Geometry of Surfaces - Exercises

Solutions to exercises marked with $*$ are to be submitted online through the link on the Keats page for this module.

56.* Describe the region of the unit sphere covered by the Gauss map of the paraboloid $\sigma : \mathbb{R}^2 \to \mathbb{R}^3$, $(x, y) \mapsto (x, y, x^2 + y^2)$.

57.* Let S^2 be the sphere of radius 1 centred at the origin. Prove that the equator, i.e., the intersection of S^2 with the plane given by $z = 0$, is a geodesic.

58. A curve $\gamma(t)$ is an asymptotic curve in a surface if $\dot{\gamma}(t)$ is an asymptotic direction for any t. Show that if a unit speed curve in a surface is an asymptotic curve and a geodesic, then it is (part of) a straight line.

59. Let γ be a unit speed curve in $\mathbb{R}^3$ with nowhere vanishing curvature and consider the surface $\sigma(u, v) = \gamma(u) + v\mathbf{b}(u)$, where $\mathbf{b}$ is the binormal of γ. Prove that γ is a geodesic on the surface.

60.* Let σ be a surface whose first fundamental form satisfies $E = G = 1$ and $F = 0$. What are the geodesics on the surface?

61. Let $\sigma : (0, 1) \times (0, 1) \to \mathbb{R}^3$ be a surface patch such that the first fundamental form is $E(u, v) = G(u, v) = \frac{1}{v^2}$ and $F(u, v) = 0$. Show that the curves $\gamma(t) = \sigma(c, e^t)$ with $c \in (0, 1)$ are unit speed geodesics.