

MATH 465 - INTRODUCTION TO COMBINATORICS
HOMEWORK 6

- (1) Find the number of derangements of $[12]$ in which $[6]$ is mapped onto:
 - (a) $[6]$ in some order. For example, 4, 3, 2, 6, 5, 1, 10, 9, 11, 12, 8, 7 is such a derangement.
 - (b) $\{7, 8, \dots, 12\}$ in some order.
- (2) How many two-digit positive integers are relatively prime to both 2 and 3?
- (3) Let $F(n, k)$ be the number of partitions of $\{1, \dots, n\}$ into k blocks where each block contains more than one element. Express the numbers $F(n, k)$ in terms of the Stirling numbers of the second kind.
- (4) Eight girls are seated around a carousel. In how many ways can they change seats so that each has a different girl in front of her?
- (5) Eight boys are seated around a carousel but facing inward, so that each boy faces another. In how many ways can they change seats so that each boy faces a different boy?
- (6) Determine the number of permutations of $\{1, \dots, 9\}$ in which at least one odd number is in its natural position.
- (7) We say that $w = (w_1, \dots, w_n) \in S_n$ is an *up-down permutation* if

$$w_1 < w_2 > w_3 < w_4 > \dots$$

For example, the up-down permutations in S_4 are 1324, 1423, 2314, 2413 and 3412. How many up-down permutations are there in S_7 ?