TUTORIAL 6: Residue Theorem and Applications

1. Find the residue of the functions

$$(i)\ \frac{z}{z-1}\ \text{ at }\ z=1 \qquad (ii)\ \frac{z^2-1}{z^2+1}\ \text{ at }\ z=i$$

$$(iii)\ \frac{z}{(z+1)(z+2i)}\ \text{ at }\ z=-1\ \text{ and }\ z=-2i$$

2. If C is a circle of radius 2 and centre the origin, evaluate the following integrals

$$(i)\ \oint_C \frac{z}{z-1}dz \quad (ii)\ \oint_C \frac{dz}{(z^2+1)(z-3i)}, \quad (iii)\ \oint_C \frac{z^2-1}{(z+i)^2}dz$$

3. If x is a real variable, use contour integration to evaluate the integrals:

$$(i)\ \int_0^\infty \frac{dx}{x^2+1} \quad (ii)\ \int_{-\infty}^\infty \frac{x^2+x+6}{(x^2+4)(x^2+1)}dx \quad (iii)\ \int_0^\infty \frac{dx}{x^4+x^2+1}$$

4. Use contour integration to evaluate

$$(i)\ \int_0^{2\pi} \frac{d\theta}{5-4\cos\theta} \qquad (ii)\ \int_0^{2\pi} \frac{\cos\theta}{2-\cos\theta}d\theta$$

Answers:

1.
(i) 1; (ii) i; (iii) $(1+2i)/5$ and $(4-2i)/5$;
2.
(i) $2\pi i$; (ii) $\pi i/4$; (iii) 4π;
3.
(i) $\pi/2$; (ii) $4\pi/3$; (iii) $\pi\sqrt{3}/3$
4.
(i) $2\pi/3$; (ii) $-2\pi(3-2\sqrt{3})/3 \approx 0.309\pi$