Fitting a logistic curve

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1 Fitting data to a logistic curve

The logistic function reads:

$$y(t; y_0, r, k) = \frac{y_0 k}{y_0 - (y_0 - k)e^{-rt}} \tag{1}$$

1.1 Parameters

- y_0 represents the state measured at $t = 0$
- k represents the carrying capacity (the asymptote)
- r represents the time scale (the higher, the faster)

1.2 Fitting

Our problem is to find the set of parameters (y_0, k, r) that better fits a set of observations $\{t_n, y_n\}$.

The value of y_0 can be directly read from our data (keep in mind that this requires t_0 to be 0).

The values of k and r can be calculated by minimizing the good old square distance function:

$$r^2(k, r) = \sum_{n=0}^{N-1} (y_n - y(t_n; y_0, r, k))^2 \tag{2}$$

I tried to do it analytically, but it spits quite an ugly nonlinear second order system. Building function (2) and finding the minimal pair (k_{min}, r_{min}) should do the trick.