MAST10006 Calculus 2, Semester 1, 2023

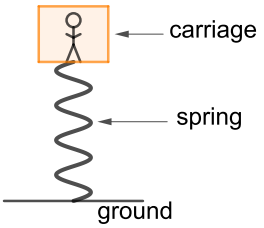
Assignment 8

School of Mathematics and Statistics, The University of Melbourne

Due at: 12pm Monday 15 May 2023

Answer all the questions below. Submit your assignment in Canvas before the deadline above. See Canvas for instructions how to submit the assignment and for the assessment criteria.

1. A ride at an amusement park consists of a carriage mounted on the top of a spring, as shown in the diagram.
The bottom of the spring is fixed to the ground.
The carriage (loaded with its passenger) has a mass of 160kg.
The carriage can only move vertically. The natural length of the spring is 6 m and its spring constant is 800 N m^{-1}.
At equilibrium the spring is compressed by a distance s metres.
Air resistance acts on the carriage with a damping constant 80 N s m^{-1}.
Assume that the gravitational constant is $g = 10$ m s^{-2}.



The ride begins with the carriage released from rest, 1 m **above the ground**.

Let $y(t)$ be the distance of the carriage below its equilibrium position at time t seconds after the start of the ride, and let $h(t)$ be the height of the carriage above ground level at time t.

The equation of motion for the system is

$$160y'' + 80y' + 800y = 0.$$

   (a) Draw a diagram of the system when the carriage is below the equilibrium position and moving up. Show all forces acting on the carriage and label them.

   (b) Express the initial conditions in terms of y and y'.

   (c) Solve for $y(t)$.

   (d) Sketch $h(t)$ vs. t.

2. Suppose that the ride from question 1 was placed underwater at the bottom of a deep river. The ride operates the same as before, except that the damping constant is multiplied by 10 due to much greater drag caused by the water. Assume that the mass, spring constant, and gravitational constant are unchanged. Explain how the system's motion will change, and sketch $h(t)$ in this case.

   Hint: you do not need to solve for $y(t)$.

End of assignment