```julia
using Plots
```

f (generic function with 1 method)

```julia
function f(x)
    return 2*x^2 + 3*x + 3
end
```

riemann (generic function with 1 method)

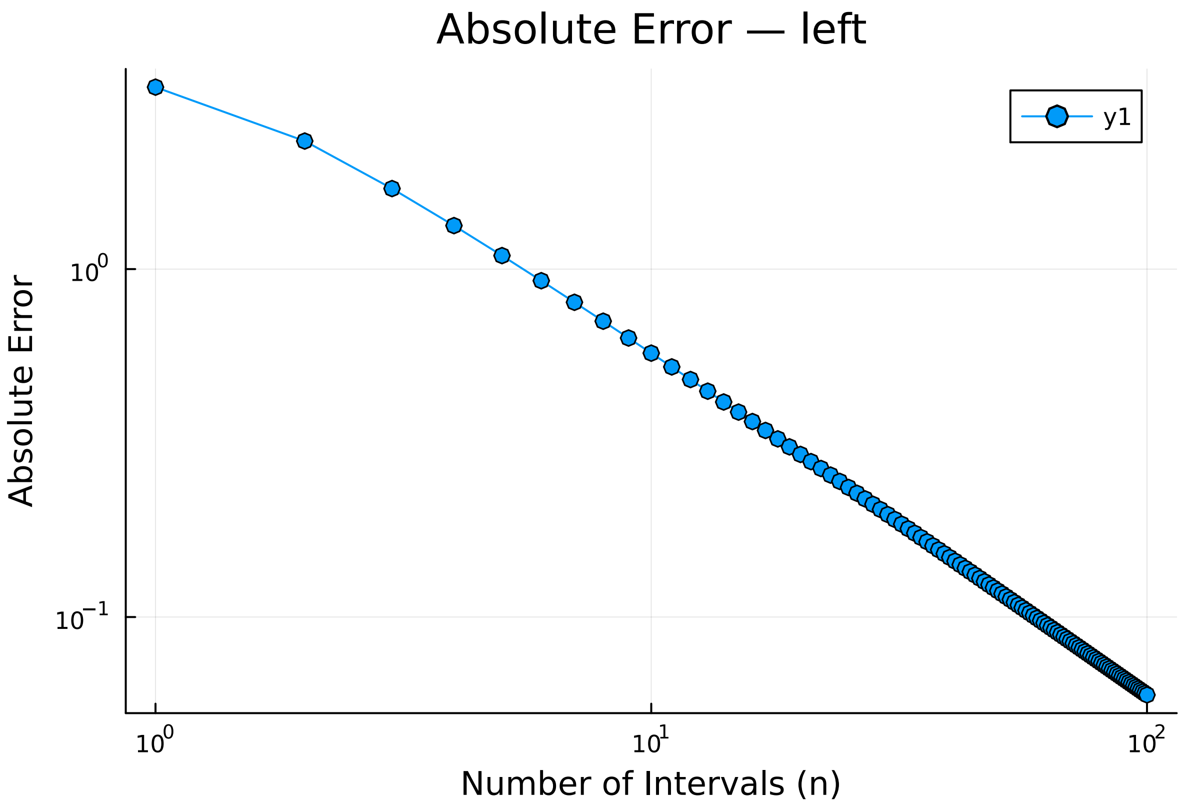
```julia
# range [a, b]
# number of sums n
# function f to integrate passed in as higher order function
function riemann(a, b, n, f, t)
    dx = (b - a) / n
    sum = 0.0
    for x in 0:(n-1)
        pt = t == "left"  ? a + x * dx :
             t == "right" ? a + x * dx + dx :
             t == "mid"   ? a + x * dx + dx / 2 :
             error("Invalid Riemann sum type")
        sum += f(pt) * dx
    end

    return sum
end
```

```julia
begin
    print("Exact integral of the function f yields 22/3 = 7.333...")

    print("\nThe left Riemann Sum with n = 5: ")
    print(riemann(-1, 1, 5, f, "left"))

    print("\nThe right Riemann Sum with n = 20: ")
    print(riemann(-1, 1, 20, f, "right"))

    print("\nThe mid-point Riemann Sum with n = 100: ")
    print(riemann(-1, 1, 100, f, "mid"))
end
```

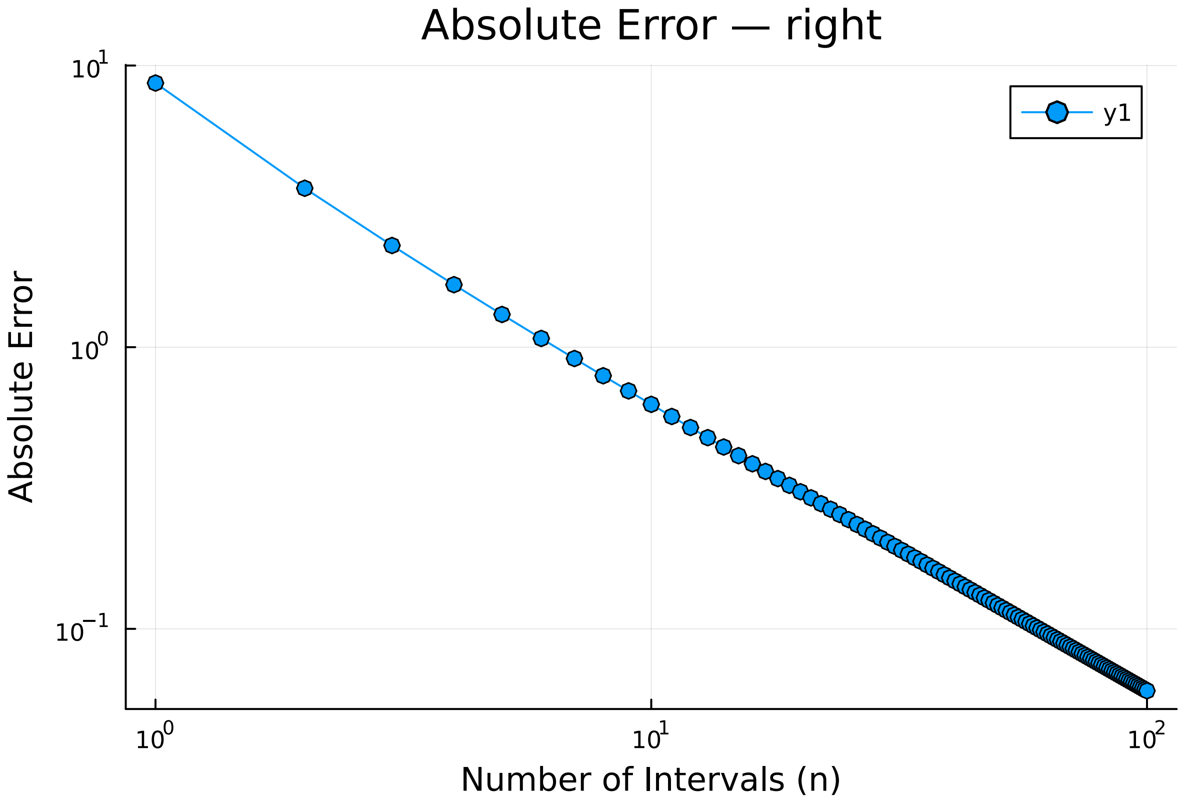
```
Exact integral of the function f yields 22/3 = 7.333...
The left Riemann Sum with n = 5: 6.24
The right Riemann Sum with n = 20: 7.640000000000002
The mid-point Riemann Sum with n = 100: 7.333200000000001
```

```
plotErr (generic function with 2 methods)
```

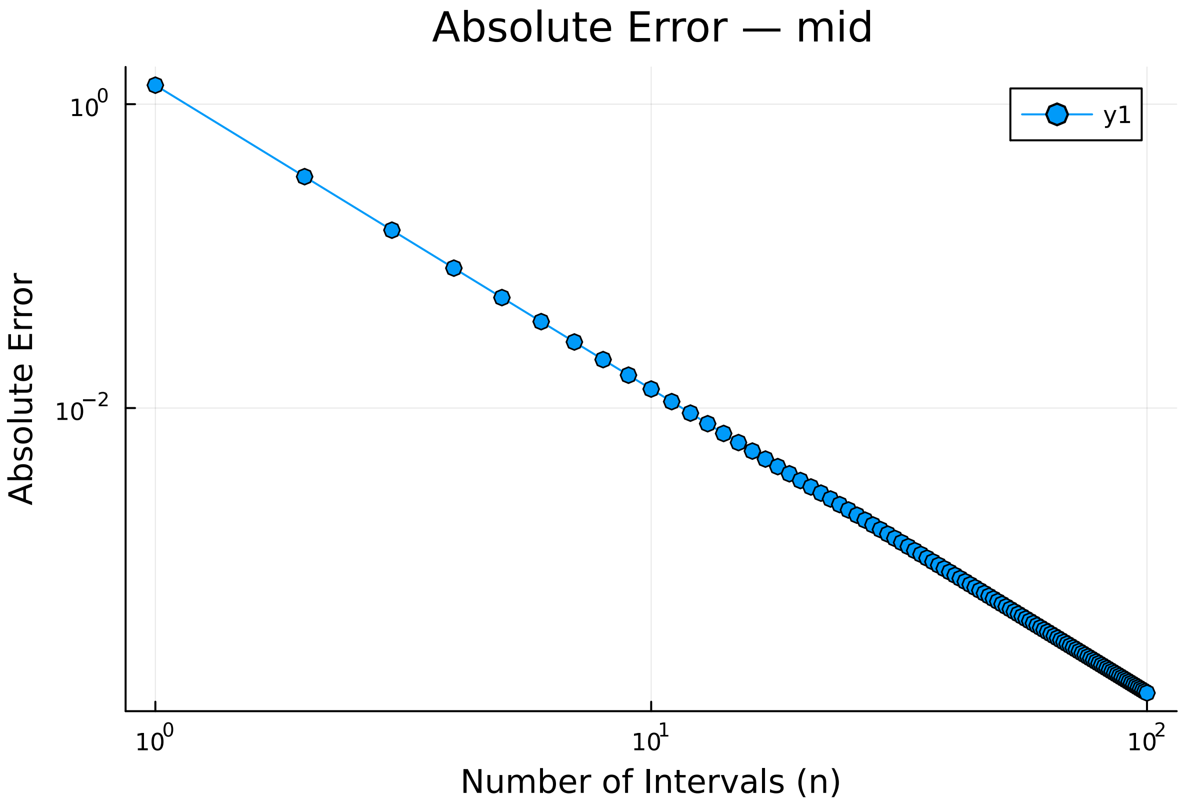
```julia
function plotErr(t="left")
    n_vals = collect(1:100)#[5, 10, 15, 100, 1000, 1250, 5000, 10000, 100000]
    errors = Float64[]  # Initialize as a Float64 array

    exact_value = (22 / 3)  # Exact value of the integral

    for n in n_vals
        approx = riemann(-1, 1, n, f, t)
        err = abs(approx - exact_value)  # Error = |sum - exact integral|
        push!(errors, err)
    end

    # Plotting the errors
    plot(n_vals, errors, marker=:o, xscale=:log10, yscale=:log10,
         xlabel="Number of Intervals (n)", ylabel="Absolute Error",
         title="Absolute Error — $t", grid=true)
end
```




```julia
plotErr("left")
```




```
plotErr("right")
```




```
plotErr("mid")
```

Q2: Taylor Polynomials for $f(x) = 2e^x + 1$ about $x_0 = 1$

First four derivatives of $f(x)$

$$f'(x) = 2e^x$$

$$f''(x) = 2e^x$$

$$f'''(x) = 2e^x$$

$$f^{(4)}(x) = 2e^x$$

1. Third Taylor Polynomial $P_3(x)$

$$f(x) \approx f(1) + f'(1) \cdot (x - 1) + \frac{f''(1) \cdot (x - 1)^2}{2!} + \frac{f'''(1) \cdot (x - 1)^3}{3!}$$

Substituting the values:

$$P_3(x) = (2e + 1) + 2e \cdot (x - 1) + \frac{2e \cdot (x - 1)^2}{2} + \frac{2e \cdot (x - 1)^3}{6}$$

2. Fourth Taylor Polynomial $P_4(x)$

$$f(x) \approx f(1) + f'(1) \cdot (x - 1) + \frac{f''(1) \cdot (x - 1)^2}{2!} + \frac{f'''(1) \cdot (x - 1)^3}{3!} + \frac{f^{(4)}(1) \cdot (x - 1)^4}{4!}$$

Substituting the values:

$$P_4(x) = (2e + 1) + 2e \cdot (x - 1) + \frac{2e \cdot (x - 1)^2}{2} + \frac{2e \cdot (x - 1)^3}{6} + \frac{2e \cdot (x - 1)^4}{24}$$

```
md"
# Q2: Taylor Polynomials for $f(x) = 2e^x + 1$ about $x_0 = 1$

### First four derivatives of $f(x)$
$f'(x) = 2e^x$
$f''(x) = 2e^x$
$f'''(x) = 2e^x$
$f^{(4)}(x) = 2e^x$

### 1. Third Taylor Polynomial $P_3(x)$

$f(x) \approx f(1) + f'(1) \cdot (x - 1) + \frac{f''(1) \cdot (x - 1)^2}{2!} +
\frac{f'''(1) \cdot (x - 1)^3}{3!}$

Substituting the values:

```

```
$P_3(x) = (2e + 1) + 2e \cdot (x - 1) + \frac{2e \cdot (x - 1)^2}{2} + \frac{2e \cdot (x - 1)^3}{6}$

### 2. Fourth Taylor Polynomial $P_4(x)$

$f(x) \approx f(1) + f'(1) \cdot (x - 1) + \frac{f''(1) \cdot (x - 1)^2}{2!} + \frac{f'''(1) \cdot (x - 1)^3}{3!} + \frac{f^{(4)}(1) \cdot (x - 1)^4}{4!}$

Substituting the values:

$P_4(x) = (2e + 1) + 2e \cdot (x - 1) + \frac{2e \cdot (x - 1)^2}{2} + \frac{2e \cdot (x - 1)^3}{6} + \frac{2e \cdot (x - 1)^4}{24}$
"
```