

**2022**



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# **AP<sup>®</sup> Physics C:**

## **Mechanics**

### **Free-Response Questions**

#### **Set 1**

## ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS	
Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol <sup>-1</sup>	Universal gravitational constant, $G = 6.67 \times 10^{-11} (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31 \text{ J}/(\text{mol}\cdot\text{K})$	Acceleration due to gravity at Earth's surface, $g = 9.8 \text{ m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23} \text{ J/K}$	
1 unified atomic mass unit, $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$	
Planck's constant, $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$	
Vacuum permittivity, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^3 \text{ eV}\cdot\text{nm}$
Coulomb's law constant, $k = 1/(4\pi\epsilon_0) = 9.0 \times 10^9 (\text{N}\cdot\text{m}^2)/\text{C}^2$	
Vacuum permeability, $\mu_0 = 4\pi \times 10^{-7} (\text{T}\cdot\text{m})/\text{A}$	
Magnetic constant, $k' = \mu_0/(4\pi) = 1 \times 10^{-7} (\text{T}\cdot\text{m})/\text{A}$	
1 atmosphere pressure, $1 \text{ atm} = 1.0 \times 10^5 \text{ N/m}^2 = 1.0 \times 10^5 \text{ Pa}$	

UNIT SYMBOLS	meter, m	mole, mol	watt, W	farad, F
	kilogram, kg	hertz, Hz	coulomb, C	tesla, T
	second, s	newton, N	volt, V	degree Celsius, °C
	ampere, A	pascal, Pa	ohm, Ω	electron volt, eV
	kelvin, K	joule, J	henry, H	

PREFIXES		
Factor	Prefix	Symbol
$10^9$	giga	G
$10^6$	mega	M
$10^3$	kilo	k
$10^{-2}$	centi	c
$10^{-3}$	milli	m
$10^{-6}$	micro	μ
$10^{-9}$	nano	n
$10^{-12}$	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin \theta$	0	1/2	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan \theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	$\infty$

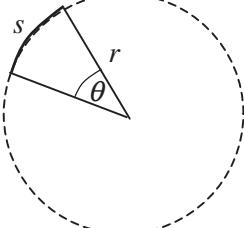
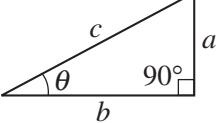
The following assumptions are used in this exam.

- I. The frame of reference of any problem is inertial unless otherwise stated.
- II. The direction of current is the direction in which positive charges would drift.
- III. The electric potential is zero at an infinite distance from an isolated point charge.
- IV. All batteries and meters are ideal unless otherwise stated.
- V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

# ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS	ELECTRICITY AND MAGNETISM
$v_x = v_{x0} + a_x t$	$a = \text{acceleration}$
$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$	$E = \text{energy}$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	$F = \text{force}$
$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{\text{net}}}{m}$	$f = \text{frequency}$
$\vec{F} = \frac{d\vec{p}}{dt}$	$h = \text{height}$
$\vec{J} = \int \vec{F} dt = \Delta \vec{p}$	$I = \text{rotational inertia}$
$\vec{p} = m\vec{v}$	$J = \text{impulse}$
$ \vec{F}_f  \leq \mu  \vec{F}_N $	$K = \text{kinetic energy}$
$\Delta E = W = \int \vec{F} \cdot d\vec{r}$	$k = \text{spring constant}$
$K = \frac{1}{2}mv^2$	$\ell = \text{length}$
$P = \frac{dE}{dt}$	$L = \text{angular momentum}$
$P = \vec{F} \cdot \vec{v}$	$m = \text{mass}$
$\Delta U_g = mg\Delta h$	$P = \text{power}$
$a_c = \frac{v^2}{r} = \omega^2 r$	$p = \text{momentum}$
$\vec{\tau} = \vec{r} \times \vec{F}$	$r = \text{radius or distance}$
$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{\text{net}}}{I}$	$T = \text{period}$
$I = \int r^2 dm = \sum mr^2$	$t = \text{time}$
$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$	$U = \text{potential energy}$
$v = r\omega$	$v = \text{velocity or speed}$
$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$	$W = \text{work done on a system}$
$K = \frac{1}{2}I\omega^2$	$x = \text{position}$
$\omega = \omega_0 + \alpha t$	$\mu = \text{coefficient of friction}$
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$\theta = \text{angle}$
	$\tau = \text{torque}$
	$\omega = \text{angular speed}$
	$\alpha = \text{angular acceleration}$
	$\phi = \text{phase angle}$
	$\vec{F}_s = -k\Delta \vec{x}$
	$U_s = \frac{1}{2}k(\Delta x)^2$
	$x = x_{\max} \cos(\omega t + \phi)$
	$T = \frac{2\pi}{\omega} = \frac{1}{f}$
	$T_s = 2\pi\sqrt{\frac{m}{k}}$
	$T_p = 2\pi\sqrt{\frac{\ell}{g}}$
	$ \vec{F}_G  = \frac{Gm_1m_2}{r^2}$
	$U_G = -\frac{Gm_1m_2}{r}$
	$ \vec{F}_E  = \frac{1}{4\pi\epsilon_0} \left  \frac{q_1q_2}{r^2} \right $
	$\vec{E} = \frac{\vec{F}_E}{q}$
	$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$
	$E_x = -\frac{dV}{dx}$
	$\Delta V = -\int \vec{E} \cdot d\vec{r}$
	$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$
	$U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$
	$\Delta V = \frac{Q}{C}$
	$C = \frac{\kappa\epsilon_0 A}{d}$
	$C_p = \sum_i C_i$
	$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$
	$I = \frac{dQ}{dt}$
	$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$
	$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$
	$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \times \hat{r}}{r^2}$
	$R = \frac{\rho\ell}{A}$
	$\vec{F} = \int I d\vec{\ell} \times \vec{B}$
	$\vec{E} = \rho\vec{J}$
	$B_s = \mu_0 nI$
	$\Phi_B = \int \vec{B} \cdot d\vec{A}$
	$I = \frac{\Delta V}{R}$
	$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$
	$R_s = \sum_i R_i$
	$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$
	$\mathcal{E} = -L \frac{dI}{dt}$
	$U_L = \frac{1}{2}LI^2$
	$P = I\Delta V$

# ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY	CALCULUS
Rectangle $A = bh$	$\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}$
Triangle $A = \frac{1}{2}bh$	$\frac{d}{dx}(x^n) = nx^{n-1}$
Circle $A = \pi r^2$ $C = 2\pi r$ $s = r\theta$	$\frac{d}{dx}(e^{ax}) = ae^{ax}$
Rectangular Solid $V = \ell wh$	$\frac{d}{dx}(\ln ax) = \frac{1}{x}$
Cylinder $V = \pi r^2 \ell$ $S = 2\pi r \ell + 2\pi r^2$	$\frac{d}{dx}[\sin(ax)] = a \cos(ax)$
Sphere $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$	$\frac{d}{dx}[\cos(ax)] = -a \sin(ax)$
Right Triangle $a^2 + b^2 = c^2$ $\sin \theta = \frac{a}{c}$ $\cos \theta = \frac{b}{c}$ $\tan \theta = \frac{a}{b}$	$\int x^n dx = \frac{1}{n+1} x^{n+1}, n \neq -1$ $\int e^{ax} dx = \frac{1}{a} e^{ax}$ $\int \frac{dx}{x+a} = \ln x+a $ $\int \cos(ax) dx = \frac{1}{a} \sin(ax)$ $\int \sin(ax) dx = -\frac{1}{a} \cos(ax)$
	<b>VECTOR PRODUCTS</b> $\vec{A} \cdot \vec{B} = AB \cos \theta$ $ \vec{A} \times \vec{B}  = AB \sin \theta$
	

Begin your response to **QUESTION 1** on this page.

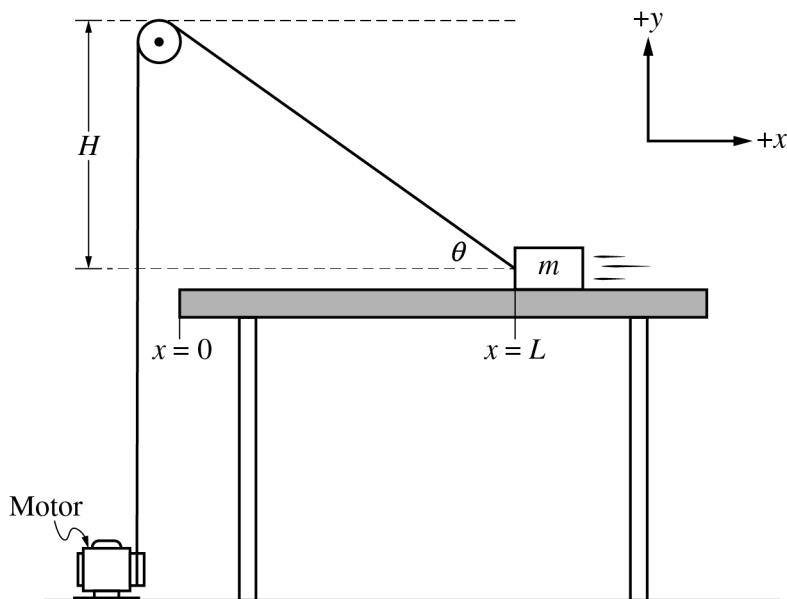
## PHYSICS C: MECHANICS

### SECTION II

**Time—45 minutes**

**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

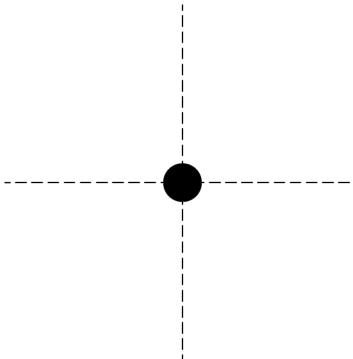


1. A block of mass  $m$  is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height  $H$  above the table. The motor exerts a constant force of tension  $F_T$  on the string, and the block remains in contact with the table at all times as the block slides across the table from  $x = L$  to  $x = 0$ . The coefficient of kinetic friction between the table and the block is  $\mu_k$ . Express all algebraic answers in terms of  $m$ ,  $H$ ,  $F_T$ ,  $x$ ,  $\mu_k$ ,  $L$ , and physical constants as appropriate.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

- (a) On the dot below that represents the block, draw and label the forces (not components) that act on the block when the block is at  $x = L$ . Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



- (b) Derive an expression for the angle  $\theta$  that the string makes with the horizontal as a function of  $x$ .

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

(c)

- i. Derive an expression for the normal force  $F_N$  exerted on the block by the table as a function of the block's position  $x$ .
  
- ii. Derive an expression for the magnitude of the net horizontal force  $F_{\text{net}}$  exerted on the block as a function of the position  $x$ .

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

- (d) Write, but do not solve, an integral expression that could be used to solve for the work  $W$  done by the string on the block as the block moves from  $x = L$  to  $x = 0$ .

- (e) Does the string do more, less, or the same amount of work on the block as the block moves from  $x = L$  to  $x = \frac{L}{2}$  compared to when the block moves from  $x = L$  to  $x = 0$ ?

More work when the block moves from  $x = L$  to  $x = \frac{L}{2}$

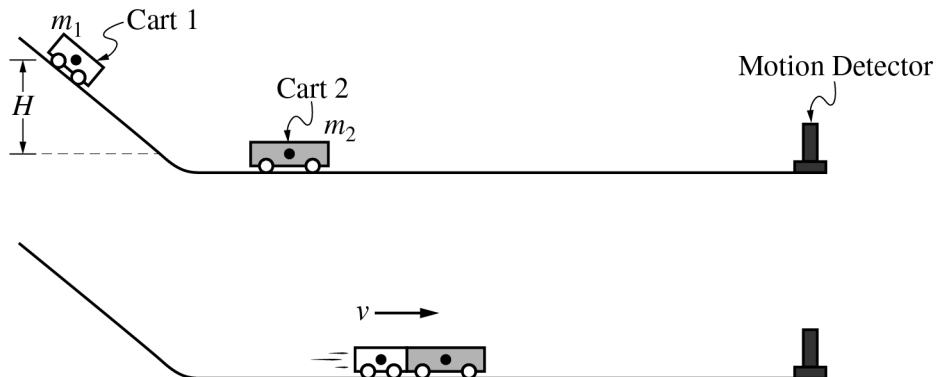
Less work when the block moves from  $x = L$  to  $x = \frac{L}{2}$

The same amount of work when the block moves from  $x = L$  to  $x = \frac{L}{2}$

Justify your answer.

**GO ON TO THE NEXT PAGE.**

Begin your response to **QUESTION 2** on this page.



2. Cart 1 of mass  $m_1$  is held at rest above the bottom of an incline. Cart 2 has mass  $m_2$ , where  $m_2 > m_1$ , and is at rest at the bottom of the incline. At time  $t = 0$ , Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of  $H$ , as shown. At time  $t_C$ , Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed  $v$ . Frictional and rotational effects are negligible.

- (a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1?

Greater than       Less than       Equal to

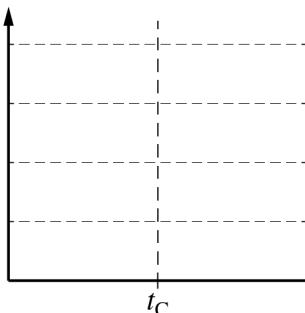
Justify your answer.

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Continue your response to **QUESTION 2** on this page.

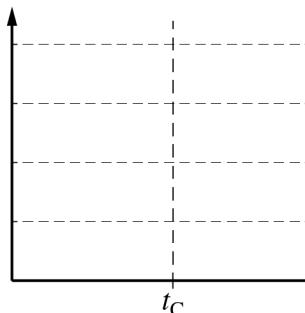
- (b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time  $t$ , before and after  $t_C$ . The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.

Momentum



Cart 1

Momentum



Cart 2

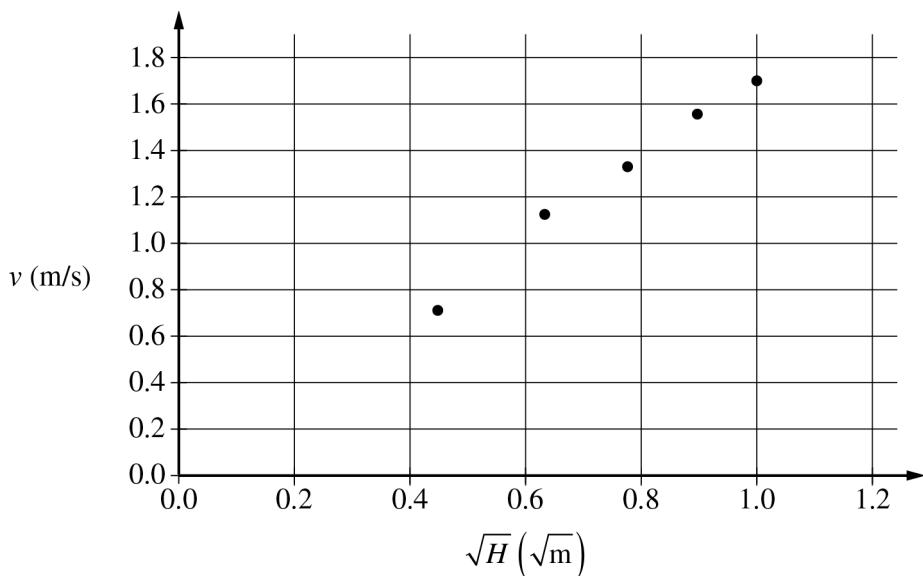
- (c) Show that the velocity  $v$  of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g} \left( \frac{m_1}{m_1 + m_2} \right) \sqrt{H}.$$

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 2** on this page.

- (d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be  $m_1 = 0.250 \text{ kg}$ . The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height  $H$  and the final velocity of the two-cart system is measured. The students graph  $v$  as a function of  $\sqrt{H}$ , as shown below.



- Draw a line that represents the best fit to the data points shown.
- Use the best-fit line to calculate the mass of Cart 2.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 2** on this page.

- (e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value,  $m_1'$ .

Should the students expect that  $m_1'$  will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg?

$m_1' > 0.250 \text{ kg}$         $m_1' < 0.250 \text{ kg}$         $m_1' = 0.250 \text{ kg}$

Justify your answer.

**GO ON TO THE NEXT PAGE.**

Begin your response to **QUESTION 3** on this page.

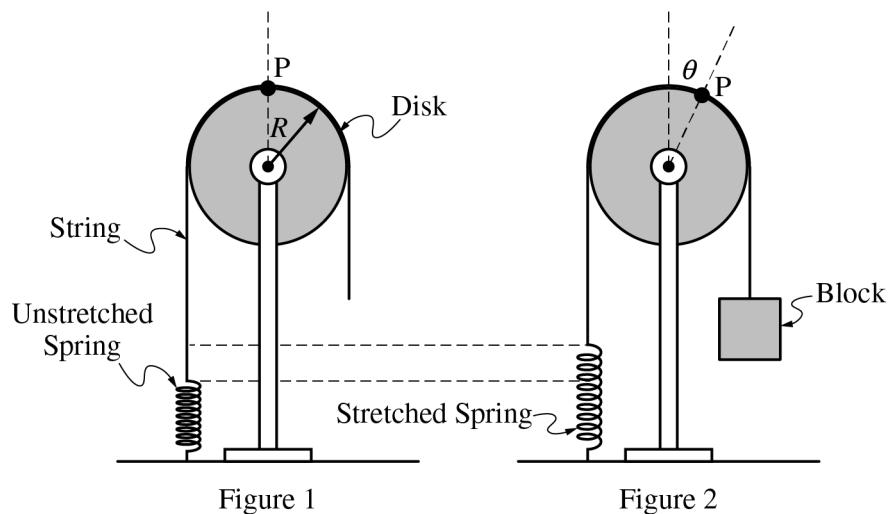


Figure 1

Figure 2

Note: Figures not drawn to scale.

3. A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by  $M_d$  and  $R$ , respectively. The rotational inertia of the disk is  $I_d = \frac{1}{2} M_d R^2$ . A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant  $k$ , which is fixed to the ground as shown in Figure 1.

A block of mass  $m_B$  is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle  $\theta$ .

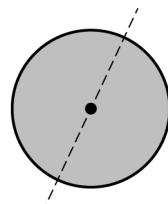
Give all algebraic answers in terms of  $M_d$ ,  $R$ ,  $k$ ,  $\theta$ , and physical constants, as appropriate.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

- (a) Derive an expression for the mass  $m_B$  of the block.

- (b) At time  $t = 0$ , the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.

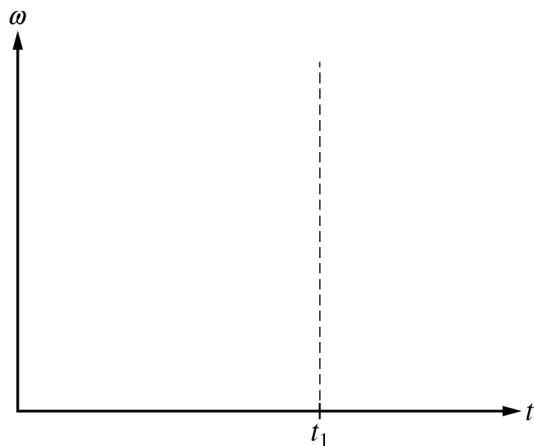


- (c) Derive an expression for the angular acceleration  $\alpha$  of the disk immediately after the string is cut.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

- (d) At  $t = t_1$ , the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity  $\omega$  of the disk as a function of time  $t$  from  $t = 0$  to  $t = t_1$ .



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Continue your response to **QUESTION 3** on this page.

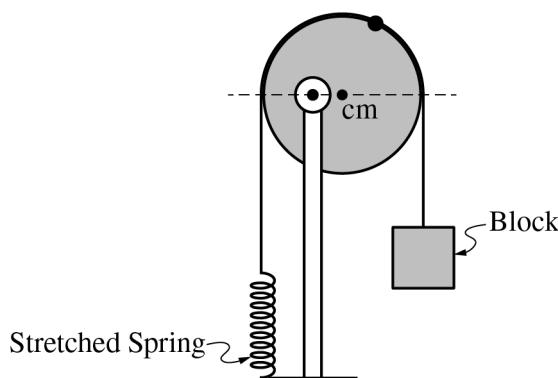


Figure 3

Note: Figure not drawn to scale.

- (e) The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

**GO ON TO THE NEXT PAGE.**

**STOP**

**END OF EXAM**