Math 153, Fall 2023

Homework 10

1. **Practice with complex exponentials.**

   (a) What are all the solutions of
   $$\frac{dx}{dt} = ix \quad ?$$

   (b) Show that a function is the real part of a solution of $\frac{dx}{dt} = ix$ if and only if it is a linear combination of $\sin t$ and $\cos t$.

   (c) Show that a function is the imaginary part of a solution of $\frac{dx}{dt} = ix$ if and only if it is a linear combination of $\sin t$ and $\cos t$.

   (d) What are the *real* (as opposed to complex) solutions of $\frac{dx}{dt} = ix$?

2. **A stable node.** Plot solutions of
   $$\begin{aligned} \frac{dx}{dt} &= -3x + y, \\ \frac{dy}{dt} &= -2x \end{aligned}$$
   near $(0,0)$ in the (x,y)-plane. Your plot should indicate what the solutions qualitatively look like.

3. **An unstable node.** Plot solutions of
   $$\begin{aligned} \frac{dx}{dt} &= 3x - y, \\ \frac{dy}{dt} &= 2x \end{aligned}$$
   near $(0,0)$ in the (x,y)-plane. Your plot should indicate what the solutions qualitatively look like. Hint: Use your work on problem 2.

4. **A saddle.** Plot solutions of
   $$\begin{aligned} \frac{dx}{dt} &= x + 4y, \\ \frac{dy}{dt} &= 2x - y \end{aligned}$$
   near $(0,0)$ in the (x,y)-plane. Your plot should indicate what the solutions qualitatively look like.

5. **When are the two eigenvalues the same?** Suppose A has two linearly independent eigenvectors with the same eigenvalue λ. What is A?

6. **A stable star.** Plot solutions of

$$\begin{aligned} \frac{dx}{dt} &= -4x, \\ \frac{dy}{dt} &= -4y \end{aligned}$$

near $(0,0)$ in the (x,y)-plane. Your plot should indicate what the solutions qualitatively look like.