TUTORIAL 5: contour integration

NOTE: The symbol $\oint_C$ is the standard contour integral around a closed contour C

1. If C is a circle of radius 2 and centre the origin oriented counterclockwise, evaluate the integrals:

$$(i)\ \oint_C z^2 dz \quad (ii)\ \oint_C \frac{1}{z} dz \quad (iii)\ \oint_C (z-i)^2 dz$$

$$(iv)\ \oint_C (z+3i)^3 dz \quad (v)\ \oint_C \frac{1}{z-3i} dz$$

2. If C is a circle of radius 3 and centre the origin oriented counterclockwise, evaluate the following integrals using Cauchy integral formula

$$(i)\ \oint_C \frac{z^2+2}{z-1} dz \quad (ii)\ \oint_C \frac{z}{(z+2)(z+4i+1)} dz$$

$$(iii)\ \oint_C \frac{z}{(z+2)^2} dz \quad (iv)\ \oint_C \frac{z+1}{(z+2i)(z-3i-4)} dz$$

3. Compute

$$\oint_C \frac{ze^z}{z^2+1} dz$$

where C is the circle of equation $|z| = 2$ oriented counterclockwise.

4. Compute

$$\oint_C \frac{z+1}{z^4+2z^3} dz$$

where C is the circle of equation $|z| = 1$, oriented counterclockwise.

Answers:

1.
(i) 0; (ii) $2\pi i$; ; (iii) 0; (iv) 0; (v) 0
2.
(i) $6\pi i$, (ii) $4\pi(-4+i)/17$; (iii) $2\pi i$, (iv) $-2\pi(13-6i)/41$
3.
$2\pi i \cos 1$
4.
$-i\pi/4$