1: The First Problem

Let $d = \gcd(x, y)$, $x = x'd, y = y'd$. Then $\gcd(x', y') = 1$ Then

$$(x'd)^2 = 2(y'd)^2 \implies x'^2 = 2y'^2$$

Then we deduced that x' and y' are even, which contradicts to that fact that $\gcd(x', y') = 1$

1.6.2:

$$n^2 + 7n + 12 = (n^2 + n) + (6n + 12)$$

1.7:

Let $n = 7m^2$, then $7n + 2 = 7(7m^2) + 2 = (7m)^2 + 2$ This satisfies that

$$(7m)^2 < (7m)^2 + 2 < (7m)^2 + 14m + 1 = (7m + 1)^2$$

2.3.4:

$$D_n = \{d|d \text{ divies } n\}$$

$$D_r \cap D_s = \{d|d \text{ divides } r \text{ and } d \text{ divides } s\}$$

Let $m = \gcd(r, s)$

$$D_m = \{d|d \text{ divides } m\}$$

We now have to show that

$$D_m \subseteq D_r \cap D_s$$

. And

$$D_r \cap D_s \subseteq D_m$$

Let $d \in D_m$, then d divides m. With m dividing r and s, we have d divides r and s. Thus,

$$D_m \subseteq D_r \cap D_s$$

On the other hand, Let $d \in D_r \cap D_s$, we have d divides both r and s, then d divides m by the definition of m. Therefore,

$$D_r \cap D_s \subseteq D_m$$

Together, we have proven that

$$D_m = D_r \cap D_s$$

2.4:

Those subsets can be divided into $n+1$ groups, and in each group, each subset has r elements for $r = 0, 1, \cdots, n$. So in total there are

$$\sum_{r=0}^{n} \binom{n}{r}$$

such subsets