COMP SCI 2LC3

Assignment #2. Due November 7 (Monday), 2022, 23:59 via Avenue. Do not hesitate to discuss with TA or instructor all the problems as soon as you discover them. This assignment is a little bit more difficult than the first one. Start early!

Total: 149 pts

Instructions: For all assignments, the students must submit their solution to Avenue → Assessments → Assignment #

Students can simply solve the exercises on a paper and use a smartphone app called CamScanner and convert their entire solution into a single PDF file and submit it to avenue. The maximum upload file size is 2Gb in avenue for each submission.

Please make sure that the final PDF file is readable.

Students, who wish to use Microsoft word and do not have Microsoft Word on their computer, are suggested to use google document editor (Google Docs). This online software allows you to convert your final file into PDF file.

There will be a mark deduction for not following the submission instruction.

Please first finish the assignment on your local computer and at the end, and attach your solution as a PDF file.

You will have unlimited number of submissions until the deadline.

Students must submit their assignments to Avenue. Any problem with Avenue, please discuss with Mahdee Jodayree <mahdijaf@yahoo.com>, the lead TA for this course.

Study of Chapters 7, 8, 9 and 10 of the Gries-Schneider textbook and Lecture Notes 4, 5, 6 and 7 is highly recommended for this assignment.

Questions.

1.[20] The *PQ-L* logic (page 9 of Lecture Notes 5 and Chapter 7 of the Gries-Schneider textbook), but with axioms from page 21 of Lecture Notes 5, is sound and complete and has a model '*A formula aPbQc means* $\#a + \#b = \#c$', where $\#x$ denotes the number of stars $(*)$ in the sequence x.

Let *PQR-L* be another 'toy' logic. For *PQR-L* we have:

- Symbols: $P, Q, R, *$
- Formulas: sequences of the form $aPbQc$, $aPbRc$, $aPbQcRd$, or $aPbRcQd$, where a, b, c, and d are finite sequences of zero or more $*$.

Provide *axioms* and *inference rules* such that *PQR-L* is:

a.[10] *sound* and *complete* logic,

b.[10] the interpretation where

- a formula $aPbQc$ means $\#a + \#b = \#c$,
- a formula $aPbRc$ means $\#a \dot{-} \#b = \#c$
- a formula $aPbQcRd$ means $(\#a + \#b) \dot{-} \#c = \#d$, and
- a formula $aPbRcQd$ means $(\#a \dot{-} \#b) + \#c = \#d$

where $\#x$ denotes the number of stars $(*)$ in the sequence x and for all natural numbers x, y, the operation '$\dot{-}$', often called 'weak subtraction', is given by:

$$x \dot{-} y = \begin{cases} x - y & \text{if } x - y \geq 0 \\ 0 & \text{if } x - y < 0 \end{cases}$$

is a *model*.

2.[8] In Standard Propositional Logic the truth tables for $p \implies q$ and $\neg p \vee q$ are identical, so we often write $p \implies q = \neg p \vee q$, i.e. these formulas are treated as equivalent. Can we treat these formulas as equivalent in *Constructive* Propositional Logic? Prove your answer.

3.[20] Exercise 7.5 (pages 135-137 of the Gries-Schneider textbook), questions (k), (l), (m) and (n).

4.[2] Exercise 8.1 (pages 155 of the Gries-Schneider textbook), questions (d) and (e).

5.[4] Exercise 8.3 (pages 155 of the Gries-Schneider textbook), questions (d) and (e).

6.[2] Exercise 8.5 (pages 155 of the Gries-Schneider textbook), question (c).

7.[4] Exercise 8.6 (pages 155 of the Gries-Schneider textbook), questions (c) and (d).

8.[2] Exercise 8.7 (pages 156 of the Gries-Schneider textbook), question (a).

9.[2] Exercise 9.4 (pages 174 of the Gries-Schneider textbook).

10.[2] Exercise 9.11 (page 174 of the Gries-Schneider textbook).

11.[2] Exercise 9.23 (page 175 of the Gries-Schneider textbook).

12.[2] Exercise 9.27 (page 175 of the Gries-Schneider textbook).

13.[6] Exercise 9.29 (page 175 of the Gries-Schneider textbook), questions (f), (g), and (i).

14.[2] Exercise 9.33 (page 176 of the Gries-Schneider textbook).

15.[4] Exercise 9.35 (page 176 of the Gries-Schneider textbook), questions (c), and (d).

16.[3] Exercise 9.36 (page 176 of the Gries-Schneider textbook).

17.[4] Le $\mathbb{N}$ denotes natural numbers (i.e. $\{0,1,2,\ldots\}$, and $PLUS(x,y,z)$ be a predicate defined as: $PLUS(x,y,z) = true \iff x+y=z$.

- a.[2] Consider a predicate logic formula (in notation from LN7a): $\Phi_1 = \exists x \forall y. R(x,y,y)$. Is $(\mathbb{N}, PLUS)$ a model of Φ_1?
- b.[2] What about the formula $\Phi_2 = \exists x \forall y. R(x,y,x)$? Is $(\mathbb{N}, PLUS)$ a model of Φ_2?

18.[6] Exercise 10.1 (page 191 of the Gries-Schneider text), questions (c), (k) and (l).

19.[2] Exercise 10.5 (page 191 of the Gries-Schneider textbook).

20.[7] Exercise 10.6 (page 192 of the Gries-Schneider text), questions (g), (i) and (k).

21.[4] Exercise 10.7 (page 192 of the Gries-Schneider textbook), questions (f) and (h).

22.[3] Exercise 10.10 (page 193 of the Gries-Schneider textbook).

23.[3] Exercise 10.14 (page 194 of the Gries-Schneider textbook).

24.[2] Show the postcondition R for the following program:

$\{y = 3\}$
$x := 2;$
$z := x + y;$
if $y > 0$ then $x := z + 1$
else $z := 0$
$\{R\}$

25.[2] Show that the following Hoare triple is valid:

$\{true\}$
if $x < y$ then $min := x$
else $min := y$
$\{(x \leq y \wedge min = x) \vee (x > y \wedge min = y)\}$

26.[3] Show the postcondition R for the following program:

$\{z = 0 \wedge y = 5\}$
for $i = 1$ to 5 do
$z := z + b[i]$;
$y := y * z$ od
$\{R\}$