Geometry of Surfaces - Exercises

Solutions to exercises marked with $*$ are to be submitted online through the link on the Keats page for this module.

24.* Find the equations of the tangent planes to the surface $\sigma(u,v) = (u, v, u^2 - v^2)$ for $(u,v) = (0,0)$ and $(u,v) = (1,1)$.

25.* Let $\gamma(s)$ be a unit speed curve in $\mathbb{R}^3$ with nowhere vanishing curvature κ. Let $a > 0$ and consider the tubular surface

$$\sigma(s,\theta) = \gamma(s) + a(\cos(\theta)\mathbf{n}(s) + \sin(\theta)\mathbf{b}(s)).$$

Prove that if $a\kappa < 1$ everywhere, then σ is a regular surface.

26. Let $\gamma(s)$ be a unit speed curve in $\mathbb{R}^3$ with nowhere vanishing curvature and consider the surface $\sigma(s,v) = \gamma(s) + v\mathbf{t}(s)$. Prove that $\sigma(s,v)$ is a regular point of the surface if and only if $v \neq 0$. Show that the tangent plane of the surface at a regular point $\sigma(s,v)$ is the osculating plane of γ at $\gamma(s)$ (and therefore does not depend on v).

27. Let $\sigma : U \to \mathbb{R}^3$, $(u,v) \mapsto \sigma(u,v)$ be a regular surface patch and N the unit normal to σ. Show that N_u and N_v are perpendicular to N. Assume that there exists a function $\alpha(u,v)$ and a point $p \in \mathbb{R}^3$ so that $\sigma + \alpha N = p$. Prove that the surface is contained in a sphere.

28. Let I be an open interval in $\mathbb{R}$ and $\alpha, \beta : I \to \mathbb{R}^3$ be regular curves. Put $U = I \times I$ and define the surface

$$\sigma : U \to \mathbb{R}^3 \ , \ (u,v) \mapsto \sigma(u,v) = \alpha(u) + \beta(v).$$

Describe the regular points of σ. Show that there exists a line that is contained in the tangent planes of all regular points of the form $\sigma(u,0)$.