1: The First Problem

(a)

$$(s \wedge e) \vee (m \wedge e) \vee (\neg s \wedge \neg m \wedge e)$$

(b)

(c)

$$\begin{aligned}
p \leftrightarrow q &\equiv (p \to q) \wedge (q \to p) \\
&\equiv (\neg p \vee q) \wedge (\neg q \vee p) \\
&\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p) \\
&\equiv ((\neg p \wedge \neg q) \vee (q \wedge \neg q)) \vee ((\neg p \wedge p) \vee (q \wedge p)) \\
&\equiv (\neg p \wedge \neg q) \vee (q \wedge p) \\
&\equiv (p \wedge q) \vee (\neg p \wedge \neg q)
\end{aligned}$$

2: The second problem

To prove

$$(p \to q) \wedge (p \to r) \to (p \to (q \wedge r))$$

3: The third problem

$$\begin{aligned}
(P \to Q) \vee (Q \to R) &\equiv (\neg P \vee Q) \vee (\neg Q \vee R) \\
&\equiv \neg P \vee ((Q \vee \neg Q) \vee R) \\
&\equiv \neg P \vee (R \vee (Q \vee \neg Q)) \\
&\equiv (\neg P \vee R) \vee (Q \vee \neg Q) \\
&\equiv (\neg P \vee R) \vee \text{True} \\
&\equiv \text{True}
\end{aligned}$$

7:

$$s_0 = a_0 + b_0$$

$$s_1 = (a_1 + b_1) + a_0 b_0$$

$$\begin{aligned}
s_2 &= (a = (11)_2 \wedge b \neq (00)_2) \vee (b = (11)_2 \wedge a \neq (00)_2) \\
&= a_1 a_0 \overline{b_0 b_1} + b_1 b_0 \overline{a_0 a_1} + \overline{a_1 a_0 \overline{b_0 b_1} + b_1 b_0 \overline{a_0 a_1}}
\end{aligned}$$

8:

$x\bar{y}$

$x\overline{yz}$

$$
\begin{aligned}
(x \wedge y \wedge z) \vee (\neg x \wedge \neg y \wedge \neg z) &= xyz \vee ((1-x)(1-y)(1-z)) \\
&= xyz + (1-x)(1-y)(1-z) - xyz(1-x)(1-y)(1-z) \\
&= xy + yz + zx - x - y - z + 1 \\
&= xy + (yz - 1) + (zx - 1) + (1-x) + (1-y) + (1-z) \\
&= xy + \overline{yz} + \overline{zx} + \bar{x} + \bar{y} + \bar{z}
\end{aligned}
$$

$$a + b + c + d$$

$$(a+b+c+d) \vee (\bar{a}+\bar{b}+\bar{c}+\bar{d}) = (a+b+c+d) + (\bar{a}+\bar{b}+\bar{c}+\bar{d}) - (a+b+c+d)(\bar{a}+\bar{b}+\bar{c}+\bar{d})$$