AP Physics C

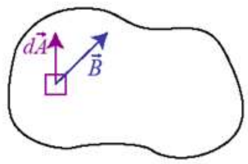
Electricity & Magnetism

Unit 11: Electromagnetism

11.1 Magnetic FLUX! (Chapter 30)

Focus Question: What is magnetic flux?

- Magnetic Flux – ϕ, Amount of magnetic field that goes through a surface.

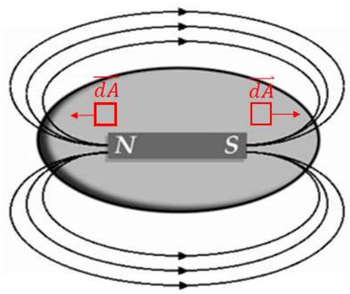


$$\phi = \int \vec{B} \cdot \overrightarrow{dA} = BA\cos\theta$$

Units: 1 weber (Wb) = 1 Tm^2

*Field vs. flux - $d\phi = B_{\perp} dA \rightarrow B_{\perp} = \frac{d\phi}{dA}$. The magnetic field equals the flux per unit area, so the magnetic field is sometimes called "flux density".

- Gauss's Law for Magnetism concerns magnetic flux through a closed surface:

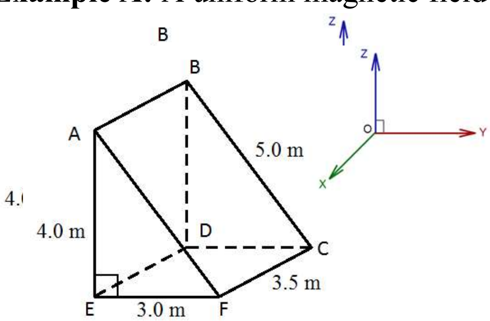


$$\phi = \int \vec{B} \cdot \overrightarrow{dA} = 0$$

The total magnetic flux through a closed surface is zero.

Gauss's Law for Magnetism is true since there are no magnetic monopoles.

Example A: A uniform magnetic field $|B| = 0.80\ T$ acts in the positive y-direction.

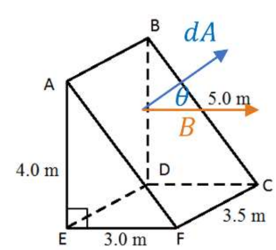


a) Calculate the flux through surface ABDE.
b) Calculate the flux through surface ABCF.

a) The flux and dA are in opposite directions since the flux to in the +y to the right, and dA is out of the surface, which is to the left:

$$\phi = BA\cos 180° = -(.8\ T)(3.5\ m)(4\ m) = -11.2\ Wb$$

b) dA points directly out of the surface as shown. The field is in the positive y-direction:

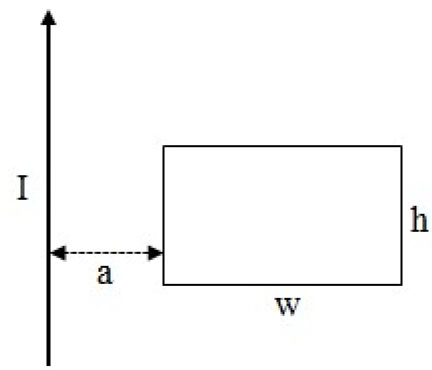


$$\phi = BA\cos\theta = (.8\ T)((3.5\ m)(5\ m)\frac{3}{5} = 8.4\ Wb$$

Example B: A uniform 2.0 T magnetic field passes through a circular loop with a radius of 0.2 m that has 4 turns. Calculate the flux through the loop.

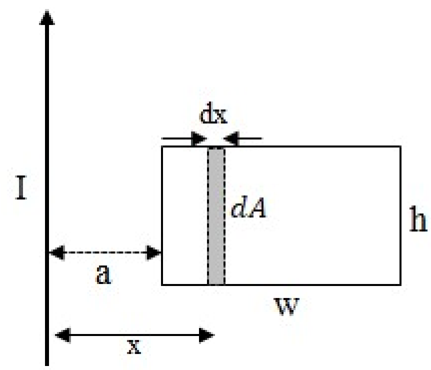
$$\phi = BA = (2\ T)(\pi(.2\ m)^2)x4 = 1\ Wb$$

Example C: An infinitely long conducting wire carries a current of I and is a distance of a from a rectangular surface as shown. Derive an expression for the magnetic flux through the rectangle.



The field is not constant through the loop, so it is broken into many dA's along its length in the x-direction since field varies in x. Each dA has a thickness of dx and an area of hdx. .

The field in each dA is constant vertically and is equal to $\frac{\mu_0 I}{2\pi r}$, where r is the distance to the wire.



$$\phi = \int B \cdot dA = \int_a^{a+w} \frac{\mu_0 I}{2\pi x} h dx = \frac{\mu_0 I h}{2\pi} \int_a^{a+w} \frac{dx}{x}$$

$$\rightarrow \frac{\mu_0 I h}{2\pi} \ln|x| \Big|_a^{a+w} = \frac{\mu_0 I h}{2\pi} (\ln|a+w| - \ln|a|)$$

$$\boldsymbol{\phi = \frac{\mu_0 I h}{2\pi} \ln\left|\frac{a+w}{a}\right|}$$

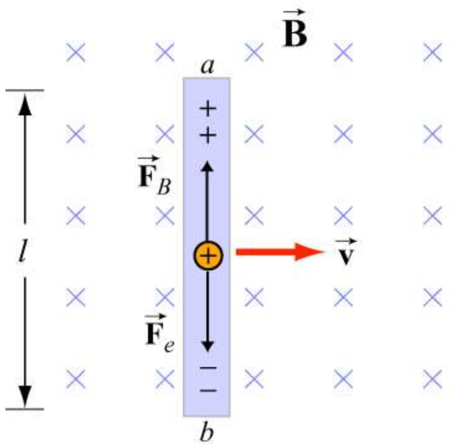
Rate your understanding: Magnetic Flux

0	1	2	3	4
I have no idea what the flux is going on.	I can apply some concepts of magnetic flux.	I can solve for magnetic flux with only a few errors.	I can solve for magnetic flux with no errors.	I can explain and teach magnetic flux.

11.2 Faraday's Law (Chapter 31)

Focus Question: What causes an induced emf?

- When the magnetic field through a loop of wire changes, a current is induced in the loop of wire. In the diagram below, the electrons in the wire experience a force, and the resulting movement induces current in the wire, even though the wire is not connected to a power source.



When the bar is moved to the right in a field that outs into the page as shown, positive charge is pushed up by the right-hand rule. Since there is current in the wire, there must be a potential difference. Charge accumulates until the electric force caused by the potential difference cancels out the magnetic force:

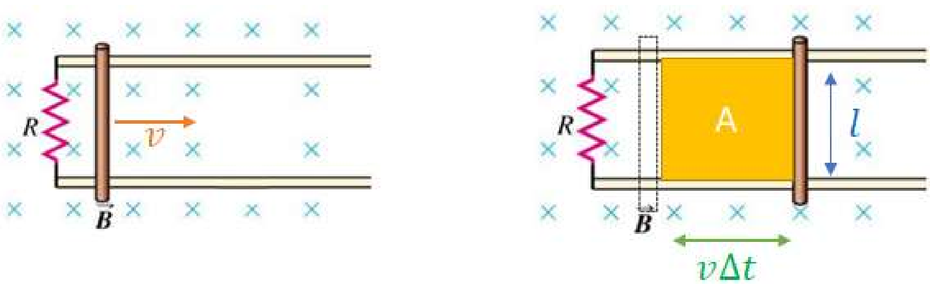
$$F_B = F_E$$

$$\cancel{q}Bv = E\cancel{q} \rightarrow Bv = \frac{\varepsilon}{l}$$

$$\rightarrow \boldsymbol{\varepsilon = Bvl}$$

*induced emf in a bar of length moving at speed in a magnetic field of B

If the bar is part of a completed circuit loop as shown, then the moving bar acts as a battery due to the induced emf in the bar.



As the bar moves to the right, it covers an area of A as shown in a time Δt.

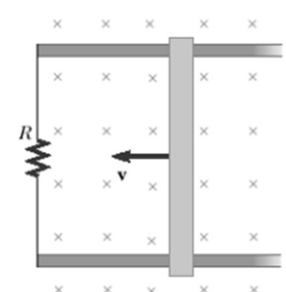
$$\phi = BA = Bv\Delta tl$$

The time rate of change of the flux is $\frac{\phi}{t} = Bvl$ since velocity is constant. This is equal to the emf.

Example A: A bar of lenth .20 m is moved to the right at a speed of v =4.0 m/s in a field of magntidue B = 2.0 T as shown. The resistor has a value of $R = 2.0\ \Omega$.

a) Find the current in the circuit.

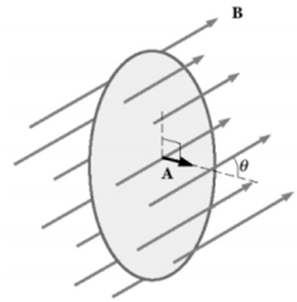
b) Calculate the force required to keep the rod moving.



a) $I = \frac{\varepsilon}{R} = \frac{Blv}{R} = \frac{(2\ T)(.2\ m)(4\frac{m}{s})}{2\ \Omega} = 0.8\ A$

b) $F = BIL = (2\ T)(.8\ A)(.2\ m) = .32\ T$

- Faraday's Law with Calculus – The induced emf due to a changing magnetic field is equal to the rate of change of the magnetic flux through the surface bounding by the circuit:



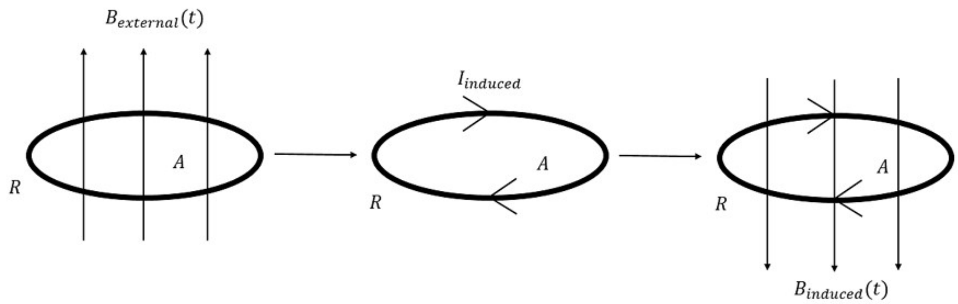
$$\boldsymbol{\varepsilon} = \frac{-d\phi}{dt}$$

$$\boldsymbol{\varepsilon} = \frac{-d}{dt}\int \vec{B}\cdot\vec{dA}\cos\theta$$

*The minus sign indicates the magnetic force produced by the induced current opposes the force that causes it.

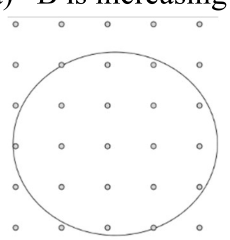
- Ways to change flux:
  - Change the magnitude of the magnitude.
  - Change the size of the area the flux is passing through.
  - Change the angle made by the field and the surface it passes through.

- Lenz's Law
  "An induced current will have a direction such that it will oppose the change in flux that produced it." (the minus sign in Faraday's law)

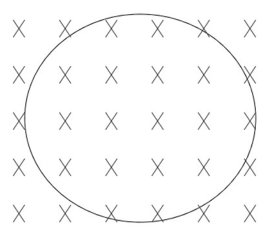


Example B: Determine the direction of the current in the wire in each of the cases below:

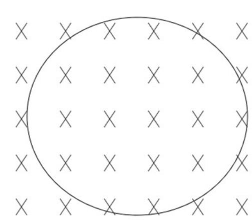
a) B is increasing



b) B is increasing



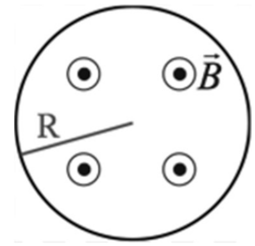
c) B is decreasing



a) The field is out of the page and increasing, so the change in flux is out of the page. The induced current resists the change in flux by inducing a current that creates a field into the page to oppose the change in flux, so the current is **clockwise**.
b) The field is into the page and increase, so the change in flux is into the page. The induced current creates a field out of the page to oppose the direction of the change in flux, so the current is **counter-clockwise**.
c) The field is into the page and decreasing, meaning the change in flux is out of the page. he induced current resists the change in flux by inducing a current that creates a field into the page to oppose the change in flux, so the current is **clockwise**.

Example C: A magnetic field has a strength given by $B(t) = 2t^2 - 2t + 2$ and acts out of the plane of this page. A circular loop of wire of radius R = 2.0 m within the plane of this page has the field pass through it.

a) Find the induced emf in this wire as a function of time.
b) Find the direction of the current in the wire at t = 2.0 s.



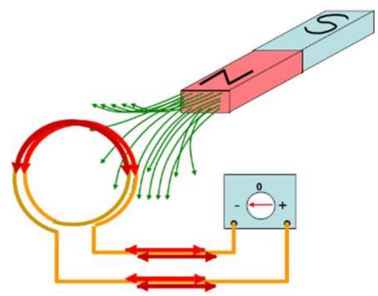
a) $\varepsilon = -\frac{d\phi}{dt} = -A\frac{dB}{dt}$ (A is constant and B changes with t)
$\rightarrow \varepsilon = -(\pi)(2\,)^2(4t - 2)$
$\rightarrow \varepsilon(t) = 16\pi t^2 - 8\pi$
b) At t = 2, $\frac{db}{dt} = 4(2) - 2 = 6$. B is increasing since dB/dt is positive. Thus the flux is increasing out of the page, so the change in flux is out of the page. The induced current creates a field into the page to the induced current is **clockwise.**

- Flux Linkage

For a coil of wire with many loops:

An emf is induced in each loop of wire, more loops = more emf

$$\varepsilon = \frac{-N\Delta\Phi}{\Delta t}$$



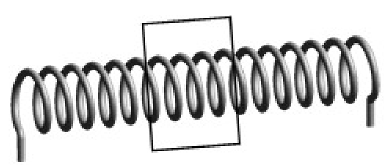
*Each loop produces its own emf, and the emfs from each loop add to the total emf.

Example D: A metal wire coil of radius 0.5 m has 1000 loops, forming a solenoid. The solenoid is placed in a 500 mT magnetic field. The field drops to 0 T in 10 s. What is the induced emf in the coil?

$$\varepsilon = -\frac{Nd\phi}{dt} = -NA\frac{dB}{dt} = -(1000)(\pi)(0.5m)^2\frac{0 - (500x10^{-3}\,T)}{10\,s} = \frac{25\pi}{2}\,V$$

Example E: A square loop (s = 0.25 m) is placed around a solenoid of 1000 loops with a diameter of .10 m and length of 0.2 m. The loop's plane is perpendicular to the axis of the solenoid. The current in the Solenoid is given by $I = 100\left(1 - e^{-\frac{t}{5}}\right)$. Calculate the induced emf in the loop as a function of time.

Using the field of a solenoid derived in 10.4: $\phi = BA = \frac{\mu_0 NI(t)}{L}\pi r^2$

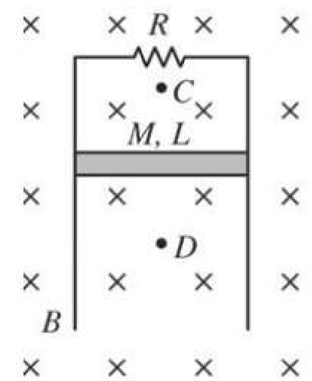


$$\rightarrow B = \frac{\mu_0 N\pi r^2}{L}100(1 - e^{-\frac{t}{5}})$$

$$\varepsilon = -\frac{dB}{dt} = s^2\frac{\mu_0 N\pi r^2}{L}20e^{-\frac{t}{5}} = (.25\,m)^2\frac{\left(4\pi x10^{-7}\,\frac{Tm}{A}\right)(1000)\pi(.05\,m)^2}{.2\,m}(20)e^{-\frac{t}{5}}$$

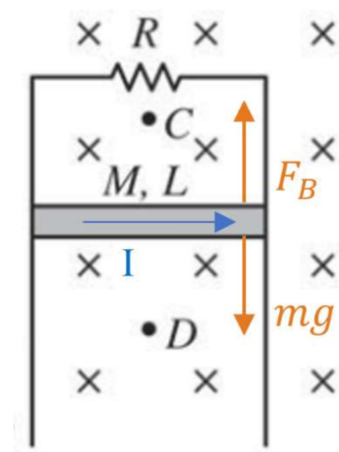
$$\rightarrow \varepsilon = .001e^{-\frac{t}{5}}$$

Example F: A conducting bar of mass M and length L is connected to two long vertical conducting rails. The two rails are connected by a resistor of resistance R at the top. The entire apparatus is in a magnetic field of magnitude B directed into this page. The bar is released from rest and slides without friction down the rails.



a) What is the direction of current in the resistor?
b) Is the magnitude of the net magnetic field above the bar at point C greater than, less than, or equal to the magnitude of the net magnetic field before the bar is released?
c) Determine an expression for the terminal velocity of the bar.
d) Derive an expression for the power dissipating in the resistor when the bar is at terminal velocity.
e) Derive an expression for the velocity of the bar as a function of time until it reaches terminal velocity.

a) Due to the downward motion due to gravity of the bar, the magnetic field into the page will push positive charge in the bar to the right due to the right-hand rule, causing a clockwise-current. The current in the resistor is to the **left.**



b) The induced current in the bar creates a magnetic field out of the page on top of the bar and into the page below the bar by the right hand-rule. Above the bar, this field goes against the existing field, so the net magnetic field in C **decreases**.

c) Due to the right hand rule again, the current to the right causes an upward magnetic force. With a downward force of gravity the forces on the bar are:

$$\sum F = mg = mg - F_B$$
$$\rightarrow ma = mg - BIL$$

The current is based on the induced emf in the bar:

$$\rightarrow ma = mg - B\left(\frac{\varepsilon}{R}\right)L = mg - B\left(\frac{BLv}{R}\right)L$$
$$\rightarrow ma = mg - \frac{B^2L^2v}{R}$$

At terminal velocity, the acceleration is zero:

$$0 = mg - \frac{B^2L^2v_T}{R} \rightarrow mg = \frac{B^2L^2v_T}{R}$$
$$\rightarrow \boldsymbol{v_T = \frac{mRg}{B^2L^2}}$$

d) Power = $I^2R = \left(\frac{\varepsilon}{R}\right)^2 R = \frac{(BL\ \ _T)^2}{R^2}R$

$$\rightarrow P = \frac{B^2L^2v_T^2}{R} = \frac{B^2L^2\left(\frac{mRg}{B^2L^2}\right)^2}{R} = \frac{m^2R^2g^2}{B^2L^2R}$$
$$\rightarrow P = \frac{m^2Rg^2}{B^2L^2}$$

e) From c) $ma = mg - \frac{B^2L^2v}{R}$

$$\rightarrow m\frac{dv}{dt} = mg - \frac{B^2L^2v}{R}$$

This is a free-fall terminal velocity question like in AP Physics C: Mechanics, just with a different drag force.

To get v positive and without and cofficients (to make integration easy), multiply both sides by $-\frac{R}{B^2L^2}$

$$\rightarrow -\frac{mR}{B^2L^2}\frac{dv}{dt} = v - \frac{mRg}{B^2L^2}$$

Separate the variables:

$$\frac{dv}{v - \frac{mRg}{B^2L^2}} = -\frac{B^2L^2}{mR}dt$$

The velocity starts at zero and goes to some value at time, v(t).

$$\int_0^{v(t)} \frac{dv}{v - \frac{mRg}{B^2L^2}} = \int_0^t -\frac{B^2L^2}{mR}dt$$

$$\rightarrow \ln\left|v - \frac{mRg}{B^2L^2}\right| \begin{matrix} v(t) \\ 0 \end{matrix} = -\frac{B^2L^2}{mR}t \rightarrow \ln\left|v(t) - \frac{mRg}{B^2L^2}\right| - \ln\left|-\frac{mRg}{B^2L^2}\right| = -\frac{B^2L^2}{mR}t \rightarrow \ln\left|\frac{v(t) - \frac{mRg}{B^2L^2}}{-\frac{mRg}{B^2L^2}}\right| = -\frac{B^2L^2}{mR}t$$

$$\rightarrow \frac{v(t) - \frac{mRg}{B^2L^2}}{-\frac{mRg}{B^2L^2}} = e^{-\frac{B^2L^2}{mR}t}$$

$$\rightarrow \boldsymbol{v(t) = \frac{mRg}{B^2L^2}\left(1 - e^{-\frac{B^2L^2}{mR}t}\right)}$$

By this equation, the velocity starts at 0 and increases asymptotically to $\frac{mRg}{B^2L^2}$, which is the terminal velocity from c).

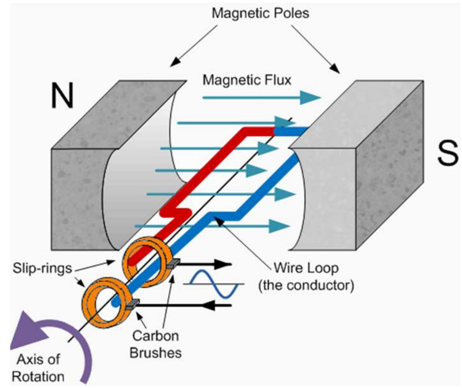
Rate your understanding: Faraday's Law

0	1	2	3	4
I know Lenz's Law.	I understand parts of Faraday's Law.	I can solve for induced emf with minor errors.	I can solve for induced emf with no errors.	I can explain and teach Faraday and Lenz's law.

11.3 Motors & Generators (Chapter 31)

Focus Question: How is Faraday's law applied in the modern world?

An AC generator converts mechanical energy into electrical energy. Suppose a wire loop is made to rotate as shown on the right by some mechanical source:



- As the loop rotates, the magnetic flux through the loop changes with time:

  $\phi = BA\cos(\omega t)$

  $*\omega = 2\pi f$ (angular frequency)

- The motional emf can be thus be found:

  $\varepsilon = \frac{-N\Delta\Phi}{\Delta t} = \frac{Nd(BA\cos\omega t)}{dt}$

  $\varepsilon = NBA\omega \sin \omega t$

- The emf is at a maximum when the plane of the loop is parallel to the magnetic field:

$$\varepsilon_{PEAK} = \omega NBA$$

Example A: Generator - A 250-turn coil of wire has dimensions 2.5 cm by 3.2 cm. It is rotating at a frequency of 60 Hz in a constant magnetic field having a strength of 1.5 T. What is its peak voltage ε_0?

$$\varepsilon_0 = \varepsilon_{PEAK} = \omega NBA = (2\pi)(60\ Hz)(250)(1.5\ T)(.025\ m)(.032\ m) = \mathbf{113\ V}$$

Example B: Motor - A motor contains a coil with a total resistance of 10 Ω and is supplied by a voltage of 120 V. When the motor is running at its maximum speed, the back emf is 70 V.

a) Find the current in the coil at the instant the motor is turned on.
b) Find the current in the coil when the motor has reached maximum speed.

a) When the motor is turned on, the available voltage is one 120 V – 70 V = 50 V, since the back emf prevents the emf from jumping to the full supply value. This is because coils prevent current from instantly jumping to their steady-state values. Back emf is due to inductance, which is the topic of 10.4

$I = \frac{50\ V}{10\ \Omega} = 5\ A$

b) $I = \frac{120\ V}{10\ \Omega} = 12\ A$

Rate your understanding: Motors & Generators

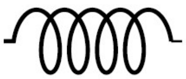
0	1	2	3	4
	I understand parts of Faraday's Law.	I can solve for induced emf with only occasional errors.	I can solve for induced emf with no errors.	I can explain and teach Faraday and Lenz's law.

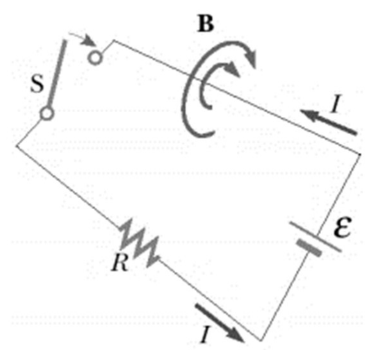
11.4 Inductance (Chapter 32)

Focus Question: What happens current runs through an inductor?

Inductance

- Inductor – A component that stores energy temporarily in a magnetic field. Inductors resist changes in current.
  *usually created by coiling wire into many loops. More loops = stronger magnetic field
  Schematic symbol:





- In a circuit with an inductor, current doesn't automatically jump to its max value, $I = \frac{\varepsilon}{R}$. As current increases, magnetic flux through the inductor increases with time, inducing an emf in the inductor; opposite the direction of the current.
- Self-Inductance – The circuit itself produces a changing magnetic flux and thus induces an emf due its own current.

$$\boldsymbol{\varepsilon_L = -N\frac{d\phi}{dt}}$$

- Inductance - Property of inductor; ratio of magnetic flux to current flow.

$$\boldsymbol{L = N\frac{\phi}{I}}$$

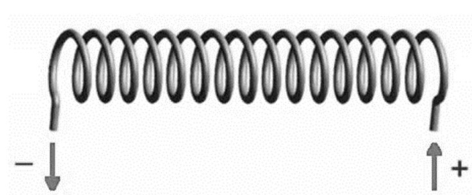
$$\boldsymbol{L = -\frac{\varepsilon}{\frac{dI}{dt}} \rightarrow \varepsilon_L = -L\frac{dI}{dt}}$$

L, inductance, is basically "electrical inertia". The second equation above is comparable to $F = m\frac{dv}{dt}$.

units of inductance: Henry (H); 1 H = 1 Vs/A
*inductance can be thought of as resistance to change in current, not to current itself.

Example A: Consider the solenoid shown with current flowing in the indicated direction.

a) Find the direction of the self-induced emf when the current is increasing.
b) Find the direction of the self-induced emf when the current is decreasing.



The current flows from right to left.
a) The current is to the left and increasing, the change in current is the left, inducing an emf to resist this change to the right. The solenoid acts as an inductor, which does not let current suddenly change, so the current will slowly start to flow to the left rather than jumping immediately when power is connect.
b) The current is to the left, but decreasing, so the change in current is the right. The solenoid will resist this change by inducing an emf with a current to the left.

Example B: Calculate the self-inductance of a solenoid of radius R and length L with N loops.

The magnetic field in a solenoid is given by $B = \frac{\mu_0 NI}{L}$ as derived in 10.4.

$$L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{N\left(\frac{\mu_0 NI}{L}\right)(\pi R^2)}{I}$$

$$\rightarrow L = \frac{N^2 \mu_0 \pi R^2}{L}$$

*The current cancels out. Inductance is a property of the inductor and is independent of current.

- Energy stored in an inductor –

  Consider the time when current in an inductor increases from 0 to I:

$$W = \int P(t)dt = \int \varepsilon I dt = \int L\frac{di}{dt} i dt = \int L i di = \boldsymbol{\frac{1}{2}LI^2}$$

*the energy is stored in the magnetic field of the inductor

Example C: A toroid solenoid with N turns has mean radius R and a cross-sectional area of A. A current of I flows through the toroid. What is the energy stored in this toroid?

The magnetic field in a toroid is given by $B = \frac{\mu_0 NI}{2\pi r}$ as derived in 10.4.

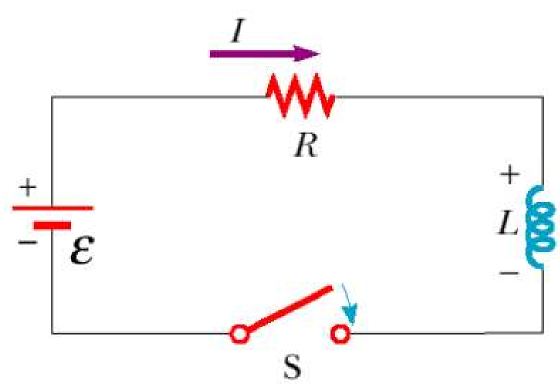
$$L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{N\left(\frac{\mu_0 NI}{2\pi R}\right)A}{I} = \frac{\mu N^2 A}{2\pi R}$$

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{2\pi R}\right)I^2$$

$$\rightarrow U = \frac{\mu_0 N^2 A I^2}{4\pi R}$$

LR Circuits

- RL Circuit – Contains a resistor and inductor in parallel.
- As the current increases towards its maximum value (when switch is closed), an emf is induced in the inductor:



The inductor prevents the current from immediately jumping to ε/R. Instead the current will build to this value over time.

- Voltage across the resistor the resistor, $V_R(t)$, will start at zero, but will build to ε over time.
- Voltage across the inductor, $V_L(t)$, will start at the battery value, ε, giving a back emf to prevent current. At steady state, this emf will drop to zero, and the inductor will act as normal current-carrying wire, but it will have energy stored in its magnetic field.
- Current, $I(t)$, will start at zero since the inductor doesn't let current immediately jump when the battery is connected. Over time, it will build to ε/R at steady state.

By Kirchoff's Law:

$$\varepsilon - V_R - V_L = 0$$

$$\rightarrow \varepsilon - IR - L\frac{dI}{dt} = 0 \rightarrow \varepsilon - IR = L\frac{dI}{dt}$$

Multiplied both sides by -1/R to get I by itself to make integration easier:

$$I - \frac{\varepsilon}{R} = -\frac{L}{R}\frac{dI}{dt}$$

Separate the variables:

$$\frac{dI}{I - \frac{\varepsilon}{R}} = -\frac{R}{L}dt$$

The current starts at 0 and goes to I at some time, t, I(t):

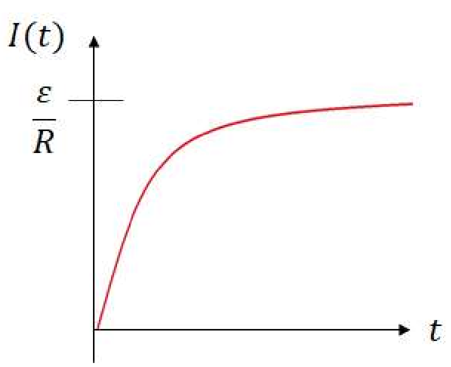
$$\int_0^{I(t)} \frac{dI}{I - \frac{\varepsilon}{R}} = \int_0^t -\frac{R}{L}dt$$

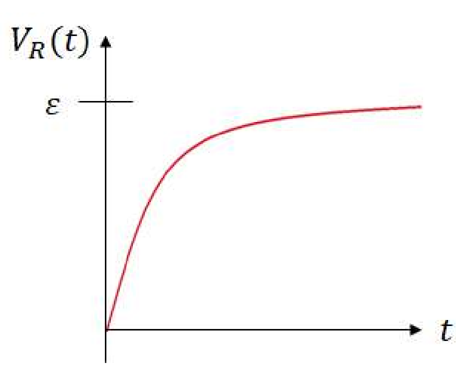
$$\rightarrow \ln\left|I - \frac{\varepsilon}{R}\right|\begin{matrix} I(t) \\ 0 \end{matrix} = -\frac{R}{L}t \rightarrow \ln\left|I(t) - \frac{\varepsilon}{R}\right| - \ln\left|-\frac{\varepsilon}{R}\right| = -\frac{R}{L}t \rightarrow= \ln\left|\frac{I(t) - \frac{\varepsilon}{R}}{-\frac{\varepsilon}{R}}\right| = -\frac{R}{L}t$$

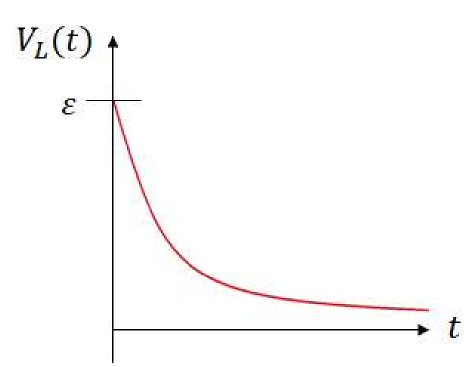
$$\rightarrow \frac{I(t) - \frac{\varepsilon}{R}}{-\frac{\varepsilon}{R}} = e^{-\frac{R}{L}t}$$

- $\boldsymbol{I(t) = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})}$ $\quad\quad$ $\boldsymbol{I(t) = \frac{\varepsilon}{R}(1 - e^{-\frac{t}{\tau}})}$
- The time constant for an LR is $\tau = \frac{L}{R}$.
- Using ohm's law for the resistor, the voltage across the resistor is: $\boldsymbol{V_R(t) = \varepsilon(1 - e^{-\frac{t}{\tau}})}$
- When the voltage is not dropped in the resistor, it is dropped in terms of back emf in the inductor: $\boldsymbol{V_L(t) = \varepsilon e^{-\frac{t}{\tau}}}$

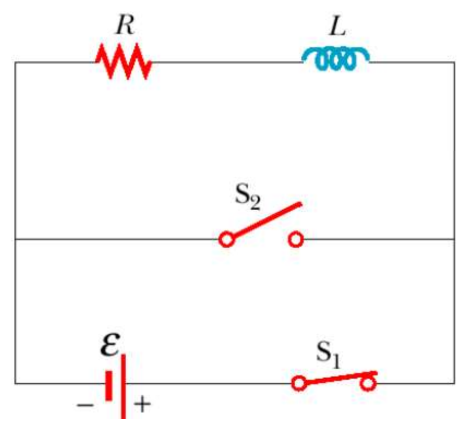
- Charging circuit graphs:







- Discharging Circuit – Suppose switch 1 is open and switch 2 is closed at t = 0 after steady state conditions have been reached.



When the circuit is charged and disconnected, the energy stored in the inductor's magnetic field acts a battery for the resistor. The current will continue flowing in the direction it was going (since the inductor resists the current decreasing) until the inductor runs out of emf.

By Kirchoff's Law:

$V_R + V_L = 0$

$-IR - L\frac{dI}{dt} = 0 \rightarrow IR = -L\frac{dI}{dt} \rightarrow I = -\frac{L}{R}\frac{dI}{dt}$

$\rightarrow \frac{dI}{I} = -\frac{R}{L}dt$

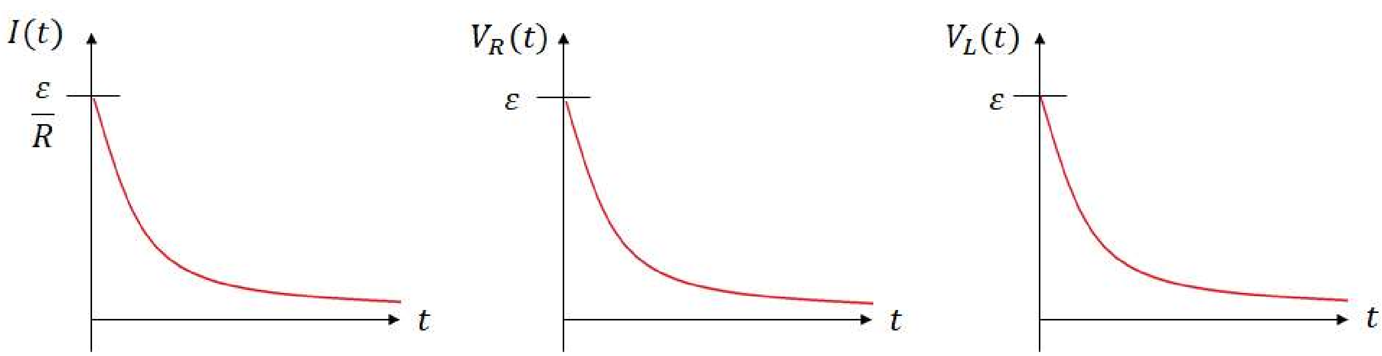
The current starts at $\frac{\varepsilon}{R}$ and decreases to some value, I(t) after time t:

$\int_{\frac{\varepsilon}{R}}^{I(t)} \frac{dI}{I} = \int_0^t -\frac{R}{L}dt$

$\rightarrow \ln|I|\Big|_{\frac{\varepsilon}{R}}^{I(t)} = -\frac{R}{L}t \rightarrow \ln|I(t)| - \ln\left|\frac{\varepsilon}{R}\right| = -\frac{R}{L}t \rightarrow \ln\left|\frac{I(t)}{\frac{\varepsilon}{R}}\right| = -\frac{R}{L}t$

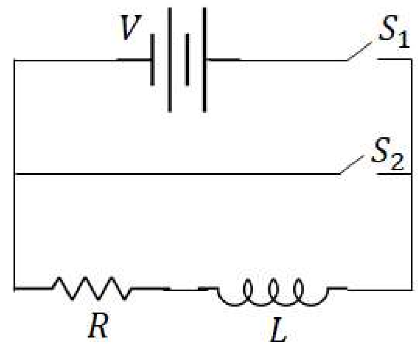
$\rightarrow \frac{I(t)}{\varepsilon/R} = -\frac{R}{L}t$

- $\boldsymbol{I(t) = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})}$ $\boldsymbol{I(t) = \frac{\varepsilon}{R}(1 - e^{-\frac{t}{\tau}})}$
- By ohm's law: $\boldsymbol{V_R(t) = \varepsilon e^{-\frac{t}{\tau}}}$
- The inductor acts as the battery, so its emf drains with the circuit: $\boldsymbol{V_L(t) = \varepsilon e^{-\frac{t}{\tau}}}$
- Disharging circuit graphs:



Example D: Appreciate the circuit given on the right.
Assume V = 100 V, R = 20 Ω and L = 4.0 H.

a) What is the current through resistor R right when the switch is closed?
b) What is the voltage across the inductor just after the switch is closed?
c) Find the potential across the inductor as a function of time.
d) At what rate is the energy stored in the inductor increasing at time t=2 s.0?



a) The current is **zero** initially, as the inductor will not allow the current to immediately jump to a value right when the power in connected.

b) The back emf is originally **100 V** against the direction of the current.

c) The time constant of the inductor is $\tau = \frac{L}{R} = \frac{4\,H}{20\,\Omega} = 0.2$

$V_L(t) = 100e^{-\frac{t}{.2}} \rightarrow \boldsymbol{V_L(t) = 100e^{-5t}}$

d) The rate of change of energy is power.

$$P = \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) = LI\frac{dI}{dt}$$

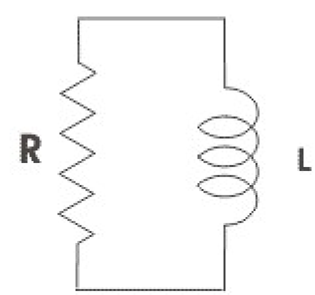
The current as a function of time is given by: $I(t) = \frac{100\,V}{20\,\Omega}(1 - e^{-5t}) = 5(1 - e^{-5t})$

$$\frac{dI}{dt} = 25e^{-5t}$$

$$\rightarrow P = LI\frac{dI}{dt} = (4H)(5(1 - e^{-5(2)})(25e^{-5(2)}) = .02\,W$$

Example E: A L = 5.0 Henry inductor carrying an initial current of 100 A is discharged through a resistor of R = 1.0 Ω.

a) Find the voltage drop across the resistor as a function of time.
b) Find the time needed for the current to reach 10 A.



a) $V_R(t) = IRe^{-\frac{R}{L}t} \rightarrow V_R(t) = (100\,A)(1\,\Omega)e^{-\frac{1\,\Omega}{5\,H}t}$

$\rightarrow V_R(t) = 100e^{-.2t}$

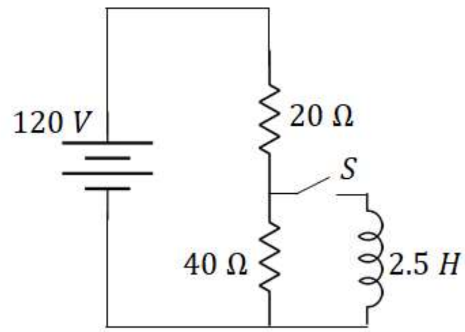
b) $I(t) = I_0e^{-\frac{R}{L}t} \rightarrow I(t) = 100e^{-\frac{1}{5}t}$

$\rightarrow 10 = 100e^{-.2t} \rightarrow .1 = e^{-.2t} \rightarrow \ln|.1| = -.2t$

$$\rightarrow t = \frac{\ln|.1|}{-.2} = 11.5\,s$$

Example F: The circuit on the right is allowed to reach a steady state. At time t = 0, the switch is closed.

a) What is the potential across the inductor at t = 0?
b) At what rate is the current increasing in the inductor at t = 0?
c) How much current flows through the inductor after the switch has been closed for a long time?



a) The inductor is in parallel with the 40 Ω resistor, so it only produces a back emf to match the voltage across the 40 Ω resistor at steady state.

$V_L = \frac{40}{20+40}(120\ V)80\ V$

b) $\varepsilon = -L\frac{dI}{dt} \rightarrow \frac{dI}{dt} = -\frac{\varepsilon}{L}$

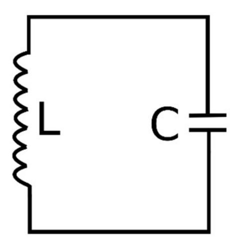
$\rightarrow \frac{dI}{dt} = \frac{80\ V}{2.5\ H} = 32\ \frac{A}{s}$

c) At steady-state, the inductor acts a wire with no resistance. As a result, the 40 Ω is "skipped" by the current, so current flows from the battery, through the 20 Ω resistor and then through the inductor, so current flows through a circuit with only 20 Ω of resistance. The current is then:

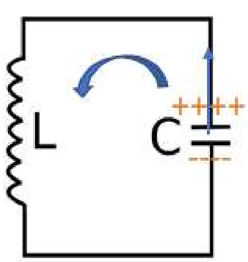
$I = \frac{120\ V}{20\ \Omega} = 6\ A$

The LC Circuit

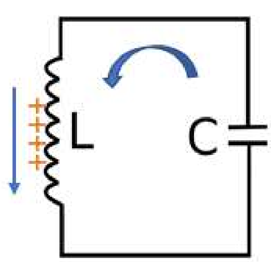
- LC Circuit – Contains a charged capacitor and inductor (with little resistance).



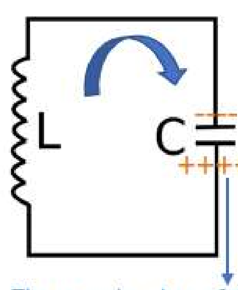
If a fully charged capacitor is connected to an inductor, it will discharge through the inductor. When the capacitor runs out of charge to discharge through the inductor, the inductor will produce an induced current to stop the current from decreasing, sending charge to the other plate of the conductor. The opposite plate eventually becomes fully charged, and then discharges again through the inductor, which does its thing again. The LC oscillates as a result.



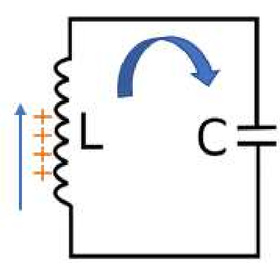
The capacitor discharges through the inductor.



When the capacitor's charge is depleted, the inductor resists the current form decreasing, "pushing" the charge to the opposite plate of the capacitor



The opposite plate of the capacitor now has the full charge and discharges in the opposite direction.



The circuit oscillates, with the current switching directions constantly.

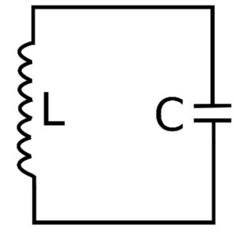
By Kirchoff's Law: $V_C + V_L = 0$

$\rightarrow \frac{q}{C} + L\frac{di}{dt} = 0$

Since $i = \frac{dq}{dt}, \frac{di}{dt} = \frac{d^2q}{dt^2}$

$\rightarrow \frac{q}{C} + L\frac{d^2q}{dt^2} = 0$

$\rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$



The energy above matches the equation for simple harmonic motion from AP Physics C: Mechanics:
$(\frac{d^2x}{dt^2} + \omega^2 x = 0)$

In this case, the variable is the charge over time and the angular frequency is $\omega = \sqrt{\frac{1}{LC}}$.

- The period of an RC circuit is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{1}{LC}}} \rightarrow \boldsymbol{T = 2\pi\sqrt{LC}}$
- The total energy in an LC circuit is $E = \frac{1}{2}LI^2 + \frac{1}{2}CV_C^2$
- From the general solution to the differential equation, the charge over time can be given by: $Q(t) = Q_0\cos(\omega t) \rightarrow Q(t) = Q_0\cos(\frac{1}{\sqrt{LC}}t)$

Rate your understanding: Inductance

0	1	2	3	4
L	I understand some concepts in inductance, but can't solve complex problems.	I understand inductance and can solve most problems involving it.	I can solve problems in inductance without any significant errors.	I can explain and teach inductance.

11.5 Maxwell's Equations (Chapter 34)

Focus Question: What is the purpose of Maxwell's Equations?

Maxwell's equations are four equations that unite electricity and magnetism and can be used (along with Lorentz force) as the basis of all phenomena in these areas.

- Gauss's Law – The total flux through any closed surface equals the net charge inside that surface divided by the permittivity of free space.

$$\oint \boldsymbol{E \cdot dA} = \frac{\boldsymbol{Q}}{\boldsymbol{\varepsilon_0}}$$

∗Charges create electric fields.

- Gauss's Law for Magnetism – The net magnetic flux through a closed surface is zero.

$$\oint \boldsymbol{B \cdot dA} = \boldsymbol{0}$$

**Isolated monopoles cannot exist.*

- Ampere's Law – The line integral of the magnetic field around any closed path is determined by the net current and the rate of change of electric flux through any surface bounded by that path.

$$\oint \boldsymbol{B \cdot dl} = \boldsymbol{\mu_0 I + \varepsilon_0 \mu_0 \frac{d\phi}{dt}}$$

**An electric current and a changing electric field create a magnetic field.*

- Faraday's Law of Induction – The line integral of the electric field around any closed path equals the rate of change of magnetic flux through any closed surface bounded by that path.

$$\oint \boldsymbol{E \cdot dr} = \boldsymbol{\varepsilon} = -\boldsymbol{\frac{d\phi}{dt}}$$

**A changing magnetic field creates an electric field.*

Rate your understanding: Maxwell's Equations

0	1	2	3	4
I ran. I ran so far away from physics.	Somebody once told E&M was gonna roll me.	I'll understand this in a day or twoooooooo!!!	We're no strangers to physics. You know the rules and so do I.	It's going to take a lot to drag me away from you.