1: The First Problem

$$\begin{aligned}
1 - C(x) + xC(x)^2 &= 1 - \sum_{n=0}^{\infty} C_n x^n + x\left(\sum_{n=0}^{\infty} C_n x^n\right)^2 \\
&= 1 - \sum_{n=0}^{\infty} C_n x^n + \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} C_{n-k}C_k\right)x^{n+1} \\
&= 1 - \sum_{n=0}^{\infty} C_n x^n + \sum_{n=1}^{\infty}\left(\sum_{k=0}^{n} C_{n-k}C_k\right)x^{n} \\
&= 1 - 1 - \sum_{n=1}^{\infty} C_n x^n + \sum_{n=1}^{\infty}\left(\sum_{k=0}^{n} C_{n-k-1}C_k\right)x^{n} \\
&= \sum_{n=1}^{\infty}\left(\left(\sum_{k=0}^{n} C_{n-k}C_k\right) - C_n\right)x^n \\
&= 0
\end{aligned}$$

2: The second Problem

Claim: there is a bijection between $\{a_i\}_{i=1}^n$ and bracket sequence. Prove

$$1 \mapsto ($$

$$-1 \mapsto)$$

3:

We prove that the sequence of non-intersection bijectively corresponds to the ballot sequence.

4:

There is a bijection from the ballot number to the $2 \times n$ matrices. Let

6:

Consider the sets

$$A_1, A_2, \cdot, A_n$$

where

$$A_i = \{a_k | a_k = i\}$$