

# 11

## Electric Circuits

### 11.3

#### Resistance, Resistivity, Ohm's Law

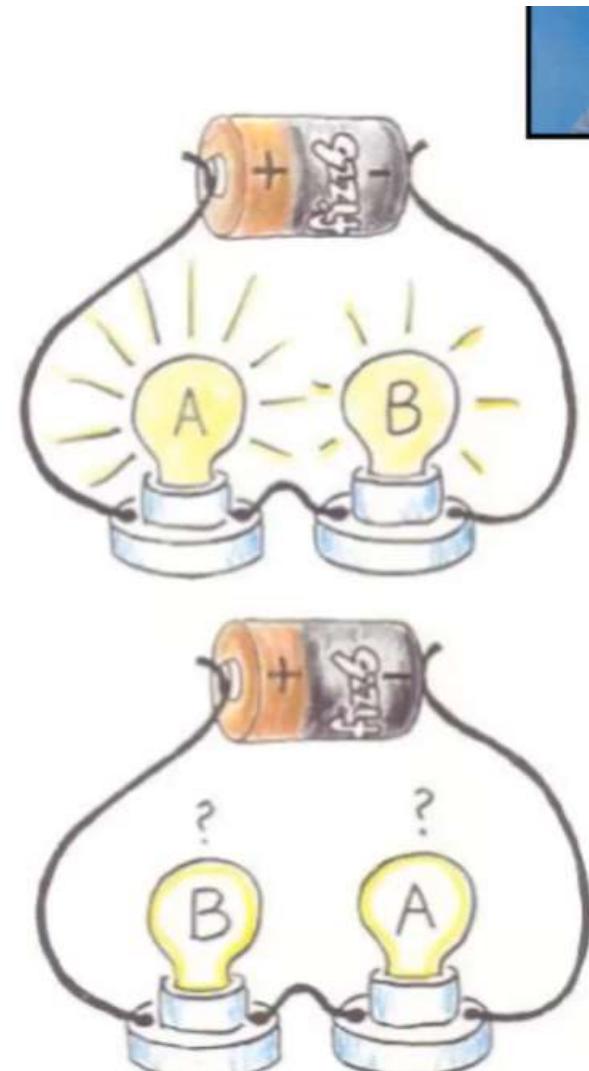
##### 11.3.1

###### Roles of Resistance and Problem-Solving strategies

## Warm Up

When the series circuit shown to the right is connected, Bulb A is brighter than Bulb B. If the positions of the bulbs were reversed:

- A Bulb A would again be brighter.
- B Bulb B would be brighter.
- C Either of the above could occur.



## Vocabulary



- **Resistance** – the opposition of charges moving through a circuit.
  - Symbol **R**, unit  **$\Omega$**  (ohm, the Greek letter capital Omega)
- **Ohm meter** – Device used to measure resistance.
  - **Note:** the resistor does not need to be part of the circuit to measure resistance.

# What is Resistance?



- For a circuit made of a 9V battery, ideal wire (0 resistance), and a fuse with a 4 A rating, there is 0 resistance in the circuit.
- This means the fuse is overloaded and breaks



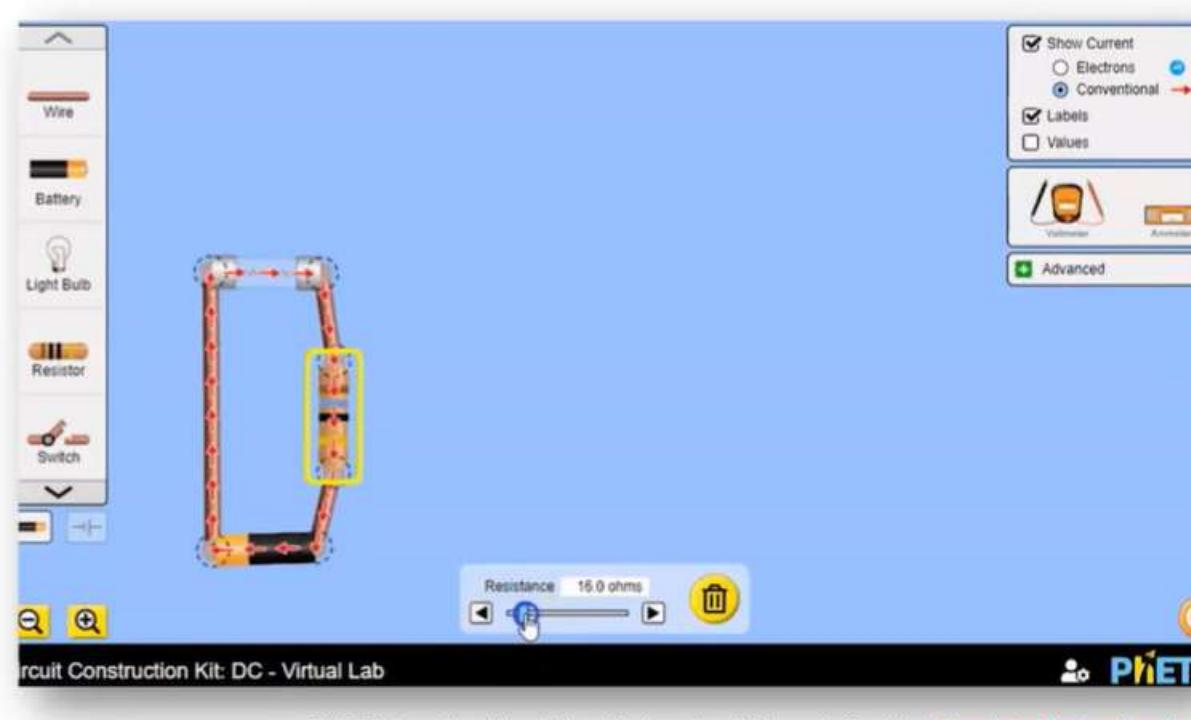
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\*Not pictured: Fuse breaks/opens at 4 amps (since  $I > 4$  amps)

# What is Resistance?



- For a circuit made of a 9V battery, wire, a  $4\Omega$  resistor and a fuse with a 4 A rating, there is now some resistance in the circuit.
- This means the fuse is no longer overloaded will not break
- **Resistance** is the opposition of charges moving through a circuit. Symbol R, unit  $\Omega$  (ohm, the Greek letter capital Omega)



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Fuse remains intact since  $I = (9/4) > 4$

# What is Resistance?



- Now we will replace the resistor with a light bulb with a tungsten filament.
- Tungsten is a material that resists the flow of charges effectively. This turns the kinetic energy of the charges into thermal energy, as we've seen friction do. Once there is enough thermal energy, the filament glows (light energy).



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# What is Resistance?



Now we will replace the resistor with a light bulb with a tungsten filament.

- The lightbulb is now limiting the flow of charges, preventing the overloading of the circuit.
- Notice how when the resistance of the bulb is increased (and everything else is constant), the brightness decreases while the charges *appear* to move slower?



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# What is Resistance?



- The charges are NOT moving slower in the resistor
- One way to think of this is the bottleneck effect that happens on a highway when there are lanes closed.
- All vehicles merge into 1 lane before moving through. This leads to a longer overall travel time.
- A smaller cross-sectional area means more resistance.



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BUT! Remember the current is constant throughout the circuit. When charges interact with the resistor, they speed up ensuring the same number of charges are crossing the same point in same time interval. So...

# What is Resistance?

- ...more resistance means the charges are moving faster and colliding inelastically with the conductor's atoms more frequently. This dissipates the kinetic energy quicker in a highly resistive material, like tungsten. So the tungsten gets hotter and glows brighter.
- Even with a resistor in a circuit, you can feel/measure the increase in temperature letting you know energy is being dissipated
- With the increased speed and collisions, there is a decrease in the potential across the resistor. You may hear this as a "Voltage Drop"

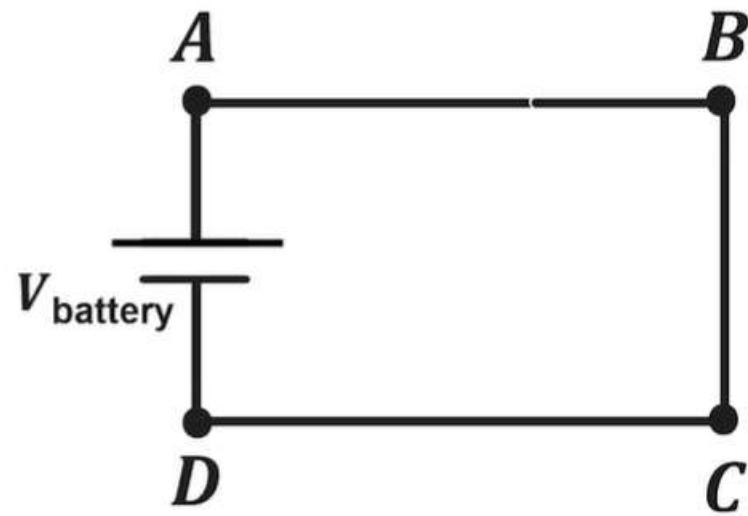


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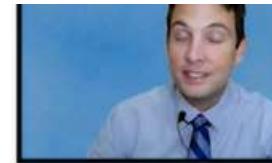
What is the consequence of having no resistor in this circuit?



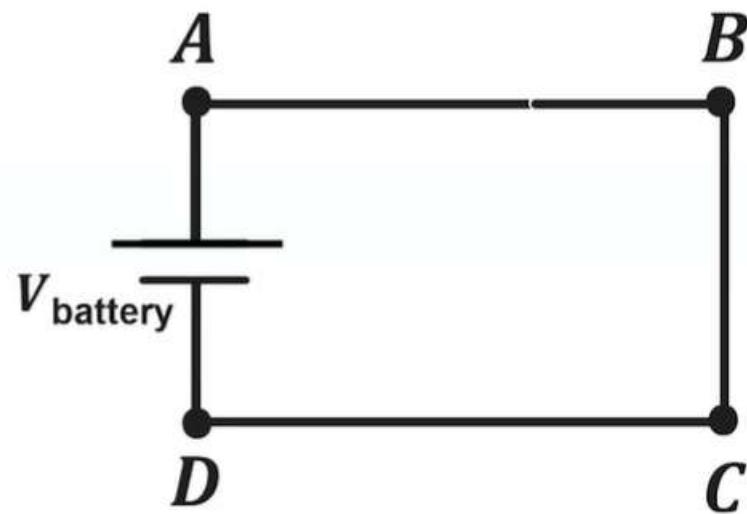
- A Nothing happens
- B The battery overheats
- C The battery becomes a wire
- D The wire falls apart



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- A Nothing happens
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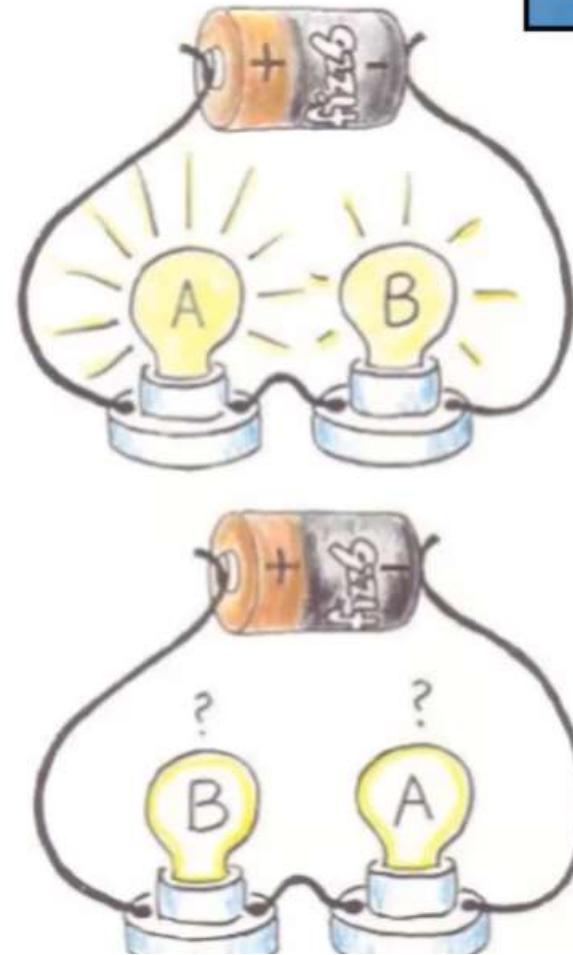


## Warm Up



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## Warm Up



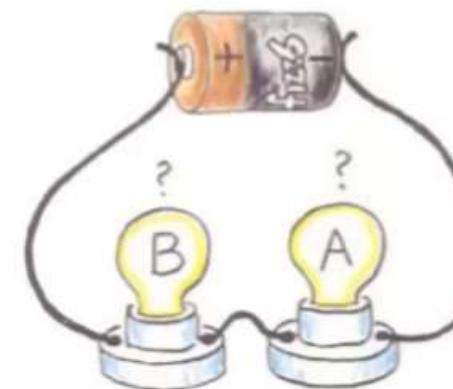
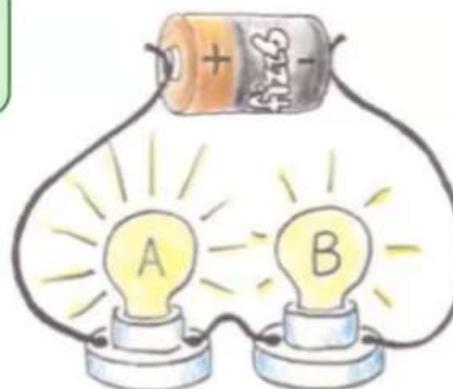
A

Bulb A would again be brighter.

The bulbs are connected in series, so the same current passes through **both of them**.

Different brightnesses indicate different filament resistances. Bulb A is **not** brighter because it is "first in line" for current from the battery!

After all, electrons deliver the energy, and they flow from negative to positive-in the opposite direction!



In which bulb is the filament resistance higher?

Which bulb would be brighter if they were connected in parallel?

# Takeaways



- Resistors clog up the flow of charges
- Current may slow down, but the charges speed up within the resistor
- Faster charges mean more collisions
- More collisions mean energy is being transformed from the circuit
- Any device that transforms electrical energy to some other type of energy can be a resistor in a circuit



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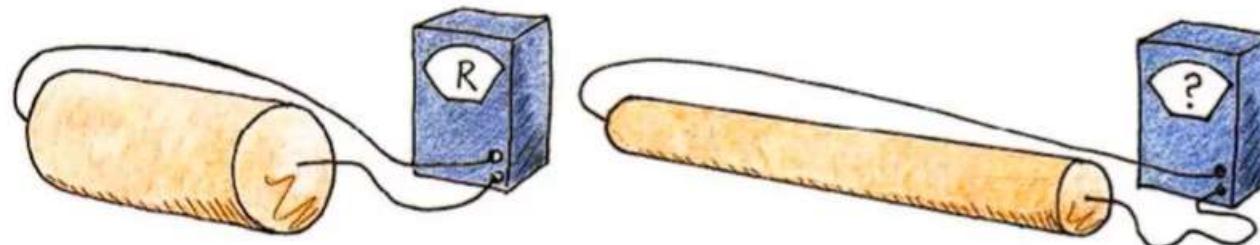
## 11.3.2

Resistance of a wire (Resistivity) and Mathematical Approaches  
for Predicting a material's Resistance

# Warm Up



Roll a piece of modeling clay into a cylinder and use an ohmmeter to measure its resistance.



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Now roll it out until it's twice as long and measure the resistance again. Compared with the initial resistance, the new resistance is:

- A unchanged
- B twice as much
- C four times as much
- D eight times as much
- E actually less

# Vocabulary



- **Cross Sectional Area** – the area of a slice of a 2-dimensional shape that was cut by a plane in the 3rd dimension.
- **Resistance** – the limiting of charges moving through a circuit. Symbol  $R$ , unit  $\Omega$  (ohm, the Greek letter capital Omega)
- **Current Density** – Total Current per unit Area. Symbol  $J$ , unit  $A/m^2$
- **Resistivity** – property of material that allows for the prediction of resistance based on the cross-sectional area and length of the conductor. Symbol  $\rho$  (the Greek letter lowercase rho), unit  $\Omega / m$ .

## Deriving an expression for resistance



**Resistance ( $R$ )** is a property of a conductor that impedes the flow of electric current.

It can be thought of as the ratio of voltage ( $V$ ) across the conductor to the current ( $I$ ) flowing through it, expressed as:



$$R = \frac{\Delta V}{I}$$

# Deriving an expression for resistance



- **Resistivity**, symbolized by the Greek letter rho ( $\rho$ ), is a fundamental property of materials that characterizes their ability to impede the flow of electric current.
- It is a measure of how strongly a material opposes the passage of electrical charge.
- In essence, resistivity determines how well a material can resist the flow of current.
- The inverse of **resistivity** is conductivity.

$\rho$

A large, stylized green Greek letter rho ( $\rho$ ) is displayed prominently on the right side of the slide.

# Deriving an expression for resistance

Recall electric field represents the force experienced by a unit positive charge at a given point in space. In mathematical terms:

So, for a conductor of length  $L$ , the voltage can be expressed as  $V = - \int E \, dr$ , integrating the electric field over the length of the conductor.

$$E_x = -\frac{dV}{dx} \quad E = \frac{\Delta V}{L}$$

Where  $E$  is the Electric Field,  $V$  represents the electric potential and  $x/r$  is a distance or Length.

## ELECTRICITY

$$|\vec{F}_E| = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} = k \frac{|q_1 q_2|}{r^2}$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$Q_{\text{total}} = \int \rho(r) dV$$

$$U_E = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

$$\Delta V = - \int_a^b \vec{E} \cdot d\vec{r}$$

$$E_x = -\frac{dV}{dx}$$

$$\Delta U_E = q \Delta V$$

$A$  = area

$C$  = capacitance

$d$  = distance

$E$  = electric field

$F$  = force

$I$  = current

$J$  = current density

$\ell$  = length

$P$  = power

$q$  = charge

$Q$  = charge

$r$  = radius, distance, or position

$R$  = resistance

$t$  = time

$U$  = potential energy

$V$  = electric potential or volume

$\epsilon$  = electric permittivity

$\rho$  = resistivity or charge density

$\kappa$  = dielectric constant

$\Phi$  = flux

# Deriving an expression for resistance

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ELECTRICITY

$$|\vec{F}_E| = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} = k \frac{|q_1 q_2|}{r^2}$$

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$\rho$  = resistivity or charge density

$\kappa$  = dielectric constant

$\Phi$  = flux

## Deriving an Expression for Resistance

Current density represents the amount of electric current passing through a unit area perpendicular to the current flow.

Mathematically,  $J = \frac{I}{A}$ , where  $I$  is the total current and  $A$  is the cross-sectional area.

# Deriving an Expression for Resistance

Current density represents the amount of electric current passing through a unit area perpendicular to the current flow. If we assume a constant current density, then mathematically,  $J = \frac{I}{A}$ , where  $I$  is the total current and  $A$  is the cross-sectional area.

Also remember that  $I = nqv_d A$ .  $n$  represents the number of charges,  $q$  is the fundamental charge,  $v_d$  is the drift velocity, and  $A$  is the cross-sectional area.

$$\vec{J} = nq\vec{v}_d$$

ELECTRICITY AND MAGNETISM	
$F_e = \frac{1}{4\pi\epsilon_0} \frac{ q_1 q_2 }{r^2} = k \frac{ q_1 q_2 }{r^2}$	$\mathcal{A}$ = area
$E = \frac{\vec{F}_e}{q}$	$C$ = capacitance
$E = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$	$d$ = distance
$\Phi_E = \int \vec{E} \cdot d\vec{A}$	$E$ = electric field
$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{r_0}$	$F$ = force
$Q_{enc} = \int \rho(r) dV$	$I$ = current
$U_I = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = k \frac{q_1 q_2}{r}$	$J$ = current density
$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$	$t$ = length
$\Delta V = - \int \vec{E} \cdot d\vec{r}$	$n$ = number of charge carriers per unit volume
$E_x = \frac{dV}{dx}$	$P$ = power
$\Delta U_e = q \Delta V$	$q$ = charge
$C = \frac{Q}{\Delta V}$	$Q$ = charge
$C = \frac{n\epsilon_0 A}{d}$	$r$ = radius, distance, or position
$U_C = \frac{1}{2} Q \Delta V$	$R$ = resistance
$\kappa = \frac{\epsilon}{\epsilon_0}$	$t$ = time
$I = \frac{dq}{dt}$	$U$ = potential energy
$I = nqv_d A$	$v$ = velocity or speed
$I = \int \vec{J} \cdot d\vec{A}$	$\mathcal{E}$ = electric potential or volume
$\vec{J} = nq\vec{v}_d$	$x$ = position
$\vec{E} = \rho \vec{J}$	$\rho$ = resistivity or charge density
$R = \frac{\rho l}{A}$	$\epsilon_0$ = permittivity
$I = \frac{\Delta V}{R}$	$\kappa'$ = dielectric constant
$P = I \Delta V$	$\Phi$ = flux
	$\omega_{oc} = \frac{1}{\sqrt{LC}}$
	$A$ = area
	$B$ = magnetic field
	$C$ = capacitance
	$E$ = electric field
	$F$ = force
	$I$ = current
	$\ell$ = length
	$L$ = inductance
	$n$ = number of loops of wire per unit length
	$N$ = number of loops
	$q$ = charge
	$r$ = radius, distance, or position
	$R$ = resistance
	$t$ = time
	$U$ = potential energy
	$v$ = velocity or speed
	$\mathcal{E}$ = emf
	$\mu$ = permeability
	$\tau$ = time constant
	$\Phi$ = flux
	$\omega$ = angular frequency

# Deriving an Expression for Resistance

When we consider the ratio of Electric Field ( $E$ ) to Charge Density ( $J$ ), we call this resistivity ( $\rho$ ).

Solved for the electric field:

$$\vec{E} = \rho \vec{J}$$

ELECTRICITY AND MAGNETISM			
$ F_e  = \frac{1}{4\pi\epsilon_0} \frac{ q_1 q_2 }{r^2} = k \frac{ q_1 q_2 }{r^2}$	$A = \text{area}$	$R_{\text{tot}} = \sum_i R_i$	$A = \text{area}$
	$C = \text{capacitance}$	$\frac{1}{R_{\text{tot}}} = \sum_i \frac{1}{R_i}$	$B = \text{magnetic field}$
	$d = \text{distance}$	$\frac{1}{C_{\text{tot}}} = \sum_i \frac{1}{C_i}$	$C = \text{capacitance}$
$E = \frac{\vec{F}_e}{q}$	$E = \text{electric field}$	$C_{\text{tot}} = \sum_i C_i$	$E = \text{electric field}$
$E = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$	$F = \text{force}$	$I = R_n C_m$	$F = \text{force}$
$\Phi_E = \int \vec{E} \cdot d\vec{A}$	$J = \text{current}$	$\int \vec{B} \cdot d\vec{A} = 0$	$I = \text{current}$
$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$	$J = \text{current density}$	$\vec{P}_s = q(\vec{r} \times \vec{B})$	$L = \text{length}$
$Q_{\text{out}} = \int \rho(r) dV$	$n = \text{number of charge carriers per unit volume}$	$d\vec{B} = \frac{\mu_0}{4\pi} I \left( \vec{d} \times \hat{r} \right)$	$L = \text{inductance}$
$U_E = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = k \frac{q_1 q_2}{r}$	$r = \text{radius, distance, or position}$	$\vec{P}_s = \int I (\vec{d} \times \vec{B})$	$n = \text{number of loops of wire per unit length}$
$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$	$R = \text{resistance}$	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{ext}}$	$N = \text{number of loops}$
$\Delta V = - \int \vec{E} \cdot d\vec{r}$	$t = \text{time}$	$\Phi = \text{flux}$	$q = \text{charge}$
$E_x = -\frac{dV}{dx}$	$U = \text{potential energy}$	$\Phi = \vec{B} \cdot \vec{A} = \int \vec{B} \cdot d\vec{A}$	$r = \text{radius, distance, or position}$
$\Delta U_E = q \Delta V$	$v = \text{velocity or speed}$	$E = \frac{1}{2} \int \vec{B} \cdot d\vec{A} = -\frac{d\Phi_B}{dt}$	$R = \text{resistance}$
$C = \frac{Q}{\Delta V}$	$V = \text{electric potential or volume}$	$[E]_{\text{tot}} = N \left[ \frac{d\Phi_B}{dt} \right]$	$t = \text{time}$
$C = \frac{\kappa \epsilon_0 A}{d}$	$x = \text{position}$	$L = \frac{\mu_0 N^2 A}{t}$	$U = \text{potential energy}$
$U_C = \frac{1}{2} Q \Delta V$	$\rho = \text{resistivity or charge density}$	$U_L = \frac{1}{2} L I^2$	$v = \text{velocity or speed}$
$\kappa = \frac{\epsilon}{\epsilon_0}$	$\epsilon = \text{permittivity}$	$\mathcal{E} = -L \frac{dI}{dt}$	$\mathcal{E} = \text{emf}$
$J = \frac{dq}{dt}$	$\kappa' = \text{dielectric constant}$	$\tau = \frac{L}{R_{\text{tot}}}$	$\mu = \text{permeability}$
$J = nqv_A A$	$\Phi = \text{flux}$	$\omega_{\text{oc}} = \frac{1}{\sqrt{LC}}$	$\tau = \text{time constant}$
$J = \int \vec{J} \cdot d\vec{A}$			$\Phi = \text{flux}$
$\vec{J} = nqv \hat{v}$			$\omega = \text{angular frequency}$
$E = \rho J$			
$R = \frac{V}{I}$			
$I = \frac{\Delta V}{R}$			
$P = I \Delta V$			

# Deriving an Expression for Resistance

Relevant equations:

$$E = \frac{\Delta V}{l} \quad J = \frac{I}{A}$$

$$R = \frac{\Delta V}{I}, \text{ so } I = \frac{\Delta V}{R}$$

$$\vec{E} = \rho \vec{J}$$

$$\frac{\Delta V}{l} = \rho J$$

$$\frac{\Delta V}{l} = \rho \frac{I}{A}$$

$$\frac{\Delta V}{l} = \rho \frac{V}{RA}$$

$$R = \frac{\rho l}{A}$$

ELECTRICITY AND MAGNETISM			
$ F_i  = \frac{1}{4\pi\epsilon_0} \frac{ q_i q_j }{r^2} = k \frac{ q_i q_j }{r^2}$	$\mathcal{A}$ = area	$R_{\text{tot}}$ = $\sum_i R_i$	$A$ = area
$E = \frac{\vec{F}_i}{q}$	$C$ = capacitance	$\frac{1}{R_{\text{tot}}} = \sum_i \frac{1}{R_i}$	$B$ = magnetic field
$E = \frac{1}{4\pi\epsilon_0} \int \frac{dq_j}{r^2} \hat{r}$	$d$ = distance	$\frac{1}{C_{\text{tot}}} = \sum_i \frac{1}{C_i}$	$C$ = capacitance
$\Phi_E = \int \vec{E} \cdot d\vec{A}$	$I$ = current	$C_{\text{tot}} = \sum_i C_i$	$E$ = electric field
$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$	$J$ = current density	$\tau = R_n C_n$	$F$ = force
$Q_{\text{out}} = \int \rho(r) dV$	$t$ = length	$\oint \vec{B} \cdot d\vec{A} = 0$	$I$ = current
$U_E = \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{r} \approx k \frac{q_i q_j}{r}$	$n$ = number of charge carriers per unit volume	$\vec{P}_s = q(\vec{v} \times \vec{B})$	$L$ = inductance
$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$	$P$ = power	$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^2}$	$n$ = number of loops of wire per unit length
$\Delta V = - \int \vec{E} \cdot d\vec{l}$	$q$ = charge	$\vec{F}_s = \int I(d\vec{l} \times \vec{B})$	$N$ = number of loops
$E_s = - \frac{dV}{dx}$	$Q$ = charge	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{ext}}$	$q$ = charge
$\Delta U_E = q \Delta V$	$r$ = radius, distance, or position	$R_{\text{ext}} = \mu_0 I$	$r$ = radius, distance, or position
$C = \frac{Q}{\Delta V}$	$t$ = time	$\oint \vec{B} \cdot d\vec{l} = \mu_0 J_{\text{ext}}$	$R$ = resistance
$C = \frac{\kappa\epsilon_0 A}{d}$	$U$ = potential energy	$\Phi_B = \vec{B} \cdot \vec{A} = \int \vec{B} \cdot d\vec{A}$	$t$ = time
$U_C = \frac{1}{2} Q \Delta V$	$v$ = velocity or speed	$E = \frac{1}{2} \int \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt}$	$U$ = potential energy
$K = \frac{\epsilon}{\epsilon_0}$	$V$ = electric potential or volume	$ E_{\text{ext}}  = N \left  \frac{d\Phi_B}{dt} \right $	$v$ = velocity or speed
$J = \frac{dq}{dt}$	$x$ = position	$L = \frac{\mu_0 N^2 A}{t}$	$\mathcal{E}$ = emf
$J = nqv_s A$	$\rho$ = resistivity or charge density	$U_L = \frac{1}{2} L J^2$	$\mu$ = permeability
$J = \int \vec{J} \cdot d\vec{A}$	$\epsilon$ = permittivity	$E = -L \frac{dI}{dt}$	$t$ = time constant
$\vec{J} = nq\vec{v}_s$	$\kappa$ = dielectric constant	$\tau = \frac{L}{R_{\text{ext}}}$	$\Phi$ = flux
$E = \rho J$	$\Phi$ = flux	$\omega_{\text{oc}} = \frac{1}{\sqrt{LC}}$	$\nu$ = angular frequency
$R = \frac{\rho l}{A}$	$R = \frac{\rho l}{A}$		
$I = \frac{\Delta V}{R}$			
$P = I \Delta V$			

A metal wire has a resistance  $R$  when it is at a temperature  $T$ . The wire is melted and all of the metal is used to reform it into a new wire 4 times as long.

What is the resistance of the new wire at temperature  $T$ ?

- A  $2R$
- B  $4R$
- C  $8R$
- D  $16R$

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$$R' = \frac{\rho l}{A}$$

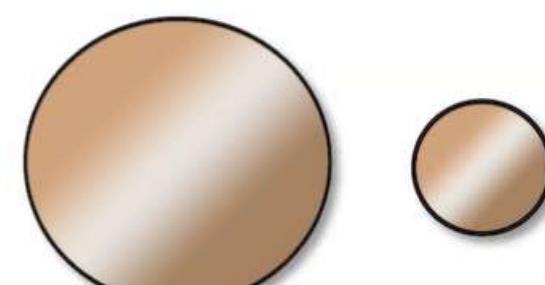
$$R' = \frac{\rho l}{A}^{16}$$
$$R' = 16R$$

$$R = \frac{\rho l}{A}$$

$$\begin{aligned}l &= l \\l' &= 4l\end{aligned}$$

$$A = A$$

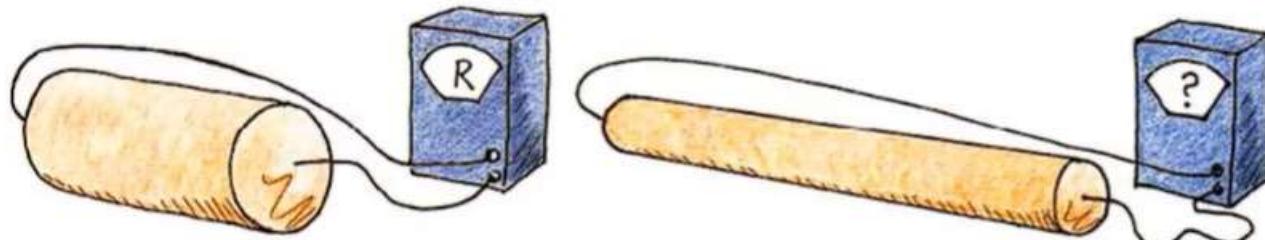
$$A' = \frac{A}{4}$$



## Warm Up



Roll a piece of modeling clay into a cylinder and use an ohmmeter to measure its resistance.



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Now roll it out until it's twice as long and measure the resistance again. Compared with the initial resistance, the new resistance is:

Defend your answer!

- A unchanged
- B twice as much
- C four times as much
- D eight times as much
- E actually less

## Warm Up



c

four times as much

Resistance is greater due to the smaller cross section of the clay, and also greater due to its greater length. Its cross section is half (increasing resistance by 2), and length doubled (increasing resistance by another 2). So resistance is four times as much.

$$R = \rho \frac{L}{A}, \text{ so } \rho \frac{2L}{A/2} = 4R$$

## Takeaways

- The electric field is the potential difference divided by the length of the conductor
- Current density is current per unit area
- Resistance is voltage over current
  - This can be rearranged to  $I = \Delta V/R$
- Resistivity ( $\rho$ ) is a characteristic of a material
  - The inverse of resistivity is conductivity
- The resistivity of a material stays the same until a chemical change has taken place.

$$E = \frac{\Delta V}{r}$$

$$J = \frac{\Delta I}{A}$$

$$R = \frac{\Delta V}{I}$$

$$E = \rho J$$

$$R = \frac{\rho L}{A}$$

### 11.3.3

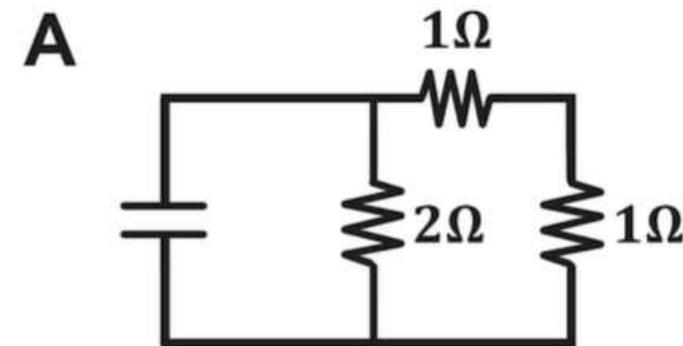
Resistance, its role in a circuit, and the calculation of total equivalent resistance for parallel and series circuits

# Warm Up

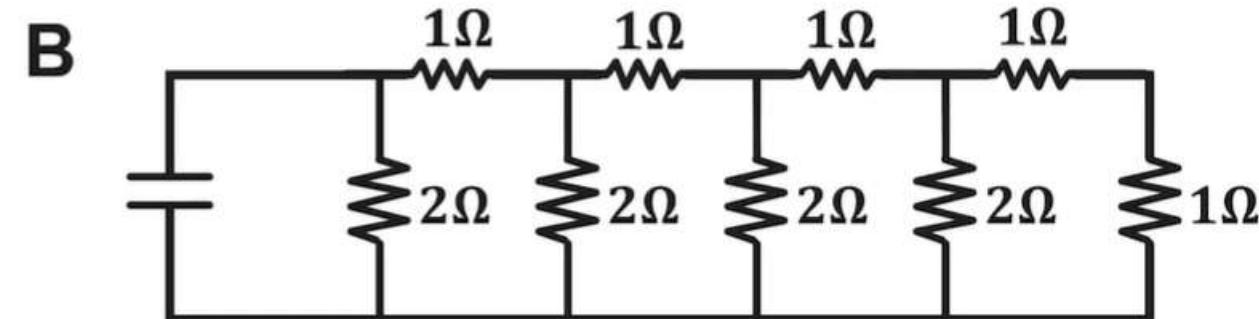


Which of these circuits draws the most current?

A Circuit A



B Circuit B



C Both the same.

# Vocabulary



- **Resistance** – the limiting of charges moving through a circuit. Symbol  $R$ , unit  $\Omega$  (ohm, the Greek letter capital Omega)
- **Resistivity** – property of material that allows for the prediction of resistance based on the cross-sectional area and length of the conductor. Symbol  $\rho$  (the Greek letter lowercase rho), unit  $\Omega / m$
- **Series Circuit** – Identifiable by charges have only one direction to choose
- **Parallel Circuit** – Identifiable by the presence of a junction, or intersection
- **Equivalent Resistance** – The total resistance for a section of a circuit

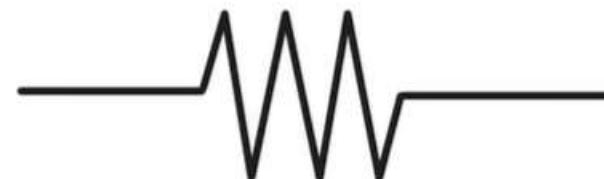
# The Resistor



In the laboratory environment, you may encounter a resistor that looks like this:



However, as part of a schematic, we commonly represent resistors using the American National Standards Institute (ANSI) symbol:



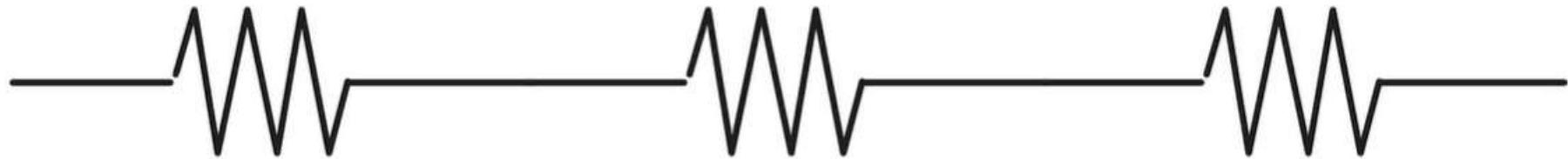
Or a lightbulb, or a variable resistor



## Series Circuits



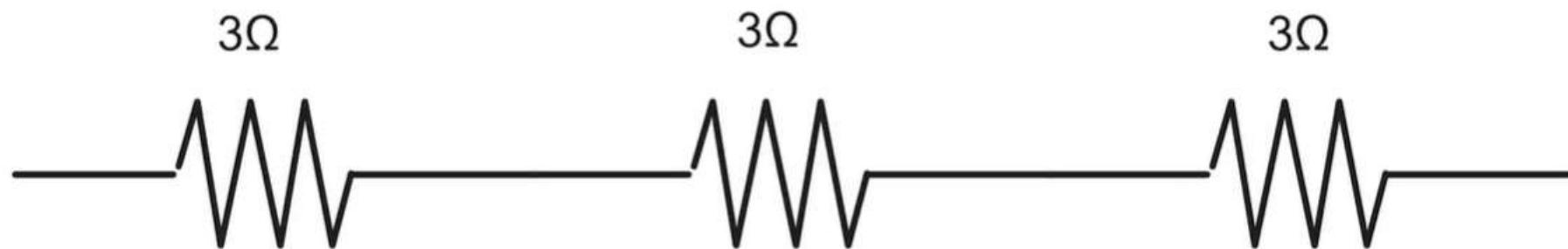
If the charges flowing through the circuit only have one path to move, we call this a **series circuit**.



# Series Circuits



When we have values for these, the equivalent resistance becomes...



# Equivalent Resistance in Series

For resistors in **series**, we use:

$$R_{eq,s} = \sum_i R_i$$

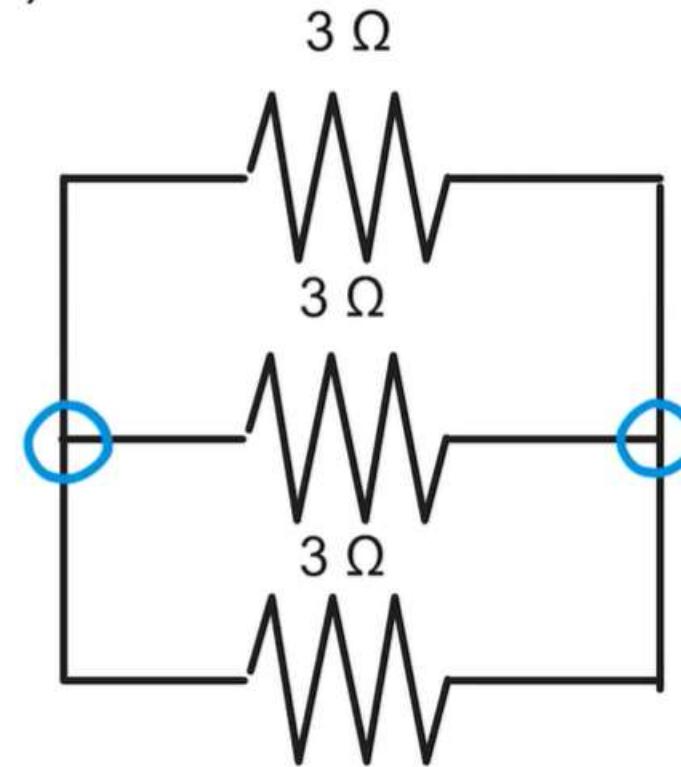
$$R_{eq} = R_1 + R_2 + R_3 \dots$$

ELECTRICITY AND MAGNETISM		
$ F_s  = \frac{1}{4\pi\epsilon_0} \frac{ q_1 q_2 }{r^2} = k \frac{ q_1 q_2 }{r^2}$	$A = \text{area}$	$R_{eq,s} = \sum R_i$
$E = \frac{\vec{F}_s}{q}$	$C = \text{capacitance}$	$\frac{1}{R_{eq,s}} = \sum \frac{1}{R_i}$
$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$	$d = \text{distance}$	$\frac{1}{C_{eq,s}} = \sum \frac{1}{C_i}$
$\Phi_B = \int \vec{E} \cdot d\vec{l}$	$E = \text{electric field}$	$C_{eq,s} = \sum C_i$
$\oint \vec{E} \cdot d\vec{l} = \frac{q_{ext}}{\epsilon_0}$	$F = \text{force}$	$\tau = R_s C_{eq}$
$Q_{ext} = \int \rho(r) dV$	$I = \text{current}$	$\oint \vec{B} \cdot d\vec{l} = 0$
$U_s = \frac{1}{4\pi\epsilon_0} \frac{ q_1 q_2 }{r} = k \frac{ q_1 q_2 }{r}$	$J = \text{current density}$	$\vec{F}_s = q(\vec{v} \times \vec{B})$
$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$	$\ell = \text{length}$	$R = \text{resistance}$
$\Delta V = - \oint \vec{E} \cdot d\vec{r}$	$n = \text{number of charge carriers per unit volume}$	$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \hat{r})}{r^2}$
$E_s = \frac{dV}{dx}$	$P = \text{power}$	$U = \text{potential energy}$
$\Delta U_e = q \Delta V$	$q = \text{charge}$	$\vec{F}_s = \int I(d\vec{l} \times \vec{B})$
$C = \frac{Q}{\Delta V}$	$Q = \text{charge}$	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{ext}$
$C = \frac{\kappa \epsilon_0 A}{d}$	$r = \text{radius, distance, or position}$	$\tau = \mu_0 N L$
$U_C = \frac{1}{2} Q \Delta V$	$R = \text{resistance}$	$\Phi = \text{flux}$
$\kappa = \frac{r}{\epsilon_0}$	$t = \text{time}$	$\omega = \text{angular frequency}$
$J = \frac{dq}{dt}$	$U = \text{potential energy}$	$\vec{F}_s = \vec{B} \cdot \vec{A} = \int \vec{B} \cdot d\vec{A}$
$I = nqv_s A$	$v = \text{velocity or speed}$	$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt}$
$J = \int \vec{J} \cdot d\vec{l}$	$V = \text{electric potential or volume}$	$ E_{ext}  = N \left  \frac{d\Phi_B}{dt} \right $
$\vec{J} = nqv_s \vec{v}_s$	$x = \text{position}$	$L = \frac{\mu_0 N^2 A}{t}$
$\vec{E} = \rho \vec{J}$	$\rho = \text{resistivity or charge density}$	$U_L = \frac{1}{2} L I^2$
$R = \frac{\rho \ell}{A}$	$\epsilon = \text{permittivity}$	$\mathcal{E} = -L \frac{dI}{dt}$
$I = \frac{\Delta V}{R}$	$\kappa' = \text{dielectric constant}$	$\tau = \frac{L}{R_{eq}}$
$P = I \Delta V$	$\Phi = \text{flux}$	$\omega_{oc} = \frac{1}{\sqrt{LC}}$

# Parallel Circuits



If there is a “junction”, or intersection, we call this being in **parallel**.



# Equivalent Resistance in Series

For resistors in **parallel**, we use:

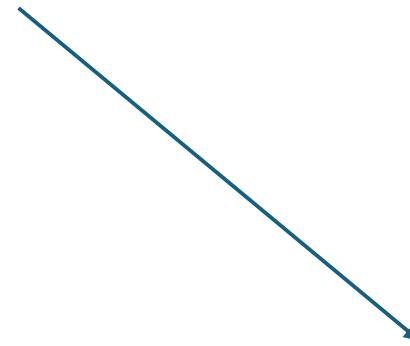
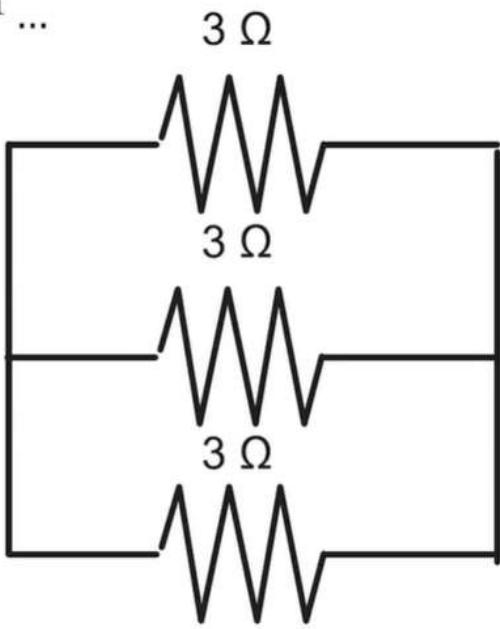
$$\frac{1}{R_{eq,p}} = \sum_i \frac{1}{R_i}$$

$$R_{eq}^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1} \dots$$

ELECTRICITY AND MAGNETISM		
$ F_e  = \frac{1}{4\pi\varepsilon_0} \frac{ q_1 q_2 }{r^2} = k \frac{ q_1 q_2 }{r^2}$	$A = \text{area}$ $C = \text{capacitance}$ $d = \text{distance}$ $E = \text{electric field}$ $F = \text{force}$ $I = \text{current}$ $J = \text{current density}$ $l = \text{length}$ $n = \text{number of charge carriers per unit volume}$ $P = \text{power}$ $q = \text{charge}$ $Q = \text{charge}$ $r = \text{radius, distance, or position}$ $R = \text{resistance}$ $t = \text{time}$ $\Delta V = -\int E \cdot d\ell$ $E_s = \frac{dV}{dx}$ $\Delta U_e = q\Delta V$ $C = \frac{Q}{\Delta V}$ $C = \frac{\kappa\varepsilon_0 A}{d}$ $U_C = \frac{1}{2} Q \Delta V$ $\kappa = \frac{\epsilon}{\epsilon_0}$ $I = \frac{dq}{dt}$ $I = nqv_d A$ $I = \int J \cdot d\ell$ $\vec{J} = nq\vec{v}_d$ $\vec{E} = \rho \vec{J}$ $R = \frac{\rho l}{A}$ $I = \frac{\Delta V}{R}$ $P = I \Delta V$	$R_{eq,s} = \sum_i R_i$ $\frac{1}{R_{eq,s}} = \sum_i \frac{1}{R_i}$ $C_{n,e} = \sum_i C_i$ $\tau = R_n C_{eq}$ $\oint \vec{B} \cdot d\vec{\ell} = 0$ $\vec{P}_s = q(\vec{v} \times \vec{B})$ $d\vec{B} = \frac{\mu_0}{4\pi} \frac{l(d\vec{\ell} \times \vec{r})}{r^3}$ $\vec{P}_s = \int l(d\vec{\ell} \times \vec{B})$ $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{ext}$ $B_{ext} = \mu_0 I$ $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \mathcal{E}_0 \frac{d\Phi_B}{dt}$ $\Phi_B = \vec{B} \cdot \vec{\lambda} = \int \vec{B} \cdot d\vec{\ell}$ $\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$ $\mathcal{E}_{emf} = N \left  \frac{d\Phi_B}{dt} \right $ $L = \frac{\mu_0 N^2 A}{l}$ $U_L = \frac{1}{2} L I^2$ $\mathcal{E} = -L \frac{dI}{dt}$ $\tau = \frac{L}{R_{eq}}$ $\omega_{oc} = \frac{1}{\sqrt{LC}}$
		$A = \text{area}$ $B = \text{magnetic field}$ $C = \text{capacitance}$ $E = \text{electric field}$ $F = \text{force}$ $I = \text{current}$ $l = \text{length}$ $L = \text{inductance}$ $n = \text{number of loops of wire per unit length}$ $N = \text{number of loops}$ $q = \text{charge}$ $r = \text{radius, distance, or position}$ $R = \text{resistance}$ $t = \text{time}$ $U = \text{potential energy}$ $v = \text{velocity or speed}$ $V = \text{emf}$ $\mu = \text{permeability}$ $\tau = \text{time constant}$ $\Phi = \text{flux}$ $\omega = \text{angular frequency}$

# Parallel Circuits

$$R_{eq}^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1} \dots$$



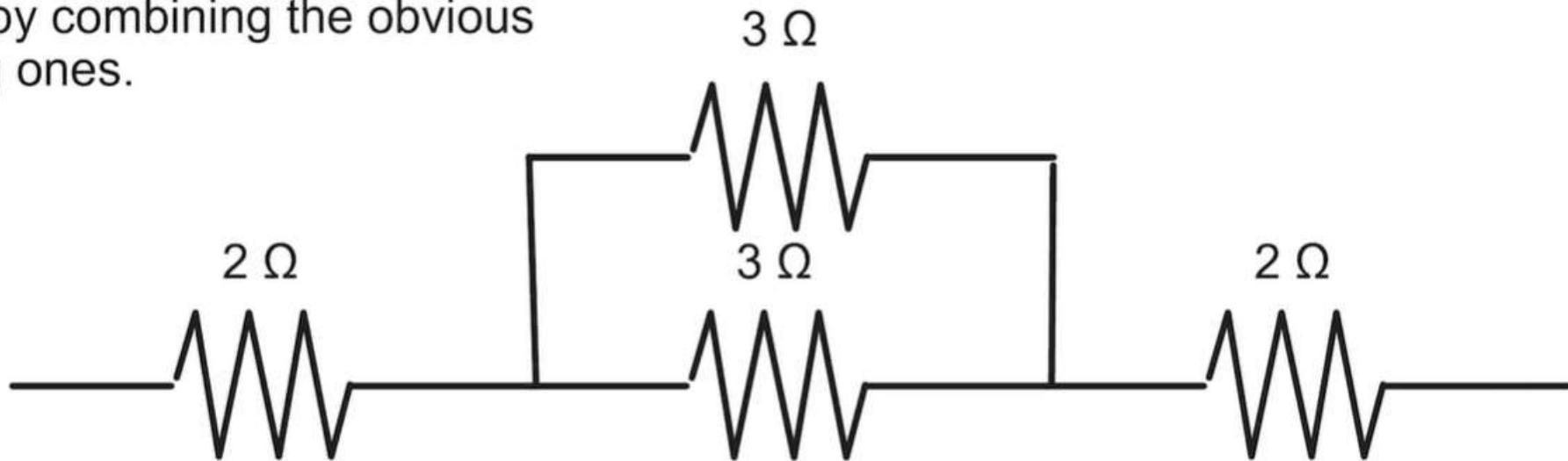
Parallel Circuits



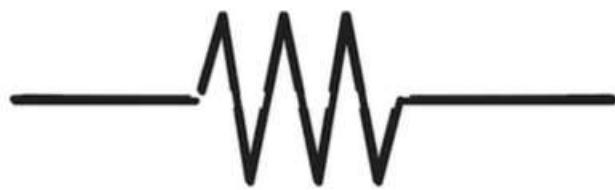
# Combination Circuits



Begin by combining the obvious looking ones.

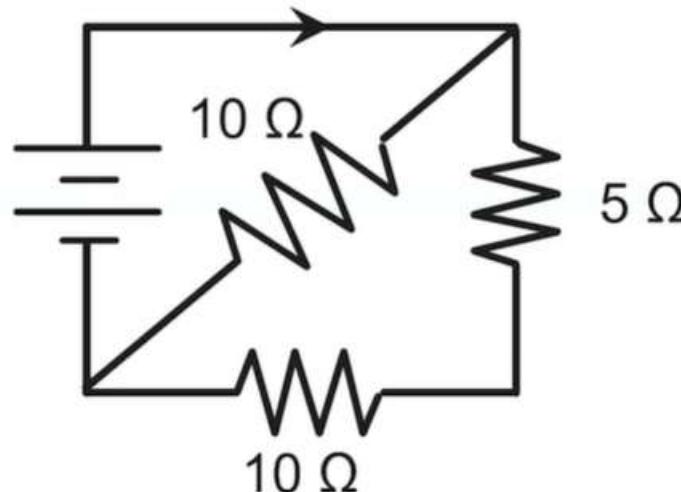


$5.5\Omega$



**What is the total resistance for the circuit shown below?**

- A 1  $\Omega$
- B 2  $\Omega$
- C 6  $\Omega$
- D 10  $\Omega$
- E 30  $\Omega$



**What is the total resistance for the circuit shown below?**

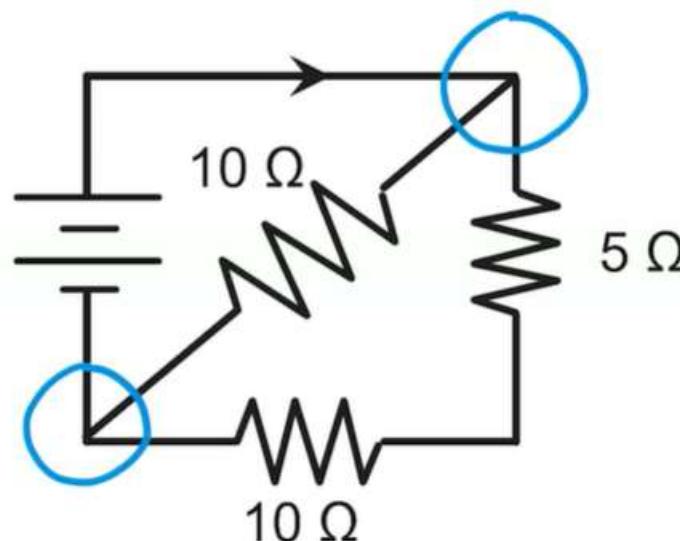
A 1 Ω

B 2 Ω

C 6 Ω

D 10 Ω

E 30 Ω

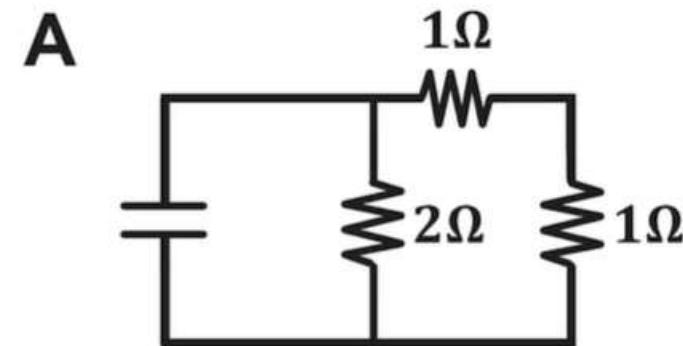


# Warm Up

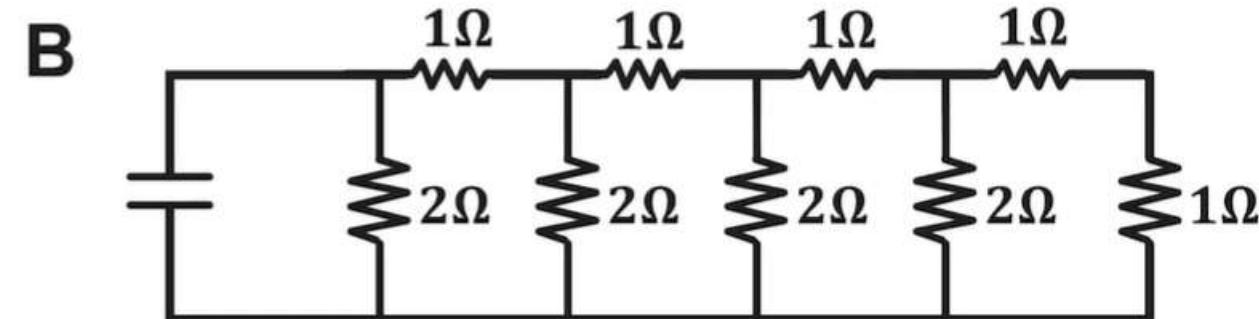


Which of these circuits draws the most current?

A Circuit A



B Circuit B



C Both the same.

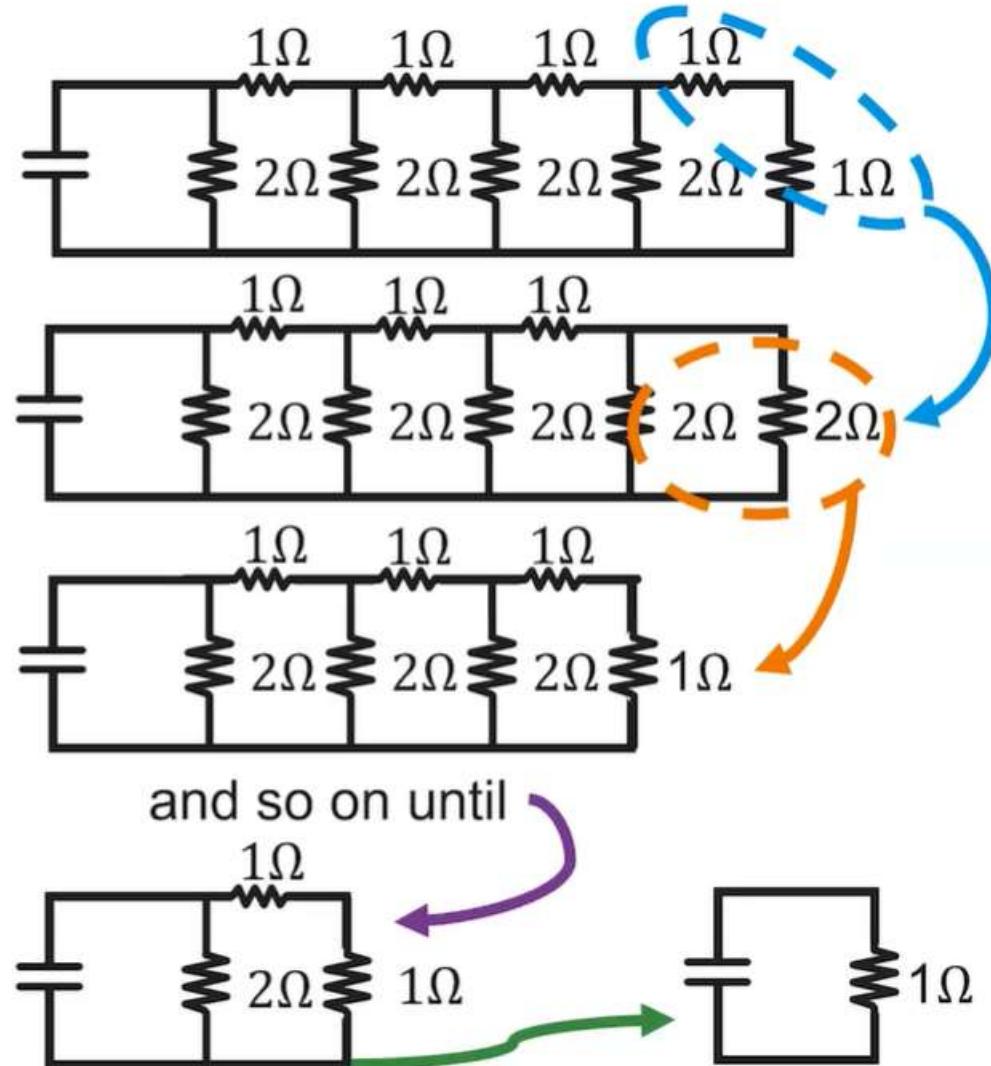
## Warm Up

C

Both the same.

This is one of those "tricky" circuits wherein the equivalent resistance for both is the same.

In fact, if you continue the sequence of a pair of 1- $\Omega$  pair resistors in series connected in parallel to a 2- $\Omega$  resistor at the right end of the circuit, the equivalent resistance will still be 1 $\Omega$ .



## Takeaways



- Combining Resistors in series, means adding all resistors in series.
  - Think of the crowded hallway the electrons must get through.
- Combining resistors in parallel means add the inverse of all the resistors, then inverse that answer.

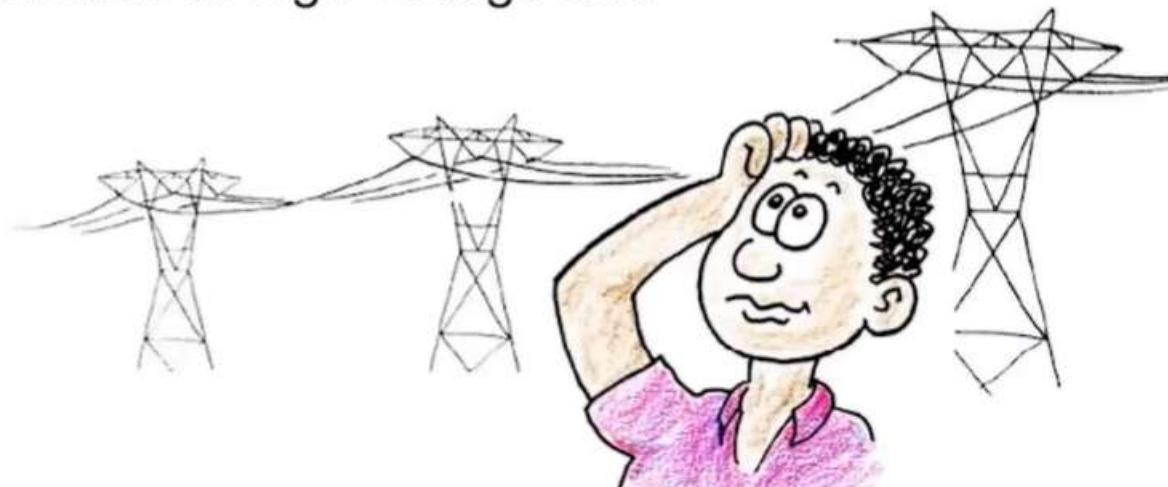
### 11.3.4

Ohm's Law and its Limitations

## Warm Up



- According to Ohm's law, high voltage produces high current.
- Does Ohm's law apply to power lines?
- If so, then how can power be transmitted at high voltage and low current in power lines?



## Vocabulary



- **Ohm's Law** – a simplistic model of electric currents in circuits that states the current flowing through a conductor is directly proportional to the voltage applied across it, and inversely proportional to its resistance.
- **Conventional Current** – the historical, or traditional, explanation of charges moving through a circuit from higher potential to a lower potential
- **Electron Flow** – the actual way charges behave – protons attract electrons

# Ohm's Law

In 1827, the German physicist **Georg Simon Ohm** made a breakthrough while studying the relationship between voltage, current, and resistance.

He presented a simplistic model of electric currents in circuits that states the current flowing through a conductor is directly proportional to the voltage applied across it, and inversely proportional to its resistance.

MECHANICS		ELECTRICITY AND MAGNETISM	
$v_x = v_{x0} + a_x t$	$a$ = acceleration	$\vec{F}_E = \frac{1}{4\pi\varepsilon_0} \frac{ q_1 q_2 }{r^2}$	$A$ = area
$x = x_0 + v_{x0}t + \frac{1}{2} a_x t^2$	$E$ = energy	$\vec{B} = \frac{\vec{F}_E}{q}$	$B$ = magnetic field
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	$F$ = force	$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{r_0}$	$C$ = capacitance
$\ddot{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{\text{net}}}{m}$	$f$ = frequency	$E_A = -\frac{dV}{dr}$	$d$ = distance
$\vec{F} = \frac{d\vec{p}}{dt}$	$h$ = height	$\Delta V = -\int \vec{E} \cdot d\vec{r}$	$E$ = electric field
$J = \int \vec{F} dt = \Delta \vec{p}$	$I$ = rotational inertia	$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$	$\mathcal{E}$ = emf
$\vec{p} = m\vec{v}$	$J$ = impulse	$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$	$F$ = force
$ \vec{F}_f  \leq \mu  \vec{F}_N $	$K$ = kinetic energy	$\Delta V = \frac{Q}{C}$	$I$ = current
$\Delta E = W = \int \vec{F} \cdot d\vec{r}$	$k$ = spring constant	$C = \kappa \epsilon_0 A$	$J$ = current density
$K = \frac{1}{2}mv^2$	$\ell$ = length	$C_p = \sum_i C_i$	$L$ = inductance
$P = \frac{dE}{dt}$	$L$ = angular momentum	$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$	$\ell$ = length
$P = \vec{F} \cdot \vec{v}$	$m$ = mass	$\vec{F}_M = q\vec{v} \times \vec{B}$	$n$ = number of loops of wire per unit length
$\Delta U_g = mg\Delta h$	$P$ = power	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$	$N$ = number of charge carriers per unit volume
$a_c = \frac{v^2}{r} = \omega^2 r$	$\theta$ = angle	$U_C = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$	$P$ = power
$\vec{r} = \vec{r} \times \vec{F}$	$\tau$ = torque	$R = \frac{\rho l}{A}$	$Q$ = charge
$\dot{a} = \frac{\sum \vec{F}}{I} = \frac{\vec{r}_{\text{net}}}{I}$	$\omega$ = angular speed	$E = \rho j$	$q$ = point charge
$I = \int r^2 dm = \sum m r^2$	$\alpha$ = angular acceleration	$I = N e \nu_d A$	$R$ = resistance
$x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i}$	$\phi$ = phase angle	$I = \frac{\Delta V}{R}$	$r$ = radius or distance
$v = \omega r$	$\vec{F}_g = -k \Delta \vec{x}$	$R_s = \sum_i R_i$	$t$ = time
$\vec{L} = \vec{r} \times \vec{p} = I \vec{\omega}$	$U_g = \frac{1}{2} k (\Delta x)^2$	$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$	$U$ = potential or stored energy
$K = \frac{1}{2} I \omega^2$	$T_x = 2\pi \sqrt{\frac{m}{k}}$	$P = I \Delta V$	$V$ = electric potential
$\omega = \omega_0 + \alpha t$	$T_p = 2\pi \sqrt{\frac{l}{g}}$		$v$ = velocity or speed
$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$	$ F_G  = \frac{G m_1 m_2}{r^2}$		$\rho$ = resistivity
	$U_G = -\frac{G m_1 m_2}{r}$		$\Phi_B = \oint \vec{B} \cdot d\vec{l}$
			$\epsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
			$\mathcal{E} = -L \frac{dl}{dt}$
			$U_L = \frac{1}{2} L I^2$

# Limitations of Ohm's Law

Assumes Constant Resistance

## 1. Non-Ohmic Materials

->Diodes or  
Transistors

## 2. Temperature Dependency

Some Metals have +/- Temp Coefficient

## 3. AC Circuits and Frequency Dependency

Current and Voltage are Constantly Changing in AC

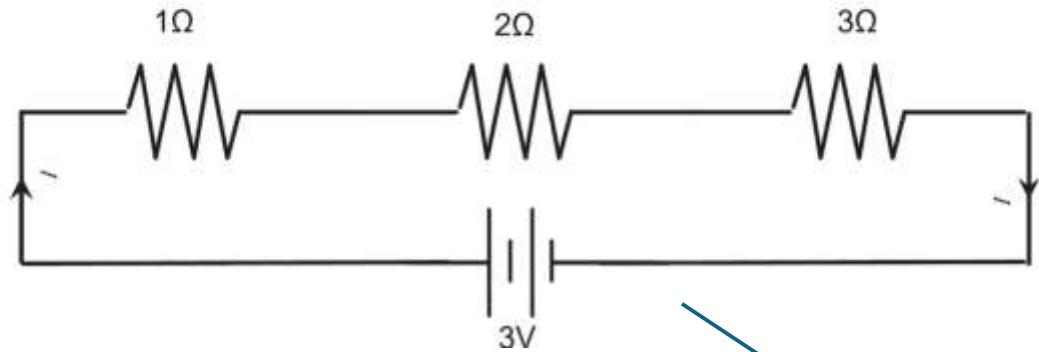
## 4. Very High Frequencies

Radio Frequency and Microwave Circuits

## V I R charts

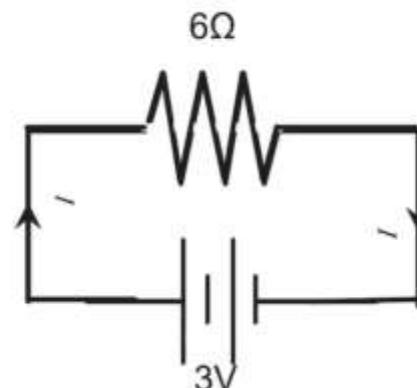


Suppose we were asked to find the current running through a  $2\Omega$  resistor, which is at a constant temperature.



## V I R charts

We would first need to combine the resistors. Redraw the circuit each time you combine a resistor.

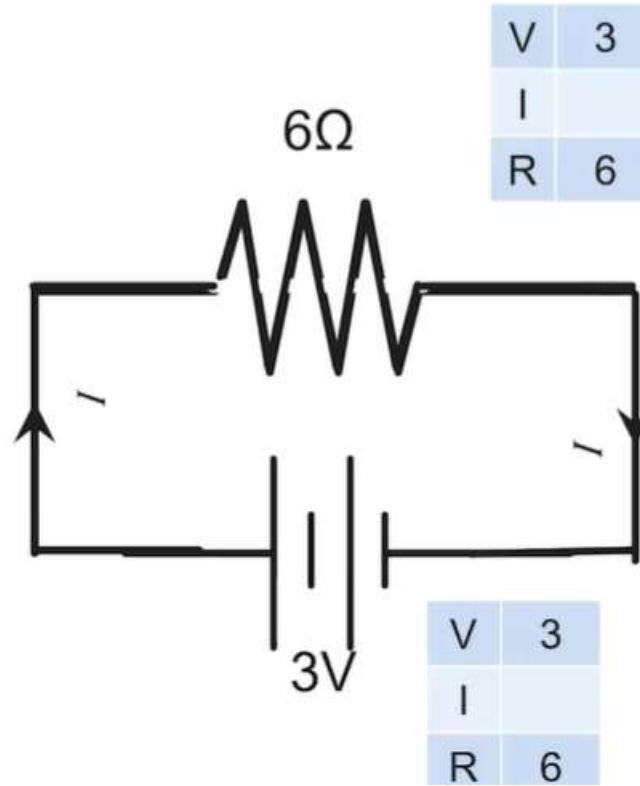




## V I R charts

Now we will use Ohm's Law, to fill in the chart:

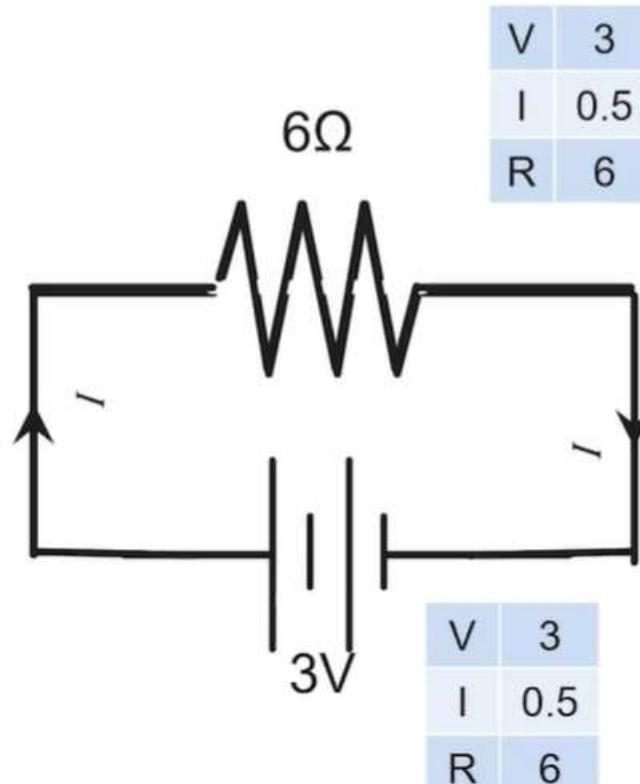
$$I = \frac{\Delta V}{R}$$



# V I R charts



Next, we will use 2 rules to fill in the charts that we will learn about more in-depth later. For now, we will simply say: when in series, current is the same and in parallel, voltage is the same.

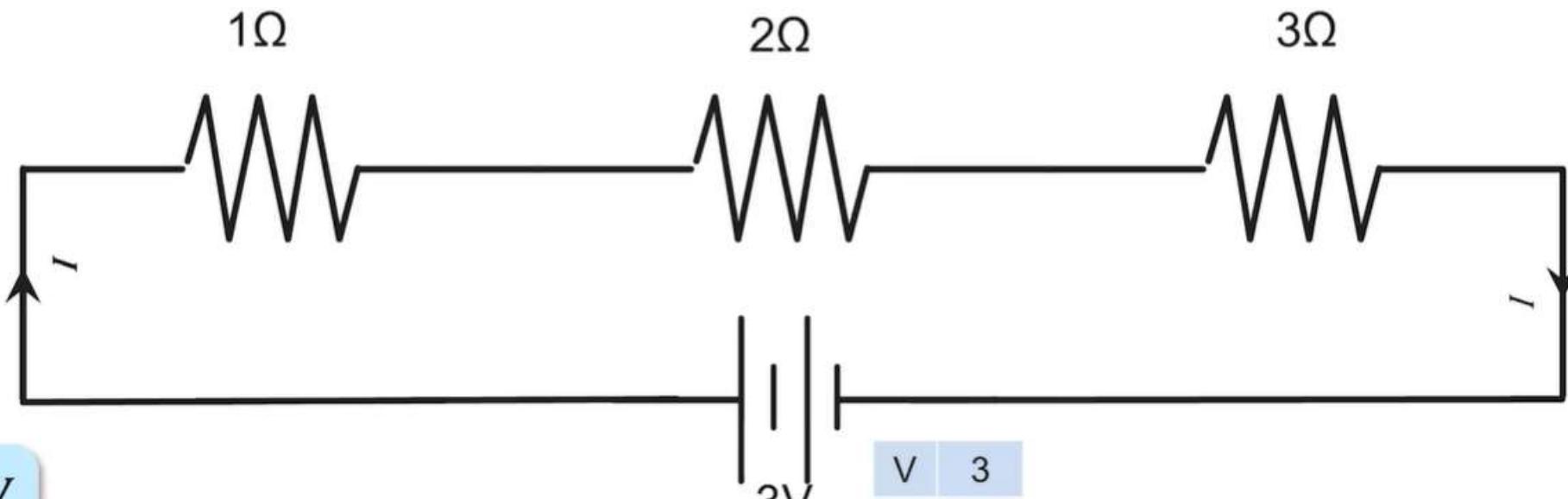


# VIR charts



Now we can “go backwards” with this saying to solve the circuit.

Because we already drew the circuit when we were combining resistors, we just need to fill in the VIR chart for each, using the phrase “in series, current is the same and in parallel, voltage is the same.”



$$I = \frac{\Delta V}{R}$$

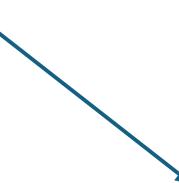
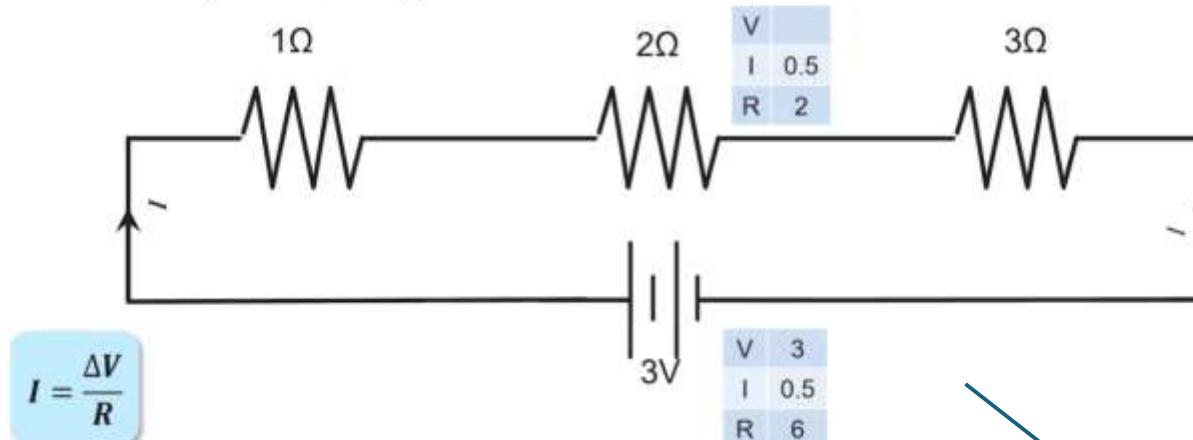
V	3
I	0.5
R	6

## VIR charts



Now we can “go backwards” with this saying to solve the circuit.

Because we already drew the circuit when we were combining resistors, we just need to fill in the VIR chart for each, using the phrase “in series, current is the same and in parallel, voltage is the same.”

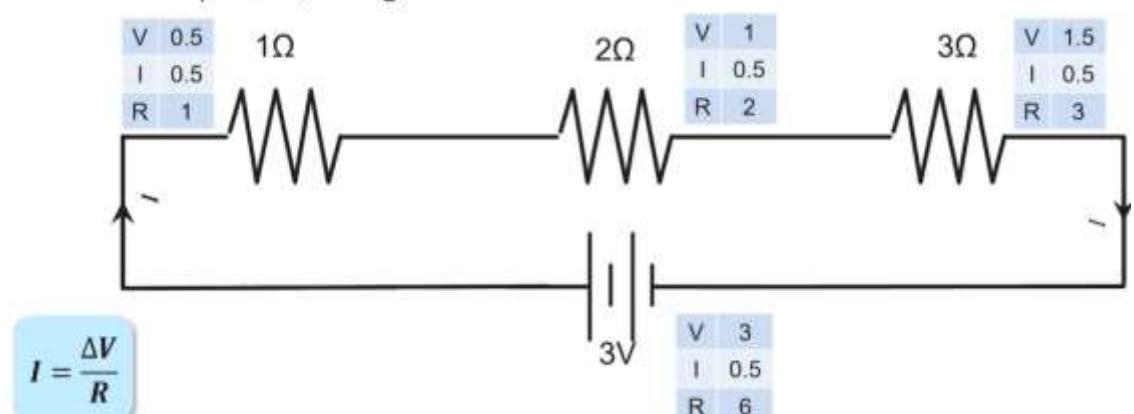


## VIR charts



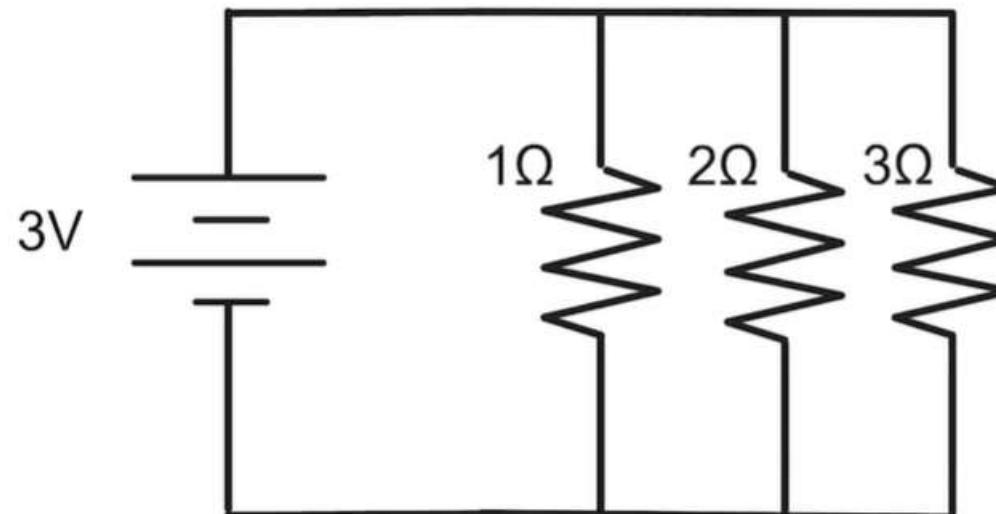
Now we can “go backwards” with this saying to solve the circuit.

Because we already drew the circuit when we were combining resistors, we just need to fill in the VIR chart for each, using the phrase “in series, current is the same and in parallel, voltage is the same.”



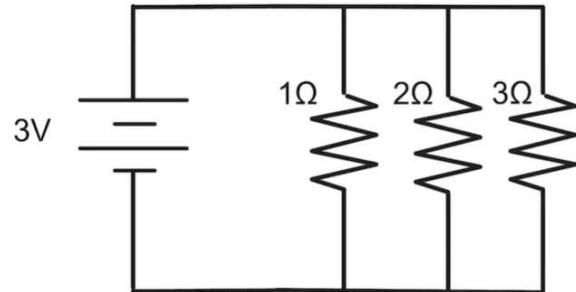
# Parallel Circuits

Our goal this time will be to solve for V, and I for each resistor.



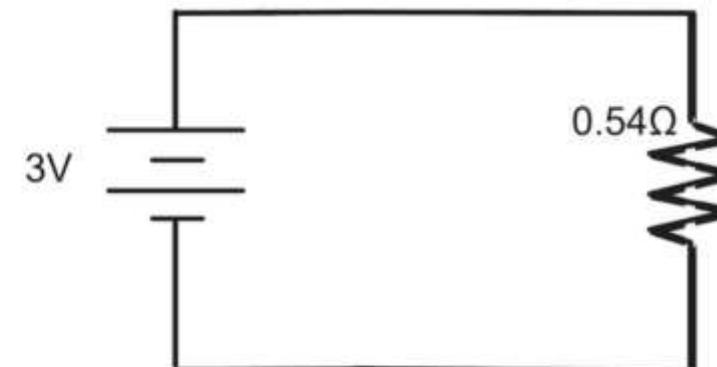
## Parallel Circuits

Our goal this time will be to solve for V, and I for each resistor.



## Parallel Circuits

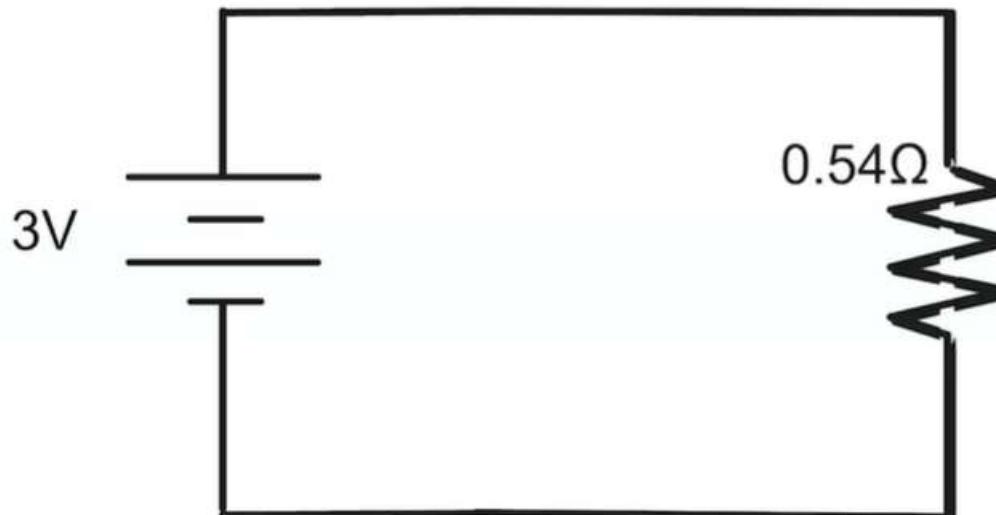
First, we will find the equivalent resistance by combining the resistors.  
Redraw the circuit after combining resistors.



## Parallel Circuits

First, we will find the equivalent resistance by combining the resistors.  
Redraw the circuit after combining resistors.

V	3
I	5.5
R	0.54

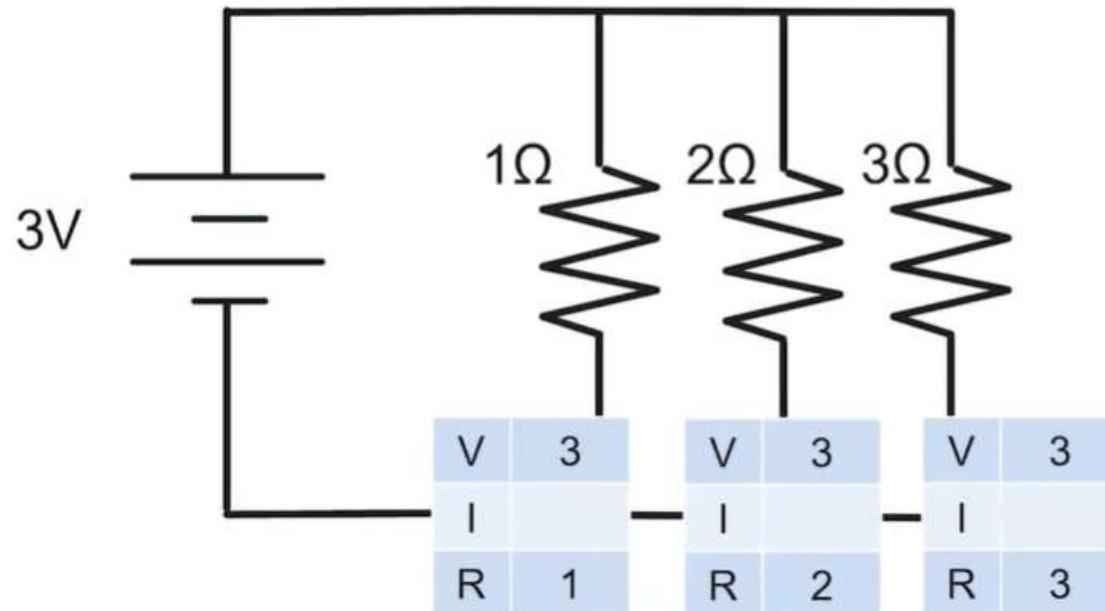


# Parallel Circuits



In series, current is the same.

In parallel, voltage is the same.



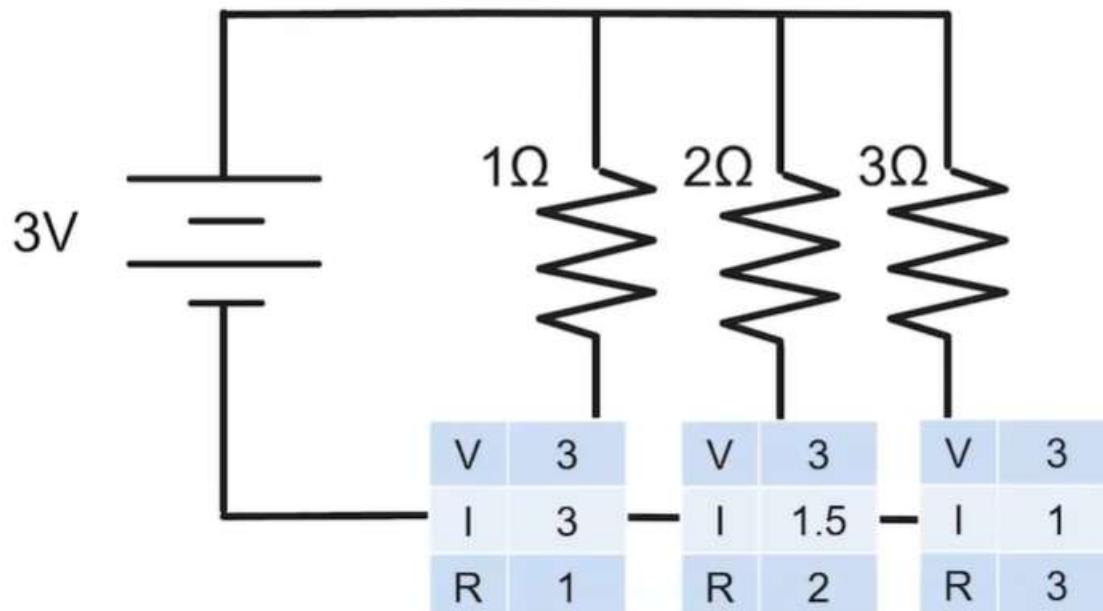
$$I = \frac{\Delta V}{R}$$

# Parallel Circuits

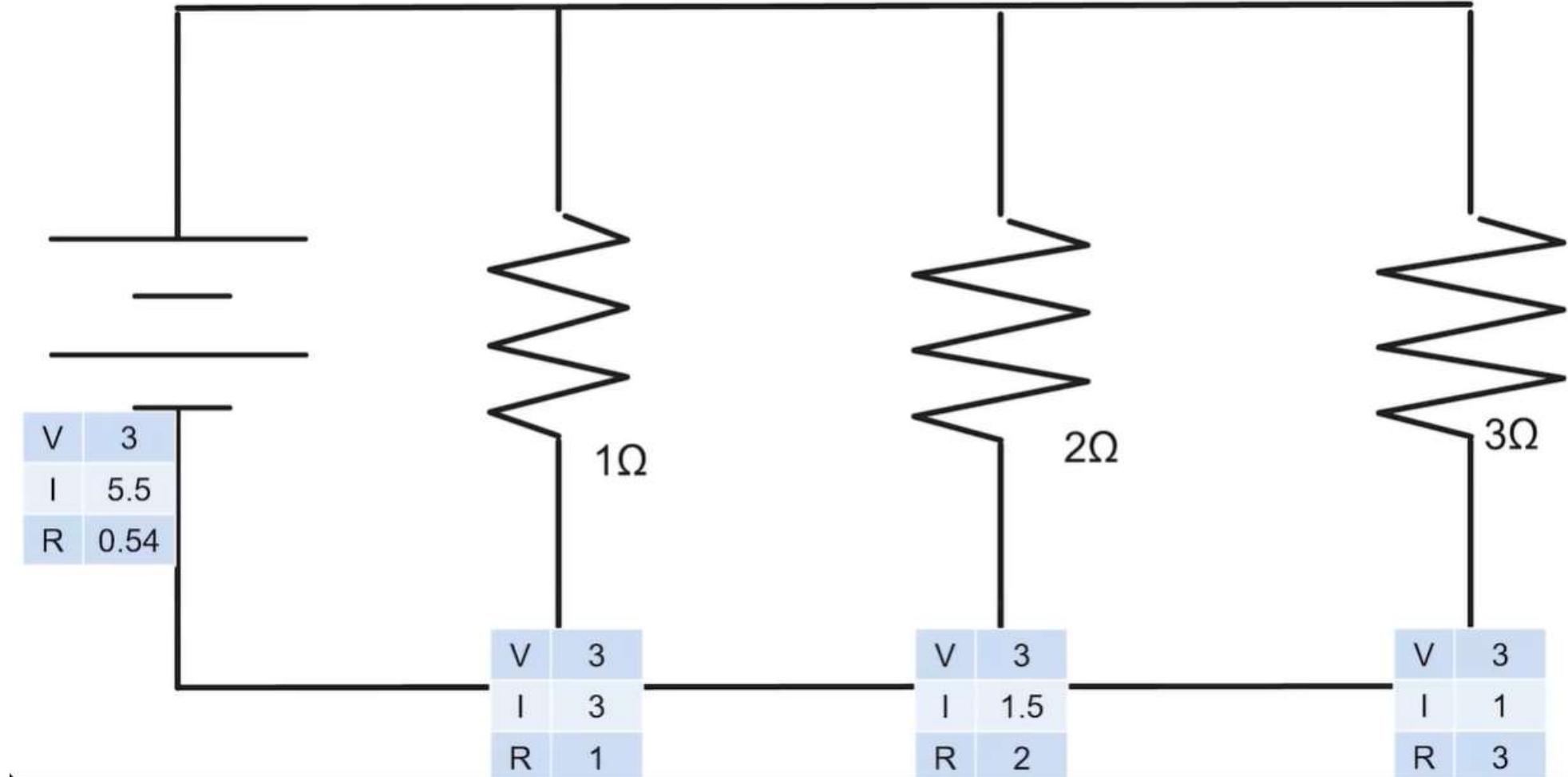


In series, current is the same.

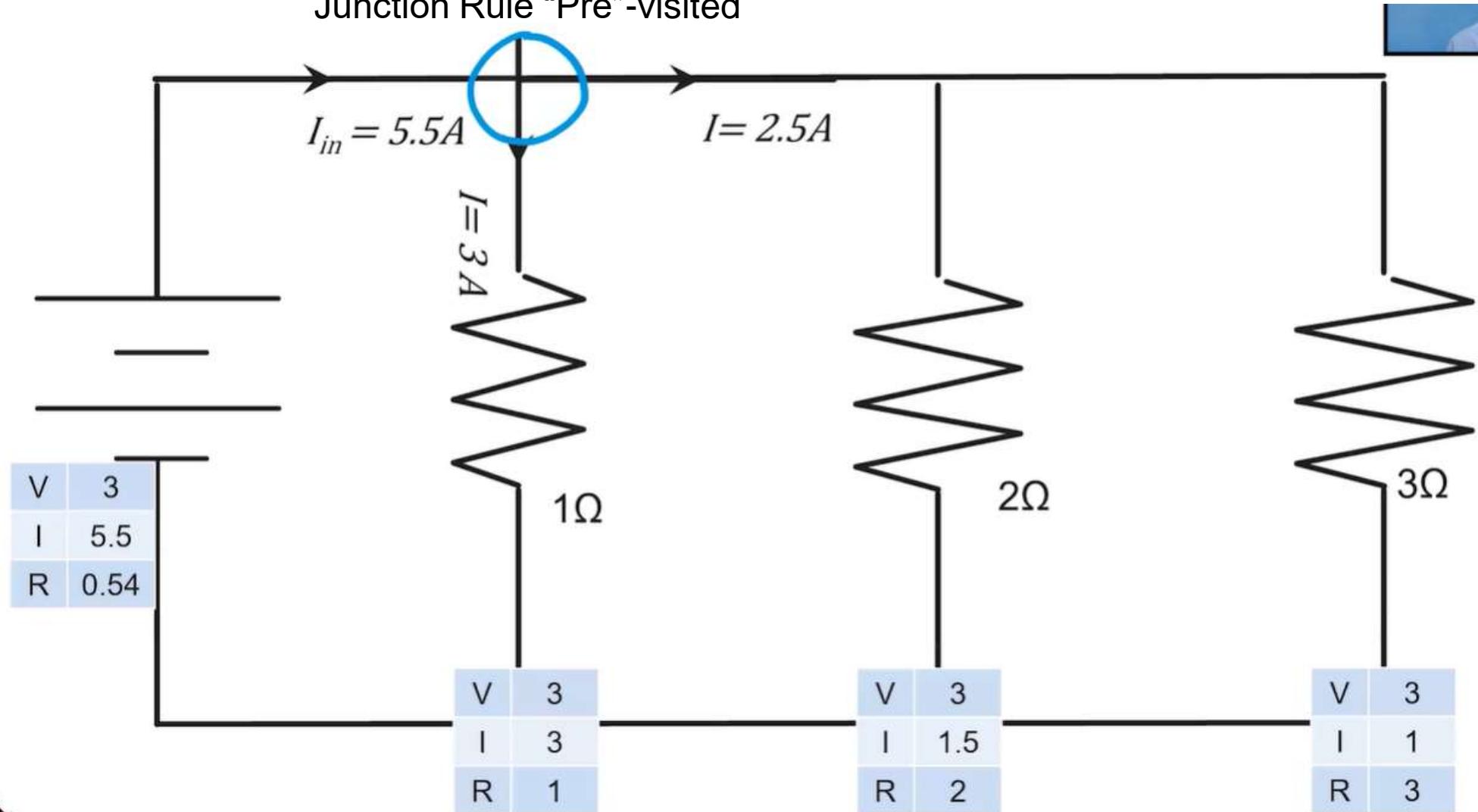
In parallel, voltage is the same.

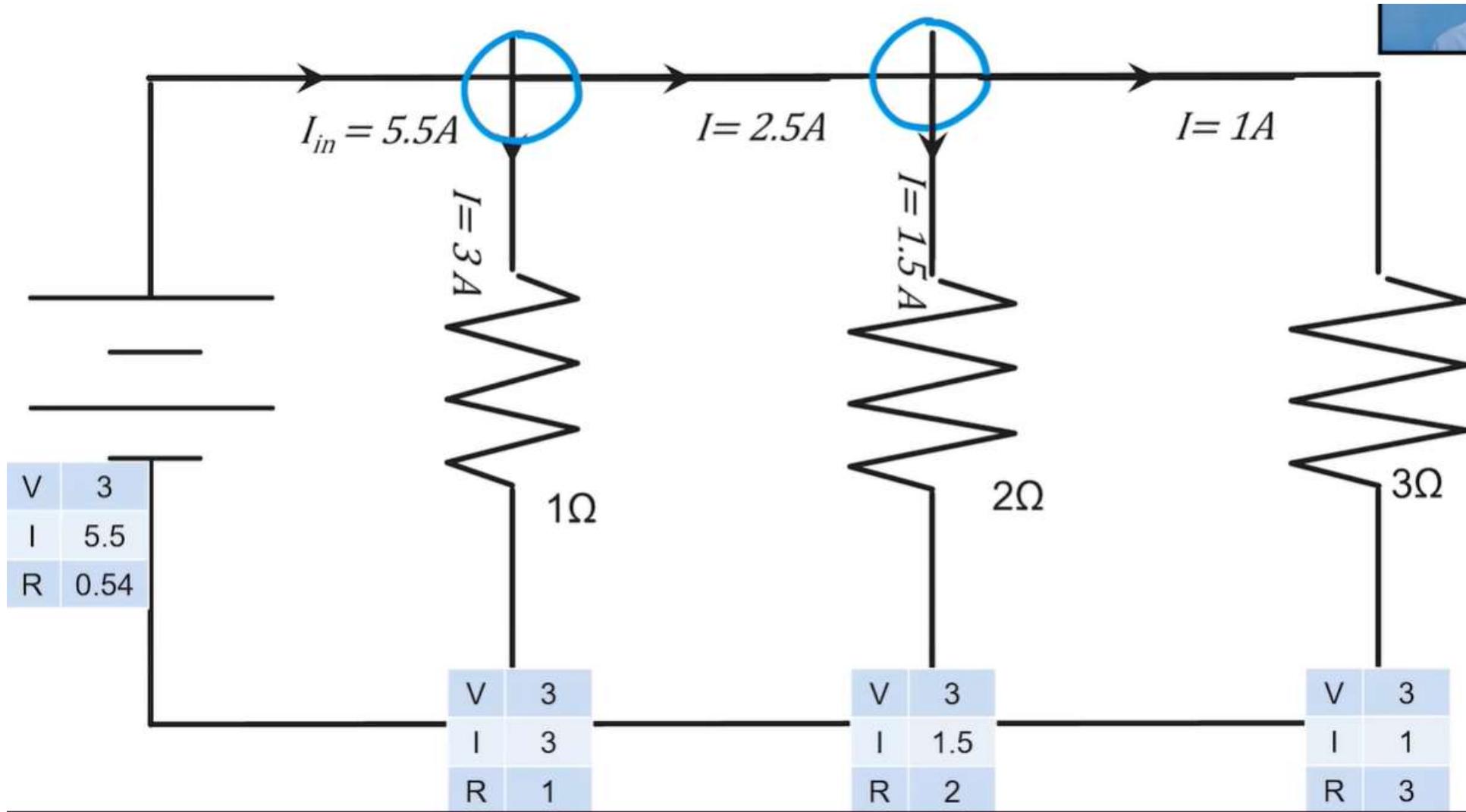


$$I = \frac{\Delta V}{R}$$



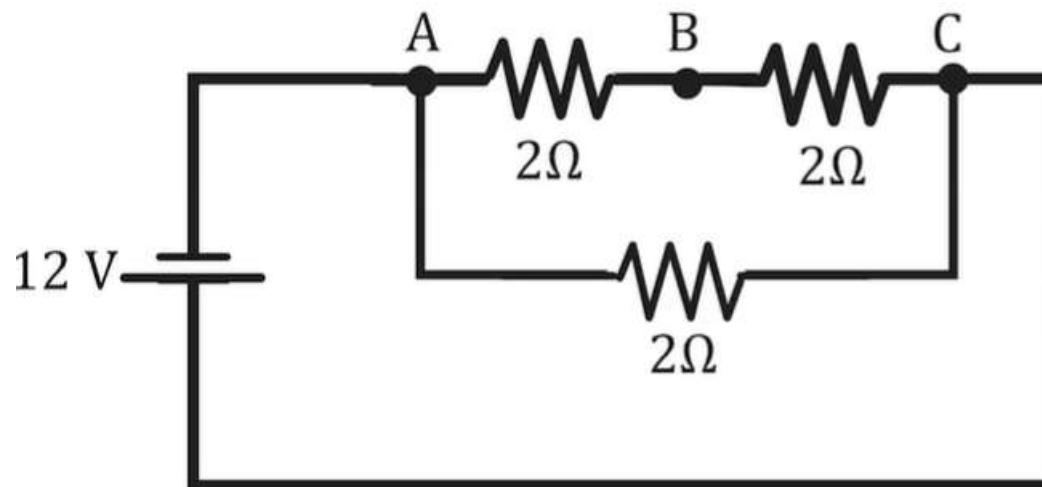
Junction Rule “Pre”-visited



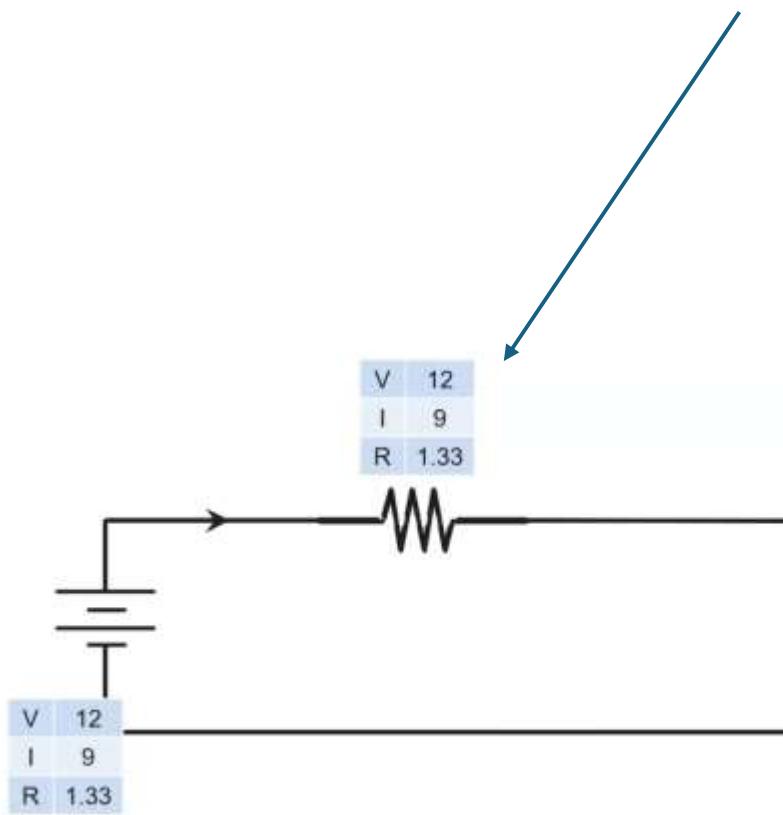
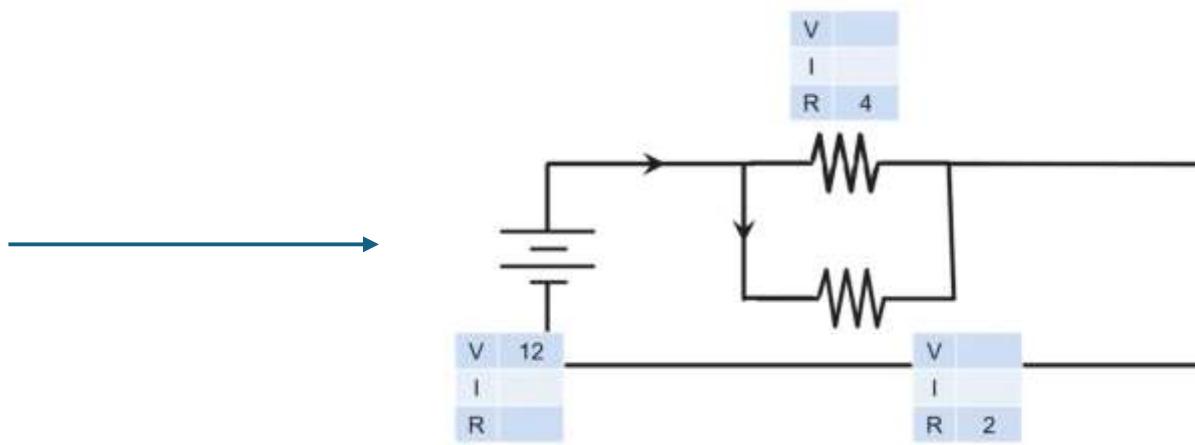
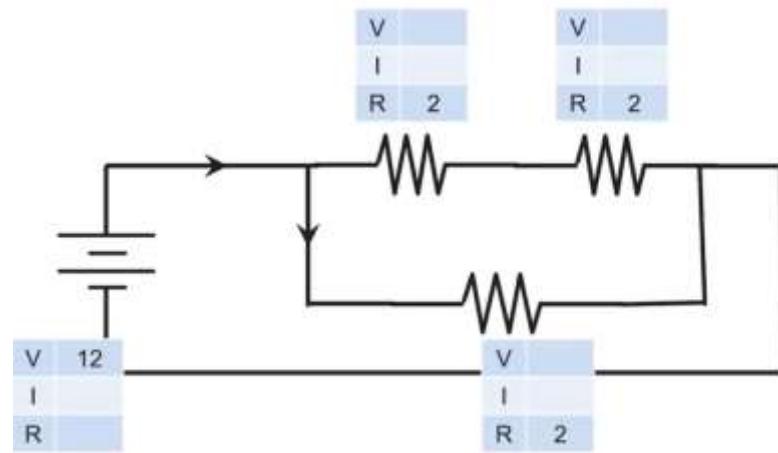


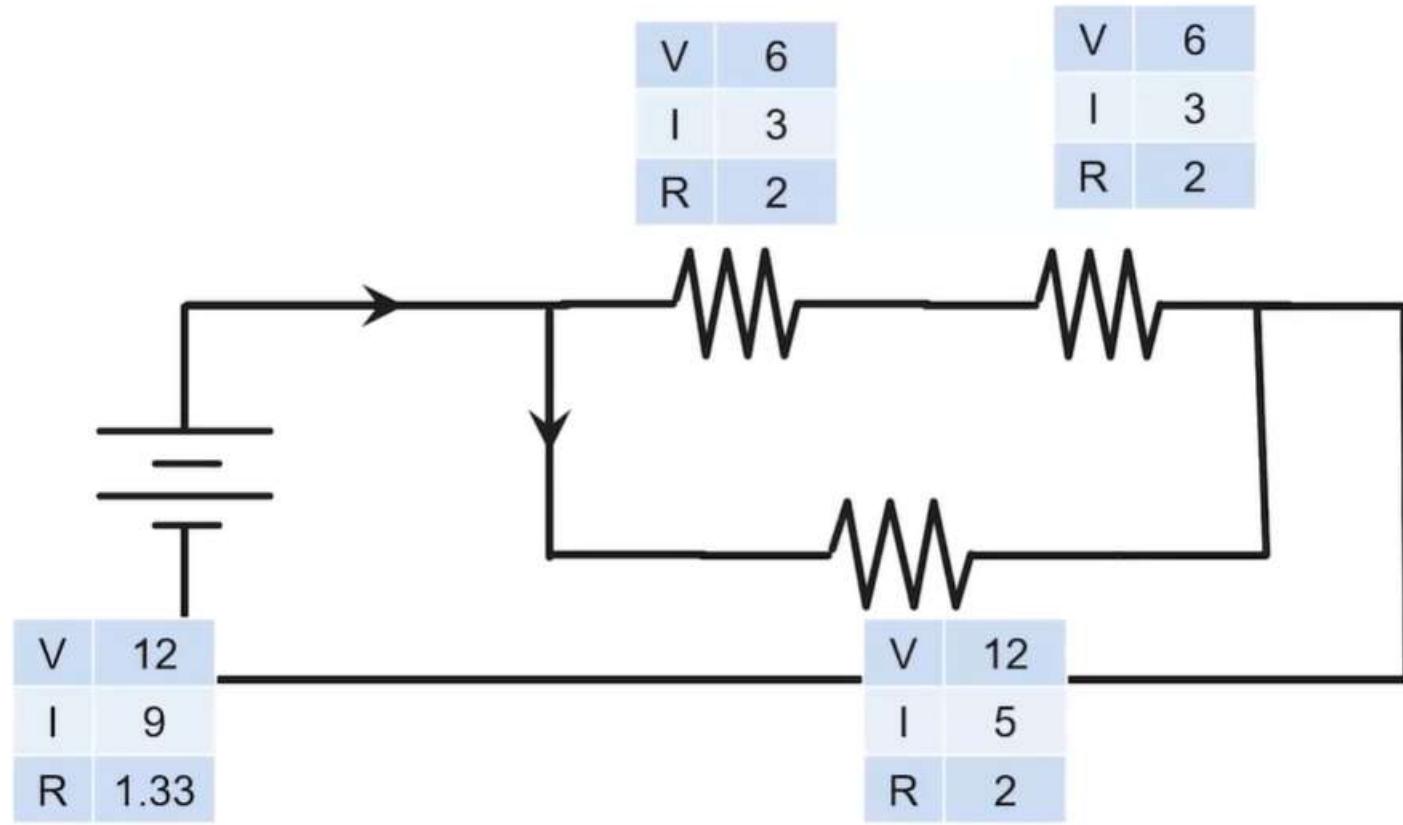
An electric circuit consists of a **12 V** battery and three  **$2 \Omega$**  resistors connected as shown below.

What would be the reading on an ammeter inserted at point **B**?



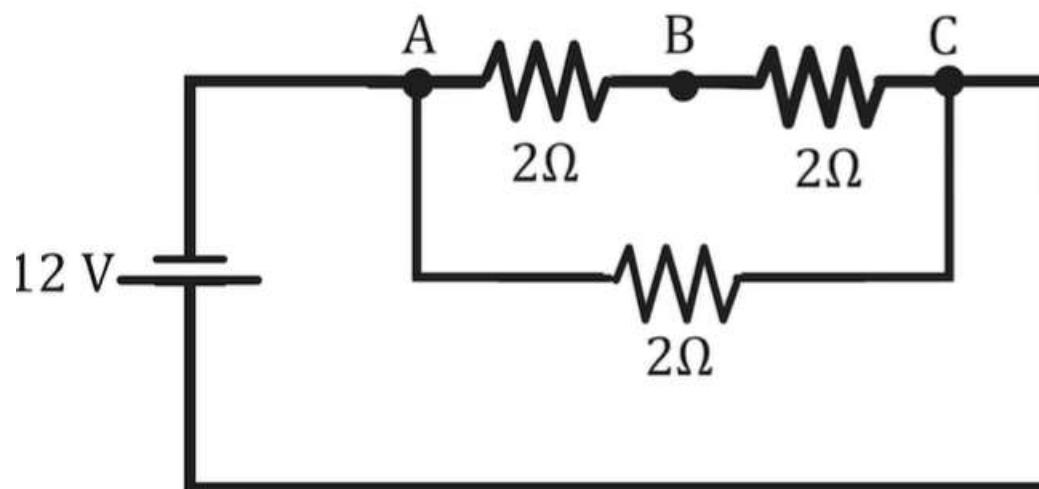
- A** 0 A
- B** 2 A
- C** 3 A
- D** 6 A





An electric circuit consists of a **12 V** battery and three  **$2 \Omega$**  resistors connected as shown below.

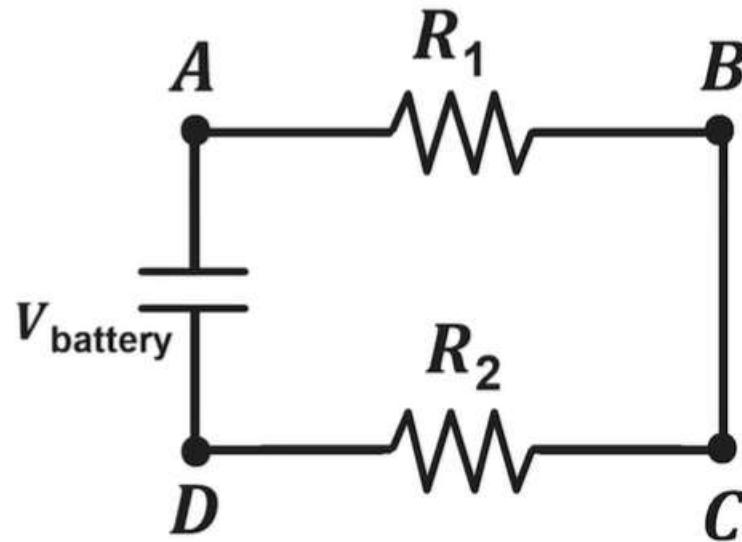
What would be the reading on an ammeter inserted at point **B**?



- A 0 A
- B 2 A
- C 3 A
- D 6 A

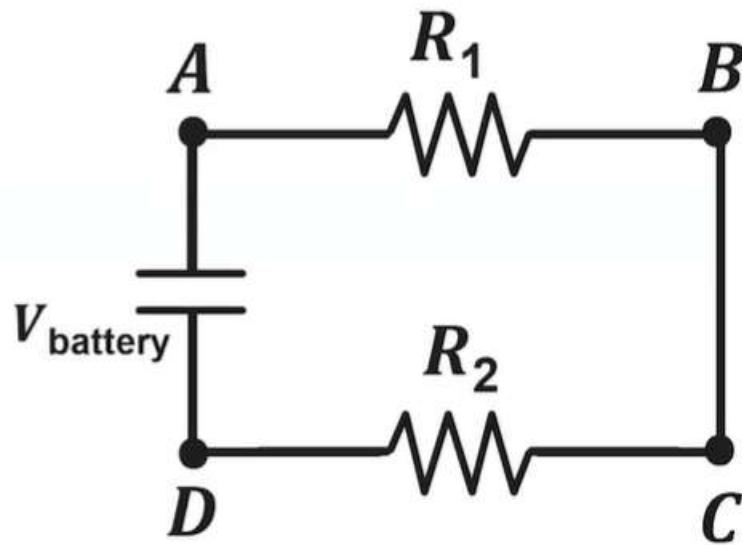
A battery with potential difference  $V_{\text{battery}}$  and two resistors are connected in series. Four points in the circuit are labeled A, B, C, and D. In terms of  $R_1$ , determine the voltage drop (potential difference) across  $R_2$ .

- A The voltage drop across  $R_2$  is  $\frac{2}{3} V_{\text{battery}}$
- B The voltage drop across  $R_2$  is  $(\frac{R_2}{R_1+R_2})V_{\text{battery}}$
- C The voltage drop across  $R_2$  is  $V_{\text{battery}} - V_{R1}$
- D The voltage drop across  $R_2$  is  $\frac{1}{2}V_{\text{battery}}$



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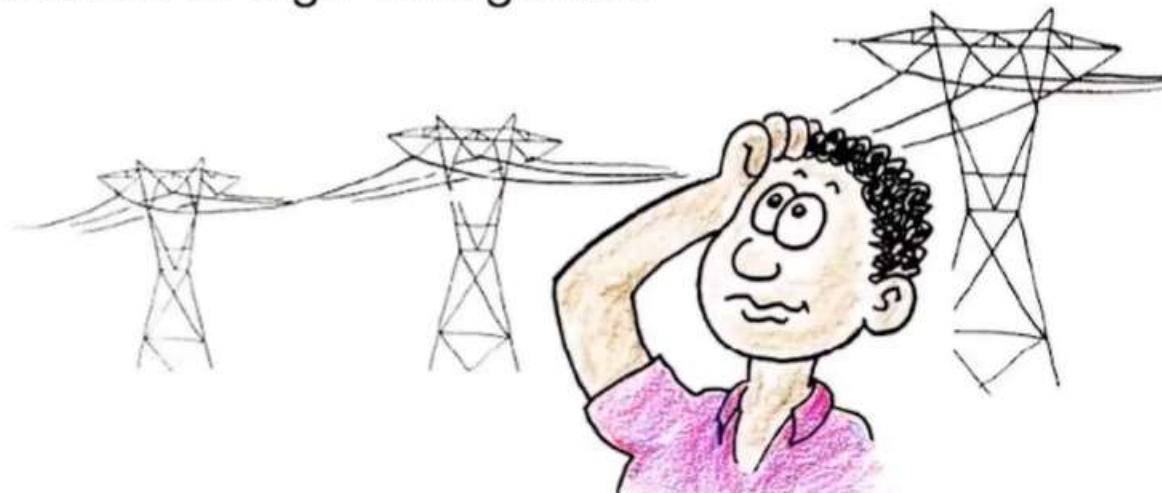
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## Warm Up



- According to Ohm's law, high voltage produces high current.
- Does Ohm's law apply to power lines?
- If so, then how can power be transmitted at high voltage and low current in power lines?



**Yes.** Ohm's law applies to power lines.



The high voltages that characterize power lines are between adjacent wires, not between the ends of the wires. The relatively low voltage across the ends of a single wire and low current accounts for small power dissipated in the wire.

The much greater power delivered by the lines, however, is the voltage difference between adjacent wires multiplied by the current in them. This power is transmitted to the load, where currents and voltages are again in accord with Ohm's law.

Ohm's law even applies in the space between lines. Resistance of air between lines is normally too high to permit a flow of charge - but sometimes arcing does occur. Hence the wide space between lines.

In applying Ohm's law, it's important that the voltage and current are applied to the same part of the circuit.

## Takeaways



- Ohm's Law, though not universal, is useful in determining potential difference, resistance, and/or current for DC circuits.
- Without keeping your work organized, simple mistakes are easily made. Re-draw your circuits and use VIR charts to assist with this.