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Solve the linearized damped pendulum equation

$$\ddot{\theta} + \frac{b}{m}\dot{\theta} + \frac{g}{L}\theta = 0 \quad (1)$$

By transforming into quadratic form, We have to solve

$$D^2 + \frac{b}{m}D + \frac{g}{L} = 0 \quad (2)$$

where $D = \frac{d}{dt}$. The solutions are

$$D = \frac{-\frac{b}{m} \pm \sqrt{\frac{b^2}{m^2} - 4\frac{g}{L}}}{2} \quad (3)$$

Therefore the solutions for the original equation are

$$\theta = e^{\frac{-b}{2m}t} \left(A_1 e^{\sqrt{\frac{b^2}{4m^2} - \frac{g}{L}}t} + A_2 e^{-\sqrt{\frac{b^2}{4m^2} - \frac{g}{L}}t} \right) \quad (4)$$

Note that

$$\begin{aligned} \frac{-b}{m} + \sqrt{\frac{b^2}{m^2} - 4\frac{g}{L}} &< \frac{-b}{m} + \sqrt{\frac{b^2}{m^2}} \\ &= \frac{-b}{m} + \frac{b}{m} \\ &= 0 \end{aligned}$$

If b is great enough, the solution will be a decaying exponential.

Physically saying, the pendulum will stop swinging after a while.

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