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```
1 using Plots
```

f (generic function with 1 method)

```
1 function f(x)
2   return 2*x^2 + 3*x + 3
3 end
```

riemann (generic function with 1 method)

```
1 # range [a, b]
 2 # number of sums n
 3 # function f to integrate passed in as higher order function
 4 function riemann(a, b, n, f, t)
       dx = (b - a) / n
       sum = 0.0
 6
 7
       for x in 0:(n-1)
           pt = t == "left" ? a + x * dx :
9
                t == "right" ? a + x * dx + dx :
                t == "mid" ? a + x * dx + dx / 2 :
10
                error("Invalid Riemann sum type")
11
12
           sum += f(pt) * dx
13
       end
14
15
       return sum
16 end
```

```
1 begin
 2
       print("Exact integral of the function f yields 22/3 = 7.333...")
 3
 4
       print("\nThe left Riemann Sum with n = 5: ")
 5
       print(riemann(-1, 1, 5, f, "left"))
6
 7
       print("\nThe right Riemann Sum with n = 20: ")
       print(riemann(-1, 1, 20, f, "right"))
8
9
10
       print("\nThe mid-point Riemann Sum with n = 100: ")
       print(riemann(-1, 1, 100, f, "mid"))
11
12 end
```

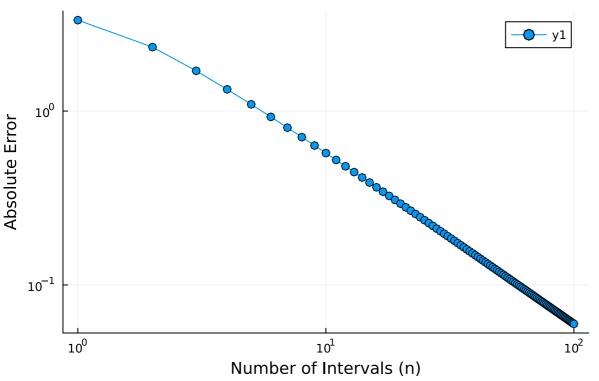
```
Exact integral of the function f yields 22/3 = 7.333... 7
The left Riemann Sum with n = 5: 6.24
The right Riemann Sum with n = 20: 7.640000000000002
The mid-point Riemann Sum with n = 100: 7.33320000000001
```

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plotErr (generic function with 2 methods)

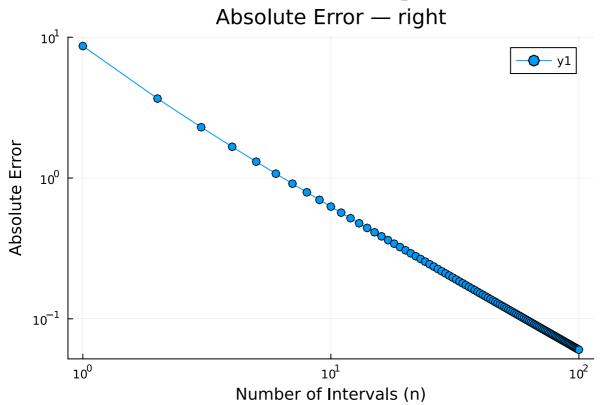
```
1 function plotErr(t="left")
       n_vals = collect(1:100)#[5, 10, 15, 100, 1000, 1250, 5000, 10000, 100000]
       errors = Float64[] # Initialize as a Float64 array
 3
 4
       exact_value = (22 / 3) # Exact value of the integral
 6
       for n in n_vals
 7
 8
            approx = \underline{riemann}(-1, 1, n, \underline{f}, t)
            err = abs(approx - exact_value) # Error = | sum - exact integral |
 9
10
            push!(errors, err)
11
       end
12
13
       # Plotting the errors
       plot(n_vals, errors, marker=:0, xscale=:log10, yscale=:log10,
14
             xlabel="Number of Intervals (n)", ylabel="Absolute Error",
15
16
             title="Absolute Error - $t", grid=true)
17 end
```

Absolute Error — left

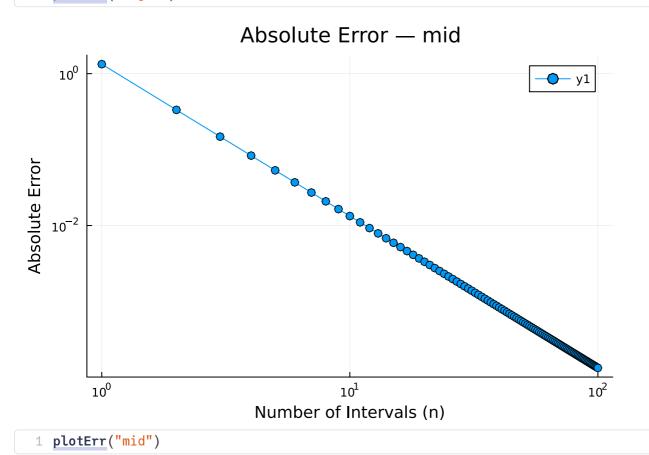


1 plotErr("left")

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1 plotErr("right")



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Q2: Taylor Polynomials for $f(x) = 2e^x + 1$ about $x_0 = 1$

First four derivatives of f(x)

$$f'(x)=2e^x \ f''(x)=2e^x \ f'''(x)=2e^x \ f^{(4)}(x)=2e^x$$

1. Third Taylor Polynomial $P_3(x)$

$$f(x)pprox f(1)+f'(1)\cdot (x-1)+rac{f''(1)\cdot (x-1)^2}{2!}+rac{f'''(1)\cdot (x-1)^3}{3!}$$

Substituting the values:

$$P_3(x) = (2e+1) + 2e \cdot (x-1) + rac{2e \cdot (x-1)^2}{2} + rac{2e \cdot (x-1)^3}{6}$$

2. Fourth Taylor Polynomial $P_4(x)$

$$f(x)pprox f(1)+f'(1)\cdot(x-1)+rac{f''(1)\cdot(x-1)^2}{2!}+rac{f'''(1)\cdot(x-1)^3}{3!}+rac{f^{(4)}(1)\cdot(x-1)^4}{4!}$$

Substituting the values:

$$P_4(x) = (2e+1) + 2e \cdot (x-1) + rac{2e \cdot (x-1)^2}{2} + rac{2e \cdot (x-1)^3}{6} + rac{2e \cdot (x-1)^4}{24}$$

```
1 md"
2 # Q2: Taylor Polynomials for $f(x) = 2e^x + 1$ about $x_0 = 1$
3
4 ### First four derivatives of $f(x)$
5 $f'(x) = 2e^x$
6 $f'''(x) = 2e^x$
7 $f''''(x) = 2e^x$
8 $f^{(4)}(x) = 2e^x$
9
10 ### 1. Third Taylor Polynomial $P_3(x)$
11
12 $f(x) \approx f(1) + f'(1) \cdot (x - 1) + \frac{f'''(1) \cdot (x - 1)^2}{2!} + \frac{f''''(1) \cdot (x - 1)^3}{3!}$
13
14 Substituting the values:
15
```

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```
16 $P_3(x) = (2e + 1) + 2e \cdot (x - 1) + \frac{2e \cdot (x - 1)^2}{2} + \frac{2e \cdot (x - 1)^3}{6}$

17

18 ### 2. Fourth Taylor Polynomial $P_4(x)$

19

20 $f(x) \approx f(1) + f'(1) \cdot (x - 1) + \frac{f''(1) \cdot (x - 1)^2}{2!} + \frac{f'''(1) \cdot (x - 1)^3}{3!} + \frac{f^{(4)}(1) \cdot (x - 1)^4}{4!}$

21

22 Substituting the values:

23

24 $P_4(x) = (2e + 1) + 2e \cdot (x - 1) + \frac{2e \cdot (x - 1)^2}{2} + \frac{2e \cdot (x - 1)^3}{6} + \frac{2e \cdot (x - 1)^4}{24}$

25 "
```