpha_g|\boldsymbol{Z}_g)$, potentially with

[6] Applying results in Chamberlain (1992), one can derive the optimal estimator of (μ_2, μ_3) attached to this system of conditional moment equalities. An issue, however, is that Chamberlain's optimality results do not apply to the estimators of $\alpha_{ate,1}$ and λ_t, the building blocks of our target parameter. Moreover, the computation of the optimal estimator requires a non-parametric first-stage estimation. To our knowledge, no data-driven method has been proposed to choose the tuning parameters involved in this first stage. Accordingly, we prefer to stick with estimators of (μ_2, μ_3) attached to unconditional moment equalities.

some negative weights, proportional to $E(\Delta D_g|\boldsymbol{Z}_g)(\Delta Z_g - E(\Delta Z_.))$ (a similar result holds for $T > 2$). Thus, θ^b may not be robust to heterogeneous effects across locations, under stronger assumptions than those under which our IV-CRC estimand identifies the ATE.

Estimator with control variables. Let X_g be a $K \times 1$ vector of time-invariant location-level control variables, with kth coordinate $X_{k,g}$. An IV-CRC estimator controlling for X_g can be obtained, replacing Assumptions 8 and 9 by the following conditions:

Assumption 12 *For all $t \in \{2, ..., T\}$, there is a real number μ_t and a $K \times 1$ vector $\mu_{X,t}$ such that $\forall g \in \{1, ..., G\}$, $E(\Delta Y_{g,t}(0)|\boldsymbol{Z}_g, X_g) = E(\Delta Y_{g,t}(0)|X_g) = \mu_t + X_g' \mu_X$.*

Assumption 13 *For all $t \in \{2, ..., T\}$ and g, $E(\alpha_g|D_{g,t}, \boldsymbol{Z}_g, X_g) = E(\alpha_g|\boldsymbol{Z}_g, X_g)$.*

Assumption 12 may be more plausible than Assumption 8: it requires that locations' outcome evolutions without treatment be mean-independent of their Bartik instruments conditional on X_g, rather than unconditionally. Then, the second equality requires that $E(\Delta Y_{g,t}(0)|X_g)$ be linear in X_g. For the same reason, Assumption 13 may be more plausible than Assumption 9. For all $t \geq 2$, let $\mu_{X,1:t} = \sum_{k=2}^{t} \mu_{X,k}$. Redefining $\tilde{D}_{g,t} \equiv E(D_{g,t}|\boldsymbol{Z}_g, X_g)$, $\boldsymbol{\theta} \equiv (\mu_{1:2}, \lambda_2, \mu_{1:3}, \lambda_3, ..., \mu_{1:T}, \lambda_T, \mu_{X,1:2}, ..., \mu_{X,1:T})'$, and

$$\mathcal{P}_g \equiv \begin{pmatrix} \mathbf{0}_{2T-2+(T-1)K} \\ 1, \tilde{D}_{g,2}, \mathbf{0}_{2T-4}, X_{1,g}, ..., X_{K,g}, \mathbf{0}_{(T-2)K} \\ \mathbf{0}_2, 1, \tilde{D}_{g,3}, \mathbf{0}_{2T-6+K}, X_{1,g}, ..., X_{K,g}, \mathbf{0}_{(T-3)K} \\ \vdots \\ \mathbf{0}_{2T-4}, 1, \tilde{D}_{g,T}, \mathbf{0}_{(T-2)K}, X_{1,g}, ..., X_{K,g} \end{pmatrix},$$

one can show that (3.2) and (3.3) still hold under Assumptions 1, 7, 12, and 13.

Estimator assuming constant effects over time. Similarly, it is easy to obtain an IV-CRC estimator assuming that treatment effects are constant over time. Then, (3.2) and (3.3) still hold, after redefining $\boldsymbol{\theta} \equiv (\mu_{1:2}, 0, \mu_{1:3}, 0, ..., \mu_{1:T}, 0)'$, and

$$\mathcal{P}_g \equiv \begin{pmatrix} \mathbf{0}_{2T-2} \\ 1, 0, \mathbf{0}_{2T-4} \\ \mathbf{0}_2, 1, 0, \mathbf{0}_{2T-6} \\ \vdots \\ \mathbf{0}_{2T-4}, 1, 0 \end{pmatrix}.$$

Testing for heterogeneous treatment effects. In the proof of Theorem 4, we show that

$$E(\alpha_g) = \left(E\left(\left(\mathcal{X}_g'\mathcal{X}_g\right)^{-1} \mathcal{X}_g' \left(\mathbf{Y}_g - \mathcal{P}_g\boldsymbol{\theta}\right)\right)\right)_2,$$

a result stronger than that in (3.3). Then one may use

$$\widehat{\alpha}_g \equiv \left(\left(\widehat{\mathcal{X}}_g'\widehat{\mathcal{X}}_g\right)^{-1} \widehat{\mathcal{X}}_g' \left(\mathbf{Y}_g - \widehat{\mathcal{P}}_g\widehat{\boldsymbol{\theta}}\right)\right)_2 \tag{3.7}$$

to estimate $E(\alpha_g)$, and $\sum_t(\widehat{\alpha}_g + \widehat{\lambda}_t)$ to estimate an effect specific to CZ g, on average across all time periods. With a fixed number of time periods T, those estimators are not consistent, and naively using them to estimate the distribution of treatment effects across locations would be misleading: one would first need to deconvolute them. Proposing a deconvolution technique goes beyond the scope of this paper. Another possibility to test for heterogeneous effects is to regress $\sum_t(\widehat{\alpha}_g + \widehat{\lambda}_t)$ on location-level covariates, and assess whether the covariates significantly predict those estimate effects (see Muris & Wacker 2022, who made a similar proposal before this paper). Inference still needs to account for the fact the $\widehat{\alpha}_g$s are estimated, which may be achieved by bootstrapping the estimation procedure.

4 Empirical application: China shock

4.1 Economic interpretation of our econometric assumptions in ADH

In this section, we start by defining the treatment, outcome, and instrument in ADH, before providing economic interpretations of our main econometric assumptions, using a gravity-based decomposition of trade flows (see, e.g., Arkolakis et al. 2012).

Treatment and outcome definitions. The main outcome variable $Y_{g,t}$ in ADH is the manufacturing employment share of the working-age population in CZ g at t, hereafter referred to as the "manufacturing employment share". Let $M_{s,t}^{US}$ denote US imports from China in sector s at t, let $E_{s,t}$ denote US employment in sector s at t, and let $Q_{s,g}$ denote the employment share of sector s in CZ g. In ADH, the treatment is a weighted average of $M_{s,t}^{US}/E_{s,t}$, China's US exports per US worker in sector s at t, with weights $Q_{s,g}$:

$$D_{g,t} \equiv \sum_{s=1}^{S} Q_{s,g} \frac{M_{s,t}^{US}}{E_{s,t}}. \tag{4.1}$$

The treatment definition in ADH stems from a micro-founded structural equation (see Equation (2) in their Web Appendix), that expresses counterfactual levels of the manufacturing employment share in US CZ g at period t as a linear function of a weighted average of $S_{s,t}/E_{s,t}$, China's export-supply capability per US worker in sector s at t, with weights $Q_{s,g}$:

$$C_{g,t} \equiv \sum_{s=1}^{S} Q_{s,g} \frac{S_{s,t}}{E_{s,t}}.$$

Linear model with heterogeneous effects. The micro-founded structural equation in ADH is linear, and it may have CZ-specific and time-varying slopes, like our Assumption 1, for instance if one allows the trade imbalance in total expenditure, ρ in their notation, to vary across CZs and/or over time (see their Equation (1)).

Instrument definition. $M_{s,t}^{US}$ is determined by China's export supply capability, but also by US demand in s at t. This may create a correlation between $D_{g,t}$ and other determinants of $Y_{g,t}$ than China's exports. Accordingly, ADH define the following instrument:

$$Z_{g,t} \equiv \sum_{s=1}^{S} Q_{s,g} \frac{M_{s,t}^{OC}}{E_{s,t}}, \tag{4.2}$$

where $M_{s,t}^{OC}$ denotes China's exports in sector s at t to eight high-income countries similar to the US, hereafter referred to as other countries.

A gravity-based decomposition of trade flows. Assume that

$$\begin{aligned} M_{s,t}^{US} &= S_{s,t} + \mathrm{Dem}_{s,t}^{US} \\ M_{s,t}^{OC} &= S_{s,t} + \mathrm{Dem}_{s,t}^{OC}, \end{aligned} \tag{4.3}$$

where $S_{s,t}$ denotes China's export-supply capabilities in s at t, and $Dem_{s,t}^{US}$ and $Dem_{s,t}^{OC}$ respectively denote demand's contribution to China's exports to the US and to other countries. Under the assumptions outlined in Arkolakis et al. (2012), if exports are in logs,[7]

$$\begin{aligned} S_{s,t} =&(1-\sigma_{s,t})(\ln(w_{s,t}^{C} + \ln(\tau_{s,t}^{C})) \\ \mathrm{Dem}_{s,t}^{US} =& \ln(\mathrm{Exp}_{s,t}^{US}) - (1-\sigma_{s,t})\ln(P_{s,t}^{US}) \\ \mathrm{Dem}_{s,t}^{OC} =& \ln(\mathrm{Exp}_{s,t}^{OC}) - (1-\sigma_{s,t})\ln(P_{s,t}^{OC}), \end{aligned} \tag{4.4}$$

where $\sigma_{s,t}$ is the elasticity of substitution in sector s at t, $w_{s,t}^{C}$ is the wage in China in s at t, $\tau_{s,t}^{C}$ is China's variable cost of trade in s at t,[8] $\mathrm{Exp}_{s,t}^{US}$ and $\mathrm{Exp}_{s,t}^{OC}$ are the expenditures in s at t in the US and in other countries, and $P_{s,t}^{US}$ and $P_{s,t}^{OC}$ are the price index in s at t in the US and in other countries (see Dixit & Stiglitz 1977).

Sufficient conditions for Assumption 2 under (4.3). With time-invariant US sectoral employments $(E_s)_{s\in\{1,\dots,S\}}$, under (4.3) one has

$$\Delta Z_{g,t} = \sum_{s=1}^{S} Q_{s,g}\frac{\Delta S_{s,t} + \Delta \mathrm{Dem}_{s,t}^{OC}}{E_s}.$$

Assume that

$$(\Delta S_{s,t}, \Delta\mathrm{Dem}_{s,t}^{OC})_{s\in\{1,\dots,S\},t\in\{2,\dots,T\}} \perp\!\!\!\perp ((Q_{s,g})_{s\in\{1,\dots,S\}}, \Delta Y_{g,t}(0))_{g\in\{1,\dots,G\},t\in\{2,\dots,T\}}, \tag{4.5}$$

meaning that the first-differences of China's export supply capabilities and of other-countries demand's shocks are independent of US CZs sectoral shares and potential employment evolutions without imports from China. Then, if $(E_s)_{s\in\{1,\dots,S\}}$ is non-stochastic and for all (s,g,t) $\mathrm{cov}(Q_{s,g}, \Delta Y_{g,t}(0)) = 0$,

$$\begin{aligned} cov(\Delta Z_{g,t}, \Delta Y_{g,t}(0)) =& \sum_{s=1}^{S} cov\left(Q_{s,g}\frac{\Delta S_{s,t} + \Delta\mathrm{Dem}_{s,t}^{OC}}{E_s}, \Delta Y_{g,t}(0)\right) \\ =& \sum_{s=1}^{S} E\left(\frac{\Delta S_{s,t} + \Delta\mathrm{Dem}_{s,t}^{OC}}{E_s}\right) cov\left(Q_{s,g}, \Delta Y_{g,t}(0)\right) \\ =&0, \end{aligned}$$

[7]ADH use exports instead of the log of exports in their empirical analysis, while the structural equation justifying their Bartik regression (Equation (2) in their Web Appendix) is a log-log equation. To keep our empirical specification as close as possible to theirs, we too use exports instead of the log of exports in our empirical analysis, despite the fact our econometric assumptions are easier to interpret in a gravity-based framework with the log of exports.

[8]This variable cost may not be the same when China exports to the US and to other countries. To account for that, one could allow China's export-supply capabilities to depend on the destination, without changing the economic interpretation of our econometric assumptions.

thus providing an economic justification of Assumption 2 in the spirit of the shares approach of Goldsmith-Pinkham et al. (2020). Similarly, one can show that if shares sum to 1, (4.5) holds, $(E_s)_{s\in\{1,\ldots,S\}}$ is non-stochastic, and

$$E\left(\frac{\Delta S_{s,t}+\Delta \text{Dem}^{OC}_{s,t}}{E_s}\right)=m_t,$$

then $\text{cov}(\Delta Z_{g,t},\Delta Y_{g,t}(0))=0$, thus providing an economic justification of Assumption 2 in the spirit of the shocks approach of Borusyak et al. (2022) and Adão et al. (2019). Note that with stochastic US sectoral employments $(E_s)_{s\in\{1,\ldots,S\}}$, rationalizing Point 2 of Assumption 2 would require replacing (4.5) by

$$(E_s,\Delta S_{s,t},\Delta \text{Dem}^{OC}_{s,t})_{s\in\{1,\ldots,S\},t\in\{2,\ldots,T\}} \perp\!\!\!\perp ((Q_{s,g})_{s\in\{1,\ldots,S\}},\Delta Y_{g,t}(0))_{g\in\{1,\ldots,G\},t\in\{2,\ldots,T\}},$$

which is much less plausible: US sectoral employments are very likely to be correlated with US CZs' counterfactual employment evolutions $\Delta Y_{g,t}(0)$. This motivates using pre-determined sectoral employments to construct the instrument, as we will do later.

Sufficient condition for Assumption 8 under (4.3). With time-invariant US sectoral employments, under (4.3) one has

$$Z_{g,t}=\sum_{s=1}^{S} Q_{s,g}\frac{S_{s,t}+\text{Dem}^{OC}_{s,t}}{E_s}.$$

Assume that

$$(S_{s,t},\text{Dem}^{OC}_{s,t})_{s\in\{1,\ldots,S\},t\in\{1,\ldots,T\}} \perp\!\!\!\perp ((Q_{s,g})_{s\in\{1,\ldots,S\}},\Delta Y_{g,t}(0))_{g\in\{1,\ldots,G\},t\in\{2,\ldots,T\}}, \tag{4.6}$$

meaning that China's export supply capabilities and demand's contribution to China's exports to other countries are independent of US CZs sectoral shares and potential employment evolutions without imports from China. Then, if $(E_s)_{s\in\{1,\ldots,S\}}$ is non-stochastic, and for all (g,t)

$$E\left(\Delta Y_{g,t}(0)|(Q_{s,g})_{s\in\{1,\ldots,S\}}\right)=\mu_t,$$

Assumption 8 holds.

Non-causal first-stage. Under (4.3), the shocks $Z_{s,t}$ do not have a direct causal effect on $D_{g,t}$. Rather, $M^{OC}_{s,t}$ and $M^{US}_{s,t}$ are co-determined by China's export-supply capabilities $S_{s,t}$, thus leading to a statistical but non-causal first-stage between $Z_{g,t}$ and $D_{g,t}$. Our decompositions of θ^b in Theorem 1 do not rely on any first-stage assumption, so they hold irrespective of whether the first-stage is causal or not. Similarly, our IV-CRC estimator does not rely on a causal first-stage model. Theorems 2 and 3 on the other hand do rely on the causal first-stage model in Assumption 3. This is not an issue, as we do not use those theorems when we revisit ADH.

4.2 Data and variables' definitions

Data. We use the replication dataset of ADH on the AEA website. In their main analysis, they use a CZ-level panel data set, with 722 CZs and 3 periods (1990, 2000, and 2007). This data set does not contain the shock and share variables. We obtained those variables from the replication dataset of Borusyak et al. (2022).

Time-invariant shares and sectoral employments. In their statistical analysis, ADH define variables directly in first-differences (see their Equations (3) and (4)). The first-differenced treatment is[9]

$$\Delta D_{g,t} = \sum_{s=1}^{S} Q_{s,g,t-1} \times \frac{\Delta M_{s,t}^{US}}{E_{s,t-1}}, \tag{4.7}$$

where $Q_{s,g,t-1}$ is the employment share of s in g at $t-1$. This first-differenced treatment does not coincide with the first-difference of $\sum_{s=1}^{S} Q_{s,g,t} \times \frac{M_{s,t}^{US}}{E_{s,t}}$, because $M_{s,t}^{US}$ and $M_{s,t-1}^{US}$ are both normalized and weighted by period-$t-1$ employment levels and shares. Similarly, the first-differenced instrument is defined as

$$\Delta Z_{g,t} = \sum_{s=1}^{S} Q_{s,g,t-2} \times \frac{\Delta M_{s,t}^{OC}}{E_{s,t-2}}.$$

We need to define a treatment variable in levels, because some weights in our decompositions of θ^b depend on it. Therefore, we use time-invariant shares and sectoral employments, to construct consistent levels and first-differences of the treatment and instrument. Shares and sectoral employment are set at their 1980 value for the instrument, and at their 1990 value for the treatment, to reflect the fact ADH use lagged shares and sectoral employments for the instrument. This also ensures that the sectoral employments used to construct the instrument are determined before the China shock, in line with the rationalizations of Assumptions 2 and 8 above, which require pre-determined employments.

Extrapolated decennial panel. The trade data used by ADH is available in 1991, 2000, and 2007. To construct first-differenced variables over a comparable time span, ADH multiply their 1991-2000 first-differenced variables by 10/9, and their 2000-2007 first-differenced variables by 10/7. We adopt the same strategy to extrapolate variables in levels. Specifically, for dest $\in \{US, OC\}$, we let $M_{s,1990}^{dest} = M_{s,2000}^{\text{dest}} - 10/9(M_{s,2000}^{\text{dest}} - \Delta M_{s,1991}^{\text{dest}})$, and $M_{s,2010}^{\text{dest}} = M_{s,2000}^{\text{dest}} + 10/7(M_{s,2007}^{\text{dest}} - M_{s,2000}^{\text{dest}})$. We adopt a similar strategy to construct CZs extrapolated employment level in 2010, as the last employment measurement in ADH uses the 2006, 2007, and 2008 American Community Survey (ACS).

Comparing our variables with those in ADH. Our 1990-to-2000 first-differenced instrument, treatment, and outcome take exactly the same values as in the original ADH dataset. Our 2000-to-2010 first-differenced outcome also takes exactly the same values as in the original data. On the other hand, the 2000-to-2010 first-differenced treatment and instrument differ slightly from those in the original ADH dataset, as we use fixed shares and sectoral employments while ADH use time-varying ones. The correlation between our and ADH's 2000-to-2010 first-differenced treatment is 0.749 (p-value<0.001), and the correlation between our and ADH's 2000-to-2010 first-differenced instrument is 0.820 (p-value<0.001). Those modifications do not change results very much: in Column (3) of Table 4 below, we obtain a Bartik 2SLS coefficient fairly close to that in Column (6) Table 3 of ADH, their preferred specification (-0.596).

[9]In this paper, $\Delta D_{g,t} = D_{g,t} - D_{g,t-1}$, while in Equation (3) in ADH $\Delta D_{g,t} = D_{g,t+1} - D_{g,t}$. Therefore, our $\Delta D_{g,t}$ coincides with $\Delta D_{g,t-1}$ in Equation (3) in ADH. The same applies to $\Delta Z_{g,t}$ defined below: it coincides with $\Delta Z_{g,t-1}$ in Equation (4) in ADH.

4.3 Tests of the identifying assumptions

4.3.1 The randomly-assigned shocks assumption is rejected

Below, we test Points 1 and 2 of Assumption 6: as they are weaker than Points 1 and 2 of Assumption 5, if we reject the former we can also reject the latter.

Shocks' first-differences are correlated to sectors' average shares. Point 1 of Assumption 6 implies that $E\left(\Delta Z_{s,t}|\frac{1}{G}\sum_{s=1}^{S} Q_{s,g}\right) = E\left(\Delta Z_{s,t}\right)$: first-differenced shocks should be mean independent of the average share of sector s across locations. We test this by regressing $\Delta Z_{s,t}$ on $\frac{1}{G}\sum_{s=1}^{S} Q_{s,g}$ in Panel A of Table 1, for $t = 2000$ in Column (1) and for $t = 2010$ in Column (2). We follow Table 3 Panel A in Borusyak et al. (2022), and cluster standard errors at the level of three-digit SIC codes, but results are very similar when one uses robust standard errors. We reject the null, with t-stats equal to -1.96 and -3.93 in Columns (1) and (2): large shocks are more likely to arise in sectors with a lower average share. Results are similar if we use the first-differenced shocks and shares defined by Borusyak et al. (2022), rather than our variables.

Shocks' first-differences are correlated to sectors' characteristics. Point 2 of Assumption 6 implies that the expectation of shocks' first-differences should not vary with sector-level characteristics. We test this by regressing shocks' first-differences on such characteristics. We use the five sector characteristics in Acemoglu et al. (2016) that are in the replication dataset of Borusyak et al. (2022). Panel B of Table 1 shows regressions of shocks' first-differences from 1990 to 2000 and from 2000 to 2010 on these characteristics. We follow Table 3 Panel A in Borusyak et al. (2022) and weight the regressions by sectors' average shares, but the results are very similar when the regressions are not weighted. We find that large shocks' first-differences tend to appear in sectors with low wages and more computer and high-tech investment. We can reject the hypothesis that shocks' first-differences are not correlated with any sectoral characteristic (p-value$<$0.001 in Column (1), p-value=0.038 in Column (2)). Results are similar if we use the first-differenced shocks defined by Borusyak et al. (2022), rather than our variables.

Conditionally randomly assigned shocks? Shocks could be as-good-as randomly assigned conditional on the sectoral characteristics in Panel B of Table 1. If that were true, shocks first-differences should be mean independent of sectors' average shares conditional on those characteristics. We can test this, by adding sectors' average shares to the regressions shown in Panel B of Table 1 (the regressions are no longer weighted by sectors' average shares). The coefficients on sectors' average share are highly significant (p-values=0.015 for $t = 2000$, =0.004 for $t = 2010$).

Comparison with the test of Assumption 6 in Borusyak et al. (2022). Our test of Assumption 6 in Panel B of Table 1 is inspired from, and related to, that in Table 3 Panel A in Borusyak et al. (2022). Regressing each sectoral characteristic on the shocks, they find no significant correlation between characteristics and shocks. As explained above, the difference between our and their results does not come from the differences in our variables' definitions. Reverting the dependent and the independent variables in their Table 3 Panel A would leave their t-stats unchanged, so the difference between our and their test is that they regress the shocks on each characteristic individually, while we regress the shocks on all the characteristics. It follows from standard OLS formulas that the null in our test is stronger than the null in their test: if the coefficients of all characteristics are equal to zero in our long regression, then the coefficients of all characteristics are equal to zero in their short regressions. The fact that we test a stronger implication of Assumption 5 may explain why our test is rejected while theirs is not, though testing a stronger null does not always imply a larger finite-sample power.

Table 1: Testing the random first-differenced shocks assumption

Variables	(1) $\Delta Z_{s,t}$: 1990-2000	(2) $\Delta Z_{s,t}$: 2000-2010
Panel A: Shocks uncorrelated to sectors' average share?		
Sector's average share	-567.488	-1,765.791
	(289.280)	(448.911)
Panel B: Shocks uncorrelated to sectors' characteristics?		
Production workers' share of employment$_{1991}$	3.447	9.481
	(5.155)	(18.260)
Ratio of capital to value-added$_{1991}$	-0.357	1.188
	(0.908)	(2.222)
Log real wage (2007 USD)$_{1991}$	-8.328	-3.815
	(2.092)	(6.117)
Computer investment as share of total investment$_{1990}$	0.173	1.058
	(0.113)	(0.457)
High-tech equipment as share of total investment$_{1990}$	0.206	0.685
	(0.131)	(0.370)
F-test P-value	0.0000	0.0383
Observations	397	397

Notes: The dependent variable in Column (1) (resp. (2)) is the change in per-worker imports from China to other high-income countries from 1990 to 2000 (resp. from 2000 to 2010). In Panel A, the independent variable is sectors' average shares across commuting zones in our Bartik instrument. In Panel B, the independent variables are five sector characteristics obtained from Acemoglu et al. (2016): sectors' share of production workers in employment in 1991, sectors' ratios of capital to value-added in 1991, sectors' log real wages in 1991, sectors' share of investment devoted to computers in 1990, and sectors' share of high-tech equipment in total investment in 1990. Standard errors clustered at the level of three-digit SIC codes are shown in parentheses. The regressions in Panel B are weighted by sectors' average shares. The F-test p-value in Panel B is the p-value of the test that the coefficients on all sector characteristics are equal to 0.

4.3.2 Assumption 8 is rejected, Assumption 12 is not.

Interpretation of the placebo tests in ADH. In their Table 2, ADH implement a placebo test. They estimate: a 2SLS regression of $\Delta Y_{g,1980}$ on the average of $\Delta D_{g,2000}$ and $\Delta D_{g,2010}$, using the average of $\Delta Z_{g,2000}$ and $\Delta Z_{g,2010}$ as the instrument; a 2SLS regression of $\Delta Y_{g,1990}$ on the same treatment, using the same instrument; a stacked 2SLS regression of $\Delta Y_{g,1980}$ and $\Delta Y_{g,1990}$ on the same treatment, using the same instrument. Those analyses yield a valid placebo test of Point 1 of Assumption 2, if $D_{g,t} = 0$ for every $t \leq 1990$. Unfortunately, as explained by

ADH, trade data with China is unavailable in 1970 and 1980, so we cannot compute $D_{g,1970}$ and $D_{g,1980}$. On the other hand, $D_{g,1990}$ can be computed, and we find that it is on average equal to 0.246: even in 1990, US CZs were on average exposed to 246 USD of imports from China per worker. The average of $D_{g,2000}$ is equal to 1.422, which is of course larger, but maybe not by a sufficiently large order of magnitude to consider that US CZs were treated in 2000 and fully untreated in 1990. Following that logic, $\Delta Y_{g,1990}$ may not be used to test Assumptions 2 and 8. Figure 1 in ADH shows that the import penetration ratio from China increased by 111% from 1987 to 1990, namely a 28.2% yearly growth rate. Extrapolating that growth rate from 1980 to 1990 would yield an average value of $D_{g,1980}$ equal to 0.021. At the other extreme, assuming that imports from China did not grow from 1980 to 1987 would yield an average value of $D_{g,1980}$ equal to 0.117. In the first scenario, one may argue that $\Delta Y_{g,1980} \approx \Delta Y_{g,1980}(0)$ is a reasonable approximation, while this approximation might be less reasonable in the second scenario. Accordingly, we report placebos using $\Delta Y_{g,1980}$ below, emphasizing that the absence of trade data with China in 1980 and 1970 complicates the interpretation of those tests.

Testing Assumptions 8 and 12. In Panel A of Table 2, we test Assumption 8 by regressing $\Delta Y_{g,1980}$ on $Z_{g,1990}$. We find that CZs with a larger value of $Z_{g,1990}$ experienced a larger employment growth from 1970 to 1980, thus suggesting a positive 1970-to-1980 pre-trend, similar to that in Table 2 Column (4) of ADH. In Panel B (resp. C), we regress $\Delta Y_{g,1980}$ on $Z_{g,2000}$ (resp. $Z_{g,2010}$) and find similar results, though the magnitude of the pre-trend is smaller. Panels D to F replicate Panels A to C, adding the same control variables as in Column (6) of Table 3 of ADH, the authors' preferred specification. Those controls include census division dummies, and six "baseline" CZ characteristics measured in 1990. Those characteristics are CZs' percentage employment in manufacturing (manufacturing employment divided by total employment), percentage college-educated population, percentage foreign-born population, female employment rate, percentage employment in routine occupations, and average offshorability index of occupations. Those controls seem to "kill" the positive 1970-1980 pre-trend. Panels G to I replicate Panels D to F, keeping CZs' percentage employment in manufacturing as the only control variable and without the census division dummies. Pre-trends are no longer statistically significant. Conducting the same exercise with the remaining five control variables, we always find very significant pre-trends: percentage employment in manufacturing seems to be the key control variable to kill the pre-trend.[10]

Implications. Without Assumption 2, Theorem 1 shows that θ^b identifies the sum of two terms: a bias term arising from the violation of Assumption 2, plus a weighted sum of treatment effects. Thus, analyzing the weights in this second term is useful even if Assumption 2 fails, as it can help analyze a bias in θ^b that may come from heterogeneous treatment effects, on top of another bias that may come from differential trends. On the other hand, it is less straightforward to assess the impact of a violation of Assumption 8 on our IV-CRC estimator. Accordingly, as a

[10]Note that percentage employment in manufacturing is closely related to ADH's main outcome variable. We acknowledge that controlling for a pre-determined outcome measure is sometimes controversial with panel data, as it can lead to mean-reversion phenomena, but this is an other methodological discussion, orthogonal to that we are interested in, so we follow ADH's specification choice.

robustness check we will recompute this estimator controlling for CZs' percentage employment in manufacturing, as Assumption 12 is not rejected with that control variable. We also note that the pre-trend test we can run is very distant in time from the China shock. CZs' employment trends from 1970 to 1980 may not be representative of their counterfactual trends from 1990 to 2010, so our tests of Assumptions 8 and 12 may not be very informative.

Table 2: Pre-trends tests of Assumptions 2, 8, and 12

Regression of $\Delta Y_{g,1980}$ on:	Estimate (1)	Standard error (2)
Panel A: $Z_{g,1990}$	1.066	0.314
Panel B: $Z_{g,2000}$	0.309	0.099
Panel C: $Z_{g,2010}$	0.117	0.034
Panel D: $Z_{g,1990}$ and all controls in ADH	0.162	0.388
Panel E: $Z_{g,2000}$ and all controls in ADH	-0.033	0.121
Panel F: $Z_{g,2010}$ and all controls in ADH	0.025	0.039
Panel G: $Z_{g,1990}$ and CZs' % employment in manufacturing	-0.008	0.336
Panel H: $Z_{g,2000}$ and CZs' % employment in manufacturing	-0.113	0.111
Panel I: $Z_{g,2010}$ and CZs' % employment in manufacturing	-0.030	0.039
Observations	722	

Notes: The table reports estimates of regressions using a US commuting-zone (CZ) level panel data set with five periods, 1970, 1980, 1990, 2000, and 2010. In all panels, the dependent variable is the change of the manufacturing employment per working-age population in CZ g, from 1970 to 1980. In Panel A, D, and G (resp. B, E, and H, C, F, and I), the main independent variable is the 1990 (resp. 2000, 2010) Bartik instrument. In Panels D to F, independent variables also include the same control variables as in Column (6) of Table 3 of Autor et al. (2013) measured in 1990 (see main text for the list of controls). In Panels G to I, independent variables also include CZs' % employment in manufacturing. Standard errors clustered at the CZ level are shown in parentheses. All regressions are unweighted.

4.3.3 Suggestive tests of Assumption 9 are conclusive

To suggestively test Assumption 9, which requires that α_g be mean independent of $D_{g,t}$ conditional on $(Z_{g,1990}, Z_{g,2000}, Z_{g,2010})$, we regress the six CZ characteristics used by ADH as controls on $(D_{g,t}, Z_{g,1990}, Z_{g,2000}, Z_{g,2010})$, for $t = 1990$, 2000 and 2010. Those characteristics are likely to be correlated with CZs' effects of imports from China on their manufacturing employment. In particular, the percentage employment in routine occupations and the average offshorability index of occupations should be good predictors of α_g. The results, shown in Table 3, are conclusive. Of the 18 coefficients in Table 3, only three are significant at the 5% level.

Table 3: Suggestive tests of Assumption 9

	(1) Manufacturing	(2) College	(3) Foreign	(4) Women	(5) Routine	(6) Offshorability
$D_{g,1990}$	-1.350	-1.206	-0.401	-0.466	-0.813	-0.091
	(2.267)	(0.861)	(0.287)	(0.692)	(0.498)	(0.061)
$D_{g,2000}$	0.060	-0.018	-0.139	0.061	-0.111	-0.018
	(0.581)	(0.160)	(0.069)	(0.147)	(0.124)	(0.014)
$D_{g,2010}$	0.603	-0.083	-0.098	0.087	0.040	0.004
	(0.296)	(0.083)	(0.031)	(0.077)	(0.063)	(0.008)
Observations	722	722	722	722	722	722

Notes: The table shows results of suggestive tests of Assumption 9. The six CZ characteristics used by Autor et al. (2013) as controls, measured in 1990, are regressed on $(D_{g,t}, Z_{g,1990}, Z_{g,2000}, Z_{g,2010})$, for $t = 1990$, $t = 2000$, and $t = 2010$. The table shows the coefficients of $D_{g,t}$ in those regressions, and their robust standard errors. The six CZ characteristics are CZs' percentage employment in manufacturing, percentage college-educated population, percentage foreign-born population, female employment rate, percentage employment in routine occupations, and average offshorability index of occupations.

4.4 Results

4.4.1 Bartik regressions

Columns (1) to (3) of Table 4 below show the results of the Bartik first-stage, reduced-form, and 2SLS regressions. In Column (1), the first-stage coefficient is 0.967. In Column (2), the reduced form coefficient is -0.545. In Column (3), the 2SLS coefficient is -0.564. In Column (4), the 2SLS regression is weighted by CZs' population in 1990, as in ADH, and the coefficient is -0.535. Standard errors clustered at the CZ level are shown between parentheses. All coefficients are statistically significant. The weighted 2SLS coefficient slightly differs from that in Table 2 Column (3) in ADH, because some of our variables' definitions differ, as explained above.

Table 4: Bartik estimates of the effect of imports from China on US manufacturing employment

	FS (1)	RF (2)	2SLS (3)	2SLS, Weighted (4)
	0.967	-0.545	-0.564	-0.535
	(0.093)	(0.071)	(0.091)	(0.061)
Observations	1,444	1,444	1,444	1,444

Notes: Columns (1) to (3) respectively report estimates of the first-stage, reduced-form, and 2SLS Bartik regressions with period fixed effects, using a US commuting-zone (CZ) level panel data set with $T = 3$ periods, 1990, 2000, and 2010. The regressions are unweighted. $\Delta Y_{g,t}$ is the change of the manufacturing employment per working-age population in CZ g, from 1990 to 2000 for $t = 2000$, and from 2000 to 2010 for $t = 2010$. $\Delta D_{g,t}$ is the change in exposure to imports from China in CZ g from 1990 to 2000 for $t = 2000$, and from 2000 to 2010 for $t = 2010$. $\Delta Z_{g,t}$ is the Bartik instrument, whose construction is detailed in the text. Column (4) reports estimates of the 2SLS Bartik regressions, weighted by CZ's share of national population in 1990. Standard errors clustered at the CZ level shown in parentheses.

4.4.2 Decompositions of the 2SLS Bartik regression

We follow Point 1 of Theorem B.1 in the Web Appendix, a generalization of Theorem 1 to weighted 2SLS Bartik regressions with more than two time periods, to estimate the weights attached to the Bartik regression in Column (4) of Table 4. The first column of Panel A of Table 5 shows that θ^b estimates a weighted sum of 2166 (722 CZs $\times 3$ periods) effects $\alpha_{g,t}$, where 1163 weights are positive, 1003 weights are strictly negative, and negative weights sum to -0.734. Therefore, θ^b is far from estimating a convex combination of effects. We do not have exactly one half of negative weights, because the regression uses three time periods, and this result is specific to the two-periods case. The weights are correlated with the year t (correlation=0.082, p-value<0.001). We also test if the weights are correlated with the six CZ-level characteristics that ADH use as controls in their preferred specification, measured in 1990. We find that the weights are correlated with CZs' percentage employment in manufacturing (correlation=0.057, p-value=0.008), percentage foreign-born population (correlation=0.075, p-value<0.001), percentage employment in routine occupations (correlation=0.043, p-value=0.044), average offshorability index of occupations (correlation=0.084, p-value<0.001) and not significantly correlated with the other characteristics. The second column of Panel A of Table 5 shows that even if one assumes constant effects over time, θ^b still estimates a weighted sum of 722 location-specific effects α_g, where 479 weights are strictly negative and negative weights sum to -0.314. As our Bartik regression is not numerically identical to the Bartik regressions in ADH, we also estimate the weights attached to the regressions in their Table 2, Column (1) and (2), under the assumption that $\alpha_{g,t} = \alpha_g$. The regressions in their Table 2 Column (1) and (2) only use two periods of data, thus allowing us to bypass the fact that their shares are time-varying (see (4.7)): with only one first-difference, their shares are time-invariant, as in our decompositions (extending our decompositions to allow for time-varying shares would not be difficult). Assuming that $\alpha_{g,t} = \alpha_g$ allows us to bypass the fact that their first-differenced treatment is hard to reconcile with a treatment in levels (see (4.7)): when assuming that $\alpha_{g,t} = \alpha_g$, our decomposition of θ^b no longer depends on $D_{g,t}$. In Panel B of Table 5, we find similar proportions and sums of negative weights as in the second column of Panel A.

Table 5: Weights attached to Bartik regressions

Panel A: Regression in Column (4) of Table 4		
Assumption on treatment effects	None	$\alpha_{g,t} = \alpha_g$
Number of strictly negative weights	1003	479
Number of positive weights	1163	243
Sum of negative weights	-0.734	-0.314
Panel B: Regressions in ADH Table 2, assuming $\alpha_{g,t} = \alpha_g$		
Column # in ADH Table 2	(1)	(2)
Number of strictly negative weights	454	429
Number of positive weights	268	293
Sum of negative weights	-0.315	-0.339

Notes: Panel A reports summary statistics on the weights attached to the 2SLS regression in Column (4) of Table 4. In the first column, no assumption is made on the treatment effects. In the second column, we assume that treatment effects do not vary over time ($\alpha_{g,t} = \alpha_g$). Panel B reports summary statistics on the weights attached to the 2SLS regressions in ADH Table 2, assuming that treatment effects do not vary over time.

4.4.3 Alternative IV-CRC estimator

Main results. In Table 6, we report IV-CRC estimates of the effects of imports from China on CZs' employment, following Theorem 4. Our baseline specification assumes that

$$E(D_{g,t}|Z_{g,1990}, Z_{g,2000}, Z_{g,2010}) = \delta_{0,t} + \delta_{t,t} Z_{g,t}, \tag{4.8}$$

and reports

$$\widehat{\alpha}_{ate,w} \equiv \frac{1}{G} \sum_{g=1}^{722} pop_g \sum_t (\widehat{\alpha}_g + \widehat{\lambda}_t),$$

where pop_g denotes CZs' populations in 1990. $\widehat{\alpha}_{ate,w}$ weights the CZ-specific effects $\sum_t (\widehat{\alpha}_g + \widehat{\lambda}_t)$ by CZs' population, consistent with the weighted 2SLS Bartik regression in Column (4) of Table 4. Our baseline estimate is positive, small, and insignificantly different from 0. It is of a different sign than the coefficient in Column (4) of Table 4. The difference between the two estimates is significant at the 5% level (t-stat=-2.086). The standard error of our IV-CRC estimate is about 5 times larger than that of the 2SLS estimate: allowing for some treatment-effect heterogeneity comes with a cost in terms of precision. In view of the positive 1970-to-1980 pre-trend shown in Table 2, which disappears once CZs' 1990 percentage employment in manufacturing is controlled for, we recompute our IV-CRC estimator controlling for that variable. The second line of Table 6 shows that with this control, the IV-CRC estimate becomes negative, but is still fairly small and insignificant. A cross-validation exercise, where we compare the out-of-sample fit of the model in (4.8) and of polynomials of order 1 to 3 in $(Z_{g,1990}, Z_{g,2000}, Z_{g,2010})$, shows that the polynomial of order 1 with all lags and leads of the Bartik instrument has the best out-of-sample fit, closely followed by (4.8), and the two models are much better than all the other models. Accordingly, we recompute our IV-CRC estimate, using a polynomial of order 1 in $(Z_{g,1990}, Z_{g,2000}, Z_{g,2010})$ as the first-stage model. The resulting estimate is positive, insignificant, and much more noisy than our baseline estimate. Finally, we compute an IV-CRC estimate assuming constant effects over time. Interestingly, this estimate is large, negative, significant, and very close to the Bartik 2SLS estimate. It is also significantly different from our baseline IV-CRC estimate (t-stat=-2.330), which implies that we can reject the null that the treatment effect is constant over time. This suggests that time-varing effects might be biasing downward the Bartik 2SLS estimate.

Table 6: IV-CRC estimates of the effect of imports from China on US manufacturing employment

	Estimate	Standard error
	(1)	(2)
Baseline estimate	0.138	0.312
Estimate controlling for CZs' percentage employment in manufacturing	-0.224	0.319
First-stage model where treatment regressed on instrument at all dates	0.492	0.685
Estimate assuming constant effects over time	-0.501	0.225
Observations	722	

Notes: Columns (1) and (2) report IV-CRC estimates of the effect of imports from China on US manufacturing employment, computed using a US commuting-zone (CZ) level panel data set with $T = 3$ periods, 1990, 2000, and 2010. $Y_{g,t}$ is the manufacturing employment per working-age population in CZ g in year t. $D_{g,t}$ is the exposure to imports from China in CZ g in year t. $Z_{g,t}$ is the Bartik instrument, whose construction is detailed in the text. Column (1) reports IV-CRC estimates computed following Theorem 4. Column (2) reports bootstrapped standard errors.

Testing for heterogeneous effects across CZs. To test for heterogeneous effects across CZs, we regress the CZ-specific estimated effects $\sum_t(\widehat{\alpha}_g + \widehat{\lambda}_t)$ on the six 1990-CZ-level characteristics used by ADH as controls in their preferred specification, weighting the regression by CZs' population. To obtain standard errors, we bootstrap the whole estimation procedure, clustering the bootstrap at the CZ level. Panel A of Table 7 below shows that CZ-specific effects are significantly negatively correlated with CZs' percentage employment in manufacturing and in routine occupations, but only the latter remains significant at the 5% level after a Bonferroni adjustment accounting for the six dimensions of heterogeneity tested in Table 7. It makes intuitive sense that CZs with a larger employment rate in routine occupations may be more negatively affected by imports from China. To test this hypothesis directly, we average $\sum_t(\widehat{\alpha}_g + \widehat{\lambda}_t)$ across CZs with an employment rate in routine occupations above and below the median, weighting the median by CZs population. It turns out that CZs' employment rate in routine occupations is highly correlated with their population, so the 72 CZs with the largest employment rate in routine occupations account for 50% of CZs' population. Panel B of Table 7 shows that in those 72 CZs, our IV-CRC estimate is very slightly negative, but still insignificant. In the remaining CZs, our IV-CRC estimate is positive and insignificant. The difference between the IV-CRC estimates in the two subgroups is highly significant (t-stat=-3.694).

Table 7: Heterogeneous treatment effects

	Estimate	Standard error
	(1)	(2)
Panel A: Predictors of CZs' treatment effects		
Percentage employment in manufacturing	-0.033	0.015
Percentage college-educated population	-0.012	0.012
Percentage foreign-born population	0.011	0.006
Female employment rate	0.023	0.015
Percentage employment in routine occupations	-0.127	0.049
Average offshorability index of occupations	-0.036	0.210
Observations	722	
Panel B: Subgroup analysis		
Above median % employment in routine occupations	-0.059	0.297
Observations	72	
Below median % employment in routine occupations	0.336	0.338
Observations	650	

Notes: Panel A shows results from a regression of $\sum_t(\widehat{\alpha}_g + \widehat{\lambda}_t)$, the estimated average effect of imports from China on employment of commuting-zone (CZ) g, on six CZ characteristics measured in 1990. The six characteristics are CZs' percentage employment in manufacturing, percentage college-educated population, percentage foreign-born population, female employment rate, percentage employment in routine occupations, and average offshorability index of occupations. Column (1) shows the coefficient of each variable in the regression, Column (2) shows a standard error, computed by bootstrapping the whole estimation procedure, clustering the bootstrap at the CZ level. The regression is run in the sample of 722 CZs used by ADH. Column (1) Panel B shows IV-CRC estimates of the effect of imports from China on the manufacturing employment share, separately for CZs above and below the median of percentage employment in routine occupations, where the median is weighted by CZs population. Panel B Column (2) shows the bootstrapped standard error of effects in Column (1).

5 Recommendations for practitioners

First, we recommend that practitioners start their analysis by testing the random-shocks assumption. To do so, they can regress the shocks $Z_{s,t}$ on sectors' average share across locations $Q_{s,.}$, and/or sectoral characteristics, controlling for period fixed effects.

When $Q_{s,.}$ and/or sectoral characteristics do not significantly predict the shocks, this is evidence that shocks are as-good-as randomly assigned. Then, practitioners may either use a slightly modified panel Bartik estimator, where shocks are standardized by their period-specific standard deviation when constructing the instrument, or a pooled-cross-section 2SLS Bartik regression of $Y_{g,t}$ on $D_{g,t}$ using $Z_{g,t}$ as the instrument, with period fixed effects but no location fixed effects. With randomly assigned shocks, both regressions estimate a convex combination of effects, even if effects vary over time and across locations.

On the other hand, when $Q_{s,.}$ and/or sectoral characteristics significantly predict the shocks, this is evidence that shocks are not as-good-as randomly assigned. Then, practitioners can start by estimating the weights in the decomposition of the Bartik coefficient we give in our Theorem 1. If most or all weights are positive, this means that this coefficient is robust to heterogeneous treatment effects under a fairly minimal exogeneity assumption, so using that estimator may be a reasonable choice. If many weights are negative, and if weights are correlated with characteristics likely to be correlated with treatment effects, the Bartik coefficient may be biased. In such instances, practitioners may consider using our IV-CRC estimator instead. If the data contains a period t_0 such that all locations are untreated at t_0 and t_0-1, we recommend that practitioners test the exogeneity condition underlying our IV-CRC estimator, by regressing $\Delta Y_{g,t_0}$ on $Z_{g,t'}$ for $t' \neq t_0$. Our estimator also requires that locations' treatment effects be independent of $D_{g,t}$, conditional on $\boldsymbol{Z}_g$. We recommend that practitioners also suggestively test that assumption, by regressing covariates likely to be correlated with locations' treatment effects on $(D_{g,t}, \boldsymbol{Z}_g)$.

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6 Proofs

6.1 Proof of Theorem 1

$$
\begin{aligned}
& E\left(\sum_{g=1}^{G} \Delta Y_g \left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right) \\
=& E\left(\sum_{g=1}^{G} \left(\Delta Y_g(0) + \alpha_{g,2} D_{g,2} - \alpha_{g,1} D_{g,1}\right)\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right) \\
=& \sum_{g=1}^{G} E\left(\Delta Y_g(0)\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right) + E\left(\sum_{g=1}^{G}\sum_{t=1}^{2}(1\{t=2\}-1\{t=1\}) D_{g,t}\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\alpha_{g,t}\right).
\end{aligned} \tag{6.1}
$$

The first equality follows from Assumption 1. Similarly,

$$
\begin{aligned}
& E\left(\sum_{g=1}^{G} \Delta D_g \left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right) \\
=& E\left(\sum_{g=1}^{G}\sum_{t=1}^{2}(1\{t=2\}-1\{t=1\}) D_{g,t}\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right).
\end{aligned} \tag{6.2}
$$

Then, plugging (6.1) and (6.2) into (2.1) yields Point 1 of the theorem. Point 2 directly follows from Point 1. **QED.**

6.2 Theorem 2

$$
\begin{aligned}
& E\left(\sum_{g=1}^{G} \Delta Y_g \left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right) \\
=& E\left(\sum_{g=1}^{G} \left(\Delta Y_g(D_g(\mathbf{0})) + \alpha_{g,2}\sum_{s=1}^{S} Q_{s,g}\beta_{s,g,2}Z_{s,2} - \alpha_{g,1}\sum_{s=1}^{S} Q_{s,g}\beta_{s,g,1}Z_{s,1}\right)\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\right) \\
=& \sum_{g=1}^{G} E\left(\Delta Y_g(D_g(\mathbf{0}))\left(\Delta Z_g - E\left(\Delta Z_g\right)\right)\right) \\
&+ E\left(\sum_{g=1}^{G}\sum_{t=1}^{2}(1\{t=2\}-1\{t=1\})\sum_{s=1}^{S} Q_{s,g}\beta_{s,g,t}Z_{s,t}\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\alpha_{g,t}\right) \\
=& E\left(\sum_{g=1}^{G}\sum_{t=1}^{2}(1\{t=2\}-1\{t=1\})\sum_{s=1}^{S} Q_{s,g}\beta_{s,g,t}Z_{s,t}\left(\Delta Z_g - E\left(\Delta Z_.\right)\right)\alpha_{g,t}\right).
\end{aligned} \tag{6.3}
$$

The first equality follows from (2.5). The second equality follows from Point 3 of Assumption

4. The third equality follows from Point 1 of Assumption 4. Similarly, one can show that

$$\begin{aligned}
&E\left(\sum_{g=1}^{G}\Delta D_g\left(\Delta Z_g-E\left(\Delta Z_.\right)\right)\right)\\
=&E\left(\sum_{g=1}^{G}\sum_{t=1}^{2}(1\{t=2\}-1\{t=1\})\sum_{s=1}^{S}Q_{s,g}\beta_{s,g,t}Z_{s,t}\left(\Delta Z_g-E\left(\Delta Z_.\right)\right)\right). \qquad (6.4)
\end{aligned}$$

Then, plugging (6.3) and (6.4) into (2.1) yields Point 1 of the theorem. Points 2 and 3 directly follows from Point 1.

6.3 Theorem 3

First, as shares sum to one, it follows from Assumption 5 that $E(\Delta Z_g|\mathcal{F}) = \Delta m$ for all g, so

$$E(\Delta Z_.) = \Delta m. \qquad (6.5)$$

Then,

$$\begin{aligned}
&E(\Delta Y_g(\Delta Z_g-\Delta m))\\
=&E\left((\Delta Y_g(0)+\alpha_{g,2}D_{g,2}-\alpha_{g,1}D_{g,1})(\Delta Z_g-\Delta m)\right)\\
=&E\left((\Delta Y_g(0)+\alpha_{g,2}D_{g,2}(\mathbf{0})-\alpha_{g,1}D_{g,1}(\mathbf{0}))\left(E(\Delta Z_g|\mathcal{F})-\Delta m\right)\right)\\
+&E\left(\left(\sum_{s=1}^{S}Q_{s,g}\beta_{s,g,2}Z_{s,2}(\Delta Z_g-\Delta m)\right)\alpha_{g,2}\right)-E\left(\left(\sum_{s=1}^{S}Q_{s,g}\beta_{s,g,1}Z_{s,1}(\Delta Z_g-\Delta m)\right)\alpha_{g,1}\right)\\
=&E\left(\left(\sum_{s,s'}Q_{s,g}Q_{s',g}\beta_{s,g,2}E\left(Z_{s,2}(Z_{s',2}-m_2)|\mathcal{F}\right)-\sum_{s,s'}Q_{s,g}Q_{s',g}\beta_{s,g,2}E\left(Z_{s,2}(Z_{s',1}-m_1)|\mathcal{F}\right)\right)\alpha_{g,2}\right)\\
+&E\left(\left(\sum_{s,s'}Q_{s,g}Q_{s',g}\beta_{s,g,1}E\left(Z_{s,1}(Z_{s',1}-m_1)|\mathcal{F}\right)-\sum_{s,s'}Q_{s,g}Q_{s',g}\beta_{s,g,1}E\left(Z_{s,1}(Z_{s',2}-m_2)|\mathcal{F}\right)\right)\alpha_{g,1}\right)\\
=&E\left(\sum_{s=1}^{S}Q_{s,g}^2\beta_{s,g,2}\left(V\left(Z_{s,2}\right)-\mathrm{cov}\left(Z_{s,1},Z_{s,2}\right)\right)\alpha_{g,2}\right)\\
+&E\left(\sum_{s=1}^{S}Q_{s,g}^2\beta_{s,g,1}\left(V\left(Z_{s,1}\right)-\mathrm{cov}\left(Z_{s,1},Z_{s,2}\right)\right)\alpha_{g,1}\right). \qquad (6.6)
\end{aligned}$$

The first equality follows from (1.3). The second equality follows from (2.3) and the law of iterated expectations. The third equality follows from the fact that $E(\Delta Z_g|\mathcal{F}) = \Delta m$, by Assumption 5 and as shares sum to one, and from the law of iterated expectations. The fourth equality follows from the fact that by Assumption 5, for all $s \neq s'$ and $(t,t') \in \{1,2\}^2$, $E\left(Z_{s,t}(Z_{s',t'}-m_{t'})|\mathcal{F}\right) = E\left(Z_{s,t}|\mathcal{F}\right)E\left(Z_{s',t'}-m_{t'}|\mathcal{F}\right) = 0$. Moreover, as for all s and $(t,t') \in \{1,2\}^2$, $E\left(Z_{s,t}Z_{s,t'}|\mathcal{F}\right) = E\left(Z_{s,t}Z_{s,t'}\right)$ and $E\left(Z_{s,t}|\mathcal{F}\right) = m_t$, $E\left(Z_{s,t}(Z_{s',t'}-m_{t'})|\mathcal{F}\right) = \mathrm{cov}\left(Z_{s,t},Z_{s,t'}\right)$.

Similarly, one can show that

$$
\begin{aligned}
&E(\Delta D_g(\Delta Z_g-\Delta m))\\
=&E\left(\sum_{s=1}^{S}Q_{s,g}^2\beta_{s,g,2}\left(V\left(Z_{s,2}\right)-\text{cov}\left(Z_{s,1},Z_{s,2}\right)\right)\right)\\
+&E\left(\sum_{s=1}^{S}Q_{s,g}^2\beta_{s,g,1}\left(V\left(Z_{s,1}\right)-\text{cov}\left(Z_{s,1},Z_{s,2}\right)\right)\right).
\end{aligned}
\tag{6.7}
$$

The result follows plugging (6.5), (6.6), and (6.7) into (2.1).

6.4 Theorem 4

For all g and t,

$$
\begin{aligned}
E(Y_{g,t}|\boldsymbol{Z}_g)=&E(Y_{g,t}(0)|\boldsymbol{Z}_g)+E(\alpha_g D_{g,t}|\boldsymbol{Z}_g)+1\{t\geq 2\}\lambda_t E(D_{g,t}|\boldsymbol{Z}_g)\\
=&E(Y_{g,1}(0)|\boldsymbol{Z}_g)+1\{t\geq 2\}\mu_{1:t}+(E(\alpha_g|\boldsymbol{Z}_g)+1\{t\geq 2\}\lambda_t)E(D_{g,t}|\boldsymbol{Z}_g)\\
=&1\{t\geq 2\}(\mu_{1:t}+\lambda_t\tilde{D}_{g,t})+E(Y_{g,1}(0)|\boldsymbol{Z}_g)+E(\alpha_g|\boldsymbol{Z}_g)\tilde{D}_{g,t}
\end{aligned}
$$

The first equality follows from Assumptions 1 and 7, the second equality follows from Assumption 8 and from the law of iterated expectations and Assumption 9.The previous display and the law of iterated expectations imply that

$$E(Y_{g,t}|\tilde{\boldsymbol{D}}_g)=1\{t\geq 2\}(\mu_{1:t}+\lambda_t\tilde{D}_{g,t})+E(Y_{g,1}(0)|\tilde{\boldsymbol{D}}_g)+E(\alpha_g|\tilde{\boldsymbol{D}}_g)\tilde{D}_{g,t}.$$

Let $\boldsymbol{\gamma}_g=(E(Y_{g,1}(0)|\tilde{\boldsymbol{D}}_g),E(\alpha_g|\tilde{\boldsymbol{D}}_g))'$. It follows from the previous display that

$$E(\boldsymbol{Y}_g|\tilde{\boldsymbol{D}}_g)=\mathcal{P}_g\boldsymbol{\theta}+\mathcal{X}_g\gamma_g. \tag{6.8}$$

As $M(\mathcal{X}_g)\mathcal{X}_g=0$, left-multiplying (6.8) by $\mathcal{P}_g'M(\mathcal{X}_g)$,

$$E(\mathcal{P}_g'M(\mathcal{X}_g)\boldsymbol{Y}_g|\tilde{\boldsymbol{D}}_g)=\mathcal{P}_g'M(\mathcal{X}_g)\mathcal{P}_g\boldsymbol{\theta}.$$

Therefore, by the law of iterated expectation and averaging across locations:

$$E\left(\frac{1}{G}\sum_{g=1}^{G}\mathcal{P}_g'M(\mathcal{X}_g)\boldsymbol{Y}_g\right)=E\left(\frac{1}{G}\sum_{g=1}^{G}\mathcal{P}_g'M(\mathcal{X}_g)\mathcal{P}_g\right)\boldsymbol{\theta}.$$

(3.2) follows from the previous display and the fact $E\left(\frac{1}{G}\sum_{g=1}^{G}\mathcal{P}_g'M(\mathcal{X}_g)\mathcal{P}_g\right)$ is invertible.

Then, we left-multiply (6.8) by $\mathcal{X}_g'$, and it follows that

$$E(\mathcal{X}_g'\boldsymbol{Y}_g|\tilde{\boldsymbol{D}}_g)=\mathcal{X}_g'\mathcal{P}_g\boldsymbol{\theta}+\mathcal{X}_g'\mathcal{X}_g\gamma_g.$$

(3.3) follows from: rearranging; the fact $\mathcal{X}_g'\mathcal{X}_g$ is invertible with probability one; the law of iterated expectations; and averaging across locations.

Web Appendix: not for publication

A Empirical application: canonical Bartik design

In this section, we revisit the canonical application in Bartik (1991), where the Bartik instrument is used to estimate the inverse elasticity of labor supply.

A.1 Data

Our data construction closely follows Goldsmith-Pinkham et al. (2020). We construct a decennial continental US commuting-zone (CZ) level panel data set, from 1990 to 2010, with CZ wages and employment levels. For 1990 and 2000, we use the 5% IPUMS sample of the U.S. Census. For 2010, we pool the 2009-2011 ACSs (Ruggles et al. 2019). Sectors are IND1990 sectors. We follow Autor & Dorn (2013) to reallocate Public Use Micro Areas level observations of Census data to the CZ level. We also follow ADH to aggregate the Census sector code ind1990 to a balanced panel of sectors for the 1990 and 2000 Censuses and the 2009-2011 ACS, with new sector code ind1990dd.[A.1] In our final dataset, we have 3 periods, 722 CZs and 212 sectors.

The outcome variable $\Delta Y_{g,t}$ is the change in log wages in CZ g from $t-10$ to t, for $t \in \{2000, 2010\}$. The treatment variable $\Delta D_{g,t}$ is the change in log employment in CZ g from $t-10$ to t. We use people aged 18 and older who are employed and report usually working at least 30 hours per week in the previous year to generate employment and average wages. We define $Q_{s,g}$ as the employment share of sector s in CZ g in 1990, and then construct the Bartik instrument using 1990-2000 and 2000-2010 sectoral employment growth rates.[A.2]

A.2 Results

A.2.1 Bartik regressions

Columns (1) to (3) of Table A.1 below show the results of the first-stage, reduced-form, and 2SLS Bartik regressions. In Column (1), the first-stage coefficient is 0.824. In Column (2), the reduced form coefficient is 0.391. Finally, in Column (3), the 2SLS coefficient is 0.475. If interpreted causally, this 2SLS coefficient means that a 1% increase in employment leads to a 0.475% increase in wages. Robust standard errors clustered at the CZ level are shown between parentheses. All coefficients are statistically significant.

[A.1]Crosswalk files are available online at https://www.ddorn.net/data.htm. The original crosswalk file for sector code only creates a balanced panel of sectors up to the 2006-2008 ACSs. We extend the crosswalk approach to one additional sector (shoe repair shops, crosswalked into miscellaneous personal services) to create a balanced panel of sectors up to the 2009-2011 ACSs.

[A.2]We do not use leave-one-out growth rates, because doing so would lead to inconsistent Bartik and first-difference Bartik instruments. Adão et al. (2019) and Goldsmith-Pinkham et al. (2020) recommend using leave-one-out to construct the national growth rates, in order to avoid the finite sample bias that comes from using own-observation information. In practice, because we have 722 locations, whether one uses leave-one-out or not to estimate the national growth rates barely changes the results.

Table A.1: Bartik estimates of the canonical setting

	FS	RF	2SLS
	(1)	(2)	(3)
	0.824	0.391	0.475
	(0.055)	(0.031)	(0.039)
Observations	1,444	1,444	1,444

Notes: Columns (1) to (3) respectively report estimates of Bartik regressions with period fixed effects, using a decennial US commuting-zone (CZ) level panel data set from 1990 to 2010. $\Delta Y_{g,t}$ is the change in log wages in CZ g from $t-10$ to t, for $t \in \{2000, 2010\}$. $\Delta D_{g,t}$ is the change in log employment in CZ g from $t-10$ to t. $\Delta Z_{g,t}$ is the Bartik instrument, whose construction is detailed in the text. Standard errors clustered at the CZ level shown in parentheses.

A.2.2 Decompositions of the 2SLS Bartik regression

We follow Theorem B.1 in the Web Appendix, a straightforward generalization of Theorem 1 to more than two periods, to estimate the weights attached to the Bartik 2SLS regression under Assumptions 1 and B.1 (the latter is a generalization of Assumption 2 to more than two periods). Column (1) of Table A.2 shows that under those assumptions, θ^b estimates a weighted sum of 2166 (722 CZs $\times 3$ periods) effects $\alpha_{g,t}$, where 1035 weights are positive, 1131 weights are strictly negative, and where negative weights sum to -163.495. Therefore, θ^b is extremely far from estimating a convex combination of effects. Column (2) shows that even if one further assumes constant effects over time, θ^b still estimates a weighted sum of 722 location-specific effects α_g, where 519 weights are positive, 203 weights are strictly negative, and where negative weights sum to -0.282.

Table A.2: Summary statistics on the weights attached to Bartik regressions in Table A.1

Assumption on treatment effects	None	$\alpha_{g,t} = \alpha_g$
	(1)	(2)
Number of strictly negative weights	1035	203
Number of positive weights	1131	519
Sum of negative weights	-163.495	-0.282

Notes: The table reports summary statistics on the weights attached to the 2SLS regression in Column (3) of Table A.1. The weights are estimated following Theorem B.1. In Column (1), no assumption is made on the first-stage and treatment effects. Column (2) assumes that the treatment effects do not vary over time ($\alpha_{g,t} = \alpha_g$).

A.2.3 Alternative IV-CRC estimator

Estimation procedure. In Table A.3, we follow Theorem 4 and present IV-CRC estimates. We assume that Assumption 10 holds with $K = 1$ and $K = 2$, based on a cross-validation exercise, where we compare the out-of-sample fit of the models with polynomials of order 1 to 5. A standard error is obtained by bootstrapping the whole estimation procedure, clustering at the CZ level.

Results. Our IV-CRC estimate is equal to 0.316 when $K = 1$ and equal to 0.407 when $K = 2$, slightly below the 2SLS coefficient in Column (3) of Table A.1. Estimates' standard errors are respectively 67% and 33% larger than that of the 2SLS Bartik estimator.

Table A.3: IV-CRC estimates of the canonical setting

	IV-CRC
Polynomial of order 1 in instruments at all dates	0.316
	(0.065)
Polynomial of order 2 in instruments at all dates	0.407
	(0.052)
Observations	722

Notes: The table reports IV-CRC estimates of the effect of employment on wages, computed using a US commuting-zone (CZ) level panel data set with $T = 3$ periods, 1990, 2000, and 2010. $Y_{g,t}$ is the log wages in CZ g in year t. $D_{g,t}$ is the log employment in CZ g in year t. $Z_{g,t}$ is the Bartik instrument, whose construction is detailed in the text. The estimates are computed following Theorem 4. Bootstrapped standard error are shown in parentheses. We use two first-stage models, one where the treatment is regressed on a first-order polynomial of instruments at all dates, and one where the treatment is regressed on a second-order polynomial of instruments at all dates.

B Weighted panel Bartik regressions with multiple periods

In this section, we use the same notation and definitions as in Section 3 of the paper and we extend our decompositions of Bartik regressions in Theorem 1 to weighted regressions, with multiple periods.

Estimator and estimand. With several periods, the analog of the 2SLS first-differenced regression with a constant in the paper is a first-differenced 2SLS regression with period fixed effects. Let $w_{g,t}$ be the positive weights used in the regression, which are treated as non-stochastic quantities in what follows. For every t, let $\Delta Z^w_{.,t} = \frac{\sum_{g=1}^G w_{g,t}\Delta Z_{g,t}}{\sum_{g=1}^G w_{g,t}}$ denote the weighted average of $\Delta Z_{g,t}$ at period t.

Definition B.1 *2SLS Bartik regression with multiple periods: let*

$$\hat{\theta}^b = \frac{\sum_{g=1}^G \sum_{t=2}^T w_{g,t}\Delta Y_{g,t}\left(\Delta Z_{g,t} - \Delta Z^w_{.,t}\right)}{\sum_{g=1}^G \sum_{t=2}^T w_{g,t}\Delta D_{g,t}\left(\Delta Z_{g,t} - \Delta Z^w_{.,t}\right)} \tag{B.1}$$

$$\theta^b = \frac{\sum_{g=1}^G \sum_{t=2}^T w_{g,t}E\left(\Delta Y_{g,t}\left(\Delta Z_{g,t} - E\left(\Delta Z^w_{.,t}\right)\right)\right)}{\sum_{g=1}^G \sum_{t=2}^T w_{g,t}E\left(\Delta D_{g,t}\left(\Delta Z_{g,t} - E\left(\Delta Z^w_{.,t}\right)\right)\right)}. \tag{B.2}$$

Identifying assumptions. The following assumption generalizes Assumption 2 to the case with multiple periods.

Assumption B.1 *1. For all $g \in \{1, ..., G\}$, $t \in \{2, ..., T\}$, $cov(\Delta Z_{g,t}, \Delta Y_{g,t}(0)) = 0$.*

2. For all $t \in \{2, ..., T\}$, $E\left(\Delta Z_{g,t}\right)$ does not depend on g.

Decompositions of θ^b under Assumptions 1 and B.1.

Theorem B.1 *Suppose Assumptions 1 and B.1 hold.*

1. *Then,*

$$\theta^b = E\left(\sum_{g=1}^{G}\sum_{t=1}^{T}\frac{D_{g,t}\left(1\{t\geq 2\}w_{g,t}\left(\Delta Z_{g,t}-E\left(\Delta Z^w_{.,t}\right)\right)-1\{t\leq T-1\}w_{g,t+1}\left(\Delta Z_{g,t+1}-E\left(\Delta Z^w_{.,t+1}\right)\right)\right)}{E\left(\sum_{g'=1}^{G}\sum_{t'=1}^{T}D_{g',t'}\left(1\{t'\geq 2\}w_{g',t'}\left(\Delta Z_{g',t'}-E\left(\Delta Z_{.,t'}\right)\right)-1\{t'\leq T-1\}w_{g',t'+1}\left(\Delta Z_{g',t'+1}-E\left(\Delta Z_{.,t'+1}\right)\right)\right)\right)}\alpha_{g,t}\right)$$

2. *If one further assumes that for all (g,t), there exists α_g such that $\alpha_{g,t}=\alpha_g$, and if the weights $w_{g,t}$ are time invariant, then*

$$\theta^b = E\left(\sum_{g=1}^{G}\frac{w_g\sum_{t=2}^{T}\Delta D_{g,t}\left(\Delta Z_{g,t}-E\left(\Delta Z^w_{.,t}\right)\right)}{E\left(\sum_{g'=1}^{G}w_{g'}\sum_{t=2}^{T}\Delta D_{g',t}\left(\Delta Z_{g',t}-E\left(\Delta Z^w_{.,t}\right)\right)\right)}\alpha_g\right).$$

C Proofs of results in Web Appendix

C.1 Theorem B.1

$$
\begin{aligned}
& E\left(\sum_{g=1}^{G}\sum_{t=2}^{T} w_{g,t}\Delta Y_{g,t}\left(\Delta Z_{g,t}-E\left(\Delta Z^{w}_{.,t}\right)\right)\right)\\
=& E\left(\sum_{g=1}^{G}\sum_{t=2}^{T} w_{g,t}\left(\Delta Y_{g,t}(0)+\alpha_{g,t}D_{g,t}-\alpha_{g,t-1}D_{g,t-1}\right)\left(\Delta Z_{g,t}-E\left(\Delta Z^{w}_{.,t}\right)\right)\right)\\
=& \sum_{g=1}^{G}\sum_{t=2}^{T} w_{g,t}E\left(\Delta Y_{g,t}(0)\left(\Delta Z_{g,t}-E\left(\Delta Z_{g,t}\right)\right)\right)\\
+& E\left(\sum_{g=1}^{G}\sum_{t=2}^{T} w_{g,t}\alpha_{g,t}D_{g,t}\left(\Delta Z_{g,t}-E\left(\Delta Z^{w}_{.,t}\right)\right)-\sum_{g=1}^{G}\sum_{t=2}^{T} w_{g,t}\alpha_{g,t-1}D_{g,t-1}\left(\Delta Z_{g,t}-E\left(\Delta Z^{w}_{.,t}\right)\right)\right)\\
=& E\left(\sum_{g=1}^{G}\sum_{t=1}^{T}\left(1\{t\geq 2\}w_{g,t}\left(\Delta Z_{g,t}-E\left(\Delta Z^{w}_{.,t}\right)\right)-1\{t\leq T-1\}w_{g,t+1}\left(\Delta Z_{g,t+1}-E\left(\Delta Z^{w}_{.,t+1}\right)\right)\right)D_{g,t}\alpha_{g,t}\right).
\end{aligned}
\tag{C.1}
$$

The first equality follows from Assumption 1. The second equality follows from Point 2 of Assumption B.1. The third equality follows from Point 1 of Assumption B.1. Similarly,

$$
\begin{aligned}
& E\left(\sum_{g=1}^{G}\sum_{t=2}^{T} \Delta D_{g,t}\left(\Delta Z_{g,t}-E\left(\Delta Z^{w}_{.,t}\right)\right)\right)\\
=& E\left(\sum_{g=1}^{G}\sum_{t=1}^{T}\left(1\{t\geq 2\}w_{g,t}\left(\Delta Z_{g,t}-E\left(\Delta Z^{w}_{.,t}\right)\right)-1\{t\leq T-1\}w_{g,t+1}\left(\Delta Z_{g,t+1}-E\left(\Delta Z^{w}_{.,t+1}\right)\right)\right)D_{g,t}\right).
\end{aligned}
\tag{C.2}
$$

Then, plugging (C.1) and (C.2) into (B.2) yields the result. Point 2 follows from Point 1.