CS 131 – Problem Set 5

Problems must be submitted by Monday February 27, 2023 at 11:59pm, on Gradescope.

Problem 1. [20 points, 4 each]

Let S be the set of all students at BU. Let C be the set of all the classes offered at BU. Let CAS(x) result in $True$ if student x is enrolled in the College of Arts and Sciences. Let T(x, y) result in $True$ if student x is registered in class y. Let L(x, y) result in $True$ if student x is always late for class y.

Given the following sentences, write the correct quantified expression.

i) Every student at BU is registered for a class.

ii) Every student at BU that is enrolled in CAS is registered for a class.

iii) There is a student at BU who is enrolled in CAS who is never late for any of the classes they are taking.

iv) There exists a class for which all the students registered in it are always late.

v) All classes at BU have at least one student registered in it who is always late.

Problem 2. [17 points]

a) [10 points] Let $A = \mathbb{R}^3$, and let $a, b \in A$ (they are 3-d points). Define the new relation $a \sim b$ that a and b have the same z coordinate, i.e. $(1, -3, 1)$ $(10, 11, 1)$ but $(1, -3, 1) \not\sim (1, -3, 0)$. Show $\sim$ is an equivalence relation. Let $p \in \mathbb{R}^3$. Describe $[p]$ geometrically.

b) [7 points] When Chuhan was taking CS 131 three years ago, he thought *reflexivity* is unnecessary since he though it can be derived from *symmetry* and *transitivity.* He wrote down a piece of proof shown below. Do you think are there any mistakes in Chuhan's proof? If you think Chuhan's proof is correct, justify your reasoning; if you think Chuhan's proof is wrong, please help him point it out, and Chuhan will appreciate it.

> **Chuhan's Claim**: *Suppose $\sim$ is a relation on A. If $\sim$ is symmetric and transitive, then $\sim$ is reflexive.*
>
> *Chuhan's Proof:* Let a be an arbitrary element in A. Let b be an element in A such that $a \sim b$. By symmetry, $b \sim a$; Since $a \sim b$ and $b \sim a$, $a \sim a$ by transitivity. Hence, since x is arbitrarily selected, by the insight of universal generalization, we've proven $\forall x, x \sim x$. Hence, *reflexivity* is totally unnecessary. ■

What's wrong with this argument?

Problem 3. [21 points, 7 each] In each case, check whether or not R is reflexive, symmetric, and transitive (**Note:** you have to check ALL 3 properties), then determine if R is a equivalence relation on A. If it is an equivalence relation, what are the equivalence class? And how many equivalence classes are there? If it is not an equivalence relation, provide a counterexample.
Denote: W is the set of all words in 2022 edition of the Oxford English Dictionary.

a) $R := \{(x, y) \in W \times W \mid \text{the words } x \text{ and } y \text{ start with the same letter}\}$

b) $R := \{(x, y) \in W \times W \mid \text{the words } x \text{ and } y \text{ have at least one letter in common}\}$

c) $R := \{(x, y) \in W \times W \mid \text{the word } x \text{ comes before the word } y \text{ alphabetically}\}$

Problem 4. [18 points] You are a lonely logician in desperate need of a friend. However, knowing that it would be impossible to make one, you decide to focus your time on something much more achievable: creating a general AI (artificial intelligence) who will then be your friend!

Amazingly, you succeed! You make yourself a friend whom you name Alan. Now all you need to do is teach Alan what "friendship" is. You decide to model "friendship" as a relation on a set of people. Rather than explicitly define the relation, you decide to explain some rules that any relation modelling friendship must follow. These are your three laws of robotic friendships:

1. "Not everyone is friends with everyone" (as you harshly know from your lack of human friends).
2. "The enemy of my enemy is my friend" (this is a good saying you've heard before! For simplicity, you tell Alan "enemy" just means "not friend").
3. "The enemy of my friend is my enemy."

With these three laws defining friendship, what will Alan learn?

a) [6 points] Draw the "directed graph" of a relation on a set of 3 people which satisfies the three properties above (i.e. a dot for each person and an arrow **from** person a **to** person b if friend(a, b)). Explain how you arrived at your relation.

b) [12 points] Does Alan think "friendship" is an equivalence relation? Explain why or why not. If it is, how many equivalence classes does it define on a set of 100 people? Explain in English how you arrived at your answer.

Problem 5. [24 points, 12 each] Write a paragraph proof of each of the following arguments:

a) Every BU students who takes CS 131 is a CS major student. There is a BU student who is a bird-watcher and not a CS major student. Therefore, there is a BU student who is a bird-watcher and not taking CS 131.

b) Everyone either likes Doritos or doesn't like Cheetos (inclusive). It is not true that someone doesn't like both Cheetos and Fritos. Every one who likes Fritos likes popcorn. Show that if you don't like Doritos, then you do like popcorn.