

**From physics:**

### Force

Consider an object of a mass  $m$  moving along a straight line.

The position of the particle at time  $t$  is  $f(t)$ .

According to Newton's Second Law of Motion, the force,  $F$ , acting on the object (in the direction of movement) is given by

$$F = ma = m \frac{d^2s}{dt^2}$$

When a force is applied to an *entire* object to move it, the energy expended is called **work**.

### Units of measurement

System	Force	Distance	Work
SI	Newtons ( $N = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$ )	meters (m)	joules ( $J = N \cdot m$ )
US Customary	pounds (lb)	feet (ft)	ft-lb

*Notes:* 1. The kilogram (kg) is a unit of mass, but the pound (lb) is a unit of force.  
2.  $1 \text{ ft-lb} \approx 1.36 \text{ J}$

### Case I

In the case of *constant acceleration* the force  $F$  is also constant.

The work,  $W$ , done in moving an object on a distance  $d$  is defined by

$$W = Fd$$

**Ex.1** a) How much work is done in lifting a 10-kg box off the floor to put it on a shelf that is 2 meters above the floor? (Recall that the acceleration due to gravity is  $g = 9.8 \text{ m/s}^2$ .)

b) How much work is done in lifting a 20-lb box 6 feet from the ground?

## Case II

One or both of the following applies:

- The force isn't constant — instead, it varies throughout the physical process being considered.
- **Different parts of the object are moved different distances.**

We will focus only on doing work against the force of gravity by lifting straight up.

Therefore, **the force in question will always be the object's weight.**

Computing the work done in lifting a rope ( or cable, or chain) of certain length (with or without additional weight) or in pumping liquid into or out of a tank of certain shape.

In these situations there is no single object moved a fixed distance.

Each layer is moved a different distance.

**The total work is computed by adding the work done for moving each layer.**

**Ex.2** A 200-lb cable is 100 feet long and hangs vertically from the top of a tall building.

- How much work is required to lift the cable to the top of the building?
- How much work is required to lift the cable to the top of the building if a 40-lb weight is attached to the bottom of the cable?

**Ex.3** A tank has a shape of the inverted cone with base radius 4 meters and height 10 meters.

It is filled with water to the height of 8 meters.

Recall that the density of the water is  $1000 \text{ kg/m}^3$ .

Find the work required to empty the tank by pumping all the water to the top of the tank.

**Ex.4** Consider that the tank in Ex.3 is filled with water to the top.

Find the work required to empty the tank by pumping all the water to the top of the tank.

**Ex.5** Consider that the tank in Ex.3 is filled with water to the top.

Find the work required to empty the tank by pumping all the water out a spout 1 meter above the top of the tank.

**Ex.6** a) An above the ground backyard swimming pool has a shape of a right circular cylinder with radius of 10 meters and height of 6 meters.

The depth of the water in the pool is 5 meters.

Find the work done in emptying the pool by pumping the water over the top edge of the pool.

b) Suppose that the pool from part a) is in the ground and its top is at the ground level.

How will the expression for the work change?

c) Suppose that the pool from part a) is filled with water to the top.

How will the expression for the work change?