Date:

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Roll # 2k23-BSSE-101
Section:
A
Semester: $3^{rd}$
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Session: 2k23
Subject:
Multivariate Calculus
Course Instructor:
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Program : Software Engg.

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Assignment.

Date:

Change in polar form
& solve.

1) $$\int_{\sqrt{2}}^{2}\int_{\sqrt{4-y^2}}^{y} dx\,dy$$

Solv.

$x = r\cos\theta$
$y = r\sin\theta$
$r = \sqrt{x^2+y^2}$

$x = r\cos\theta$
$y = r\cos\theta$
$r\sin\theta = r\cos\theta$
$\sin\theta = \cos\theta$
$\frac{\sin\theta}{\cos\theta} = 1$
$\tan\theta = 1$ $\quad \because \tan\theta = \frac{\sin\theta}{\cos\theta}$

Date:

$$\theta = \tan^{-1}(1)$$

$$\boxed{\theta = \frac{\pi}{4}}$$

$$x = r\cos\theta$$

$$\sqrt{4-y^2} = r\cos\theta$$

$$\sqrt{(4-r^2\sin^2\theta)}^2 = (r\cos\theta)^2$$

$$4-r^2\sin^2\theta = r^2\cos^2\theta$$

$$4 = r^2\sin^2\theta + r^2\cos^2\theta$$

$$4 = r^2(\sin^2\theta + \cos^2\theta)$$

$$4 = r^2(1)$$

$$r^2 = 4$$

$$\sqrt{r^{\cancel{2}}} = \sqrt{4}$$

$$\boxed{r = 2}$$

$$y = r\sin\theta$$

$$2 = r\sin\theta$$

$$\frac{2}{r} = \sin\theta$$

$$\frac{2}{2} = \sin\theta$$

$$1 = \sin\theta$$

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$$\sin\theta = 1$$
$$\theta = \sin^{-1}(1)$$
$$\boxed{\theta = \frac{\pi}{2}}$$

$$y = r\sin\theta$$
$$2 = r\sin\theta$$
$$2 = r\sin\left(\frac{\pi}{4}\right)$$
$$2 = r\left(\frac{1}{\sqrt{2}}\right)$$
$$\boxed{r = 2\sqrt{2}}$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_{2}^{2\sqrt{2}} r\,dr\,d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left[\left(\frac{r^2}{2}\right)\Big|_{2}^{2\sqrt{2}}\right] d\theta$$

te: 

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[ \frac{(2\sqrt{2})^2}{2} - \frac{(2)^2}{2} \right] d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (4 - 2)\, d\theta$$

$$= 2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta$$

$$= 2(\theta)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= 2\left[\frac{\pi}{2} - \frac{\pi}{4}\right]$$

$$= 2\left(\frac{\pi}{4}\right)$$

$$= \frac{\pi}{2} \quad \text{Ans.}$$

Date:

2) $$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2+y^2+1)\,dx\,dy$$

Solv.
$$= 4\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}} \ln(x^2+y^2+1)\,dx\,dy$$

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

$x = r\cos\theta$

$\sqrt{1-y^2} = r\cos\theta$

$\left[\sqrt{1-r^2\sin^2\theta}\right]^2 = (r\cos\theta)^2$

$1 - r^2\sin^2\theta = r^2\cos^2\theta$

$1 = r^2\sin^2\theta + r^2\cos^2\theta$

$1 = r^2(\sin^2\theta + \cos^2\theta)$

$1 = r^2(1)$

$\sqrt{1} = \sqrt{r^2}$

$\boxed{r = 1}$

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$x = r\cos\theta$

$0 = r\cos\theta$

$0 = (1)\cos\theta$

$0 = \cos\theta$

$\theta = \cos^{-1}(0)$

$\boxed{\theta = \frac{\pi}{2}}$

$y = r\sin\theta$

$0 = r\sin\theta$

$0 = (1)\sin\theta$

$0 = \sin\theta$

$\theta = \sin^{-1}(0)$

$\boxed{\theta = 0}$

$y = r\sin\theta$

$0 = r\sin\theta$

$0 = r\sin\left(\frac{\pi}{2}\right)$

$0 = r(1)$

$\boxed{r = 0}$

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$$= 4 \int_0^{\frac{\pi}{2}} \int_0^1 \underbrace{\ln(r^2+1)}_{\mathrm{I}} \underbrace{r}_{\mathrm{II}} \, dr \, d\theta$$

$$= 4 \int_0^{\frac{\pi}{2}} \left[ \ln(r^2+1)\left(\frac{r^2}{2}\right)\Big|_0^1 - \int_0^1 \frac{r^2}{2}\left(\frac{2r}{r^2+1}\right) dr \right] d\theta$$

$$= 4 \int_0^{\frac{\pi}{2}} \left[ \frac{r^2}{2} \ln(r^2+1)\Big|_0^1 - \frac{2}{2} \int_0^1 \frac{r^3}{r^2+1} dr \right] d\theta$$

$$= 4 \int_0^{\frac{\pi}{2}} \left[ \frac{r^2}{2} \ln(r^2+1)\Big|_0^1 - \int_0^1 \frac{r(r^2+1-1)}{r^2+1} dr \right] d\theta$$

$$= 4 \int_0^{\frac{\pi}{2}} \left[ \frac{1}{2}\ln(2) - \int_0^1 \left( \frac{r\cancel{(r^2+1)}}{\cancel{r^2+1}} - \frac{r}{r^2+1} \right) dr \right] d\theta$$

$$= 4 \int_0^{\frac{\pi}{2}} \left[ (0.3465) - \int_0^1 \left( r - \frac{r}{r^2+1} \right) dr \right] d\theta$$

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$$= 4\int_0^{\frac{\pi}{2}}\left[(0.3465) - \int_0^1 r\,dr + \int_0^1 \frac{r}{r^2+1}\,dr\right]d\theta$$

$$= 4\int_0^{\frac{\pi}{2}}\left[(0.3465) - \left(\frac{r^2}{2}\right)\Big|_0^1 + \frac{1}{2}\int \frac{2r\,dr}{r^2+1}\right]d\theta$$

$$= 4\int_0^{\frac{\pi}{2}}\left[(0.3465) - \frac{1}{2} + \frac{1}{2}\left[\ln(r^2+1)\right]\Big|_0^1\right]d\theta$$

$$= 4\int_0^{\frac{\pi}{2}}\left[(0.3465) - \frac{1}{2} + \frac{1}{2}\left[\ln(2)\right]\right]d\theta$$

$$= 4\int_0^{\frac{\pi}{2}}\left[0.3465 - \frac{1}{2} + 0.3465\right]d\theta$$

$$= 4\int_0^{\frac{\pi}{2}}\left(0.693 - \frac{1}{2}\right)d\theta$$

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$$= 4\int_{0}^{\frac{\pi}{2}} 0.193\, d\theta$$

$$= 0.772 \int_{0}^{\frac{\pi}{2}} d\theta$$

$$= 0.772 \left[\theta \Big|_{0}^{\frac{\pi}{2}}\right]$$

$$= 0.772 \left[\frac{\pi}{2} - 0\right]$$

$$= 0.386\,\pi \quad \text{Ans.}$$

3) $$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\, dy\, dx$$

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Solv.

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2 + y^2}$

$y = r\sin\theta$

$0 = r\sin\theta$

$\frac{0}{r} = \sin\theta$

$0 = \sin\theta$

$\sin\theta = 0$

$\theta = \sin^{-1}(0)$

$\boxed{\theta = 0}$

$y = r\sin\theta$

$-\sqrt{1 - x^2} = r\sin\theta$

$-\sqrt{(1 - r^2\cos^2\theta)}^2 = (r\sin\theta)^2$

$-(1 - r^2\cos^2\theta) = r^2\sin^2\theta$

$-1 + r^2\cos^2\theta = r^2\sin^2\theta$

$-1 = r^2\sin^2\theta - r^2\cos^2\theta$

$-1 = r^2(\sin^2\theta - \cos^2\theta)$

$-1 = -r^2(\cos^2\theta - \sin^2\theta)$

te:

$-1 = -r^2 \cos 2\theta$ $(\because \cos 2\theta = \cos^2\theta - \sin^2\theta)$

$1 = r^2 \cos 2\theta$

$1 = r^2 \cos 0$ $(\because \theta = 0)$

$1 = r^2$

$\sqrt{r^2} = \sqrt{1}$

$\boxed{r = 1}$

$x = r\cos\theta$

$-1 = r\cos\theta$

$-1 = (1)\cos\theta$ $(\because r = 1)$

$-1 = \cos\theta$

$\theta = \cos^{-1}(-1)$

$\boxed{\theta = \pi}$

$x = r\cos\theta$

$-1 = r\cos\theta$

$-1 = r\cos 0$ $(\because \theta = 0)$

$-1 = r(1)$

$\boxed{r = -1}$

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$$= \int_0^{\pi}\int_{-1}^{1}\left(\frac{2}{1+r}\right) r\,dr\,d\theta$$

$$= 2\int_0^{\pi}\int_0^{1}\left(\frac{2}{1+r}\right) r\,dr\,d\theta$$

$$= 4\int_0^{\pi}\left[\int_0^{1} \frac{r+1-1}{r+1}\,dr\right]d\theta$$

$$= 4\int_0^{\pi}\left[\int_0^{1}\left(\cancel{\frac{r+1}{r+1}} - \frac{1}{r+1}\right)dr\right]d\theta$$

$$= 4\int_0^{\pi}\left[\int_0^{1} 1\,dr - \int_0^{1}\frac{1}{r+1}\,dr\right]d\theta$$

$$= 4\int_0^{\pi}\left[r - \ln(r+1)\right]_0^1 d\theta$$

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$$= 4\int_0^{\pi} (1 - \ln 2)\, d\theta$$

$$= 4(0.306) \int_0^{\pi} d\theta$$

$$= 1.2274 \left(\theta\right)\Big|_0^{\pi}$$

$$= 1.2274 [\pi - 0]$$

$$= 1.2274\,\pi \quad \text{Ans.}$$

4) $$\int_1^2 \int_0^{\sqrt{2x - x^2}} \frac{1}{(x^2 + y^2)^2}\, dy\, dx$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r = \sqrt{x^2 + y^2}$$

$$y = r\sin\theta$$

e:

$0 = r\sin\theta$

$\frac{0}{r} = \sin\theta$

$0 = \sin\theta$

$\theta = \sin^{-1}(0)$

$\boxed{\theta = 0}$

$y = r\sin\theta$

$\sqrt{2x - x^2} = r\sin\theta$

$\left(\sqrt{2r\cos\theta - r^2\cos^2\theta}\right)^2 = (r\sin\theta)^2$

$2r\cos\theta - r^2\cos^2\theta = r^2\sin^2\theta$

$2r\cos\theta = r^2\sin^2\theta + r^2\cos^2\theta$

$2r\cos\theta = r^2(\sin^2\theta + \cos^2\theta)$

$2r\cos\theta = r^2(1)$

$2r\cos\theta = r^2$

$2\cos\theta = r$

$\boxed{r = 2\cos 0}$ $(\because \theta = 0)$

$\boxed{r = 2}$

$x = r\cos\theta$

$1 = (2\cos\theta)$

$$\frac{1}{2} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{\theta = \frac{\pi}{3}}$$

$$x = r\cos\theta$$

$$2 = r\cos\theta$$

$$2 = r\cos\left(\frac{\pi}{3}\right) \qquad \left(\because \theta = \frac{\pi}{3}\right)$$

$$2 = r\left(\frac{1}{2}\right)$$

$$\boxed{r = 4}$$

$$= \int_0^{\frac{\pi}{3}} \int_2^4 \frac{1}{(r^2)^2} \, r \, dr \, d\theta$$

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te: ______

$$= \int_{0}^{\frac{\pi}{3}}\int_{2}^{4} \left(\frac{r}{r^{4}}\right) dr\,d\theta$$

$$= \int_{0}^{\frac{\pi}{3}}\int_{2}^{4} \left(\frac{1}{r^{3}}\right) dr\,d\theta$$

$$= \int_{0}^{\frac{\pi}{3}}\int_{2}^{4} r^{-3}\, dr\,d\theta$$

$$= \int_{0}^{\frac{\pi}{3}} \left[\frac{r^{-2}}{-2}\Bigg|_{2}^{4}\right] d\theta$$

$$= \int_{0}^{\frac{\pi}{3}} \left(\frac{4^{-2}}{-2} + \frac{2^{-2}}{2}\right) d\theta$$

$$= \int_0^{\frac{\pi}{3}} \left( -\frac{1}{2 \times (4)^2} + \frac{1}{2 \times 2^2} \right) d\theta$$

$$= \int_0^{\frac{\pi}{3}} \left( -\frac{1}{32} + \frac{1}{8} \right) d\theta$$

$$= \int_0^{\frac{\pi}{3}} \frac{3}{32} \, d\theta$$

$$= \frac{3}{32} \int_0^{\frac{\pi}{3}} d\theta$$

$$= \frac{3}{32} (\theta) \Big|_0^{\frac{\pi}{3}}$$

$$= \frac{3}{32} \left( \frac{\pi}{3} \right)$$

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$$= \frac{\pi}{32} \quad \text{Ans.}$$

5) $$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{2}{(1+x^2+y^2)^2}\, dy\, dx$$

Solv. $$= 4 \int_0^1 \int_0^{\sqrt{1-y^2}} \frac{2}{(1+x^2+y^2)^2}\, dy\, dx$$

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

$y = r\sin\theta$

$0 = r\sin\theta$

$\frac{0}{r} = \sin\theta$

$\sin\theta = 0$

$\theta = \sin^{-1}(0)$

$\boxed{\theta = 0}$

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$$\left(\sqrt{1-y^2}\right. = r\sin\theta$$
$$\left(\sqrt{1-r^2\sin^2\theta}\right)^2 = (r\sin\theta)^2$$
$$1-r^2\sin^2\theta = r^2\sin^2\theta$$
$$1 = r^2\sin^2\theta + r^2\sin^2\theta$$
$$1 = 2r^2\sin^2\theta$$
$$1 = 2r^2 \times$$

$$x = r\cos\theta$$
$$0 = r\cos\theta$$
$$\frac{0}{r} = \cos\theta$$
$$0 = \cos\theta$$
$$\theta = \cos^{-1}(0)$$
$$\boxed{\theta = \frac{\pi}{2}}$$
$$x = r\cos\theta$$
$$0 = r\cos\theta$$
$$0 = r\cos 0 \qquad (\because \theta = 0)$$
$$0 = r(1)$$
$$\boxed{r = 0}$$

$x = r\cos\theta$

$1 = r\cos\theta$

$1 = r\cos\theta$ $\quad (\because \theta = 0)$

$1 = r(1)$

$\boxed{r = 1}$

$$= 4\int_0^{\frac{\pi}{2}}\int_0^{1} \frac{2}{(1+r^2)^2}\, r\, dr\, d\theta$$

$$= 8\int_0^{\frac{\pi}{2}}\int_0^{1} \left(\frac{r}{1+r^4+2r^2}\right) dr\, d\theta$$

$$= 8\int_0^{\frac{\pi}{2}}\int_0^{1} \frac{r}{(r^2+1)^2}\, dr\, d\theta \rightarrow (i)$$

Let

$u = r^2 + 1$

$du = 2r\, dr$

$$\frac{du}{2} = r\,dr$$

$$r\,dr = \frac{du}{2}$$

$$= \int_0^1 \frac{du}{2u^2}$$

$$= \frac{1}{2}\int_0^1 u^{-2}\,du$$

$$= \frac{1}{2}\left(\frac{u^{-1}}{-1}\right)\Bigg|_0^1$$

$$= -\left(\frac{1}{2u}\right)\Bigg|_0^1 \rightarrow \text{(ii)}$$

Now, put eq. (ii) in (i).

e: ______

$$= 8\int_0^{\frac{\pi}{2}} \left. \frac{-1}{2(r^2+1)} \right|_0^1 d\theta \quad (\because u = r^2+1)$$

$$= -\frac{8}{2}\int_0^{\frac{\pi}{2}} \left(\frac{1}{2} - 1\right) d\theta$$

$$= -4\int_0^{\frac{\pi}{2}} \left(-\frac{1}{2}\right) d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} d\theta$$

$$= 2(\theta)\Big|_0^{\frac{\pi}{2}}$$

$$= 2\left(\frac{\pi}{2}\right)$$

$$= \pi$$ Ans.