School of Mathematics and Statistics

MAST10007 Linear Algebra, Semester 1 2023

Written Assignment 6

Submit a single pdf file of your assignment solutions via the MAST10007 website before 12 noon on Monday 15th May.

- This assignment is worth 2.22% of your final MAST10007 mark.
- Assignments must be neatly handwritten, but this includes digitally handwritten documents using an ipad or a tablet and stylus, which have then been saved as a pdf.
- Full working must be shown in your solutions.
- Direct questions by email to Nora, who will post answers to frequently asked questions on the Ed Board.
- Part of your overall mark is for quality of exposition.

Note: You can find fully worked solutions from a previous year on Canvas.

Question 1: Matrix Multiplication Revisited

Let A be an $m \times k$-matrix, and let B be a $k \times n$-matrix.

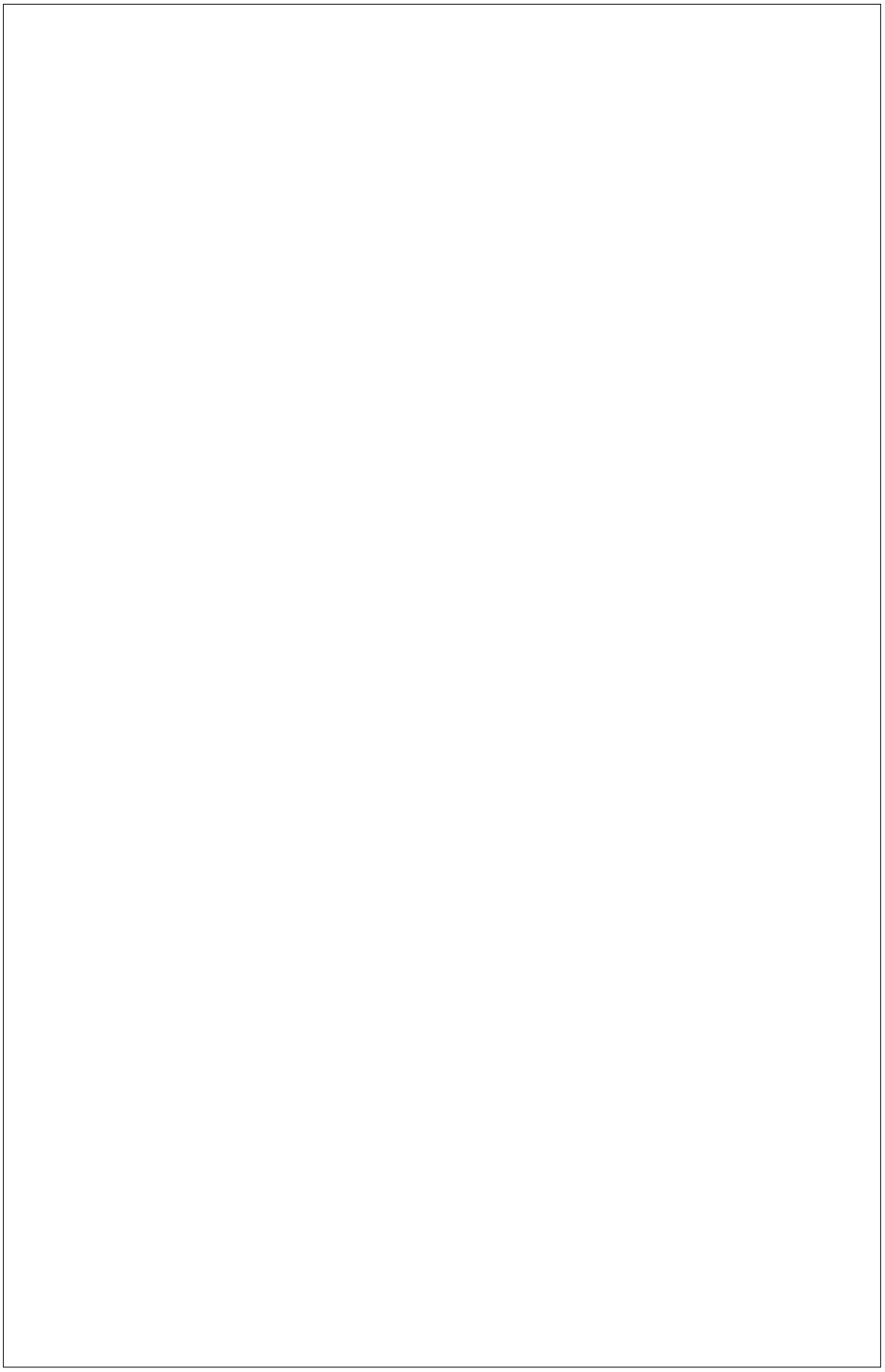
(a) Prove that the column space of AB is contained in the column space of A.

Subset proof format: Prove the implication $\vec{v} \in Col(AB) \implies \vec{v} \in Col(A)$.

(b) Assume that $k = m$ and that A is invertible. Prove that the null space of AB is equal to the null space of B.

Proof format for equality of sets: prove $\vec{x} \in Null(AB) \iff \vec{x} \in Null(B)$.

(c) In the situation of question part (b), prove that the rank of AB is equal to the rank of B.



(d) Revisit Tutorial 4, and use your above findings to give a proper proof for Challenge Question 1(h). You may use without proof that there are 100 non-zero singular values and that the matrices U and V in the singular value decomposition are invertible matrices. **Hint:** Use the fact that the row-rank equals the column rank, i.e., $rank(A^T) = rank(A)$.

Question 2: Given

$$A = \begin{bmatrix} 1 & 2 & 0 & -4 & 5 & 11 \\ -3 & -6 & 2 & 18 & -5 & 9 \\ 5 & 10 & 6 & -2 & 55 & 3 \\ 0 & 0 & -2 & -6 & -10 & 8 \\ -1 & -2 & 4 & 16 & 15 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & -4 & 5 & 0 \\ 0 & 0 & 1 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the column space of A.

(b) Write A as a product of a 5×3 matrix with a 3×6 matrix.

Question 3: Least Squares Line of Best Fit

(a) Find the line of best fit for the data points

x	1	2	4	5
y	6	3	2	3

Use the method taught in class.

(b) Produce a high-quality drawing of the data points and your fitted line on the grid paper below.