COMP SCI 2LC3

Assignment #3. Due December 6 (Tuesday), 2022, 23:59 via Avenue. Do not hesitate to discuss with TA or instructor all the problems as soon as you discover them. This assignment is a little bit more difficult than the first one. Start early!

Total: 67 pts

Instructions: For all assignments, the students must submit their solution to Avenue → Assessments → Assignment #

Students can simply solve the exercises on a paper and use a smartphone app called CamScanner and convert their entire solution into a single PDF file and submit it to avenue. The maximum upload file size is 2Gb in avenue for each submission.

Please make sure that the final PDF file is readable.

Students, who wish to use Microsoft word and do not have Microsoft Word on their computer, are suggested to use google document editor (Google Docs). This online software allows you to convert your final file into PDF file.

There will be a mark deduction for not following the submission instruction.

Please first finish the assignment on your local computer and at the end, and attach your solution as a PDF file.

You will have unlimited number of submissions until the deadline.

Students must submit their assignments to Avenue. Any problem with Avenue, please discuss with Mahdee Jodayree <mahdijaf@yahoo.com>, the lead TA for this course.

Study of Chapters 11, 12, 13, 14 and 18 of the Gries-Schneider textbook and Lecture Notes from 9 to the last one is highly recommended for this assignment.

Questions.

1.[2] Exercise 11.6 (page 213 of the Gries-Schneider textbook).

2.[2] Exercise 11.13 (page 214 of the Gries-Schneider textbook), question (e).

3.[2] Exercise 11.17 (page 215 of the Gries-Schneider textbook).

4.[3] Consider the following proof by induction.

Theorem. For all natural number $n \geq 1$, we have $(*)$ $\sum_{i=1}^{n} i = \frac{1}{2}(n+\frac{1}{2})^2$.

Proof: We prove the claim by induction.

Base step: When $n = 1$, $(*)$ holds.

Induction step: Let $k \in \mathbb{N}$ and suppose $(*)$ holds for $n = k$. Then

$$\begin{aligned}
\sum_{i=1}^{k+1} i &= \sum_{i=1}^{k} i + (k+1) \\
&= \frac{1}{2}\left(k+\frac{1}{2}\right)^2 + (k+1) \quad \text{(by ind. hypothesis)} \\
&= \frac{1}{2}\left(k^2+k+\frac{1}{4}+2k+2\right) \quad \text{(by algebra)} \\
&= \frac{1}{2}\left(\left(k+1+\frac{1}{2}\right)^2 - 3k - \frac{9}{4} + k + \frac{1}{4} + 2k + 2\right) \quad \text{(more algebra)} \\
&= \frac{1}{2}\left((k+1)+\frac{1}{2}\right)^2 \quad \text{(simplifying).}
\end{aligned}$$

Thus, $(*)$ holds for $n = k+1$, so the induction step is complete.

Conclusion: By the principle of induction, $(*)$ holds for all $n \in \mathbb{N}$.

Where is an error?

5.[4] Consider the following proof by induction.

Theorem. For every nonnegative integer n, $(*)$ $5n = 0$.

Proof: We prove that $(*)$ holds for all $n = 0, 1, 2, \ldots$, using strong induction with the case $n = 0$ as base case.

Base step: When $n = 0$, $5n = 5 \cdot 0 = 0$, so $(*)$ holds in this case.

Induction step: Suppose $(*)$ is true for all integers n in the range $0 \leq n \leq k$, i.e., that for all integers in this range $5n = 0$. We will show that $(*)$ then holds for $n = k+1$ as well, i.e., that $(**)$ $5(k+1) = 0$.

Write $k+1 = i+j$ with integers i, j satisfying $0 \leq i, j \leq k$. Applying the induction hypothesis to i and j, we get $5i = 0$ and $5j = 0$. Then

$$5(k+1) = 5(i+j) = 5i + 5j = 0 + 0 = 0,$$

proving $(**)$. Hence the induction step is complete.

Conclusion: By the principle of strong induction, $(*)$ holds for all nonnegative integers n.

Where is an error?

6.[3] Exercise 12.6 (page 244 of the Gries-Schneider textbook).

7.[3] Exercise 12.14 (page 244 of the Gries-Schneider textbook).

8.[4] Exercise 12.32 (page 246 of the Gries-Schneider textbook).

9.[4] Exercise 12.37 (page 246 of the Gries-Schneider textbook).

10.[2] Exercise 12.40 (page 247 of the Gries-Schneider textbook).

11.[6] Exercise 12.42 (page 247 of the Gries-Schneider textbook), question (a).

12.[6] In the textbook questions, preconditions, postconditions and loop invariants are usually given. When applying this method in practice, one has to define them, so the problems look usually as the below one:

Problem:

Assume that some processor (which uses multiplication very seldom and must use as less energy as possible) does not have hardware implementation of multiplication. Instead, when needed, it uses the following procedure (written in an easily understood pseudocode, for easier proof of correctness):

procedure *multiply*$(m.n : integers,$ **return** $product : integers)$
  if $n < 0$ **then** $a := -n$ **else** $a := n$;
  $k := 0; x := 0$;
  while $k < a$ **do**
  begin
  $x := x + m; k := k + 1$;
  end;
  if $n < 0$ **then** $product := -x$ **else** $product := x$
  end of procedure

Prove the correctness (i.e. $product = m \cdot n$) and termination using Hoare triples (i.e. Lecture Notes 10 and Chapter 12 of the textbook).

13.[8] (a) Prove the correctness of the program from Question 12 using relational method presented in Lecture Notes 12.
(b) Discuss advantages and disadvantages of both methods. Be honest, express your true feelings.

14.[2] Exercise 14.26 (page 300 of the Gries-Schneider textbook).

15.[2] Exercise 14.29 (page 301 of the Gries-Schneider textbook).

16.[4] Do we necessarily get an equivalence relation when we form the transitive closure of the symmetric closure of the reflexive closure of a relation? Prove your answer, just ‘yes’ or ‘no’ is not enough.

17.[4] Assume that Theorem from page 25 of Lecture Notes 13 is a definition of interval orders.
Prove that the partial order $\prec_4$ from page 26 of Lecture Notes 13 is not an *interval* order.

18.[2] Suppose that $(S_1, \preceq_1)$ and $(S_1, \preceq_2)$ are posets. Show that $(S_1 \times S_2, \preceq)$ is a poset where $(s,t) \preceq (u,v)$ iff $s \preceq_1 u \wedge t \preceq_2 v$.

19.[2] Exercise 18.16 (page 418 of the Gries-Schneider textbook).

20.[2] Exercise 18.60 (page 420 of the Gries-Schneider textbook).