CS 131 – Problem Set 2

Problems must be submitted by Monday February 6, 2023 at 11:59pm, on Gradescope.

Problem 1. (20 points)

Chuhan plans to sign up for a tour at the Arnold Arboretum in Jamaica Plain. However, he's not sure whether he can enter the arboretum without special permission from Harvard. He goes to the arboretum's website and there are three strange rules below:

The rules: (must satisfy at least one of these)

- One is a student AND they can enter
- One has a valid membership AND they can enter
- One is not a student and does not have a valid membership, AND they can not enter

Now, consider three statements:

- Let s denotes Chuhan is a student
- Let m denotes Chuhan has a valid membership
- Let e denotes Chuhan can enter the arboretum

a) (3 points) Convert the above three rules into propositional formula.

b) (5 points) Prove that the rules are equivalent to "he can enter if and only if he is a student or he has a valid membership" by using truth tables

c) (8 points) Prove the following statement: *a bi-implication is true if and only if the hypothesis and the conclusion have the same truth value*, i.e., use propositional logic (NOT TRUTH TABLE) to prove:

$$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$$

d) (4 points) Now, use part (c) to prove part (b), i.e., use propositional logic (NOT TRUTH TABLE) to prove:

$$e \leftrightarrow (s \lor m) \equiv (s \land e) \lor (m \land e) \lor (\neg s \land \neg m \land \neg e)$$

Problem 2. (10 points)

a) (6 points) Prove using laws of propositional logic (not truth tables), the following: "if you take CS 131, you can take CS 330 and if you take CS 131, you can take CS 365" is equivalent to "if you take CS 131, you can take CS 330 and CS 365".

Let p denote "you take CS 131," q denote "you can take CS 330" and "r" denote "you can take CS 365".

b) (4 points) Prove, now using truth tables, that if $(p \to q)$ and $(q \to r)$, then $(p \to r)$. That is, prove that

$$((p \to q) \land (q \to r)) \to (p \to r)$$

is a tautology. This result (often used as a law of logic) is known as the "hypothetical syllogism". Please show your work by including all columns of the truth table.

Note: A tautology is an expression that will always result in True. Put differently, the entire column for this expression would be True in a truth table.

Problem 3. (16 points)

a) (6 points) Prove using laws of propositional logic (NOT truth tables) that $(P \to Q) \vee (Q \to R)$ is a tautology.

Hint: To prove using the laws of propositional logic, you must start with the entire compound proposition and apply the laws of propositional logic until you have simplified the statement to the literal **True**.

b) (10 points) Prove using laws of propositional logic (NOT truth tables) that $(P \leftrightarrow Q) \wedge (P \vee Q) \wedge (\neg P \wedge \neg Q)$ is a contradiction.

Note: A contradiction is an expression that will always result in False.

Problem 4. (12 points, 2 each)

Evaluate the following logical expressions with $x = y = 1$ and with $w = z = 0$. (This problem was taken from zyBooks, problem 2.1.1).

a) $xy\overline{z}\overline{w}$

b) $x\overline{y} + z(\overline{w + z})$

c) $\overline{z}y\overline{x}(1 + w)$

d) $xy\overline{z} + z\overline{w}$

e) $\overline{(z + y)(w + x)}$

f) $x\overline{y} + (\overline{\overline{x} + w + \overline{y}\overline{z}})$

Problem 5. (6 points, 3 each)

Apply De Morgan's law repeatedly to each Boolean expression until the complement operations in the expression only operate on a single variable. For example, there should be no $\overline{xy}$ or $\overline{x+y}$ in the expression. Then apply the double complement law to any variable where the complement operation is applied twice. That is, replace $\bar{\bar{x}}$ with x. (This problem was taken from zyBooks, problem 2.1.2)

a) $\overline{\overline{x+yz}+u}$

b) $\overline{x\overline{(y+z)}u}$

Problem 6. (6 points, 3 each)

Give an input/output table for each Boolean function. (This problem was taken from zyBooks, problem 2.2.3)

a) $f(x, y) = \bar{x}\bar{y} + xy$

b) $f(x, y) = \bar{x}\bar{y} + \bar{x}y + x\bar{y}$

Problem 7. (14 points)

Continuing from problem 3 from lab 2: We have 2 two-bit numbers `a` and `b`: `a` consists of two bits `a1 a0` and `b` consists of two bits `b1 b0`. We want to add them, which will give us the result `s`, consisting of three bits `s2 s1 s0`. We want to write a Python formula in Boolean logic for each bit in `s`.

(a reminder that it is possible for numbers to start with one or more 0 bits; also note that while you were not allowed to use the $\oplus$ operator last week, please feel free to use it from now on for problems. The Python equivalent for $x \oplus y$ is

```
x ^ y
```

Hint 1: think about the addition algorithm you learned in elementary school, and apply it base two instead of base ten; note that in this algorithm, you will be adding two bits in column 0 and three bits in column 1.

a) (6 points) Write a boolean formula for `s1`. Submit a file named `p7a.py`.

b) (8 points) Write a boolean formula for `s2`. Submit a file named `p7b.py`.

Problem 8. (16 points, 4 each)

For each description below, give a Boolean expression that is a sum of minterms.

a) A boolean function that accepts 2 variables and results in 1 if there is an odd number of ones

b) A boolean function that accepts 3 variables and results in 1 if and only if the first variable is one and but none of the others are.

c) A boolean function that accepts 3 variables and results in 1 if all three of the variables are of the same value

d) A boolean function that accepts 4 variables and results in 1 if there is an odd number of ones.