1.6 Questions

Problem 1. Evaluate the limit $\lim_{x\to 3} \dfrac{x-3}{x^2+7x-30}$

Problem 2. Evaluate the limit $\lim_{x\to 25} \dfrac{25-x}{\sqrt{x}-5}$

Problem 3. Evaluate the limit $\lim_{x\to 3^-} \dfrac{|x-3|}{3-x}$

Problem 4. Evaluate the limit $\lim_{x\to 3^+} \dfrac{|x-3|}{3-x}$

Problem 5. Evaluate the limit $\lim_{x\to 3} \dfrac{|x-3|}{3-x}$

Problem 6. Evaluate the limit $\lim_{h\to 0} \dfrac{(-5+h)^2 - 25}{h}$

Problem 7. Evaluate the limit $\lim_{h\to 0} \dfrac{\dfrac{4}{7+h} - \dfrac{4}{7}}{h}$

Challenge Problems: Note that the final page of the recitation activity will usually have problems such as these, which go beyond the sorts of problems that you are expected to know on quizzes or exams. Depending on time, your TA may choose to cover these problems, or might opt to skip them. Some students may find them interesting to do on their own, but they are not required.

These final two questions are related. They involve some review of trigonometry and the second one is more challenging. Struggling with these questions is a great way to deepen your understanding of the course material and gain mastery of these topics.

Problem 8. This problem has an associated Desmos graph here https://www.desmos.com/calculator/snqy6xi6hs. You can use the slider to choose different values for b and the calculator will automatically compute the value of $\sin\left(\frac{\pi}{\theta}\right)$ at some of the values of θ listed below.

1. Estimate $\lim_{\theta\to 0}\sin\left(\frac{\pi}{\theta}\right)$ by plugging in $\theta = .1, .01, .001, -.1, -.01, -.001$
2. Now estimate $\lim_{\theta\to 0}\sin\left(\frac{\pi}{\theta}\right)$ by plugging in $\theta = \frac{2}{5}, \frac{2}{9}, \frac{2}{13}, \frac{2}{17}, -\frac{2}{5}, -\frac{2}{9}, -\frac{2}{13}, -\frac{2}{17}$
3. What do you notice about your answers from the previous two parts? After doing both of them do you know the value of $\lim_{\theta\to 0}\sin\left(\frac{\pi}{\theta}\right)$? What limitations of computing limits via plugging in values does this reveal?

Problem 9. Billy is trying to evaluate the following limit

$$\lim_{x\to 0}(x^2+4)\sin\left(\frac{\pi}{x}\right).$$

He explains "The limit is equal to 4 by the squeeze theorem. First we know that

$$\lim_{x\to 0}\left(x^2+4\right) = \lim_{x\to 0}x^2 + \lim_{x\to 0}4 \qquad \text{(sum limit rule)}$$

$$\lim_{x\to 0}x^2 + \lim_{x\to 0}4 = \left(\lim_{x\to 0}x\right)^2 + \lim_{x\to 0}4 \qquad \text{(power limit rule)}$$

$$\left(\lim_{x\to 0}x\right)^2 + \lim_{x\to 0}4 = 0 + \lim_{x\to 0}4 \qquad \text{(basic limit rule)}$$

$$0 + \lim_{x\to 0}4 = 4 \qquad \text{(basic limit rule).}$$

So $\lim_{x\to 0}(x^2+4) = 4$. We know that $\sin(\theta)$ is always trapped between -1 and 1, so $\sin\left(\frac{\pi}{x}\right)$ is too. Therefore by the squeeze theorem

$$\lim_{x\to 0}(x^2+4)\sin\left(\frac{\pi}{x}\right) = 4."$$

Is Billy right? If not what mistakes did he make?