3. **Visualisation of a Newton fractal**

[10 marks total]

The convergence of Newton's method from different initial guesses is very unpredictable. This can be illustrated by visualising *Newton fractals*. A Newton fractal for a complex-valued polynomial function, which we choose here as

$$f(z) = z^3 - z^2 + z - 1, \quad z \in \mathbb{C},$$

can be computed by solving the equation $f(z) = 0$ with Newton's method starting from different initial guesses $z_0 \in \mathbb{C}$ in the complex plane and visualising either the number of iterations required for convergence or the specific root that is found (see for example Figure 1 of the Newton fractal for $f(z) = z^3 - 1$).

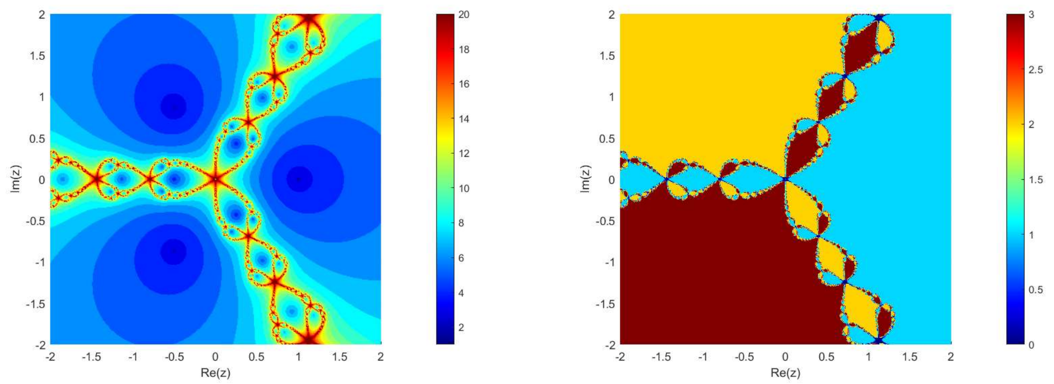

Figure 2: Newton fractals plotted by number of iterations (left) and the root reached (right)

(a) Write a MATLAB/Python code to solve the equation $z^3 - z^2 + z - 1 = 0$ using Newton's method for initial values z_0 in the rectangle $R := [-2, 2] \times [-2, 2] \subset \mathbb{C}$. Discretise the rectangle R using a regular grid of size 401×401 and for each grid point z_0 solve the problem using Newton's method with the initial guess z_0, stopping tolerance 10^{-6} and a maximum of 20 Newton iterations.

(b) Plot the Newton fractal for $z^3 - z^2 + z - 1$ in the rectangle R *coloured by the number of iterations required* $iters(z_0)$ to converge from each grid point z_0. Provide a colourbar indicating how many Newton iterations the method requires to converge from each point z_0. Indicate in the figure where the roots of the function $f(z)$ are located.

(c) Plot the Newton fractal for $z^3 - z^2 + z - 1$ in the rectangle R *coloured by which of the roots was reached* $z^*(z_0)$ to converge from each grid point z_0. Provide a colourbar indicating which of the roots was reached. Indicate in the figure where the roots of the function $f(z)$ are located.