



NANYANG  
TECHNOLOGICAL  
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**CZ3005**

**Artificial Intelligence**

**Knowledge  
Representation and  
Reasoning**

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# Knowledge Representation

## □ Knowledge representation (KR)

- KB: set of sentences  $\rightarrow$  need to
- Express knowledge in a (computer-) tractable form

## □ Knowledge representation language

- Syntax – implementation level
  - Possible configurations that constitute sentences
- Semantics – knowledge level
  - Facts of the world the sentences refer to
  - e.g. language of arithmetics:  $x, y$  numbers  
sentence: “ $x \geq y$ ”, semantics: “greater or equal”

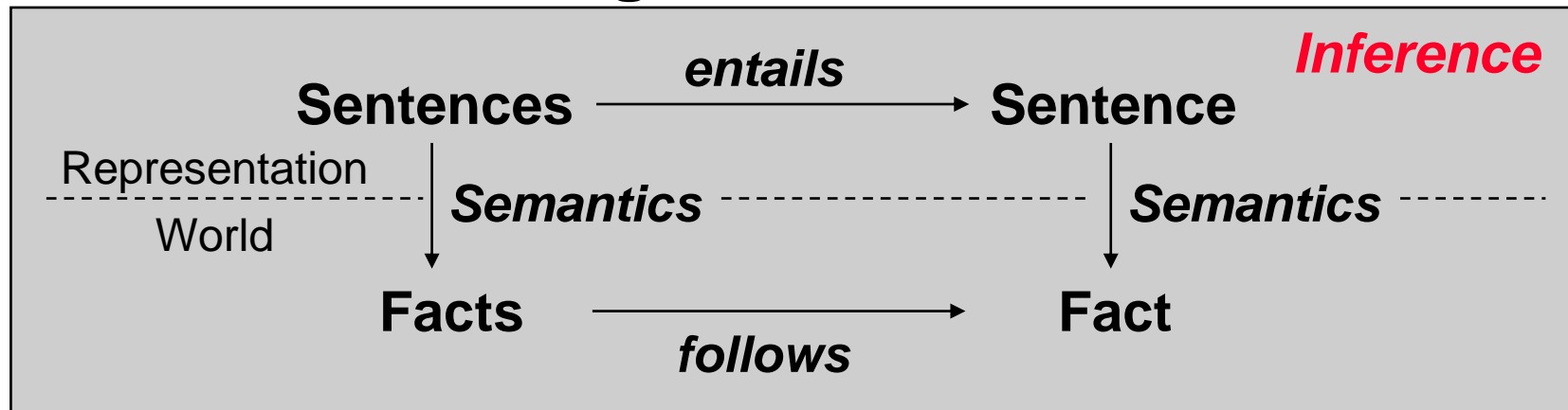
## ❑ Logic

- Representation + Inference = Logic
  - Where representation = syntax + semantics

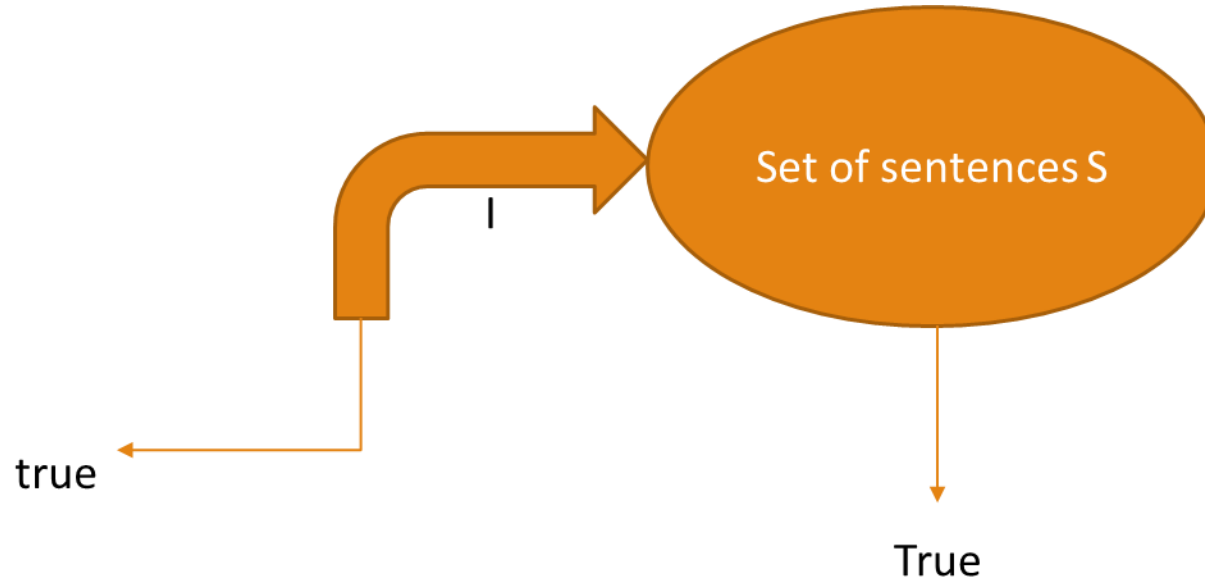
## ❑ Reasoning

- Construction of new sentences from existing ones

## ❑ Entailment as logical inference



# Entailment





# Entailment and Inference

## □ Entailment

- Generate sentences that are necessarily true, given that the existing sentences are true
- Notation:  $KB \models \alpha$ 
  - e.g. Wumpus world:  
 $\{ \neg S(1,1), \neg B(1,1) \} \models \text{OK}(2,1)$
  - Arithmetics:  
 $\{ x \geq y, y \geq z \} \models x \geq z$

## □ Inference

- **Tell**, given KB:  $(KB \models \alpha) !$
- **Ask**, given KB and  $\alpha$ :  $(KB \models \alpha) ?$

- *Can be described by the sentences it derives,  $KB \models \alpha_I$*

## □ Soundness

- Generate only entailed sentences
- Proof: sequence of operations of a sound inference
  - Record of operations that generate a specific entailed sentence  
e.g. “Smoke  $\Rightarrow$  Fire” and “Smoke”  $\models$  “Fire”  
“Fire  $\Rightarrow$  Call\_911” and “Fire”  $\models$  “Call\_911”

## □ Completeness

- A proof can be found for any entailed sentence

## □ Proof theory

- Specify the reasoning operations that are sound

## □ Definition

- *Inference (reasoning) is the process by which conclusions are reached*
- Logical inference (deduction) is the process that implements entailment between sentences

## □ Useful properties

- Valid sentence (tautology)
  - iff TRUE under all possible interpretations in all possible worlds.
    - e.g. “ $S$  or  $\neg S$ ” is valid, “ $S(2,1)$  or  $\neg S(2,1)$ ”, etc.
- Satisfiable sentence
  - iff there is some interpretation in some world for which it is TRUE
    - e.g. “ $S$  and  $\neg S$ ” is unsatisfiable

# Quiz



**Kahoot.it**







# An Example of Sound Inference

- ❑ Sentence:  $x$ 
  - Semantics: an expression; can be a single symbol or number, the concatenation of 2 expressions, etc.
- ❑ Sentence:  $x y$ 
  - Semantics: an expression which refers to a quantity that is the product of the quantities referred to by each of the expressions
- ❑ Sentence:  $x = y$ 
  - Semantics: the 2 expressions on each side of “=” refer to the same quantity

- ❑ A sound inference: from  $E = mc^2$

$$T_1 \geq T_2 \quad = \quad E \quad T_1 \geq mc^2 \quad T_2$$

# □ Interpretation (meaning)

- *Correspondence between sentences and facts*
- Arbitrary meaning, fixed by the writer of the sentence
  - e.g. Natural languages: meaning fixed by usage (cf. dictionary)  
exceptions: encrypted messages, codes (e.g. Morse)  
*chat*
- Systematic relationship: compositional languages
  - *The meaning of a sentence is a function of the meaning of its parts.*
- Truth value
  - A sentence make a claim about the world → TRUE or FALSE
  - *Depends on the interpretation and the state of the world*
    - e.g. Wumpus world:  $S(1,2)$  true if means “Stench at  $[1,2]$ ” and the world has a wumpus at either  $[1,3]$  or  $[2,2]$ .

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# □ Formal logic

- Syntax
  - A set of rules for writing sentences
- Semantics
  - A set of rules (constraints) for relating sentences to facts
- Proof theory / inference procedure
  - A set of rules for deducing entailments of sentences

# □ Propositional logic

- Symbols, representing propositions (facts)
- Boolean connectives, combining symbols
  - e.g. “Hot” or “Hot and Humid”

## □ First-order logic

- Objects and predicates, representing properties of and relations between objects
- Variables, Boolean connectives and quantifiers
  - e.g. “Hot(x)”, “Hot(Air)” or “Hot(Air) and Humid(Air)”

## □ Temporal logic

- World ordered by a set of time points (intervals)

## □ Probabilistic and fuzzy logic

- Degrees of belief and truth in sentences
  - e.g. “Washington is a state” with belief degree 0.4, “a city” 0.6, “Washington is a large city” with truth degree 0.6



# Commitments of Logics

Formal (KR) Language	Ontological commitment (what exists in the world)	Epistemological commitment (what an agent believes about facts)
Propositional logic	facts	true / false / unknown
First-order logic	facts, objects, relations	true / false / unknown
Temporal logic	facts, objects, rel., times	true / false / unknown
Probability logic	facts	degree of belief 0...1
Fuzzy logic	degrees of truth 0...1	degree of belief 0...1



# Propositional Logic

□ P is “It rains on Tuesday”

□ Q is “John likes chocolate”

P and Q are either TRUE or FALSE.



# First order Logic

□ If  $x$  is a man, then  $x$  is a mortal

- $\text{man}(x) \Rightarrow \text{mortal}(x)$
- $\neg \text{man}(x) \vee \text{mortal}(x)$

□ If  $n$  is a natural number, then  $n$  is either even or odd.

- $\text{Natural}(n) \Rightarrow \text{even}(n) \vee \text{odd}(n)$

# Fuzzy Logic



## Sets with fuzzy boundaries

**A = Set of tall people**

