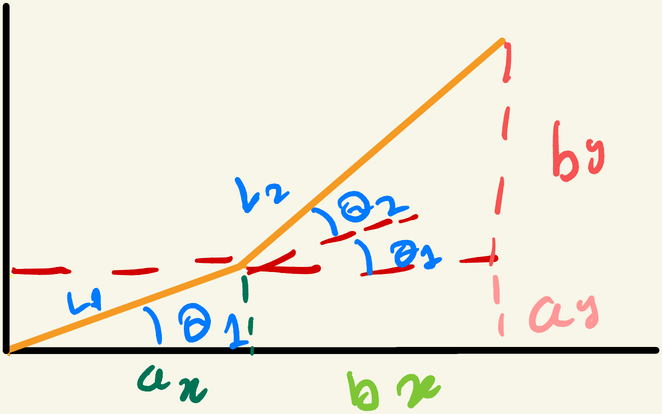


$$a_x = L_1 \cos(\theta_1), \quad b_x = L_2 \cos(\theta_1 + \theta_2)$$

$$a_y = L_1 \sin(\theta_1), \quad b_y = L_2 \sin(\theta_1 + \theta_2)$$

$$\Rightarrow x = a_x + b_x = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2)$$

$$\Rightarrow y = a_y + b_y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2)$$

$$\Rightarrow x^2 = L_1^2 \cos^2(\theta_1) + L_2^2 \cos^2(\theta_1 + \theta_2) + 2 L_1 L_2 \cos(\theta_1) \cos(\theta_1 + \theta_2)$$

$$\boxed{\text{We have: } \cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]}$$

$$x^2 = L_1^2 \cos^2(\theta_1) + L_2^2 \cos^2(\theta_1 + \theta_2) + 2 L_1 L_2 \left[\frac{1}{2}\left[\cos(2\theta_1 + \theta_2) + \cos(\theta_2)\right]\right]$$

$$x^2 = L_1^2 \cos^2(\theta_1) + L_2^2 \cos^2(\theta_1 + \theta_2)$$
$$+ L_1 L_2 [\cos(2\theta_1 + \theta_2) + \cos(\theta_2)]$$

$$y^2 = L_1^2 \sin^2(\theta_1) + L_2^2 \sin^2(\theta_1 + \theta_2)$$
$$+ 2 L_1 L_2 \sin(\theta_1) \sin(\theta_1 + \theta_2)$$

$$\sin(a) \sin(b) = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$y^2 = L_1^2 \sin^2(\theta_1) + L_2^2 \sin^2(\theta_1 + \theta_2)$$
$$+ 2 L_1 L_2 \left[\frac{1}{2} [\cos(\theta_2) - \cos(2\theta_1 + \theta_2)]\right]$$

$$y^2 = L_1^2 \sin^2(\theta_1) + L_2^2 \sin^2(\theta_1 + \theta_2)$$
$$+ L_1 L_2 [\cos(\theta_2) - \cos(2\theta_1 + \theta_2)]$$

$$\Rightarrow x^2 + y^2 = L_1^2 \cos^2(\theta_1) + L_1^2 \sin^2(\theta_2) + L_2^2 \cos^2(\theta_1 + \theta_2)$$
$$+ L_2^2 \sin^2(\theta_1 + \theta_2) + L_1 L_2 [\cos(\theta_2) + \cos(2\theta_1 + \theta_2)]$$
$$+ L_1 L_2 [\cos(\theta_2) - \cos(2\theta_1 + \theta_2)]$$

$$x^2+y^2 = L_1^2\left[(\cos^2(\theta_1)+\sin^2(\theta_1))\right.$$

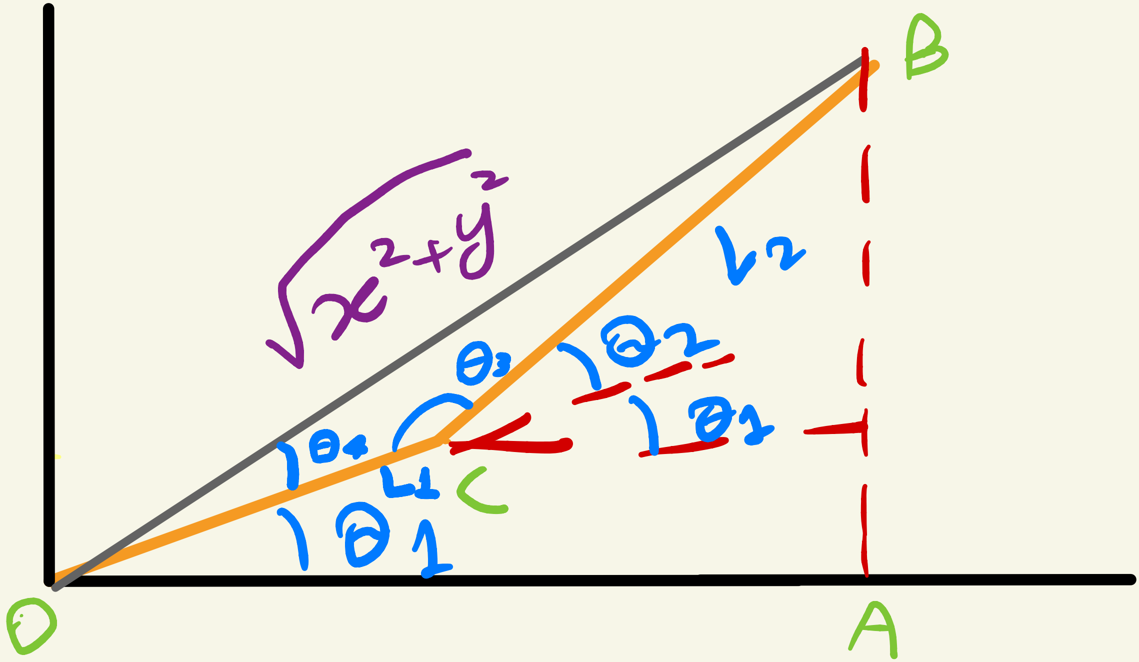
$$+L_2^2\left.(\cos^2(\theta_1+\theta_2)+\sin^2(\theta_1+\theta_2))\right]$$

$$+L_1L_2\left[\cos(\theta_2)+\cos(2\theta_1+\theta_2)+\cos(\theta_2)-\cos(2\theta_1+\theta_2)\right]$$

$$\Rightarrow x^2+y^2 = L_1^2+L_2^2+2L_1L_2\cos(\theta_2)$$

$$\Rightarrow \theta_2 = \cos^{-1}\left[\frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}\right]$$

Now θ_2 is <u>known</u>



$$\theta_3 = 180 - \theta_2$$

by cosine law in Δ BOA

$$L_2^2 = (x^2+y^2) + L_1^2 - 2L_1(\sqrt{x^2+y^2})\cos(\theta_4)$$

$$\Rightarrow \cos(\theta_4) = \frac{x^2+y^2+L_1^2-L_2^2}{2L_1(\sqrt{x^2+y^2})}$$

$$\theta_4 = \cos^{-1}\left(\frac{x^2+y^2+L_1^2-L_2^2}{2L_1(\sqrt{x^2+y^2})}\right)$$

in $\triangle$ OBA we have:

$$\cos(\theta_1+\theta_4)=\frac{x}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \theta_1+\theta_4=\cos^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right)$$

$$\Rightarrow \theta_1=\cos^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right)-\theta_4$$

$$\Rightarrow \theta_1=\cos^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right)-\cos^{-1}\left(\frac{x^2+y^2+L_1^2-L_2^2}{2L_1\left(\sqrt{x^2+y^2}\right)}\right)$$