

1.8/2.1 Questions

Problem 1. Determine where the function $f(x) = \frac{1}{1 + \sin x}$ is continuous on $[0, 2\pi]$

Problem 2. Sketch 3 examples of graphs that have different types of discontinuities. Explain how each example violates the definition of continuity, being sure to identify the specific violation.

Problem 3. 1. Find a value of c so that $f(x)$ is continuous everywhere, where

$$f(x) = \begin{cases} x^2 - 10, & x \leq c \\ 8x - 26 & x > c \end{cases}$$

2. Fill in the blanks with digits 1-9, so that $f(x)$ is continuous everywhere. You can use the same digit multiple times. How many solutions can you find?

$$f(x) = \begin{cases} x^2 - 17, & x \leq \square \\ \square x - 29 & x > \square \end{cases}$$

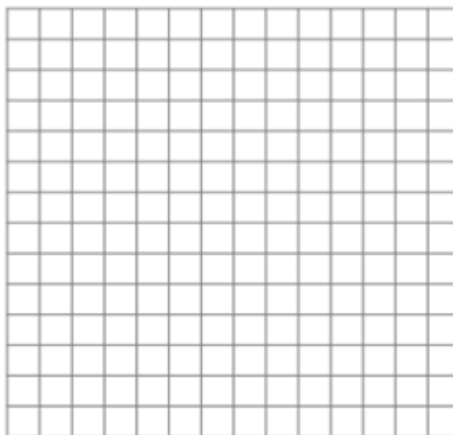
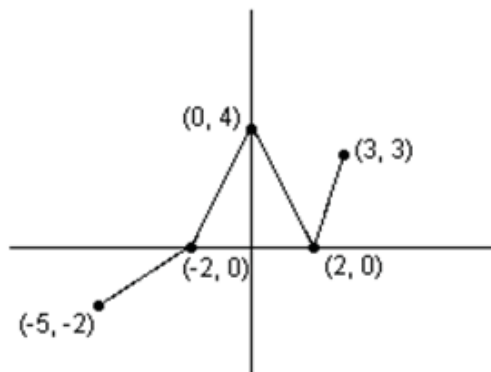
Problem 4. The Intermediate Value Theorem is a useful tool for showing that an equation has a solution, even if it is impossible to calculate directly.

1. Claim: (Almost) all of you were at some point in your life exactly $e = 2.718\dots$ feet tall. Why is this true and what would be true about someone who was never e feet tall?
2. It is sometimes said that a broken clock is right twice a day. Explain why this is true for an analog clock whose hands have stopped moving (but is otherwise unbroken).

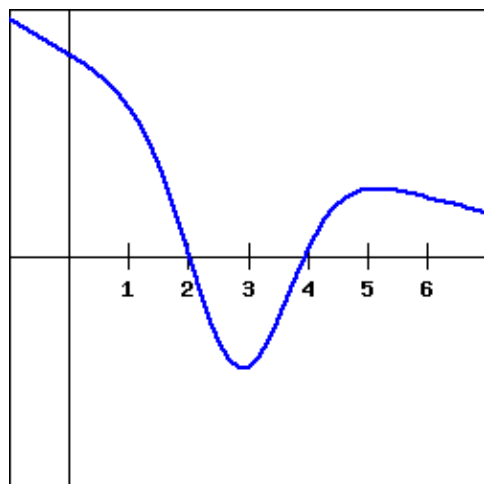
Problem 5. Find the interval on which the intermediate value theorem guarantees a root for the function $f(x) = x^2 - \frac{4}{x} + 1$.

- (a) $[-2, -1]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) $[1, 2]$

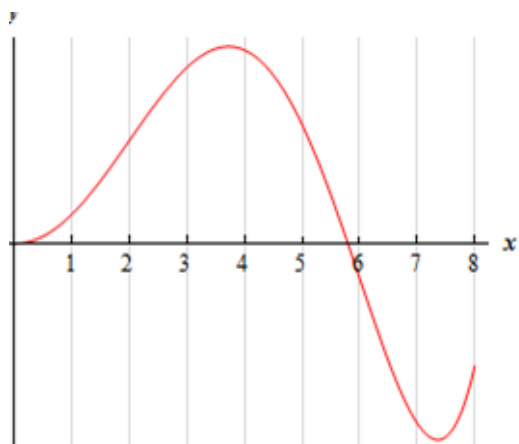
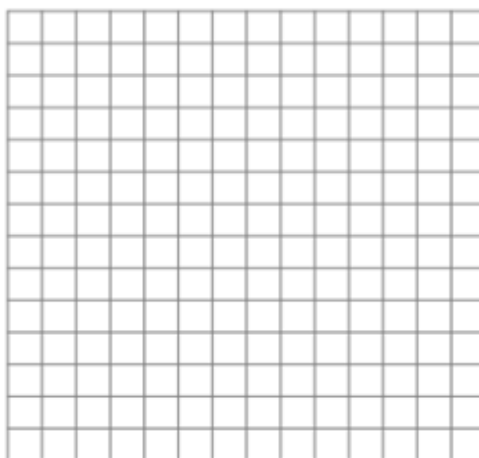
Problem 6. Graph the derivatives of each of the functions shown in the graphs below. Indicate if the derivative fails to exist at a point.
Questions? This is difficult at first!



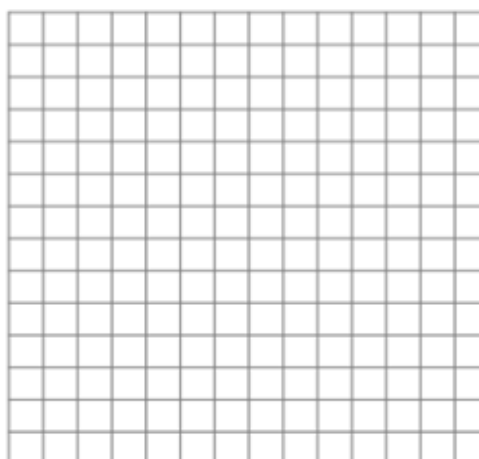
(a)



(b)



(c)



Problem 7. Use the definition of the derivative to find $f'(4)$ for the function $f(x) = 3x + 4$

Problem 8. Use the definition of the derivative to find an equation for the tangent line to the curve $y = \sqrt{x}$ at the point $(1, 1)$

Problem 9. Use the definition of the derivative to find an equation for the tangent line to the hyperbola $y = \frac{3}{x}$ at the point $(3, 1)$