TUTORIAL 3: complex series II
Laurent series

1. Find the Laurent series expansions of

$$f(z) = \frac{1}{2i + (2 - i)z - z^2} \quad \text{in the region } 1 < |z| < 2$$

2. Find the Laurent series expansion of the function

$$f(z) = \frac{z + 1}{z^3(z^2 + 1)}$$

valid when $|z| > 1$.

3. Find the Laurent series expansion of

$$f(z) = \frac{1}{(z + 1)(z + 3)}$$

valid for (a) $1 < |z| < 3$, (b) $0 < |z + 1| < 2$, (c) $0 < |z + 3| < 2$.

Answers

1.
(i)
$$\frac{2-i}{5}\left[\frac{1}{2}+\sum_{n=1}^{\infty}\left(\frac{z^n}{2^{n+1}}-\frac{1}{i^{n+1}z^n}\right)\right]$$

2.
$$-\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{i^{n+1}z^{n+3}}\left[1+i-(1-i)(-1)^n\right]$$

3.
(a)
$$\frac{1}{2}\left[\frac{1}{3}+\sum_{n=1}^{\infty}\left(\frac{(-z)^n}{3^{n+1}}-\frac{1}{(-z)^n}\right)\right]$$

(b)
$$\sum_{n=0}^{\infty}\frac{(-1)^n(z+1)^{n-1}}{2^{n+1}}$$

(c)
$$-\sum_{n=0}^{\infty}\frac{(z+3)^{n-1}}{2^{n+1}}$$