MIU-Math 221 Lecture schedule

The following lecture schedule assumes there are 12 weeks in the semester, whereas there are 14 weeks. This allows time for in-class exams, and time to catch up if one falls behind schedule.

Week 1

- Chapter 1. (things)
- What is a number?
- Functions
- Implicit functions
- Inverse functions
- Inverse trigonometric functions

A big difference between 221 and 275 (honors calculus) is that we do not try to give a good description of the real numbers in 221 (no least upper bound axiom). Instead we just assume that "everyone knows" what a real number is. Here we point out to the students that there are problems with this attitude, but that we will ignore them this semester. To show some of the problems, you can begin by explaining that ∞ is not a number. Then ask the class if they think $0.9999\ldots = 1$? (i.e. if infinitely large numbers don't exist, do infinitely small numbers such as $1 - 0.999\ldots$ exist?)

The rest of this chapter ought to be review. I plan to explain/remind students of what a function is, describe implicit and inverse functions, and review the arcsine and arctangent. The inverse trig functions are covered in a course which is a prerequisite for math 221, so this should not be new material. Still, students may not know what the graphs of $y = \sin(\arcsin(x))$ and $y = \arcsin(\sin(x))$ look like.

Week 2

- Chapter 2. (derivs)
- The tangent to a curve
- An example – tangent to a parabola
- Instantaneous velocity
- Rates of change
- Examples of rates of change

We introduce the derivative as a rate of change. We don't get into the details of how to compute derivatives, but this chapter motivates why we need to understand limits.

Week 3

- Chapter 3. (limits)
- Informal definition of limits
- "The formal, authoritative, definition of limit"
- Variations on the limit theme
- Properties of the Limit
- Examples of limit computations
- When limits fail to exist
- Limits that equal ∞
- What's in a name? – Free Variables and Dummy variables
- Limits and Inequalities
- Continuity
- Substitution in Limits

There is a fair amount of text devoted to the ε/δ definition of the limit. Opinions on whether this is useful for the students and on how much they will actually learn from this vary. In other years I have put a fair amount of time and energy into this topic, but this year I plan to spend only half a (75 minute) lecture on the definition, and skip it otherwise. My current motives for this are that the ε/δ definition provides an answer to a question most students won't think to ask. It also leads to exam questions that test the students' algebraic skills but not their understanding of the ε/δ definition. For instance, if you cover this subject, then students should be able to answer the question "show that $\lim_{x\to 1} x^2 \neq 3$." I think that would be reasonable in math 275 or necessary in math 421, but not here.

Instead of the ε/δ definition I do intend to use the limit properties, and ask students to derive limits showing step by step how they use those properties.

Week 4

- Two Limits in Trigonometry
- Asymptotes
- Chapter 4. (derivs)
- Derivatives Defined
- Direct computation of derivatives
- Differentiable implies Continuous
- Some non-differentiable functions

The trigonometric limits have a nice proof. I will briefly describe what asymptotes are (they should do the examples themselves for homework and/or in discussion.)

The main example of direct computation of derivatives is dx^n/dx. The derivation in the text uses the geometric sum formula. You could also use the (first two terms of) the binomial formula.

There are some examples of non differentiable functions. I like to mention Brownian motion as a continuous but nowhere differentiable function (no proof of course) and I like to point out that this kind of "pathological function" actually occurs in physics (Brownian motion) but is also fundamental to mathematical finance (there's a link to finance.yahoo.com for the prospective econ students the class).

Week 5

- The Differentiation Rules

- Differentiating powers of functions
- Higher Derivatives
- Differentiating Trigonometric functions
- The Chain Rule
- Implicit differentiation

I would prove the product rule (straight forwardly, and also using the picture proof.) For the quotient rule I would give the "implicit differentiation proof." It is not completely rigorous, but it has the advantage of being a first example of implicit differentiation (while doing the proof I tell them that this is an easier case of a method they should be able to use on an exam.)

For the derivative of trig functions I think I'll do the more geometric derivation in problem 16 on page 72/73 this year. It avoids the addition formulas, and makes the answer perhaps less mysterious. On the other hand it would be nicer if the students did that problem themselves.

The "related rate problems" at the end are important because they force the students to relate derivatives to the so-called real world, i.e. they test if students can do more than "use the formulas."

Week 6

- Chapter 5. (graphsketching)
- Tangent and Normal lines to a graph
- The Intermediate Value Theorem
- Finding sign changes of a function
- Increasing and decreasing functions
- Examples
- Maxima and Minima
- Must a function always have a maximum?
- Examples – functions with and without maxima or minima

Depending on how fast you go, and on how much attention you want to pay to the Intermediate and Mean Value Theorems, this could actually be not enough material for a whole week. On the other hand, the example of the weird function on page 96 shows that "$f'(x) > 0$" perhaps doesn't mean all that some people think it means.

Week 7

- General method for sketching the graph of a function
- Convexity, Concavity and the Second Derivative
- Optimization Problems
- Parametrized Curves

Optimization problems are again important to see if students know more than "$dx^n/dx = nx^{n-1}$." To prove that the graph of a convex function always lies below any of its chords you need the Mean value Theorem. In math 275 I would make this exam material. Here I will explain it in lecture if there is time.

The subject of parametrized curves can be shortened if necessary by skipping the description of curvature and osculating circle.

Week 8

- l'Hopital's rule

- Chapter 7. (expANDlog)
- Exponents
- Logarithms
- Properties of logarithms
- Graphs of exponential functions and logarithms
- The derivative of a^x and the definition of e
- Derivatives of Logarithms
- Limits involving exponentials and logarithms
- Exponential growth and decay

The proof of l'Hopital's rule (or "reason why it works") requires the MVT and parametrized curves. A number of other departments have told us they like to use l'Hopital, so it doesn't look good when we send them students who don't know the rule.

There are many topics here, but the description of logarithms should be review. The derivation of $de^x/dx = e^x$ is rigorous if you are willing to assume that a^x is defined for all $x \in \mathbb{R}$ and $a > 0$, and if you are willing to assume that at least 2^x is differentiable at $x = 0$.

The topic of exponential growth and decay is important for many reasons: exponential growth just shows up very often while $dX/dt = kX$ is the first differential equation they run into. They will see more of them in 222.

Week 9

- Chapter 8. (integration)
- Area under a Graph
- When f changes its sign
- The Fundamental Theorem of Calculus
- The summation notation
- The indefinite integral
- Properties of the Integral

The treatment here is a bit streamlined compared with more traditional approaches to the integral that begin with long discourses on finding a formula for $1^2 + 2^2 + \cdots + n^2$. The point of the fundamental theorem is that you don't need to do that to find the area under a parabola.

Week 10

- The definite integral as a function of its integration bounds
- Method of substitution
- Chapter 9. (intapps)
- Areas between graphs

This and the next two weeks seem straightforward. Students should get a lot of practice finding simple antiderivatives. The point of the chapter on applications of integrals is to solidify the students' intuition for the integral. A complaint we have heard from departments that use a lot of calculus is that their students know how to find $\int(x^2+3x)dx$, but they have no idea of the intuitive meaning of the integral as a sum of "infinitely many infinitely small pieces."

The formulas in the last chapter are less important than their intuitive "derivation," where derivation does not mean rigorous mathematical proof, but rather the simple intuitive arguments that physicists and engineers use to reconstruct the formulas.

Week 11

- Cavalieri's principle and volumes of solids
- Three examples of volume computations of solids of revolution
- Volumes by cylindrical shells
- Distance from velocity

Week 12

- The length of a curve
- Velocity from acceleration
- Work done by a force