MATH 221 – The Limit properties

1. Special Limits

$$\lim_{x\to a} c = c \tag{1}$$

$$\lim_{x\to a} x = a \tag{2}$$

$$\lim_{x\to a} \frac{\sin x}{x} = 1 \tag{3}$$

2. Sums, Products and Quotients

If $\lim_{x\to a} f(x) = A$ and $\lim_{x\to a} g(x) = B$ both exist, then

(4) $\lim_{x\to a} f(x) + g(x)$ exists, and $\lim_{x\to a} f(x) + g(x) = A + B$

(5) $\lim_{x\to a} f(x) - g(x)$ exists, and $\lim_{x\to a} f(x) - g(x) = A - B$

(6) $\lim_{x\to a} f(x)g(x)$ exists, and $\lim_{x\to a} f(x)g(x) = AB$

(7) $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ exists, and $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{A}{B}$ provided $B \neq 0$.

3. Inequalities

If $\lim_{x\to a} f(x) = A$ and $\lim_{x\to a} g(x) = B$ both exist, and if $f(x) \le g(x)$ for all $x \neq a$, then $A \le B$, i.e.

$$\lim_{x\to a} f(x) \le \lim_{x\to a} g(x). \tag{8}$$

If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist ***and are equal***, and if there is a third function h for which you know that

$$f(x) \le h(x) \le g(x) \text{ for all } x \neq a,$$

then, first of all, the limit $\lim_{x\to a} h(x)$ exists, and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = \lim_{x\to a} g(x).$$

4. Substitution

If $\lim_{x\to a} f(x) = A$ exists, and if the function $g(u)$ is continuous at $u = A$ then

$$\lim_{x\to a} g\big(f(x)\big) = \lim_{u\to A} g(u) = g(A),$$

or, written differently,

$$\lim_{x\to a} g\big(f(x)\big) = g\big(\lim_{x\to a} f(x)\big). \tag{9}$$