Effect of Diversification GUIDE

slide 2

Title: Portfolio Variance

Remember from PortfolioPresentation.pptx slide 23 that the risk of a portfolio,
that is, the variance to be minimized, is expressed as in (1) on this slide.
Remember from PortfolioPresentation.pptx .pptx slide 17 that the covariance matrix
has variances along the diagonal and covariances off the diagonal.
Variances are usually denoted by sigma-sub-i-comma-i
Covariances are usually denoted by sigma-sub-i-comma-j,
We have replaced the c-sub-i-j's in (1) by sigmas in (2).

slide 3

Title: Portfolio Variance

In harmony with this notation,
(1) can be re-written reorganizing the terms into two groups as in (2):
one group comes from the variance of the eligible investment assets,
i.e. from the diagonal of C, when n=m;
the other comes from the covariance effects,
i.e. from the off-diagonal elements of C,
when n is different from m.
To illustrate the effect of diversifying one's capital among many assets,
assume that:
1/Nth of the capital is invested in each of the N assets in the eligible investment universe.
Using this assumption in (2) leads to expression (1) on the next slide...

slide 4

Title: Portfolio Variance

As you can see in (1)
1/Nth of the capital is invested in each of the N assets.
In (2) we do a factorization of (1/N) outside the summation.
This leaves the remaining part of the first term equal to the average variance,
indicated with the red arrow on the slide.
Also, the factorization of (N−1)/N outside the double summation in the second term of (2)
leaves the remainder equal to the average covariance.
indicated by the green arrow on the slide.
This is because there are N times (N−1) off-diagonal terms in the covariance matrix,
so dividing the sum of the covariances by this factor
gives the average covariance.
What happens when we invest in an infinite number of assets (when N grows to infinity)?

slide 5

Title: Infinite Asset Investment

Letting N approach infinity gives the limits in (1) and (2).
In other words, when the investor's capital is well distributed among the eligible investment vehicles, the risk that stems from the individual assets is diversified away.
This is indicated by the expression in (1) approaching zero at the limit.
What remains approaches the average covariance in (2).
It is seen that the covariance terms, in this case the average covariance, cannot be diversified away.

This part of the risk is also referred to as the systematic risk of the portfolio.

slide 6

Title: Effect of Diversification

In reality it is (naturally) not possible to distribute the capital among an infinite number of assets, nor is it necessary for achieving diversification benefits.
In most cases, the diversification effect is considerable
even when only 15 assets are included in the portfolio and
the marginal benefit of including more than 50 assets
is for all practical purposes equal to zero.
Actually, this statement is only true when dealing with an investment universe where
there is one major risk factor, as is the case for equities and government bonds
but not when investigating an asset universe comprising corporate bonds.

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Title: EffectOfDiversification.py

To illustrate the issue of diversification the Python script in EffectOfDiversification.py
simulates a covariance matrix and
calculates the variance of portfolios with an increasing number of assets included.

slide 8

Title: EffectOfDiversification.py (script)

This slide explains how the script is programmed.

slide 9

Title: EffectOfDiversification.py (graph)

The output of the script is shown in this slide, which
confirms the above-mentioned diversification gains when including additional assets in the portfolio.
It is worth remembering, though, that
the Python script uses a simulated covariance matrix
which may not fully reflect covariance matrices estimated from real-life data.

Still, the example proves the point and shows how important it is to be aware of diversification
benefits and limits.

slide 9

Title: How much data is needed?

Markowitz portfolios need to be solved by inverting the covariance matrix,
so it is important to avoid dealing with a singular covariance matrix that cannot be inverted.
Some guidelines for avoiding this are given on the slide.
On the other hand,
too much data (more than a year)
will prevent the portfolio from benefiting from momentum,
so a sweet spot needs to be reached.

The general rule to prevent the covariance matrix from becoming singular is
to require the amount of data to be at least 1/2*N*(N+1) independent and identically distributed observations,
where N is the number of assets.
This means that if we want the window to be quite short (e.g. 60 days) with the objective of
allowing the portfolio to benefit from the most relevant momentum information, then
the number of assets must be 11 or less; otherwise, the covariance matrix could become singular.
But we saw in the previous slide that we want to have between 15 to 30 assets and no more than 50 assets
to diversify optimally.
One way around this problem is to keep the lookback window short and
the number of assets at 11 but use a mixture of ETFs and stocks or
to use only 11 ETFs.
Another way around this is to keep the lookback window quite short and
increase the number of assets to between 15 and 30 but
use algorithms like the Critical Line Algorithm and Hierarchical Clustering
that are not affected if the covariance matrix is singular.