1. $[x(0), y(0), \theta(0)]^T = [10, 20, 0.5]^T$

$[x(T), y(T), \theta(T)]^T = [0, 0, 0]^T$

$T = 10$

At $t = 0$

$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

$y(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3$

$x(0) = a_0 = 10 \quad - (1)$

$y(0) = b_0 = 20 \quad - (2)$

At $t = 10$

$x(t) = a_0 + 10a_1 + 100a_2 + 1000a_3 \quad - (3)$

$y(t) = b_0 + 10b_1 + 100b_2 + 1000b_3 \quad - (4)$

For initial state

$\dot{x}\cos\theta + \dot{y}\sin\theta = v$

At $v = 0 \Rightarrow t = 10$

$\dot{x}\cos\theta + \dot{y}\sin\theta = 0 \quad - (5)$

At $v = 10 \Rightarrow t = 0$

$\dot{x}\cos\theta + \dot{y}\sin\theta = 10 \quad - (6)$

Nonholonomic constraint

$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$

~~$\sin(0.5) - 20\cos(0.5) = 0$~~ ~~(7)~~

$\dot{x}_0 \sin\theta_0 - \dot{y}_0 \cos\theta = 0 \quad - (7)$

$\dot{x}_f \sin\theta_0 - \dot{y}_f \cos\theta = 0 \quad - (8)$

Solve these 8 equations

(2) $z_1 = x$

$z_2 = y$

$z_3 = \dot{x} = v\cos\theta$

$z_4 = \dot{y} = v\sin\theta$

Differentiating

$\dot{z}_1 = \dot{x} = z_3$

$\dot{z}_2 = \dot{y} = z_4$

$\dot{z}_3 = \ddot{x} = \dot{v}.\cos\theta - v\sin\theta.\dot{\theta}$

$\dot{z}_4 = \ddot{y} = \dot{v}.\sin\theta + v\cos\theta.\dot{\theta}$

(3) $u_1 = \dot{z}_3$ $\quad\quad \dot{z}_1 = z_3$

$u_2 = \dot{z}_4$ $\quad\quad \dot{z}_2 = z_4$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Reachability check: calculate $[A^3B \quad A^2B \quad AB \quad B]$

From MATLAB

$$\begin{bmatrix} 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

The system is reachable.

Rank = 4

Full rank matrix = No of state variables

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(4) From the previous question

$$\dot{z} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

error dynamics

$$e = z - z\_d$$

$$k_p = \begin{bmatrix} k_p & 0 \\ 0 & k_p \end{bmatrix} \qquad k_D = \begin{bmatrix} k_D & 0 \\ 0 & k_D \end{bmatrix}$$

$$k = [k_p, k_D]$$

$$u = -k[z - z_d] + a$$

$$\dot{z} = Az + Bu$$

(5) $x_a = [x, y, \theta, v]$

$\dot{v} = [a, \omega]$

$$\dot{x}_a = f(x_a, a, \omega) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \cancel{\omega} \\ \cancel{a} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \\ a \end{bmatrix}$$

$$= \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{v} \end{bmatrix}$$

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(6) $u_1 = \ddot{x}$

$u_2 = \ddot{y}$

$\dot{x} = v\cos\theta$

$\ddot{x} = \dot{v}\cos\theta - v\sin\theta . \dot{\theta}$

$u_1 = a\cos\theta \quad - v\sin . \omega \qquad -(1)$

$\dot{y} = v\sin\theta$

$\ddot{y} = \dot{v}\sin\theta \quad + v\cos\theta . \dot{\theta}$

$u_2 = a\sin\theta + v\cos\theta . \omega \qquad -(2)$

solve (1) & (2)

$$u_1 \sin\theta = \quad a\cos\theta\sin\theta \quad - v\sin^2\theta\,\omega \qquad (+)$$

$$-u_2\cos\theta \quad = -\ a\cos\theta \ \sin\theta - v \quad \cos^2\theta\,\omega$$

$$u_1\sin\theta - u_2\cos\theta = \quad -v\omega$$

$$\boxed{\omega \quad = \frac{u_2\cos\theta - u_1\sin\theta}{v}}$$

Similarly

$$\ddot{x}\cos\theta \quad = a\cos^2\theta \ - v\cos\theta . \sin\theta . \omega \qquad (+)$$

$$\ddot{y}\sin\theta \quad = a\sin^2\theta \quad + v\cos\theta . \sin\theta . \omega$$

$$u_1\cos\theta + u_2\sin\theta = \quad a$$

$$\boxed{a = u_1\cos\theta \quad + u_2\sin\theta}$$