MATH/STAT 455: Mathematical Statistics

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Table of contents

Welcome to Mathematical Statistics!

This book contains the course notes for *MATH/STAT 455: Mathematical Statistics* at Macalester College, as taught by Prof. Taylor Okonek. These notes draw from course notes created by Prof. Kelsey Grinde, and heavily from the course textbook, *An Introduction to Mathematical Statistics and Its Applications* by Richard Larsen and Morris Marx (6th Edition). Each chapter will contain (at a minimum): Learning Goals, Associated Readings, Definitions, Theorems, and Worked Examples.

I will be editing and adding to these notes throughout Spring 2024, so please check consistently for updates!

If you find any typos or have other questions, please email tokonek@macalester.edu.

1 Probability: A Brief Review

MATH/STAT 455 builds directly on topics covered in *MATH/STAT 354: Probability*. You're not expected to perfectly remember everything from *Probability*, but you will need to have sufficient facility with the following topics covered in this review Chapter in order to grasp the majority of concepts covered in *MATH/STAT 455*.

1.1 Learning Objectives

By the end of this chapter, you should be able to…

- Distinguish between important probability models (e.g., Normal, Binomial)
- Derive the expectation and variance of a single random variable or a sum of random variables
- Define the moment generating function and use it to find moments or identify pdfs

1.2 Associated Readings

1.3 Definitions

You are expected to know the following definitions:

- Probability density function (discrete, continuous)
    - Note: I don't care if you call a pmf a pdf… I will probably do this continuously throughout the semester. We don't need to be picky about this in *MATH/STAT 455*.
- Cumulative distribution function (discrete, continuous)
- Joint probability density function
- Conditional probability density function
- Independence

- Random Variable
- Expected Value / Expectation
- Variance
- r^{th} moment
- Covariance
- Random Sample
- Moment Generating Function

You are expected to know the following probability distributions:

Table 1.1: Table of main probability distributions we will work with for *MATH/STAT 455*.

Distribution	PDF/PMF	Parameters
Uniform	$\pi(x) = \frac{1}{\beta-\alpha}$	$\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$
Normal	$\pi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{1}{2\sigma^2}(x-\mu)^2)$	$\mu \in \mathbb{R}$, $\sigma > 0$
Multivariate Normal	$\pi(\mathbf{x}) - (2\pi)^{-k/2}\|\Sigma\|^{-1/2} \exp(-\frac{1}{2}(\mathbf{X} - \mu)^\top \Sigma^{-1}(\mathbf{X} - \mu)))$	$\mu \in \mathbb{R}^k$, $\Sigma \in \mathbb{R}^{k \times k}$, positive semi-definite
Gamma	$\pi(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$	α (shape), β (rate) > 0
Chi-square	$\pi(x) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2})$	$\nu > 0$
Exponential	$\pi(x) = \beta e^{-\beta x}$	$\beta > 0$
Student-\$t\$	$\pi(x) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{\nu\pi}}(1 + \frac{x^2}{\nu})^{-(\nu+1)/2}$	$\nu > 0$
Beta	$\pi(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$	$\alpha, \beta > 0$
Poisson	$\pi(x) = \frac{\lambda^k e^{-\lambda}}{k!}$	$\lambda > 0$
Binomial	$\pi(x) = \binom{n}{x} p^x (1-p)^{n-x}$	$p \in [0,1], n = \{0, 1, 2, \dots\}$
Multinomial	$\pi(\mathbf{x}) = \frac{n!}{x_1! \dots x_k!} p_1^{x_1} \dots p_k^{x_k}$	$p_i > 0$, $p_1 + \cdots + p_k = 1$, $n = \{0, 1, 2, \dots\}$
Negative Binomial	$\pi(x) = \binom{k+r-1}{k}(1-p)^k p^r$	$r > 0$, $p \in [0,1]$

1.4 Theorems

- Law of Total Probability
- Bayes' Theorem: $p(A \mid B) = \frac{p(B|A)p(A)}{p(B)}$
- Relationship between pdf and cdf

- Expectation and variance of linear transformations of random variables
- Relationship between mean and variance
- Finding a marginal pdf from a joint pdf
- Independence of random variables and joint pdfs
- Expected value of a product of independent random variables
- Covariance of independent random variables
- Using MGFs to find moments
- Using MGFs to identify pdfs
- Central Limit Theorem

1.5 Worked Examples

2 Maximum Likelihood Estimation

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3 Method of Moments

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4 Properties of Estimators

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5 Consistency

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6 Asymptotics & the Central Limit Theorem

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7 Computational Optimization

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8 Bayesian Inference

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9 Decision Theory

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10 Hypothesis Testing

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References