

1.4: Average Rate of Change

Problem 1. What is the **average rate of change** (A.R.o.C) and how do you calculate it?

Problem 2. How is the average rate of change related to secant and tangent lines?

Problem 3. Find the average rate of change of $y = \sqrt{x-3}$ between $x = 4$ and $x = 7$.

Problem 4. Find the average rate of change of $f(x) = \sin x + 3$ on the interval $[\pi/6, \pi/3]$.

Problem 5. A climber is on a hike. After 1 hour, he is at an altitude of 200 feet. After 5 hours, he is at an altitude of 600 feet. Find his average rate of change.

Problem 6. Use the table given below to calculate the average rate of change over the interval $[2,6]$.

Time (s)	0	2	6	9	15
Distance (ft)	3	8	20	10	10

Problem 7. Let $f(x) = x^2 - 6x + 5$.

(a) Find the average rate of change of $f(x)$ over the interval $[3,5]$.

(b) Sketch a graph of part (a) using the top graph.

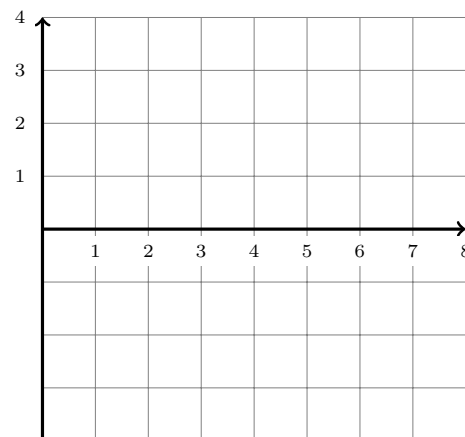
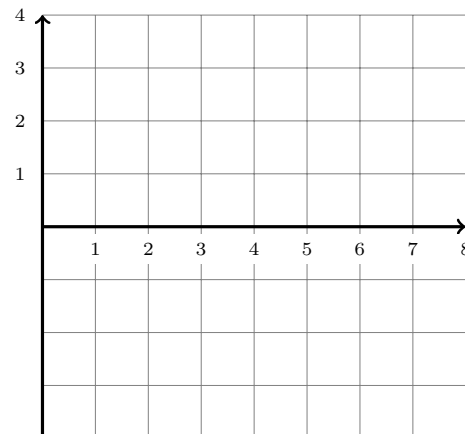
(c) Find slope of the secant line joining $P(4, f(4))$ and $Q(4 + h, f(4 + h))$.

(d) What is the slope of the secant line if $h = 1$? $h = -1$?

(e) Sketch a graph of part (d) using the bottom graph.

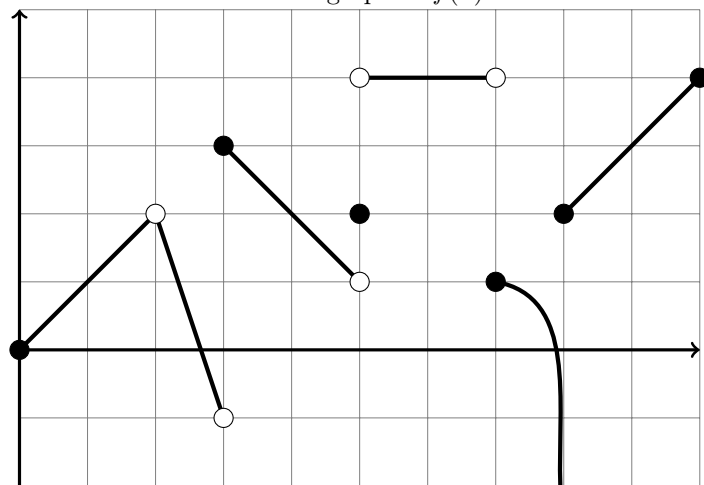
(f) What is the slope of the secant line if h is very close to 0?

(g) Write an equation of the tangent line at $x = 4$.



1.5: Limits and Vertical Asymptotes

Problem 8. Consider the graph of $f(x)$ below. Evaluate the following:



(a) $\lim_{x \rightarrow 1} f(x) =$

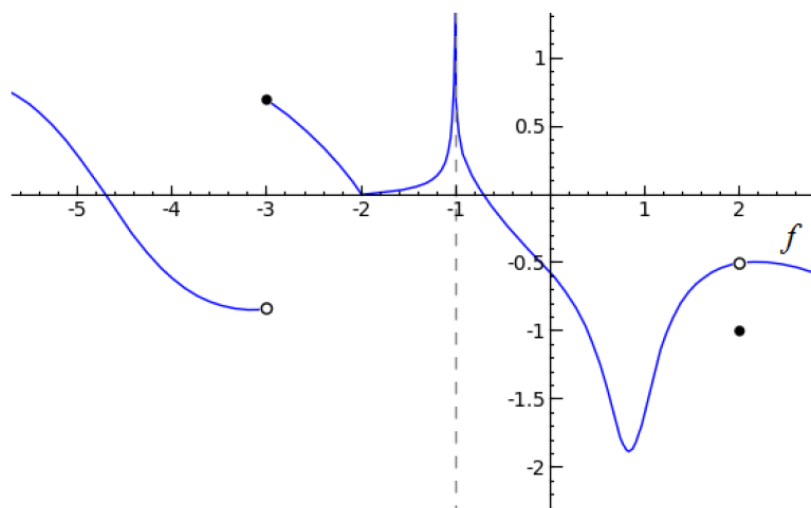
(b) $\lim_{x \rightarrow 2} f(x) =$

(c) $\lim_{x \rightarrow 3} f(x) =$

(d) $\lim_{x \rightarrow 4} f(x) =$

(e) $\lim_{x \rightarrow 3^-} f(x) =$

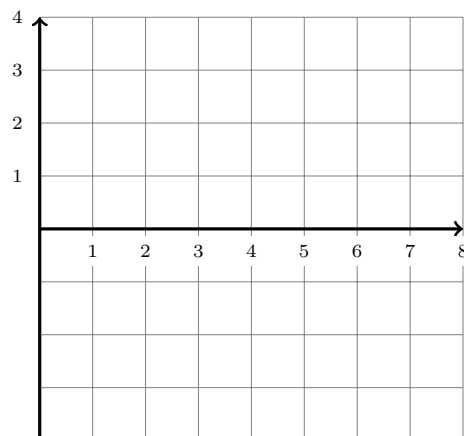
Problem 9. Describe the function $f(x)$ shown below using at least 3 one-sided and 3 two-sided limit statements.



Problem 10. Sketch the graph of the following function and find $\lim_{x \rightarrow 3} g(x)$.

$$g(x) = \begin{cases} -x & , \quad x < 3 \\ -1 & , \quad x = 3 \\ x^2 - 13 & , \quad x > 3 \end{cases}$$

What is $g(3)$? Does that value matter when finding $\lim_{x \rightarrow 3} g(x)$?



Problem 11. How do you find the vertical asymptotes of a rational function?

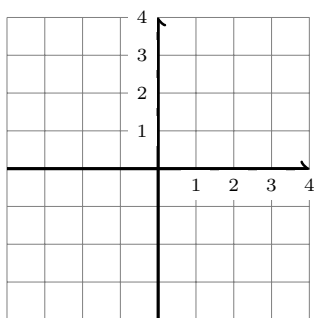
Problem 12. Find the vertical asymptotes of the following functions:

(a) $f(x) = \frac{1}{(x-1)(x+3)}$

(b) $g(x) = \frac{x-1}{(x-1)(x+3)}$

(c) $h(x) = \frac{x^2+3x+2}{x^2-4}$

Problem 13. Graph $f(x) = \frac{1}{x^2}$.

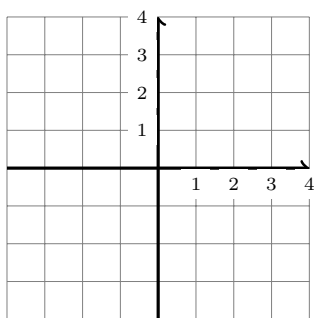


(a) $\lim_{x \rightarrow 0^-} \frac{1}{x^2} =$

(b) $\lim_{x \rightarrow 0^+} \frac{1}{x^2} =$

(c) $\lim_{x \rightarrow 0} \frac{1}{x^2} =$

Problem 14. Graph $f(x) = \frac{1}{x-5}$.



(a) $\lim_{x \rightarrow 5^-} \frac{1}{x-5} =$

(b) $\lim_{x \rightarrow 5^+} \frac{1}{x-5} =$

(c) $\lim_{x \rightarrow 5} \frac{1}{x-5} =$

Problem 15. Webwork Help! 1.5 #12: Consider the function $f(x) = x^3 \sec x$ on the interval $[0, \pi]$.

(a) When does f have a vertical asymptote?

(b) Evaluate $\lim_{x \rightarrow \frac{\pi}{2}^-} x^3 \sec x$ and $\lim_{x \rightarrow \frac{\pi}{2}^+} x^3 \sec x$.