11

Electric Circuits

11.1

Electric Current

11.1.1

Causes of Electric Currents

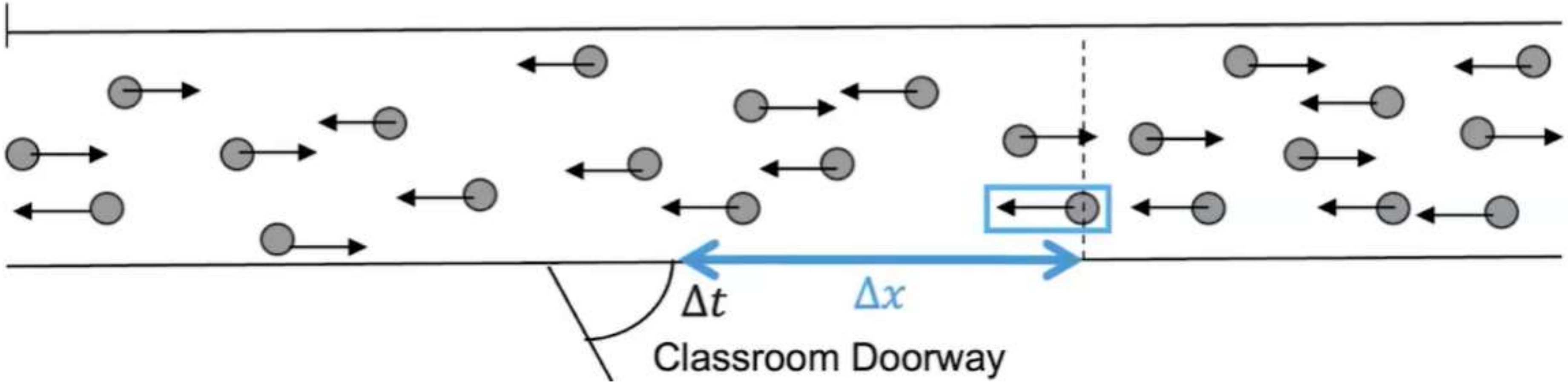
What is Electric Current?

- Many times, when a person hears the word “current”, the idea of speed comes to mind.
- For example, a river that has a high current can sometimes be interpreted as fast moving water.
- Let’s use students moving through a school hallway to help understand the concept of current.
- Suppose a teacher is standing in her classroom doorway and observing students as they walk by during passing time.

What is Electric Current?

Let's use the diagram below for this analogy:

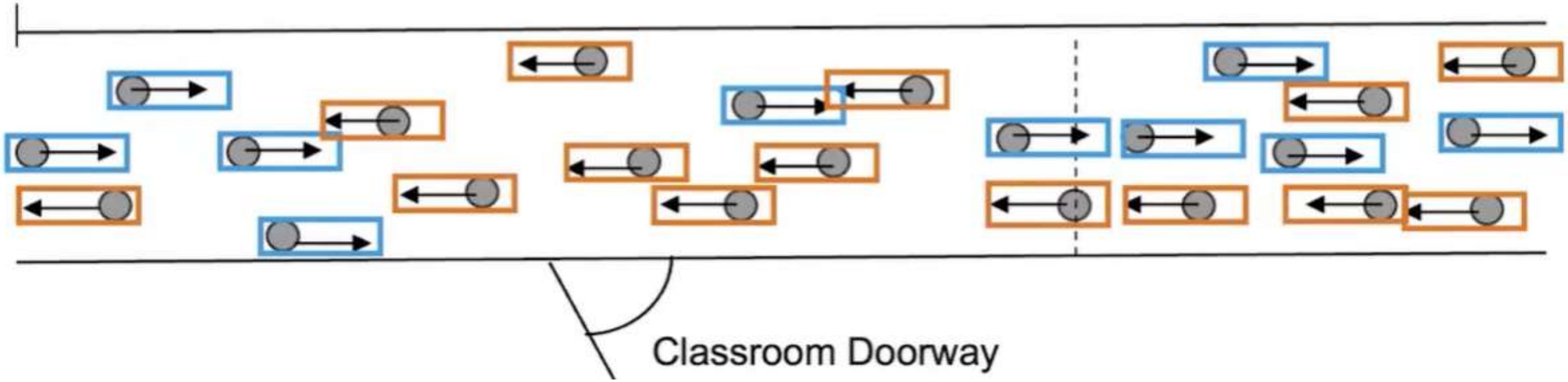
- Each dot represents students moving through a hallway.
- The arrows represent the velocities of the students and let's assume each student is walking with the same speed.
- Let's choose one of the students to measure the speed.
- This student's speed can be calculated using the distance the student moves from an initial position to the classroom door using the time measured by the teacher and the equation $v = \frac{\Delta x}{\Delta t}$.



What is Electric Current?

However, this is not the ***current*** of the students moving through the hallway.

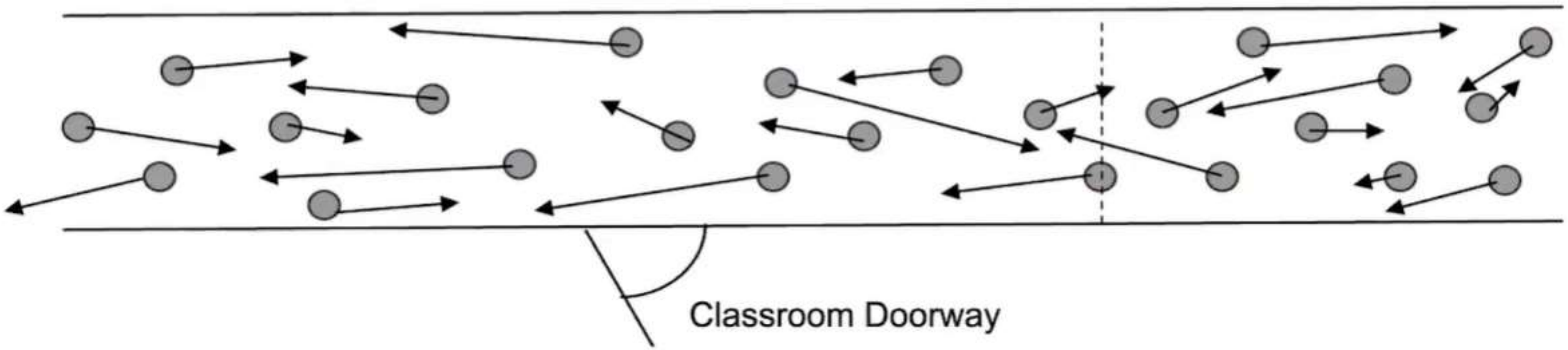
- Instead of timing how long it takes a student to move a set distance, the teacher now counts how many students pass by her door during the passing time.
- She counts **10 students walking to the right**.
- She also counts **14 students walking to the left**.
- So, there is a net movement of **4 students to the left**.



What is Electric Current?

Now, let's no longer assume that the speed of each student is the same.

- In the diagram below, the arrows represent the speed and direction of each student as they are moving.
- If the teacher again measured the distance and time of one student, it would no longer be the same as the previous measurement and it could be different for each student.
- However, if the teacher counted the number of students moving right and left during the passing time, this would still be the same.

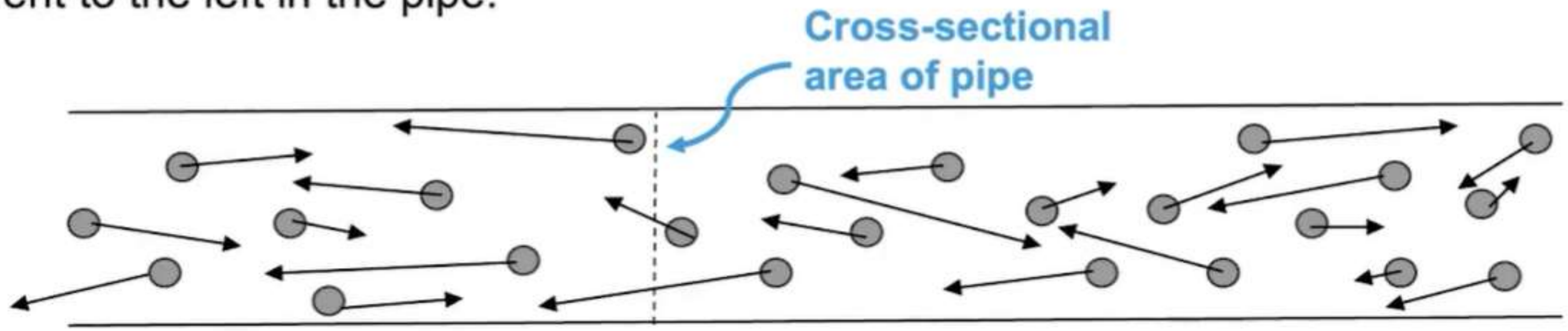


What is Electric Current?

- We can define this new concept of number of students during passing time as the ***current*** of the students.
- For example, there is a **current of 10 students moving to the right** during passing time and there is a **current of 14 students moving to the left** during the same passing time.
- The **net current would be 4 students moving to the left** during passing time.
- So, the current of the students depends on the number of students and the time during which they are moving, instead of the distance traveled by each student during a period of time, which would be the speed of each student.
- Notice, that if the hallway were wider, more students could be moving down the hallway, which would increase the current.
- How does this relate to water current?

What is Electric Current?

- Let the dots in the diagram below represent individual water molecules in a pipe through which water is flowing.
- The dashed line replaces the classroom door and now represents the **cross-sectional area** of the pipe.
- Notice that each water molecule has a **different speed and direction**, just like the students in the hallway.
- If the number of water molecules that pass the dashed line were counted over a specified period of time, this would be known as the current of the water with the net current to the left in the pipe.



What is Electric Current?

- Obviously, counting water molecules to find the current of water is crazy because there are so many molecules.
- Instead, the number of liters or gallons of water that move through an area in a given time are used to measure water current.
- Notice that water current is still a measurement of a quantity (number) over a period of time.
- Also notice that if the pipe were wider, more liters/gallons of water could flow through the pipe increasing the water current.
- This raises several questions:
  - In the example of the water current, what makes the water move through the pipe?
  - Is there a relationship between current and speed? If so, what is the relationship?
  - How does all this relate to electric current?

What is Electric Current?

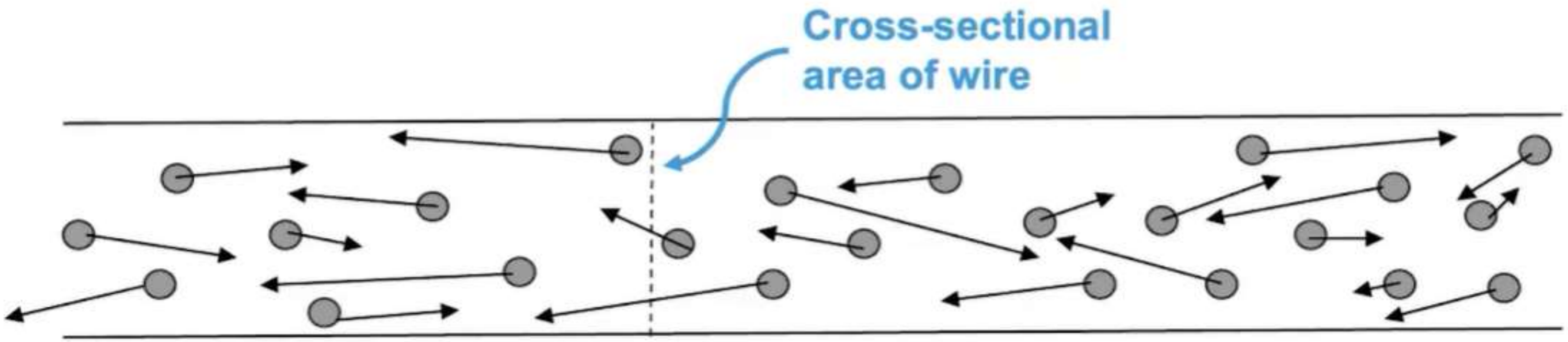
- What makes water move through a pipe?
- The answer is known as a pressure difference, which causes the pressure at one end of the pipe to be greater than the other end.
- This can be accomplished in a variety of ways, one of which is using a water pump.
- As long as there is a pressure difference between the two ends of a pipe, the net motion of the water molecules will be from the high-pressure end to the low-pressure end of the pipe.
- The larger the pressure difference, the greater the number of liters/gallons of water can pass through an area in the same amount of time.
- So, the larger the pressure difference, the greater the water current.

What is Electric Current?

- Are speed and current related?
- The quick answer is, **yes**.
- If water current is a measurement of how much water can pass through a cross-sectional area in a specific amount of time, can this current be affected by the speed of the water?
- To help answer this question, think of two rivers of the same width.
    - Suppose the speed of water in one river is faster than the speed of the other.
    - Would the currents of the two rivers be the same?
    - No, because the river with the faster moving water can move a greater amount of water through the same cross-sectional area in the same amount of time.
    - So, faster moving water would result in a greater water current.
    - It works out that there is a direct relationship between the speed and water current.

What is Electric Current?

- How does what we have discussed so far have to do with electric current?
- Using the same diagram as before, let the dots now represent electrically charged particles moving in a wire.
- As with the other examples, the speed and direction of each charged particle is different.
- But the number of charged particles passing by the dashed line can be counted over a specified time to find the current of the charged particles.
- This is known as **electric current**.



What is Electric Current?

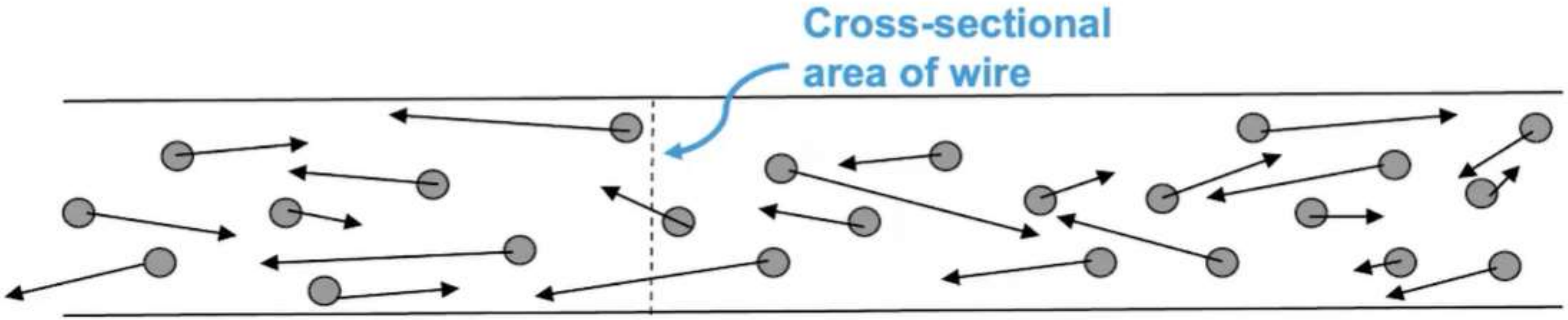
- Obviously, counting charged particles to find the electric current is crazy because there are so many particles.
- Instead, the number of coulombs of charge that move through an area in a given time is used to measure electric current.
- Notice that the electric current is still a measurement of a quantity (number) over a period of time.
- Electric current (I) is the amount of charge (Q) moving through a cross-sectional area of a wire in time an interval (Δt).

$$I = \frac{Q}{\Delta t} = \frac{dq}{dt}$$

- Electric current is measured in Amps or Amperes (A), which is the number of coulombs of charge per second (C/s)

What is Electric Current?

- In our example diagram from before, the number of particles moving left and right can be counted, but the net movement was found to be to the left.
- The speed and direction of each particle is very different than the net motion of all the particles.
- Overall, the motion of all the particles "drifts" to the left.
- This net charged particle motion has been given the term ***drift velocity***.
- This drift velocity, or speed if no direction is included, directly affects the current in the wire much like the speed of water in a river affects the current of that river.



What is Electric Current?

- In addition to drift velocity, the electric current depends on the following:
  - Number of charged particles per cubic meter of volume
  - Quantity of charge
  - Cross-sectional area of wire

- Putting all this together, we get the equation for the electric current in a wire:

$$I = nqv_dA$$

- In the equation, n represents the number of charged particles per cubic meter of volume, q is the quantity of the charge, v_d is the drift velocity of the charged particles, and A is the cross-sectional area of the wire.

- Notice that if the net motion of the charged particles in the wire is zero, the **average drift velocity** is zero. This average zero-drift velocity would result in no net current even though each charged particle has its own velocity.

What is Electric Current?

- If we return to our analogy of water moving through a pipe, we can understand how electric charges can move through a wire.
- Remember that in order for water to flow in a pipe, there must be a pressure difference between the ends of the pipe.
- To have a current in a wire, there must also be a pressure difference, otherwise known as an electric potential difference, between the ends of the wire.
- This electric potential difference is sometimes also referred to as an electromotive force, or emf (ε).
- Much like a water pump provides the pressure difference in a pipe, there are many devices that provide an electric potential difference in a wire, such as a battery or generator.

Which of the following would increase the amount of current flowing through a wire?

- A. Decrease the cross-sectional area of the wire.
- B. Decrease the charge of each particle moving through the wire.
- C. Decrease the number of charged particles moving through the wire.
- D. Decrease the time for the net charge to move through the wire.

Which of the following would increase the amount of current flowing through a wire?

- **A** Decrease the cross-sectional area of the wire.
- **B** Decrease the charge of each particle moving through the wire.
- **C** Decrease the number of charged particles moving through the wire.
- **D** Decrease the time for the net charge to move through the wire.

According to the equation $I = nqv_dA$ decreasing the cross-sectional area, the charge of each particle, and the number of charged particles would decrease the current in the wire. Decreasing the time, would increase the drift velocity, which would increase the current.

Which of the following is a correct statement about the speed of each charged particle in a current carrying wire and the net speed of the charged particles in the same wire?

- A. The speed of each particle and net speed are the same.
- B. The speed of each particle is greater than the magnitude of the net speed.
- C. The speed of each particle is less than the magnitude of the net speed.
- D. The speed of each particle is directly related to the net speed.

Which of the following is a correct statement about the speed of each charged particle in a current carrying wire and the net speed of the charged particles in the same wire?

- **A** The speed of each particle and net speed are the same.
- **B** The speed of each particle is greater than the magnitude of the net speed.
- **C** The speed of each particle is less than the magnitude of the net speed.
- **D** The speed of each particle is directly related to the net speed.

The net speed of the charged particles is also known as the drift speed, which is a result of all the motions of the charged particles combined, both in the direction of current and perpindicular to it.

Therefore, the drift speed through the wire is less than each individual particles speed.

Takeaways

- The concept of current is not the same as the concept of speed, but they are related to each other.
- Electric current is defined as the quantity of charge moving through the cross-sectional area of a wire in a time interval.
- This is given in equation form as

$$I = \frac{Q}{\Delta t} = \frac{dq}{dt}$$

- The amount of current in a wire depends on several quantities given by

$$I = nqv_dA$$

- The speed of each charged particle is not the same as the drift speed, but is actually much greater.

11.1.2

Electric Current Density and its relationship to an Electric Field

Review

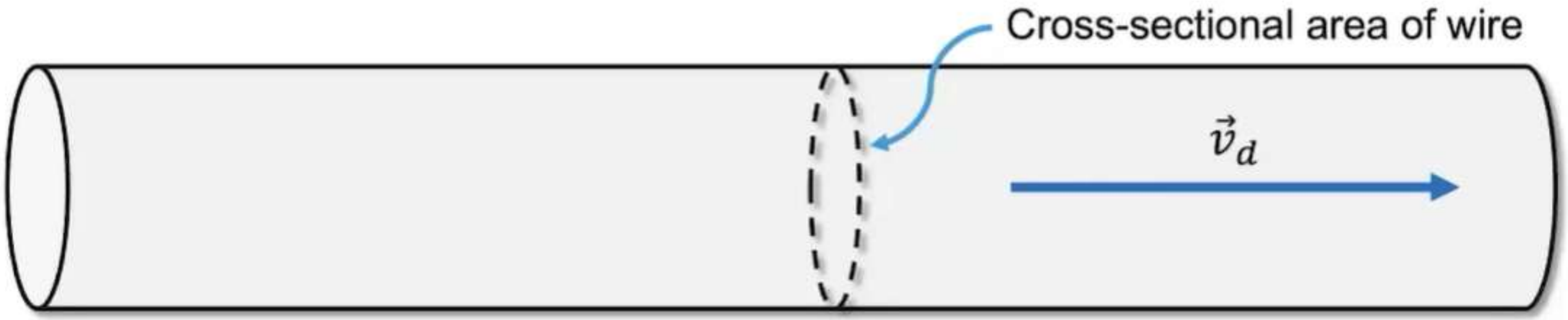
- Electric current is the movement of electric charge carriers through a material.
- In a wire, the electric charges move with many different velocities.
- The net motion of the electric charge carriers in a wire is called the drift velocity v_d.
- The electric current in a wire depends on the drift velocity according to the equation $I = nqv_dA$ where n represents the number of charged particles per cubic meter of volume, q is the quantity of the charge, v_d is the drift velocity of the charged particles, and A is the cross-sectional area of the wire
- Electric current can also be calculated with the equation $I = \frac{Q}{\Delta t} = \frac{dq}{dt}$ because current is a measurement of electric charge moving in a wire per time.

What is Current Density?

- In a prior unit, we learned about electric charge densities.
- We learned that a linear charge density is an amount of electric charge over a length, such as along a wire.
- We learned that a surface charge density is an amount of electric charge over an area, such as on a capacitor plate.
- We learned that a volume charge density is an amount of electric charge over a volume, such as a charged sphere.
- So, it would make sense that an electric current density would be very similar to these.

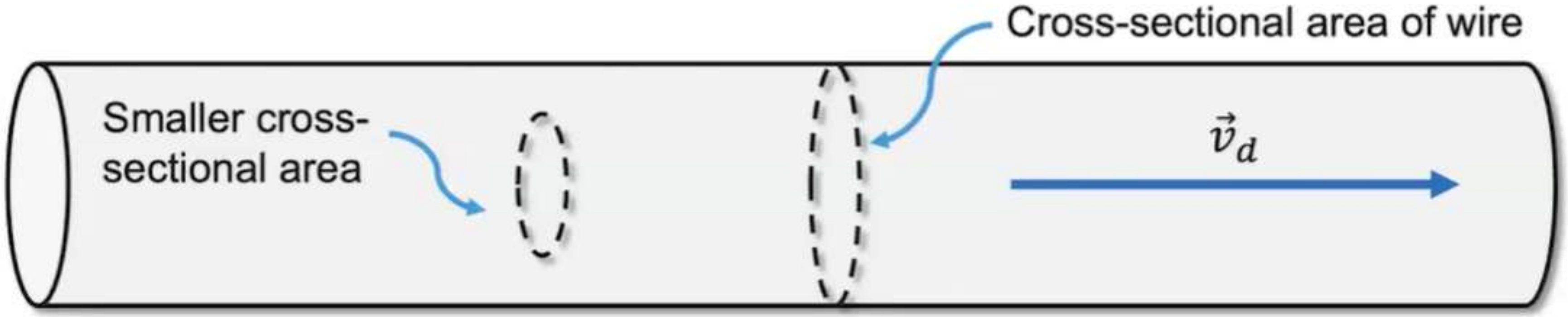
What is Current Density?

- The diagram below represents a segment of wire with a cross-sectional area shown near the middle and a drift velocity v_d to the right.
- Remember that the drift velocity is the net velocity of all the individual charge velocities moving in the wire.
- As a result, there is an electric current in the wire with a magnitude of $I = nqv_dA$.
- Notice that the electric current directly depends on the cross-sectional area of the wire.



What is Current Density?

- Would the electric current be the same if a smaller area inside the wire were measured instead of the entire cross-sectional area?
- According to the equation $I = nqv_dA$ the smaller the area, the lower the electric current.
- However, there is a quantity that does stay the same for both areas.
- This quantity is the ratio of electric current and the cross-sectional area or $\frac{I}{A}$.
- This ratio is known as the electric current density in the wire.



What is Current Density?

- The symbol used for electric current density is J and the equation is $J = \frac{I}{A}$.
- The units for electric current density are A/m^2.
- If we combine the above equation for electric current density with the equation for the electric current, we get the following $J = \frac{I}{A} = \frac{nqv_d A}{A} = nqv_d$.
- Since the drift velocity is a vector quantity and the other two quantities in the equation are scalar quantities, then the electric current density is also a vector quantity.
- This makes the above equation $\vec{J} = nq\vec{v}_d$.
- If the electric charges are positive, then the electric current density is in the same direction as the drift velocity.
- However, if the electric charges are negative, then the electric current density is in the opposite direction of the drift velocity.

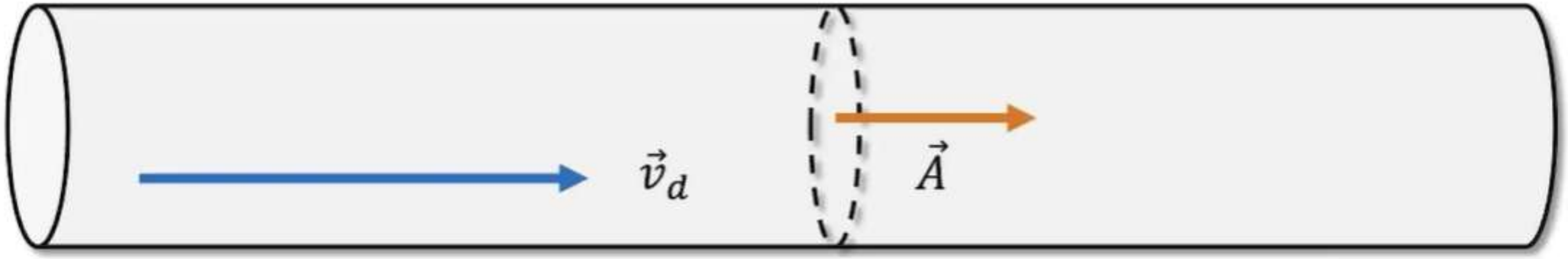
What is Current Density?

- So, how does this correlate with the other equation for current density?

If $J = \frac{I}{A}$, then $I = JA$.

- We learned in a prior unit that a unit of area has an area vector that is perpendicular to the surface of the area.

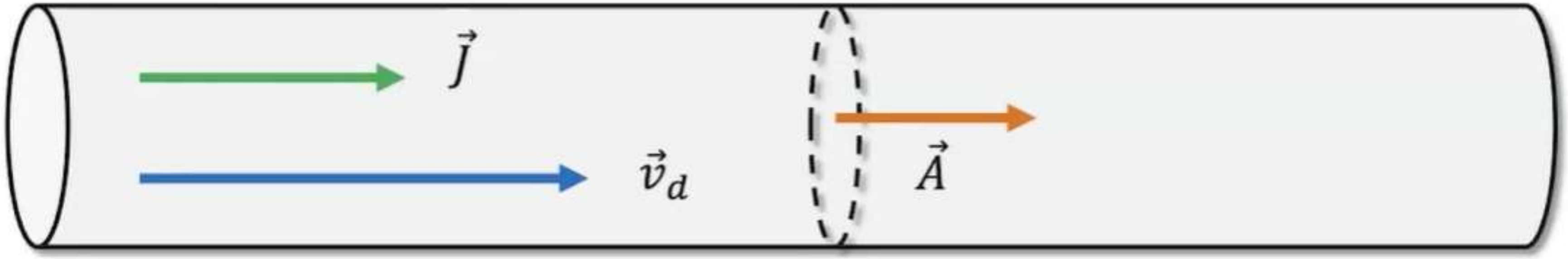
- So, if current is the product of electric current density and the cross-sectional area, is that product a vector/cross product or a scalar/dot product?



What is Current Density?

- If the current density is in the same direction as the drift velocity, then the current density is also in the same direction as the area vector in this example.
- If current was the vector/cross product of current density and area, then the current would be zero because the angle between the current density and area vectors is zero degrees since they are in the same direction.
- Therefore, current must be the scalar/dot product.

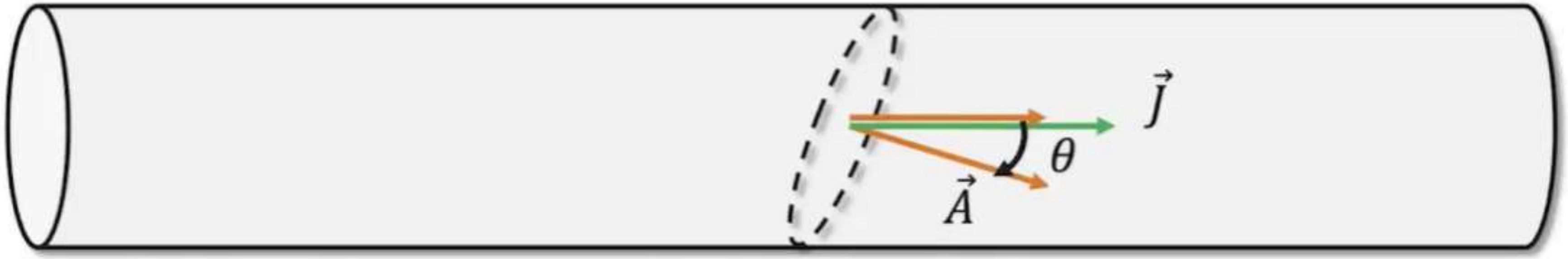
So, $I = \vec{J} \cdot \vec{A}$



What is Current Density?

- What if the area vector was not parallel to the electric current density vector?
- Then only a component of the area vector would be in the direction of the electric current density vector.
- This component is the parallel component of the area vector, or $\boldsymbol{A\cos\theta}$.
- This fits with the scalar/dot product equation determined earlier.

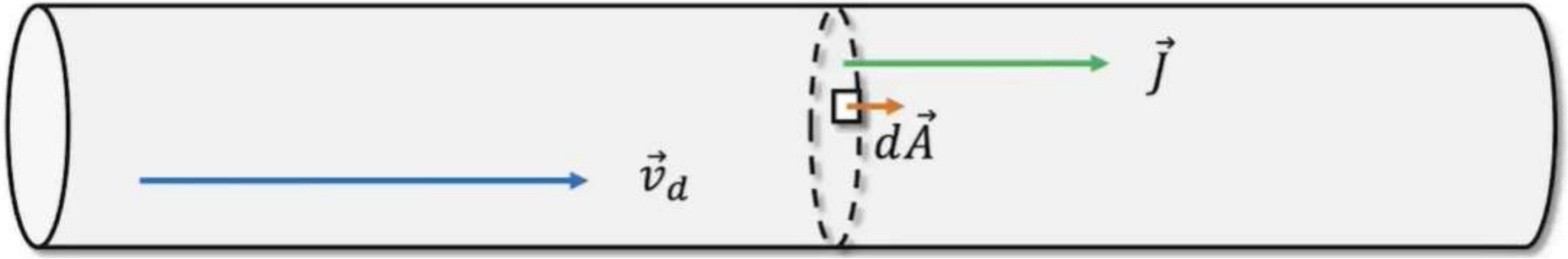
$$I = \vec{J} \cdot \vec{A} = JA\cos\theta$$



What is Current Density?

- What happens if the current passing through an infinitely small area needs to be determined?
- Let the area vector of an infinitely small area be $d\vec{A}$.
- The electric current equation would become:

$$I = \int \vec{J} \cdot d\vec{A}$$

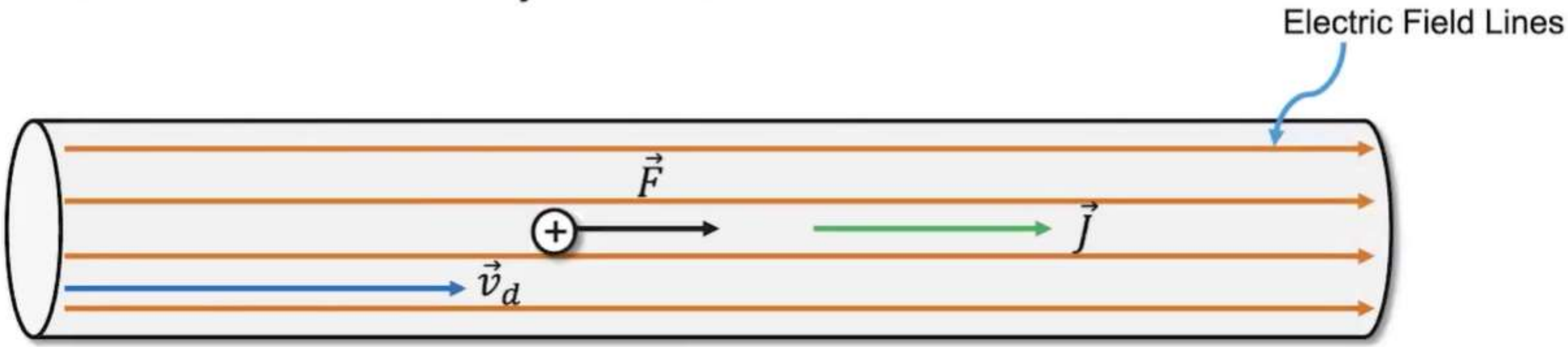


What Causes Charge Flow?

- If current density is a vector quantity, then there must be something that causes the charges to flow in a particular direction.
- In the last video we learned that much like a water pump causes water to move in a pipe, an electric potential difference, or emf, causes charges to move in a wire.
- We have also learned from a prior unit that an electric potential difference sets up an electric field that points from the area of high electric potential to an area of low electric potential.
- Additionally, we have learned that a charge located within an electric field will experience a force.
- This force will be in the same direction as the electric field if the charge is positive and in the opposite direction if the charge is negative.

What Causes Charge Flow?

- So, how does this connect to current density?
- If there is an electric field in a wire, then the charges in that wire will be forced to move.
- If the charge is positive, the force is in the same direction as the electric field.
- This force is what causes the charges to begin to drift down the wire, which results in a current density in the wire.



What Causes Charge Flow?

- If the strength of the electric field is increased, the force is increased, which increases the electric current density.
- It works out that the electric field and electric current density are directly related to each other.
- They differ from each other by a factor known as the resistivity of the material from which the wire is made.
- The concept of resistivity will be covered in more detail in a later video.
- For now, understand that it is a property of the material that affects the movement of charges in the material.
- The greater the resistivity, the lower the electric current density.
- The equation that puts these things together is $\vec{E} = \rho\vec{J}$ where ρ is the resistivity of the material.

A copper wire with a cross-sectional area of 4×10^{-6} m^2 has a free electron density of 8.4×10^{28} m^{-3}. The electrons in the wire have a drift velocity of 3.4×10^{-6} m/s to the right. What is the electric current density in the wire?

A $0.183\ A/m^2$

B $-0.183\ A/m^2$

C $45{,}696\ A/m^2$

D $-45{,}696\ A/m^2$

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C $45{,}696\ A/m^2$

D $-45{,}696\ A/m^2$

Electric current density is calculated using the equation

$$\vec{J} = nq\vec{v}_d.$$

$\vec{J} = nq\vec{v}_d =$ (8.4 x 10^{28})(1.6 x 10^{-19})(3.4 x 10^{-6}) = 45,696 A/m^2.

Because the charges are electrons, the electric current density will be in the opposite direction of the drift velocity, which is to the left.

The first two answer options calculate the electric current in the wire instead of the electric current density.

Which of the following must be present within a wire where the charges are located to cause electric charges to move through that wire?

- A Electric Field
- B Electric Potential Difference
- C Electric Force
- D Electric Current

Which of the following must be present within a wire where the charges are located to cause electric charges to move through that wire?

- **A** Electric Field
- **B** Electric Potential Difference
- **C** Electric Force
- **D** Electric Current

The electric field is produced by the electric potential difference, which causes a force to be applied to the charges in the wire. However, forces and potential differences do not exist in a region of space like the electric field. Electric current is the movement of charges, not the cause of the movement.

Remember: $q = e = -1.6 \times 10^{-19}\ C$

Takeaways

- Electric current density is the ratio of current moving through a cross-sectional area in a conductor and the equation is $J = \frac{I}{A}$.
- The magnitude of the electric current density in a conductor can also be calculated using the equation $J = nqv_d$.
- Electric current density is a vector quantity with a direction equal to the drift velocity direction if the charges are positive and in the opposite direction if the charges are negative.
- Electric current is the scalar/dot product of the electric current density and cross-sectional area vectors. The equation is $I = \int \vec{J} \cdot d\vec{A}$.
- Electric current density is directly related to the electric field according to the equation $\vec{E} = \rho\vec{J}$ where ρ is the resistivity of the conductor.

11.1.3

Direction as it pertains to electric current

Review

- Electric current is the flow of electric charges through a conducting material.

- Electric current is the result of the scalar/dot product of the electric current density and area vectors.

$$I = \vec{J} \cdot \vec{A} = JA\cos\theta.$$

Electric Current Direction

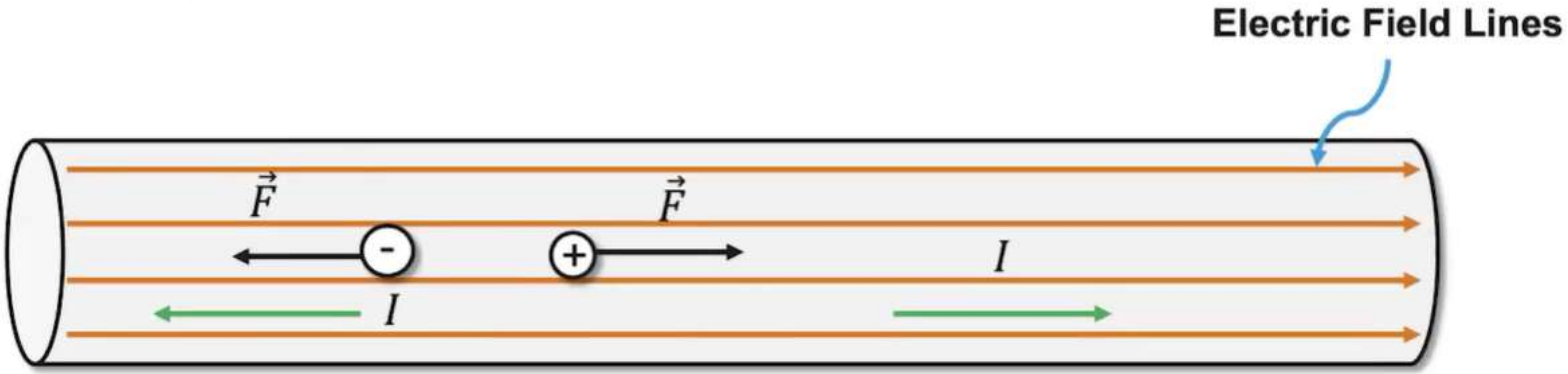
- In the previous lesson and video, we learned that electric current density and drift velocity were vectors.
- We also learned that an electric current is a scalar quantity because it is the dot product of the electric current density and drift velocity vectors.
- So, why even concern ourselves with whether or not current has a direction?
- If electric current is the flow of an electric charge in a conductor, then wouldn't that imply that the charges would flow in a particular direction?
- Also, in a previous lesson we learned that current and drift velocity are directly related, much like water current is directly related to the speed of the water.
- So, wouldn't this also imply that electric current has a direction?

Electric Current Direction

- These ideas do indicate that current has a direction, but not like drift velocity and electric current density, which are vector quantities.
- In other words, electric current does not have a direction like a vector quantity does.
- Vector quantities obey the laws of vector addition and can have vector components if an angle is involved.
- Electric current is based on the charges that are moving in a conductor, which does have a direction.
- However, this electric current direction is relative to the current carriers and does not obey the laws of vector addition.
- Then what is the direction of electric current?

Electric Current Direction

- The movement of charges is what makes up electric current.
- If an electric field is set up in a wire, then the positive charges in that wire will be forced to move in the direction of the electric field.
- Therefore, in this example, the current would be to the right in the wire.
- However, charges can also be negative, which would mean that the current in this example would be to the left.



Electric Current Direction

- Because there is more than one type of charge and the current depends on that charge, a choice had to be made.
- This choice was made by Ben Franklin in 1747.
- He understood that there were two types of charges and he assigned them the + and – signs/symbols.
- No one knew anything about electrons until 1897, which means Ben Franklin had no idea about which charge moves in a wire.
- So, he made a choice about which charge moves in a wire.
- Unfortunately, he chose wrong by choosing the positive charges as the ones moving in a wire.
- Today we know that the actual charges moving in a wire are the electrons, which have a negative charge.

Electric Current Direction

- So, by convention, the direction of current in a wire is based on the movement of positive charges.
- However, the actual direction that the electrons move in a wire is in the opposite direction of conventional current.
- You might wonder why it hasn't been changed now that we know what charges really move in a wire.
- Since electrons were not discovered until 150 years after Ben Franklin made the choice, everything that we understood about wires and circuits were already based on positive charge flow.
- So, scientists decided to just keep it the way it has been and just remember that electrons in a wire move in the opposite direction of conventional current.

On which of the following does the direction of conventional current in a wire depend?

- A The direction of the electric field.
- B The direction of the drift velocity.
- C The direction in which the negative charge carriers are moving.
- D The direction in which the positive charge carriers are moving.

On which of the following does the direction of conventional current in a wire depend?

- A: The direction of the electric field.
- B: The direction of the drift velocity.
- C: The direction in which the negative charge carriers are moving.
- D: The direction in which the positive charge carriers are moving.

The drift velocity and electric field are in the same direction as the positive charge flow. However, by definition, conventional current is based on the movement of the positive charge carriers, as chosen by Ben Franklin.

Takeaways

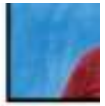
- The electric current in a wire is based on the movement of positive charges, which is called conventional current.

- Electrons in a wire move in the opposite direction of conventional current.

11.1.4

Problem Solving for non-Constant electric current density in a conductor

Review



- Electric current is the flow of electric charges through a conducting material.

- Electric current is the result of the scalar/dot product of the electric current density and area vectors.

$$I = \vec{J} \cdot \vec{A} = JA\cos\theta$$

- The electric current can also be calculated for an infinitely small area $d\vec{A}$.

$$I = \int \vec{J} \cdot d\vec{A}$$

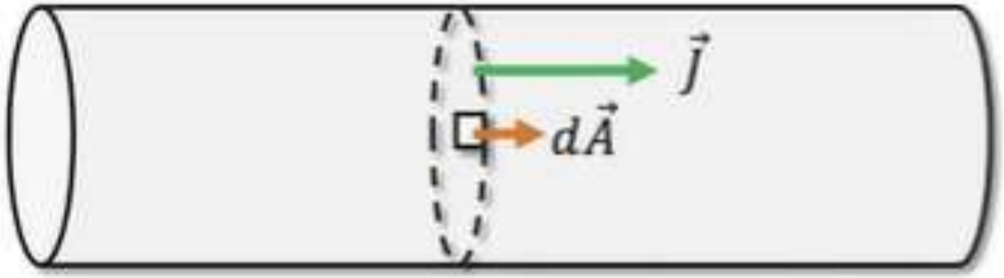
Electric Current Calculation

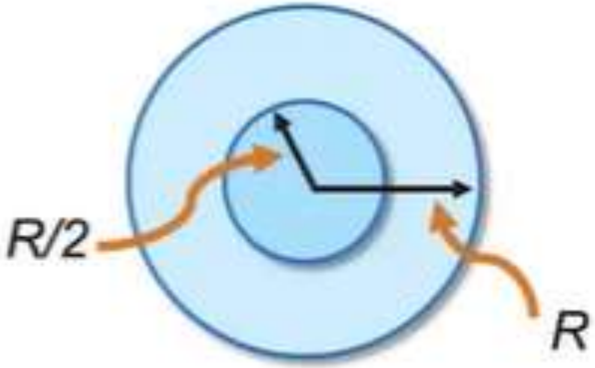
- In a previous lesson, we learned how to calculate electric current using drift velocity and electric current density.
- But what if the electric current density is not constant throughout the wire?
- Then we would need to use the equation $I = \int \vec{J} \cdot d\vec{A}$.
- Suppose a wire with a radius of 2 mm has an electric current density that varies with radial distance r as $J = ar^2$, in which $a = 1.5 \times 10^{10}$ A/m^4 and r is in meters.

What would be the current through the outer portion of the wire between R/2 and R?

Electric Current Calculation

- In the equation $I = \int \vec{J} \cdot d\vec{A}$ the vectors have the same direction, as shown in the diagram below.
- So, the equation simplifies to $I = \int J dA$
- The diagram below on the right shows the cross-section of the wire with $R/2$ and R shown in the diagram.
- We need to replace the differential area dA with something we can integrate between the limits $r = R/2$ and $r = R$.

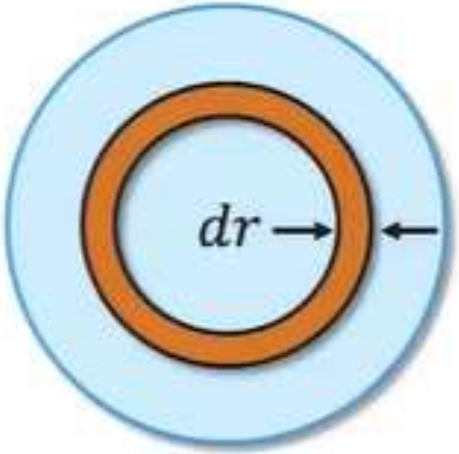




Electric Current Calculation

- The simplest replacement is the area $2\pi r dr$ of a thin ring of circumference $2\pi r$ and width dr.
- We can now integrate with r as the variable of integration.

$$I = \int J dA = \int_{\frac{R}{2}}^{R} \boxed{ar^2 2\pi r dr}$$



Current Density Practice Problem

A wire has a radius of 3 mm and a current density that varies with radial distance r as $J = \frac{b}{r}$, in which $b = 200$ A/m and r is in meters.

Calculate the electric current between $R/4$ and $3R/4$.

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$$I = \int \vec{J} \cdot d\vec{A} = \int J dA = \int_{\frac{R}{4}}^{\frac{3R}{4}} \frac{b}{r} 2\pi r dr$$

$$I = 2\pi b \int_{\frac{R}{4}}^{\frac{3R}{4}} dr = 2\pi b [r] \Big|_{\frac{R}{4}}^{\frac{3R}{4}} = 2\pi b \left[\frac{3R}{4} - \frac{R}{4}\right] = \pi b R$$

$$I = \pi(200)(0.003) = 1.88\ A$$

Takeaways

- The electric current in a wire with a varying electric current density can be calculated using the equation $I = \int \vec{J} \cdot d\vec{A}$.

- If the electric current density and area vectors are in the same direction, the equation becomes $I = \int J dA$.

- To complete the calculation, the dA must be replaced with something that can be integrated in terms of the provided electric current density function.