Straight lines and circles in the complex plane, Line integrals

. 1. Determine the equation of straight lines (including the parametric form) in the complex plane that connects the points

(a) $z_1 = -1$; $z_2 = i$; (b) $z_1 = 1 + i$, $z_2 = -2$; (c) $z_1 = 1 - i$ $z_2 = 2 + 2i$

2. Determine the equation of circle in the complex plane (including parametric form) whose centre and radius are given by

$$(a)\ z_0 = 0,\ \ R = 1/2; \quad (b)\ z_0 = 1 - i,\ \ R = 2$$

3. Let C be the line from $z_1 = 1$ to $z_2 = -1 - i$. Calculate

$$\int_C |z|^2 dz$$

and express your result in the form $a + ib$.

4. Evaluate $\int_C z^2 dz$ along the straight line C that joins the origin and the point $P(2, 1)$ in the complex plane.

5. Integrate

$$\int_C z^2 dz$$

where C is the unit $|z| = 1$ traversed in an anticlockwise direction.

6. Integrate

$$\int_C \overline{z} dz$$

where C is the unit $|z| = 1$ traversed in an anticlockwise direction.

7. Let C be the top half of a circle of radius 2 centred at $z_0 = 1 + i$, oriented in the anti-clockwise direction. Calculate

$$\int_C Re(z)\ dx$$

Solutions:

3. $-\frac{4}{3} - \frac{2i}{3}$

4. $\frac{2}{3} + \frac{11i}{3}$

5. 0

6. $2i\pi$

7. $2i\pi - 4$