Geometry of Surfaces - Exercises

Solutions to exercises marked with $*$ are to be submitted online through the link on the Keats page for this module.

62. Consider the unit sphere S^2 with the parametrization

$$\sigma(\theta, \varphi) = (\cos(\theta)\cos(\varphi), \cos(\theta)\sin(\varphi), \sin(\theta))$$

($\frac{\pi}{2} < \theta < \frac{\pi}{2}$, $0 < \varphi < 2\pi$). Determine the geodesics on S^2 by solving the geodesic equations.

63. Let $\gamma(t) = \sigma(u(t), v(t))$ be a unit speed curve on a surface of revolution

$$\sigma(u, v) = (f(u)\cos(v), f(u)\sin(v), g(u))$$

with $\dot{f}^2 + \dot{g}^2 = 1$. Denote by $\rho(u, v)$ the distance between $\sigma(u, v)$ and the axis of rotation and by $\psi(t)$ the angle between $\dot{\gamma}(t)$ and the meridian through $\gamma(t)$; thus $\rho(u, v) = f(u)$ and $\cos(\psi(t)) = \dot{\gamma}(t) \cdot \sigma_u(u(t), v(t))$. Prove the following statements:

(i)* If γ is a geodesic, then $\rho\sin(\psi)$ is constant along γ.

(ii) If $\rho\sin(\psi)$ is constant along γ, and if no part of γ is part of some parallel of the surface, then γ is a geodesic.