MATH 465 - INTRODUCTION TO COMBINATORICS
HOMEWORK 4

(1) Solve the recurrence

$$h_n = h_{n-1} + 4h_{n-2} - 4h_{n-3}, \quad \text{for } n \geq 3,$$

with initial values $h_0 = 0, h_1 = 1$, and $h_2 = 2$.

(2) Solve the recurrence

$$h_n = 6h_{n-1} - 9h_{n-2}, \text{ for } n \geq 2,$$

with initial conditions $h_0 = -2$ and $h_1 = 0$.

(3) Show that for any non-negative integer n, the number $h_n = (1+\sqrt{7})^n + (1-\sqrt{7})^n$ is an integer. *Hint:* Find and prove a recurrence satisfied by h_n.

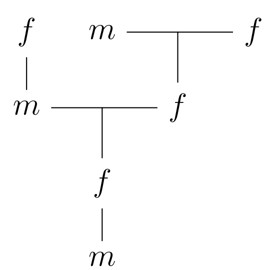
(4) The following table gives several values of a polynomial $f(x)$ of degree 4:

x	1.0	1.1	1.2	1.3	1.4
$f(x)$	15	-2	10	34	-4

Find $f(1.5)$.

(5) Let a_n denote the number of strings of length n over the alphabet $\{0, 1, 2\}$ in which the substrings $00, 01, 10$, and 11 (consecutive entries) never occur. Prove that $a_n = a_{n-1} + 2a_{n-2}$ (for $n \geq 2$), with $a_0 = 1$ and $a_1 = 3$. Then find a formula for a_n.

(6) Find the recurrence relation for the number a_n of bees in the nth previous generation of a male bee, if a male bee is born asexually from a single female and a female bee has the normal male and female parents. The ancestral chart below shows that $a_1 = 1, a_2 = 2, a_3 = 3$.



(7) Show that the number h_n of n-digit binary sequences with at least one instance of consecutive 0s satisfies the recurrence

$$h_n = 2h_{n-1} + 2^{n-3} - h_{n-3}.$$

(8) Let h_n denote the number of different ways in which the squares of a $1 \times n$ chessboard can be colored, using the colors red, white and blue so that no two squares that are colored red are adjacent. Find and verify a recurrence relation that h_n satisfies, and then find a formula for h_n.