Skills, Tasks and Technologies: Implications for Employment and Earnings

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4. A Ricardian Model of the Labor Market

4. A Ricardian model of the labor market

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Production technology

- Static environment with a unique final good, Y.
- Y produced with a continuum of tasks on the interval $[0, 1]$.
- Cobb-Douglas technology mapping tasks into the final good:

$$Y = \exp\left[\int_0^1 \ln y(i)\, di\right] \tag{11}$$

  where $y(i)$ is the service or production of task i.

- The final good is the numeraire, $P \equiv 1$.

Supply of skills to tasks

- There are 3 types of labor: low, medium and high skilled. Their supply is inelastic and given by L, M, H.
- Each task on the continuum is produced using:

$$y(i) = A_L \alpha_L(i) l(i) + A_M \alpha_M(i) m(i) + A_H \alpha_H(i) h(i) + A_K \alpha_K(i) k(i) \tag{12}$$

where:

- A_L, A_M, A_H are skill productivity parameters.
- $\alpha_L(i), \alpha_M(i), \alpha_H(i)$ are task productivity schedules determining comparative advantage in task production.
- $l(i), m(i), h(i)$ are the number of workers allocated to task i.

Comparative advantage in task production

Assumption 1

$\alpha_L(i)/\alpha_M(i)$ and $\alpha_M(i)/\alpha_H(i)$ are continuously differentiable and strictly decreasing in i.

- This is an assumption about the *relative* productivity of workers in doing different tasks.
- This assumption results in the allocation of workers across tasks based on their comparative advantage (as in Roy [51]).
- The rationale is that higher i correspond to "more complex" tasks for which more skilled workers have a comparative advantage.

Clearing factor markets

- Labor markets clearing requires:

$$\int_0^1 l(i)di = L \text{ and } \int_0^1 m(i)di = M \text{ and } \int_0^1 h(i)di = H \tag{13}$$

- The role of capital as a factor of production:
    - Equilibrium without machines (sections 4.2-4.4): $\alpha_K(i) = 0$ for all i.
    - Task replacing technologies (section 4.5): $\alpha_K(i) \neq 0$ for some i.

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Task thresholds

Lemma 1

In any equilibrium there exist I_L and I_H such that $0 < I_L < I_H < 1$ and for any $i < I_L$ we have that $m(i) = h(i) = 0$, for any $i \in (I_L, I_H)$ we have that $l(i) = h(i) = 0$, and for any $i > I_H$ we have that $l(i) = m(i) = 0$.

Thresholds I_L, I_H determine the allocation of tasks to skill:

- $i < I_L$ will be performed by L workers (Manual).
- $I_L \leq i \leq I_H$ will be performed by M workers (Routine).
- $i > I_H$ will be performed by H workers (Abstract).

Task thresholds I_L and I_H must exist

- Given that $\alpha_L(i)/\alpha_M(i)$ is strictly decreasing in i, there must exist a task I_L such that the unit cost of producing I_L is the same for L and M workers:

$$\frac{w_L}{A_L \alpha_L(I_L)} = \frac{w_M}{A_M \alpha_M(I_L)}$$

- Given that $\alpha_M(i)/\alpha_H(i)$ is strictly decreasing in i, there must exist a task I_H such that the unit cost of producing I_H is the same for M and H workers:

$$\frac{w_M}{A_M \alpha_M(I_H)} = \frac{w_H}{A_M \alpha_M(I_H)}$$

Three equilibrium conditions

E1. Law of one price for skill.

E2. Equal task cost shares.

E3. No arbitrage across skills at task thresholds.

E4. Price of the final good equals marginal cost.

E1. Law of one price for skill

- Denote $p(i)$ as the price of task i.
- All tasks by $j = L, M, H$ workers must pay the same wage w_j:

$$\boxed{w_j = p(i) A_j \alpha_j(i) \text{ for } j = L, M, H}$$

- This has a convenient implication:

$$p(i)\alpha_L(i) = p(i')\alpha_L(i') \equiv P_L \text{ for any } i, i' < I_L \quad (14)$$

$$p(i)\alpha_M(i) = p(i')\alpha_M(i') \equiv P_M \text{ for any } I_L < i, i' < I_H \quad (15)$$

$$p(i)\alpha_H(i) = p(i')\alpha_H(i') \equiv P_H \text{ for any } i > I_H \quad (16)$$

E2. Equal task cost shares within skill groups

- Cobb-Douglas technology implies:

$$p(i)y(i) = p(i')y(i') \text{ for any } i, i' \in [0,1] \tag{17}$$

- Within the group of L workers this implies:

$$p(i)\alpha_L(i)l(i) = p(i')\alpha_L(i')l(i') \text{ for any } i, i' < I_L$$

- From the law of one price for skill we had that:

$$p(i)\alpha_L(i) = p(i')\alpha_L(i') \equiv P_L$$

- This implies:

$$l(i) = l(i') \text{ for any } i, i' < I_L$$

E2. Equal task cost shares within skill group

- Within skill groups, we had that:

$$l(i) = l(i') \text{ for any } i, i' < I_L$$

$$m(i) = m(i') \text{ for any } I_L < i, i' < I_H$$

$$h(i) = h(i') \text{ for any } i, i' > I_H$$

- Using the market clearing conditions, this gives:

$$l(i) = \frac{L}{I_L} \text{ for any } i < I_L \tag{18}$$

$$m(i) = \frac{M}{I_H - I_L} \text{ for any } I_L < i < I_H \tag{19}$$

$$h(i) = \frac{H}{1 - I_H} \text{ for any } i > I_H \tag{20}$$

E2. Equal task cost shares between skill group

- Cobb-Douglas technology implies:

$$p(i)y(i) = p(i')y(i') \text{ for any } i, i' \in [0, 1] \tag{17}$$

- Between M and H workers this implies:

$$p(i)\alpha_M(i)A_M m(i) = p(i')\alpha_H(i')A_H h(i')$$

- Using definitions and results from above:

$$\frac{P_M A_M M}{I_H - I_L} = \frac{P_H A_H H}{1 - I_H}$$

$$\frac{P_H}{P_M} = \left[\frac{A_H H}{1 - I_H}\right]^{-1} \left[\frac{A_M M}{I_H - I_L}\right] \tag{21}$$

E2. Equal task cost shares between skill group

- We had that:

$$\frac{P_H}{P_M} = \left[\frac{A_H H}{1 - I_H}\right]^{-1} \left[\frac{A_M M}{I_H - I_L}\right] \tag{21}$$

- Similarly, comparing tasks between L and M workers gives:

$$\frac{P_M}{P_L} = \left[\frac{A_M M}{I_H - I_L}\right]^{-1} \left[\frac{A_L L}{I_L}\right] \tag{22}$$

- We can use these equations to solve for relative wages as a function of task thresholds.

E2. Equal task cost shares between skill group

- Using eq. (21), we get that:

$$\frac{w_H}{w_M} = \frac{P_H A_H}{P_M A_M} = \left[\frac{1 - I_H}{I_H - I_L}\right] \left[\frac{M}{H}\right] \qquad (26)$$

- Using eq. (22), we get that:

$$\frac{w_M}{w_L} = \frac{P_M A_M}{P_L A_L} = \left[\frac{I_H - I_L}{I_L}\right] \left[\frac{L}{M}\right] \qquad (27)$$

- Given I_L and I_H, these equations solve for relative wages.

E3. No arbitrage across skills at task thresholds

- Unit cost to produce I_H must be the same for M or H:

$$\frac{w_H}{w_M} = \frac{A_H \alpha_H(I_H)}{A_M \alpha_M(I_H)} = \left[\frac{1 - I_H}{I_H - I_L}\right] \left[\frac{M}{H}\right] \tag{23}$$

- Unit cost to produce I_L must be the same for L or M:

$$\frac{w_M}{w_L} = \frac{A_M \alpha_M(I_L)}{A_L \alpha_L(I_L)} = \left[\frac{I_H - I_L}{I_L}\right] \left[\frac{L}{M}\right] \tag{24}$$

- These equations solve for I_L and I_H.
- This solves for $l(i), m(i), h(i)$, P_H/P_M, P_M/P_L and w_H/w_M, w_M/w_L.

Summary of equilibrium

Proposition 1

There exists a unique equilibrium summarized by $(I_L, I_H, P_L, P_M, P_H, w_L, w_M, w_H)$.

- There exist values of I_L and I_H. Still required is proof that these values are unique.
- Knowing I_L and I_H directly solves for $l(i), m(i), h(i)$, $P_H / P_M, P_M / P_L$ and $w_H / w_M, w_M / w_L$.
- Solving for P_H, P_M, P_L and w_H, w_M, w_L requires solving for the equilibrium price of the final good P.

Uniqueness of I_L and I_H

- Rewrite the no arbitrage condition for I_H as:

$$\frac{A_M \alpha_M(I_H) M}{A_H \alpha_H(I_H) H} \left[\frac{1 - I_H}{I_H - I_L} \right] = 1 \tag{23'}$$

- Rewrite the no arbitrage condition for I_L as:

$$\frac{A_L \alpha_L(I_L) L}{A_M \alpha_M(I_L) M} \left[\frac{I_H - I_L}{I_L} \right] = 1 \tag{24'}$$

- Both are upward sloping in (I_L, I_H)-space but eq. (23') is everywhere steeper than eq. (24') given Assumption 1.

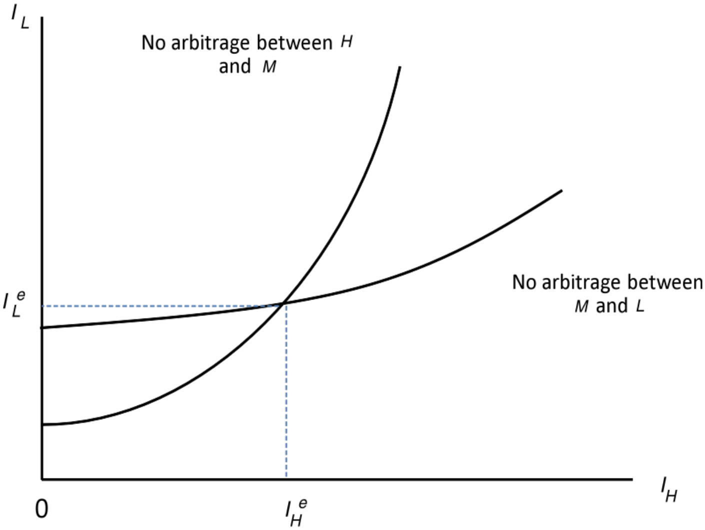

Figure 22 ***Determination of equilibrium threshold tasks.***

Equilibrium allocation of skills to tasks

- Rewrite the no arbitrage condition for I_H as:

$$\frac{A_H H}{A_M M} = \frac{1 - I_H}{I_H - I_L} \frac{\alpha_M(I_H)}{\alpha_H(I_H)} \tag{29}$$

  which is decreasing in I_H for given I_L.

- Rewrite the no arbitrage condition for I_L as:

$$\frac{A_M M}{A_L L} = \frac{I_H - I_L}{I_L} \frac{\alpha_L(I_L)}{\alpha_M(I_L)}$$

  which is decreasing in I_L for given I_H.

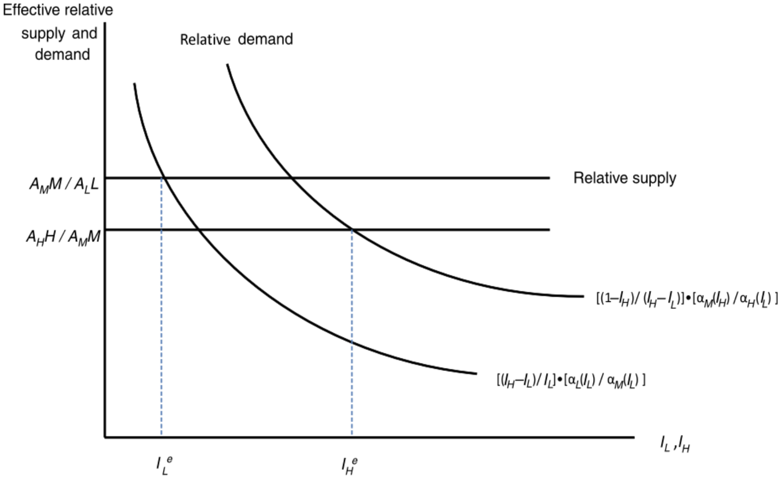

Figure 23 ***Equilibrium allocation of skills to tasks.***

E.4 Price of the final good equals marginal cost

- Profit maximization implies:

$$P = MC = \exp\left[\int_0^1 \ln(p(i))di\right] \equiv 1$$

- Rewriting gives:

$$\ln P = \int_0^{I_L} [\ln(P_L) - \ln(\alpha_L(i))]di + \int_{I_L}^{I_H} [\ln(P_M) - \ln(\alpha_M(i))]di$$
$$+ \int_{I_H}^{1} [\ln(P_H) - \ln(\alpha_H(i))]di = 0 \qquad (28)$$

which, together with eqs. (21)-(22), solves for P_L, P_M, P_H.

- This solves for $w_j = P_j A_j$ for $j = L, M, H$.

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Special cases

- We discuss 2 special cases:
  1. A model with a fixed task set for L-type workers.
  2. Assuming two skill groups and simple versions of comparative advantage schedules (as in Acemoglu & Zilibotti [01]).
- There are other interesting cases:
  - Assuming a discrete set of tasks instead and a continuum of skills (as in Autor, Levy & Murnane [03]).
  - Assuming CES instead of Cobb-Douglas task production function (as in Acemoglu & Restrepo [18,19])

Special case 1: Fixed task set for L-type workers

- Assume 3 types of workers: L, M, H.
- Assume the following comparative advantage schedule for low skill workers:
$$\alpha_L(i) = \begin{cases} \tilde{\alpha}_L \text{ if } i \leq \tilde{I}_L \\ 0 \text{ if } i > \tilde{I}_L \end{cases}$$
where $\tilde{\alpha}_L$ is large and $\tilde{I}_L$ is small.
- This fixes $I_L = \tilde{I}_L$ while I_H is determined as before from an arbitrage condition.
- This is essentially a two-skill version of the model.

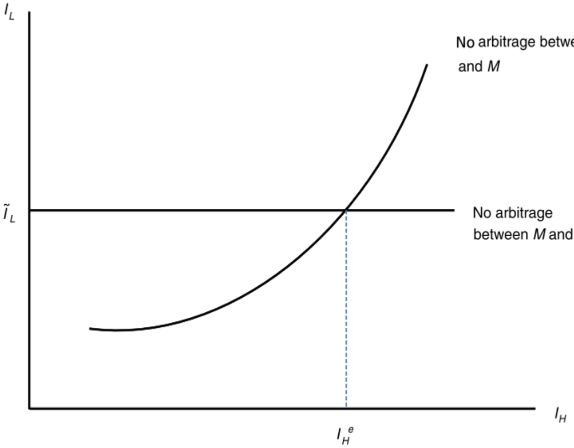

Figure 24 ***Determination of threshold high skill task (I_H) with task assignment for low skilled workers fixed.***

Special case 2: Assuming two skill groups

- Assume there are only 2 types of workers, low and high skill.
- Assume the following comparative advantage schedules:

$$\alpha_L(i) = 1 - i \text{ and } \alpha_H(i) = i$$

- This model is isomorphic to the canonical model with $\sigma = 2$:

$$\omega \equiv \frac{w_H}{w_L} = \left[\frac{A_H}{A_L}\right]^{\frac{1}{2}} \left[\frac{H}{L}\right]^{-\frac{1}{2}} \tag{5}$$
$$= \left[\frac{A_H}{A_L}\right]^{\frac{\sigma-1}{\sigma}} \left[\frac{H}{L}\right]^{-\frac{1}{\sigma}}$$

Special case 2: Assuming two skill groups

- Equal division of labor among tasks within skill groups gives:

$$l(i) = \frac{L}{I} \text{ and } h(i) = \frac{H}{1-I}$$

- Comparing tasks between skill groups gives:

$$\frac{P_H}{P_L} = \left[\frac{A_H H}{1-I}\right]^{-1} \left[\frac{A_L L}{I}\right]$$

- The relative wage is given by:

$$\frac{w_H}{w_L} = \frac{P_H A_H}{P_L A_L} = \left[\frac{H}{1-I}\right]^{-1} \left[\frac{L}{I}\right]$$

Special case 2: Assuming two skill groups

- No arbitrage across skill gives:

$$\frac{A_L \alpha_L(I) L}{I} = \frac{A_H \alpha_H(I) H}{1 - I}$$

- Re-writing and using that $\alpha_L(I) = 1 - I$ and $\alpha_H(I) = I$ gives:

$$\frac{1 - I}{I} = \left[\frac{A_H H}{A_L L}\right]^{\frac{1}{2}}$$

- Re-writing the relative wage gives using this expression gives:

$$\frac{w_H}{w_L} = \left[\frac{A_H}{A_L}\right]^{\frac{1}{2}} \left[\frac{H}{L}\right]^{-\frac{1}{2}}$$

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Basic comparative statics

- Taking logs of eq. (23') gives:

$$\begin{aligned}\ln(A_M/A_H) + \beta_H(I_H) + \ln(M/H) \\ + \ln([1 - I_H]/[I_H - I_L]) = 0\end{aligned} \tag{32}$$

  with $\beta_H(I_H) \equiv \ln(\alpha_M(I_H)/\alpha_H(I_H))$.

- Taking logs of eq. (24') gives:

$$\begin{aligned}\ln(A_L/A_M) + \beta_L(I_L) + \ln(L/M) \\ + \ln([I_H - I_L]/I_L) = 0\end{aligned} \tag{33}$$

  with $\beta_L(I_L) \equiv \ln(\alpha_L(I_L)/\alpha_M(I_L))$.

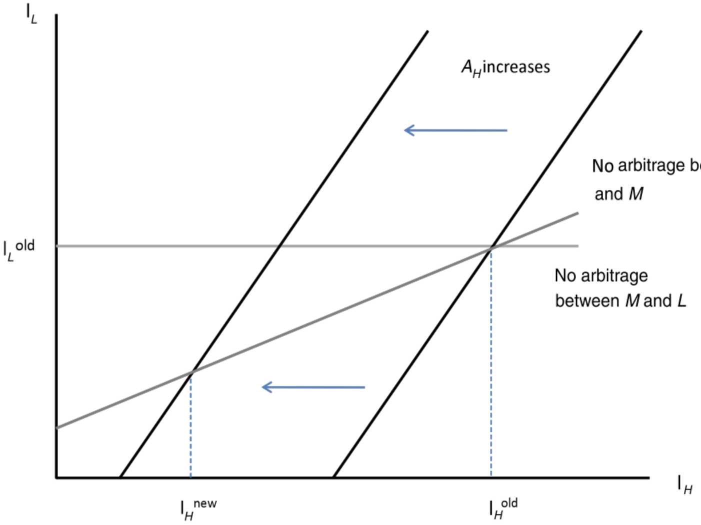

Figure 25 *Comparative statics.*

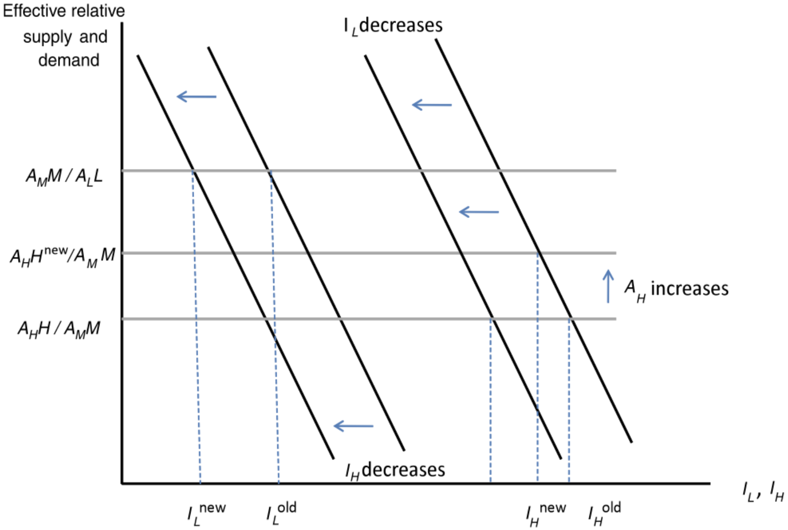

Figure 26 ***Changes in equilibrium allocation.***

Proposition 2.1: The response of task allocation to A_H, H

1. Task range of H workers:

$$\frac{dI_H}{d\ln(A_H)} = \frac{dI_H}{d\ln(H)} < 0$$

2. Task range of L workers:

$$\frac{dI_L}{d\ln(A_H)} = \frac{dI_L}{d\ln(H)} < 0$$

3. Task range for M workers:

$$\frac{d(I_H - I_L)}{d\ln(A_H)} = \frac{d(I_H - I_L)}{d\ln(H)} < 0$$

Proposition 2.1: The response of task allocation to A_L, L

1. Task range of H workers:

$$\frac{dI_H}{d\ln(A_L)} = \frac{dI_H}{d\ln(L)} > 0$$

2. Task range of L workers:

$$\frac{dI_L}{d\ln(A_L)} = \frac{dI_L}{d\ln(L)} > 0$$

3. Task range for M workers:

$$\frac{d(I_H - I_L)}{d\ln(A_L)} = \frac{d(I_H - I_L)}{d\ln(L)} < 0$$

Proposition 2.1: The response of task allocation to A_M, M

1. Task range of H workers:

$$\frac{dI_H}{d\ln(A_M)} = \frac{dI_H}{d\ln(M)} > 0$$

2. Task range of L workers:

$$\frac{dI_L}{d\ln(A_M)} = \frac{dI_L}{d\ln(M)} < 0$$

3. Task range for M workers:

$$\frac{d(I_H - I_L)}{d\ln(A_M)} = \frac{d(I_H - I_L)}{d\ln(M)} > 0$$

Proposition 2.2: Response of relative wages to skill supplies

1. An increase in H:

$$\frac{d\ln(w_H/w_M)}{d\ln(H)} < 0 \text{ and } \frac{d\ln(w_H/w_L)}{d\ln(H)} < 0$$

2. An increase in M:

$$\frac{d\ln(w_H/w_M)}{d\ln(M)} > 0 \text{ and } \frac{d\ln(w_M/w_L)}{d\ln(M)} < 0$$

3. An increase in L:

$$\frac{d\ln(w_M/w_L)}{d\ln(L)} > 0 \text{ and } \frac{d\ln(w_H/w_L)}{d\ln(L)} > 0$$

Proposition 2.3: Response of relative wages to A_H, A_M, A_L

1. An increase in A_H:

$$\frac{d\ln(w_H/w_M)}{d\ln(A_H)} > 0 \text{ and } \frac{d\ln(w_H/w_L)}{d\ln(A_H)} > 0$$

2. An increase in A_M:

$$\frac{d\ln(w_H/w_M)}{d\ln(A_M)} < 0 \text{ and } \frac{d\ln(w_M/w_L)}{d\ln(A_M)} > 0$$

3. An increase in A_L:

$$\frac{d\ln(w_M/w_L)}{d\ln(A_L)} < 0 \text{ and } \frac{d\ln(w_H/w_L)}{d\ln(A_L)} < 0$$

Propositions 2.2 and 2.3: Response of w_H / w_L to M, A_M

- This depends critically on the comparative advantages of marginal H and L workers in doing M tasks, given by:

$$\beta_H(I_H) \equiv \ln(\alpha_M(I_H)/\alpha_H(I_H))$$

$$\beta_L(I_L) \equiv \ln(\alpha_L(I_L)/\alpha_M(I_L))$$

- Define:

$$\beta'_L(I_L)I_L \equiv \partial\beta_L(I_L)/\partial I_L$$

$$\beta'_H(I_H)I_H \equiv \partial\beta_H(I_H)/\partial I_H$$

- If $\beta'_H(I_H)$ is high relative to $\beta'_L(I_L)$ in absolute value, H workers have a relatively strong comparative advantage for tasks above I_H compared to L workers for tasks below I_L.

Propositions 2.2 and 2.3: Response of w_H/w_L to M, A_M

- An increase in M:

$$\frac{d\ln(w_H/w_L)}{d\ln(M)} \gtrless 0 \text{ iff } |\beta'_H(I_H)(1-I_H)| \gtrless |\beta'_L(I_L)I_L|$$

- An increase in A_M:

$$\frac{d\ln(w_H/w_L)}{d\ln(A_M)} \gtrless 0 \text{ iff } |\beta'_H(I_H)(1-I_H)| \gtrless |\beta'_L(I_L)I_L|$$

- If $\beta'_H(I_H)$ is high relative to $\beta'_L(I_L)$ in absolute value, an increase in M or A_M displaces L workers more than H workers (and I_L falls more than I_H rises) such that w_H/w_L rises.

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Automation of routine labor tasks done by M-type workers

- Labor tasks most subject to machine displacement are routine or codifiable.
- These tasks are primarily done by M-type workers.
- Assume a range of tasks $[I', I''] \subset [I_L, I_H]$ for which $\alpha_K(i)$ increases sufficiently (with fixed cost of capital r) so that they are now more economically performed by machines.
- For all $i \notin [I', I'']$, continue to assume that $\alpha_K(i) = 0$.

Changes in task thresholds

Proposition 3

Suppose we start with an equilibrium characterized by thresholds $[I_L, I_H]$ and technical change implies that the tasks in the range $[I', I''] \subset [I_L, I_H]$ are now performed by machines.

Then:

After the introduction of machines, there exists a new equilibrium characterized by new thresholds $\hat{I}_L$ and $\hat{I}_H$ such that $0 < \hat{I}_L < I' < I'' < \hat{I}_H < 1$ and for any $i < \hat{I}_L$ we have that $m(i) = h(i) = 0$ and $l(i) = L/\hat{I}_L$; for any $i \in (\hat{I}_L, I') \cup (I'', \hat{I}_H)$, $l(i) = h(i) = 0$ and $m(i) = M/(\hat{I}_H - I'' + I' - \hat{I}_L)$; and for any $i > \hat{I}_H$, $l(i) = m(i) = 0$ and $h(i) = H/(1 - \hat{I}_H)$.

Changes in relative wages

Proposition 4

Suppose we start with an equilibrium characterized by thresholds $[I_L, I_H]$ and technical change implies that the tasks in the range $[I', I''] \subset [I_L, I_H]$ are now performed by machines.

Then:

1. w_H / w_M increases;
2. w_M / w_L decreases;
3. $w_H / w_L \gtrless 0$ if and only if $|\beta'_H(I_H)(1 - I_H)| \gtrless |\beta'_L(I_L)I_L|$.

Automation of routine labor tasks done by M-type workers

- Focal case:
  - Routine tasks are of average complexity for humans.
  - Strong comparative advantage of H relative to L workers.
- If so, there is wage and employment polarization:
  1. Wages:
     - Middle wages fall relative to top and bottom.
     - Top rises relative to bottom.
  2. Employment:
     - Declining labor input in middling routine tasks.
     - Middle-skill workers move disproportionately downward.

Ricardian model: Summary

- Model's inputs:
  1. Explicit distinction between skills and tasks.
  2. Comparative advantage among workers in different tasks.
  3. Multiple sources of competing task supplies.
- What the model delivers:
  1. A natural concept of occupations.
  2. Sorting of skill to tasks based on comparative advantage.
  3. Reallocation of skill across tasks as technology changes.
  4. Polarization of wages and employment.
  5. Technological progress can decrease wage levels.

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Endogenous choice of skill supply

- We have focussed on the substitution of skills across tasks.
- A complementary force is substitution of workers across skills.
- In response to changes in technology or factor supplies, workers may change the types of skills they supply.
- If machines replace medium skill workers, the supply to especially low skills will increase.
- This gives a richer explanation of observed wage and job polarization.

Environment

- Each worker j is endowed with some amount of low skill l^j, medium skill m^j and high skill h^j.
- Workers have one unit of time subject to a skill allocation constraint:
$$t_l^j + t_m^j + t_h^j \leqslant 1$$
- The worker's income is:
$$w_L t_l^j l_j + w_M t_m^j m_j + w_H t_h^j h_j$$
- A worker will prefer to allocate all her time to one skill.

Supply to skills

- The supply to skills is given by:

$$L = \int_{j \in E_l} l^j dj \ ; \ M = \int_{j \in E_m} m^j dj \ ; \ H = \int_{j \in E_h} h^j dj$$

  with E_l, E_m, E_h the sets of workers choosing to supply their labor to low, medium and high skills respectively.

- A worker will choose to be in set E_h only if:

$$\frac{l^j}{h^j} \leq \frac{w_H}{w_L} \text{ and } \frac{m^j}{h^j} \leq \frac{w_H}{w_M}$$

  with similar inequalities for sets E_m and E_l.

Comparative advantage in skill

Assumption 2

h^j / m^j and m^j / l^j are both strictly decreasing in j and $\lim_{j \to 0} h^j / m^j = \infty$ and $\lim_{j \to 1} m^j / l^j = 0$

- Lower indexed workers have a comparative advantage in high skill and higher indexed workers have a comparative advantage in low skill.
- At the extremes these comparative advantages are strong enough that there will always be some workers choosing to supply high and low skills.

Worker thresholds

Lemma 2

For any ratios of wages w_H/w_M and w_M/w_L, there exist $J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ such that:

- $t_h^j = 1$ for all $j < J^*(w_H/w_M)$
- $t_m^j = 1$ for all $j \in (J^*(w_H/w_M), J^{**}(w_M/w_L))$
- $t_l^j = 1$ for all $j > J^{**}(w_M/w_L)$

$J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ are both strictly increasing in their arguments.

$J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ are defined such that:

$$\frac{m^{J^*(w_H/w_M)}}{h^{J^*(w_H/w_M)}} = \frac{w_H}{w_M} \text{ and } \frac{l^{J^{**}(w_M/w_L)}}{m^{J^{**}(w_M/w_L)}} = \frac{w_M}{w_L}$$

Relative supply of skill

The supply to skills is given by:

$$L = \int_{J^{**}(w_M/w_L)}^{1} l^j \, dj; \; M = \int_{J^{*}(w_H/w_M)}^{J^{**}(w_M/w_L)} m^j \, dj; \; H = \int_{0}^{J^{*}(w_H/w_M)} h^j \, dj$$

- An increase in w_H/w_M, given w_M/w_L, increases H/M given that $J^*(w_H/w_M)$ is increasing in w_H/w_M.
- An increase in w_M/w_L, given w_H/w_M, increases M/L given that $J^{**}(w_M/w_L)$ is increasing in w_M/w_L.

We have two upward sloping relative supply curves.

Equilibrium with endogenous supply of skill

Proposition 5

In the model with endogenous supplies, there exists a unique equilibrium summarized by $(I_L, I_H,\ P_L, P_M, P_H,\ w_L, w_M, w_H,\ J^*(w_H/w_M), J^{**}(w_M/w_L),\ L, M, H)$.

Proof: To prove uniqueness of the equilibrium requires a little more work and is relegated to the Theoretical Appendix.

Comparative statics and interpretation

- Following the automation of routine tasks, the reallocation of skills across tasks resulted in wage and employment polarization.
- In response to these changes, some workers will also change their skill supply away from medium skills.
- If the more elastic margin is between medium and low skills, a significant fraction of workers previously supplying medium skills will now supply low skill.
- This gives a richer explanation of observed wage and employment polarization.

4. A Ricardian model of the labor market

4.1 Environment

4.2 Equilibrium without machines

4.3 Special cases

4.4 Comparative statics

4.5 Task replacing technologies

4.6 Endogenous choice of skill supply

4.7 Offshoring

4.8 Directed technical change

Offshoring

- Besides technical progress, international trade is believed to have a major impact on labor markets.
- Instead of simply trading finished products, there has been a greater tendency to engage in trade in tasks through offshoring.
- Our analysis of technological progress directly translates to offshoring of routine and codifiable tasks (Proposition 6).
- The task-based model provides a rich and unified framework to analyse the impacts of labor supply, technological change, and trade on labor markets.

4. A Ricardian model of the labor market

4.1 Environment
4.2 Equilibrium without machines
4.3 Special cases
4.4 Comparative statics
4.5 Task replacing technologies
4.6 Endogenous choice of skill supply
4.7 Offshoring
4.8 Directed technical change

Directed technical change

- We have rewritten the canonical model by allowing for the endogenous allocation of skill groups across tasks.
- We have further extended the model by also allowing for the endogenous allocation of workers across skill groups.
- Another significant aspect of the economic environment absent from the canonical model is the endogeneity of technological progress to changes in supply.
- This is known as endogenous or directed technical change.

Directed technical change

In our Ricardian model, there can be different types of directed technical change in response to changes in supply:

1. A_L, A_M, A_H respond to changes in skill supplies. This idea is analyzed in Acemoglu & Zilibotti [01] for the special case of our model discussed in section 4.3.
2. Comparative advantage schedules respond to changes in skill supplies:

$$\alpha_L(i|\theta), \alpha_M(i|\theta), \alpha_H(i|\theta)$$

   with θ a variable that captures the endogenous choice of technology in the economy.

Factor biases in directed technical change

- An increase in the supply of factor $f = L, M, H$ induces technical change that is **weakly biased** towards f if:

$$\frac{\partial w_f(E_{-f}, E_f|\theta)}{\partial\theta}\frac{d\theta}{dE_f} \geq 0$$

- An increase in the supply of factor $f = L, M, H$ induces technical change that is **strongly biased** towards f if:

$$\frac{dw_f(E_{-f}, E_f|\theta)}{dE_f} = \frac{\partial w_f(E_{-f}, E_f|\theta)}{\partial E_f} + \frac{\partial w_f(E_{-f}, E_f|\theta)}{\partial\theta}\frac{d\theta}{dE_f} > 0$$

Weakly biased directed technical change

Proposition 7

Under regularity conditions (which ensure the existence of a locally isolated equilibrium), an increase in the supply of factor f (for $f \in L, M, H$) will induce technical change **weakly biased** towards that factor.

- Proof: Acemoglu [07]
- This is a strong theoretical reason to expect that the increase in educational attainment has induced the development of technologies favoring skilled workers.
- It does not necessarily imply that the wage of skilled workers must have increased.

Strongly biased directed technical change

Proposition 8

Under regularity conditions (which ensure the existence of a locally isolated equilibrium), an increase in the supply of factor f (for $f \in L, M, H$) will induce technical change **strongly biased** towards that factor - thus increasing the wage of that factor - if and only if the aggregate production possibilities set of the economy is locally nonconvex in factor f and technology θ.

- Proof: Acemoglu [07]
- Local nonconvexity in f and θ is common in models of endogenous technological change.
- The endogenous technology (or "long-run") demand for skill is upward sloping.

5. Comparative Advantage and Wages: An Empirical Approach

Relative returns to skill versus tasks

- The model predicts that following automation, $w_L / w_M \uparrow$ and $w_H / w_M \uparrow$, even as workers reallocate across tasks.
- However, changes in relative wages are not the same as changes in relative task prices (e.g. they can be different for tasks that are reallocated between skill groups).
- Regarding automation, the price of automatable tasks is expected to declined relative to abstract and manual tasks.
- But tasks and task prices are not directly observed in data.

Occupations as proxies for tasks

- Let $\gamma^{A}_{sejk}, \gamma^{R}_{sejk}, \gamma^{S}_{sejk}$ be the employment shares of Abstract, Routine and Service occupations in demographic group *sejk* (gender, education, age, region) in 1959.
- By construction, we have that:

$$\gamma^{A}_{sejk} + \gamma^{R}_{sejk} + \gamma^{S}_{sejk} = 1$$

- Assume that in 1959 these groups have self-selected into task specialities according to comparative advantage, taking as given overall skill supplies and task demands (reflecting also available technologies and trade opportunities).

Wages, skill and tasks

- Let w_{sejkt} be the mean log wage of a demographic group in year t and $\Delta w_{sejk\tau}$ the change in w in decade τ.
- Estimate the regression model:

$$\Delta w_{sejk\tau} = \sum_t \beta_t^A \gamma_{sejk}^A 1[\tau = t] + \sum_t \beta_t^S \gamma_{sejk}^S 1[\tau = t] + \delta_\tau + \phi_e + \lambda_j + \pi_k + e_{sejk\tau} \quad (40)$$

- β_t^A and β_t^S are decade specific slopes on the initial occupation shares in wage changes by demographic group (relative to the omitted routine task shares with effects absorbed by δ_τ).
- We expect that β_t^A and β_t^S will rise, and that δ_t will decline.

Table 10 OLS stacked first-difference estimates of the relationship between demographic group occupational distributions in 1959 and subsequent changes in demographic groups' mean log wages by decade, 1959-2007.

	A. Males		B. Females	
	(1)	(2)	(1)	(2)
Abstract occupation share				
1959 share × 1959-1969 time dummy	0.021	0.033	0.146	0.159
	(0.044)	(0.104)	(0.041)	(0.081)
1959 share × 1969-1979 time dummy	−0.129	−0.123	−0.054	−0.032
	(0.044)	(0.105)	(0.036)	(0.079)
1959 share × 1979-1989 time dummy	0.409	0.407	0.143	0.174
	(0.046)	(0.106)	(0.033)	(0.079)
1959 share × 1989-1999 time dummy	0.065	0.060	0.070	0.107
	(0.049)	(0.109)	(0.033)	(0.079)
1959 share × 1999-2007 time dummy	0.198	0.194	0.075	0.113
	(0.051)	(0.11)	(0.033)	(0.08)
Service occupation share				
1959 share × 1959-1969 time dummy	−0.836	−1.014	0.359	0.404
	(0.278)	(0.303)	(0.064)	(0.09)
1959 share × 1969-1979 time dummy	−0.879	−0.991	0.304	0.363
	(0.295)	(0.316)	(0.065)	(0.091)
1959 share × 1979-1989 time dummy	1.007	0.917	−0.143	−0.060
	(0.332)	(0.349)	(0.074)	(0.096)
1959 share × 1989-1999 time dummy	0.202	0.143	0.117	0.221
	(0.378)	(0.39)	(0.086)	(0.104)
1959 share × 1999-2007 time dummy	0.229	0.212	−0.056	0.058
	(0.398)	(0.408)	(0.094)	(0.109)
Decade dummies				
1959-1969	0.274	0.274	0.120	0.046
	(0.031)	(0.037)	(0.021)	(0.032)
1969-1979	0.084	0.085	−0.083	−0.163
	(0.033)	(0.038)	(0.020)	(0.033)
1979-1989	−0.287	−0.283	−0.011	−0.099
	(0.036)	(0.041)	(0.021)	(0.034)
1989-1999	−0.002	0.002	0.061	−0.035
	(0.039)	(0.045)	(0.022)	(0.035)

(continued on next page)

Table 10 (continued)

	A. Males		B. Females	
	(1)	**(2)**	**(1)**	**(2)**
1999-2007	−0.157	−0.157	−0.073	−0.171
	(0.041)	(0.046)	(0.024)	(0.036)
Education, age group, and region main effects?	No	Yes	No	Yes
R-squared	0.789	0.821	0.793	0.844
N	400	400	400	400

Source: Census IPUMS 1960, 1970, 1980, 1990 and 2000, and American Community Survey 2008. Each column presents a separate OLS regression of stacked changes in mean log real hourly wages by demographic group and year, where demographic groups are defined by sex, education group (high school dropout, high school graduate, some college, college degree, post-college degree), age group (25-34, 35-44, 45-54, 55-64), and region of residence (Northeast, South, Midwest, West). Models are weighted by the mean start and end-year share of employment of each demographic group for each decadal change. Occupation shares are calculated for each demographic group in 1959 (using the 1960 Census) and interacted with decade dummies. Occupations are grouped into three exhaustive and mutually exclusive categories: (1) abstract—professional, managerial and technical occupations; (2) service—protective service, food service and cleaning, and personal services occupations; (3) routine—clerical, sales, administrative support, production, operative and laborer occupations. The routine group is the omitted category in the regression models.

6. Concluding Remarks

Conclusions

- Declining real wages for some workers and job and wage polarization are problematic for the canonical model.
- A task-based framework, in which tasks are the basic unit of production and the allocation of skills to tasks is endogenous, provides a fruitful alternative theory.
- The basic framework can be extended to include e.g. worker's choice of skill supply and directed technological change.
- These models generate new ideas that can be tested empirically, but more needs to be done.