Cont'd MPC Problem Formulation

$$\min_{\Delta U(k)} \quad \frac{1}{2}(r-Y)^T Q(r-Y) + \frac{1}{2}\Delta U^T R \Delta U$$

$$\text{s.t.} \quad \underbrace{M}_{\in\ \mathbb{R}^{2nuNp\times nuNp}} \Delta U(k) \leq \underbrace{\begin{bmatrix} -\Delta u^{min} \\ \Delta u^{max} \end{bmatrix}}_{\text{Constraints to Control Rate} \in \mathbb{R}^{2n_u N_p \times 1}}$$

$$\begin{bmatrix} -Z \\ Z \end{bmatrix} \Delta U(k) \leq \underbrace{\begin{bmatrix} -y^{min} + Wx_a(k) \\ y^{max} - Wx_a(k) \end{bmatrix}}_{\text{Constraints to output}}$$

$$\begin{bmatrix} -H \\ H \end{bmatrix} \Delta U(k) \leq \underbrace{\begin{bmatrix} -u^{min} + Eu(k-1) \\ u^{max} - Eu(k-1) \end{bmatrix}}_{\text{Constraints to control}}$$

Constrained MPC Algorithm

1. Specify your prediction horizon (Np)
2. Construct the augmented dynamics (Offset-free approach)
3. Compute W and Z to formulate Y
4. Compute H and E to formulate constraints for the control variables
5. Provide initial conditions to $x(k)$, $y(k) = Cx(k)$, and $u(k-1)$
6. Formulate Objective function ($\frac{1}{2}(r-Y)^T Q(r-Y) + \frac{1}{2}\Delta U^T R \Delta U$)
7. Formulate the optimization problem using `fmincon` format by defining
    1. Inequality constraints ($u(k)$ & $y(k)$)
    2. Lower bounds and upper bounds (Δu_k)
    3. Initial condition to the decision variable (ΔU)
8. Compute optimal control of ΔU and extract $\Delta u(k)$
9. Compute U from $U = Eu(k-1) + H\Delta U$ to extract $u(k)$ (receding horizon control)
10. Compute Y from $Y = Wx_a + Z\Delta U$
11. Measure the state variables based on the plant model, $x(k+1) = Ax(k) + Bu(k)$

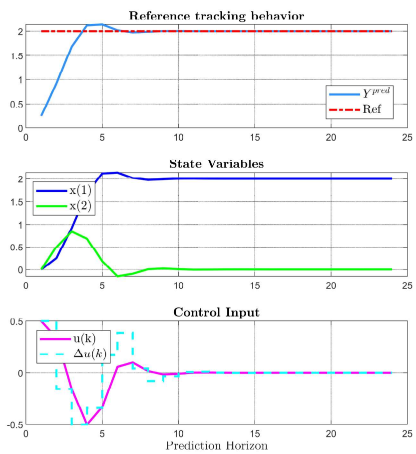
In-class assignment for MPC Problem Formulation

For the given discrete-time Linear Time Invariant (LTI) System

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

1. Let $Np = 10$, $\Delta u^{min} = -0.5$,$\Delta u^{max} = 0.5$,$u^{min} = -0.5$,$u^{max} = 1$, $y^{min} = 0$, $y^{max} = 3$
2. Select an output reference signal (r = 2I) and weight on control (R=0.1I)
3. Design the MPC Problem Formulation (by hand and MATLAB code), which includes constraints to the output and input variables
   - Define the size and the elements of W, Y, Z, H, E, A_{ineq}, b_{ineq}, LB, UB
   - Write the final MPC problem formulation in `fmincon` format

Results of Constrained MPC



Related matrices

Z — 10x10 double

	1	2	3	4	5	6	7	8	9	10
1	0.5000	0	0	0	0	0	0	0	0	0
2	2	0.5000	0	0	0	0	0	0	0	0
3	4.5000	2	0.5000	0	0	0	0	0	0	0
4	8	4.5000	2	0.5000	0	0	0	0	0	0
5	12.5000	8	4.5000	2	0.5000	0	0	0	0	0
6	18	12.5000	8	4.5000	2	0.5000	0	0	0	0
7	24.5000	18	12.5000	8	4.5000	2	0.5000	0	0	0
8	32	24.5000	18	12.5000	8	4.5000	2	0.5000	0	0
9	40.5000	32	24.5000	18	12.5000	8	4.5000	2	0.5000	0
10	50	40.5000	32	24.5000	18	12.5000	8	4.5000	2	0.5000

W — 10x3 double

	1	2	3
1	1	1	1
2	2	3	1
3	3	6	1
4	4	10	1
5	5	15	1
6	6	21	1
7	7	28	1
8	8	36	1
9	9	45	1
10	10	55	1

H — 10x10 double

	1	2	3	4	5	6	7	8	9	10
1	1	0	0	0	0	0	0	0	0	0
2	1	1	0	0	0	0	0	0	0	0
3	1	1	1	0	0	0	0	0	0	0
4	1	1	1	1	0	0	0	0	0	0
5	1	1	1	1	1	0	0	0	0	0
6	1	1	1	1	1	1	0	0	0	0
7	1	1	1	1	1	1	1	0	0	0
8	1	1	1	1	1	1	1	1	0	0
9	1	1	1	1	1	1	1	1	1	0
10	1	1	1	1	1	1	1	1	1	1

E — 10x1 double

	1
1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1

Constrained MPC Algorithm over Simulation Time

1. Specify your prediction horizon (Np)
2. Construct the augmented dynamics (Offset-free approach)
3. Compute W and Z to formulate Y
4. Compute H and E to formulate constraints for the control variables
5. Provide initial conditions to $x(k)$, $y(k) = Cx(k)$, and $u(k-1)$
6. Initialize `fmincon` by adding initial condition (x_0) to the decision variable (ΔU)
7. Formulate `for loop` to run the simulation, $t = 1 : Ts : Tfinal$. Within the `for loop`, we have to:
    1. Formulate Objective function ($\frac{1}{2}(r - Y)^T Q(r - Y) + \frac{1}{2}\Delta U^T R \Delta U$)
    2. Formulate the optimization problem using `fmincon` format by defining
        1. Inequality constraints ($u(k)$ & $y(k)$)
        2. Lower bounds and upper bounds (Δu_k)
    3. Compute optimal control of ΔU and extract $\Delta u(k)$
    4. Compute U from $U = Eu(k-1) + H\Delta U$ to extract $u(k)$ (receding horizon control)
    5. Compute Y from $Y = Wx_a + Z\Delta U$
    6. Measure the state variables based on the plant model, $x(k+1) = Ax(k) + Bu(k)$
    7. Update x_a and $x0$