NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

CZ3005
Artificial Intelligence

Propositional Logic

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Illustration

- Anil is Intelligent
- Anil is hardworking
- If Anil is Intelligent and Anil is Hardworking, then Anil scores a high mark

Propositions

Elements of Propositional Logic

- **Symbols**
  - Logical constants: TRUE, FALSE
  - Propositional symbols: P, Q, etc.
  - Logical connectives: Λ, $\vee$, $\Leftrightarrow$, $\Rightarrow$, $\neg$
  - Parentheses: ( )

- **Sentences**
  - Atomic sentences: constants, propositional symbols
  - Combined with connectives, e.g. $P \Lambda Q \vee R$
    also wrapped in parentheses, e.g. $(P \Lambda Q) \vee R$

Elements of Propositional Logic

- Sentences
  - Atomic sentences: the indivisible syntactic elements
    - constants, propositional symbols
  - Complex Sentences: constructed from simpler sentences using logical connectives.
    - Combined with connectives, e.g. $P \land Q \lor R$
    - Wrapped in parentheses, e.g. $(P \land Q) \lor R$

True is the always-true proposition and *False* is the always-false proposition.

Example

- P is “It rains on Tuesday”
- Q is “John likes chocolate”

P and Q are either TRUE or FALSE.

Logical Connective

- ❑ Conjunction $\wedge$
  - Binary op., e.g. P $\wedge$ Q, “P and Q”, where P, Q are the *conjuncts*
- ❑ Disjunction $\vee$
  - Binary op., e.g. P $\vee$ Q, “P or Q”, where P, Q are the *disjuncts*
- ❑ Implication $\Rightarrow$
  - Binary op., e.g. P $\Rightarrow$ Q, “P implies Q”, where P is the *premise* (antecedent) and Q the *conclusion* (consequent)
  - Conditionals, “if-then” statements, or <u>*rules*</u>
- ❑ Equivalence $\Leftrightarrow$
  - Binary op., e.g. P $\Leftrightarrow$ Q, “P equivalent to Q”
  - Biconditionals.
- ❑ Negation $\neg$
  - Unary op., e.g. $\neg$ P, “not P”

Syntax of Propositional Logic

Sentence	$\rightarrow$	AtomicSentence | ComplexSentence
AtomicSentence	$\rightarrow$	LogicalConstant | PropositionalSymbol
ComplexSentence	$\rightarrow$	(Sentence) | Sentence LogicalConnective Sentence | ¬Sentence
LogicalConstant	$\rightarrow$	TRUE | FALSE
PropositionalSymbol	$\rightarrow$	P | Q | R | …
LogicalConnective	$\rightarrow$	$\wedge \mid \vee \mid \Leftrightarrow \mid \Rightarrow \mid \neg$

Precedence (from highest to lowest): $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

e.g.: $\neg P \wedge Q \vee R \Rightarrow S$ (not ambiguous), eq. to: $(((\neg P) \wedge Q) \vee R) \Rightarrow S$

Example

- ❑Let P stands for Intelligent(Anil)
- ❑Let Q stands for Hardworking(Anil)
- ❑What does $P \land Q$ mean ?
- ❑What does $P \lor Q$ mean ?

$P \land Q$, $P \lor Q$ are compound proposition

Example

Use parenthesis to ensure that the syntax is completely unambiguous:

$(A \wedge B) \Rightarrow C$ and $A \wedge (B \Rightarrow C)$

- A: John likes Kate.
- B: John likes Chocolate.
- C: John buys Chocolate

If John likes Chocolate, then John buys Chocolate:

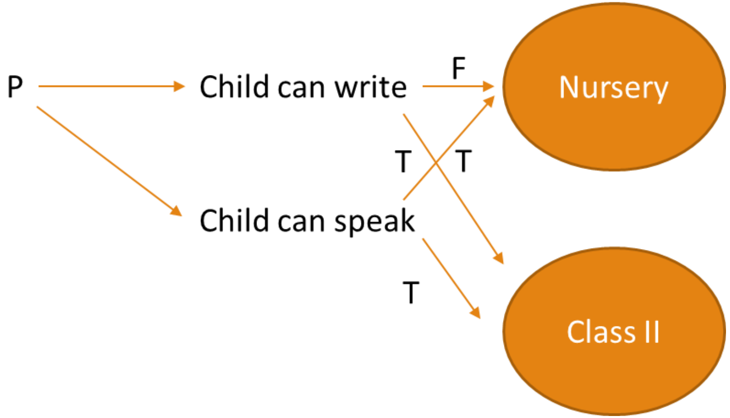
$B \Rightarrow C$

Semantic of Propositional Logic

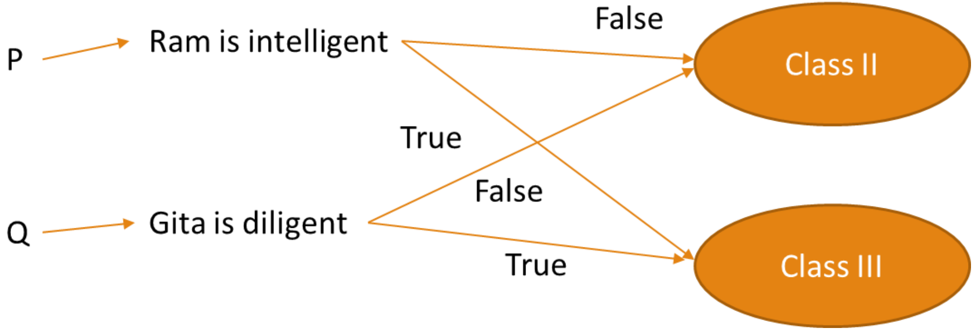
- **Interpretation of symbols**
  - Logical constants have fixed meaning
    - True: always means the fact is the case; valid
    - False: always means the fact is not the case; unsatisfiable
  - Propositional symbols mean “whatever they mean”
    - e.g.: **P** “we are in a pit”, etc.
    - Satisfiable, but not valid (true only when the fact is the case)
- **Interpretation of sentences**
  - Meaning derived from the meaning of its parts
    - Sentence as a combination of sentences using connectives
  - Logical connectives as (boolean) functions:

TruthValue ***f*** (TruthValue, TruthValue)

Example



P
Child can write
Child can speak
F
T
T
T
Nursery
Class II



P
Q
Ram is intelligent
Gita is diligent
False
True
False
True
Class II
Class III

Example

P: likes(Joyce,Richard)

Q: Know(Budi,Andi)

World: Joyce likes Richard and Budi knows Andi

P=T, Q=T

$P \wedge Q = ?$

$P \wedge \neg Q = ?$

Validity

- A sentence is **valid** if it is true in **all** models.
- Valid sentences are known as **tautologies – necessarily true or vacuously true.**
- Every valid sentence is logically equivalent to True.

Satisfiability

- A sentence is **satisfiable** if it is true in **some** models.
- Satisfiability can be checked by enumerating the possible models until one is found that satisfies the sentence.
- Most problems in computer sciences are satisfiability problems.
  - E.g., Constraint satisfaction problem, Search problems.

Interpretation

- Interpretation of symbols
  - Logical constants have fixed meaning
    - True: always means the fact is the case; valid
    - False: always means the fact is not the case; unsatisfiable
  - Propositional symbols mean “whatever they mean”
    - e.g.: P: “we are in a pit”, etc.
    - Satisfiable, but not valid (true only when the fact is the case)

Truth Table

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

Truth tables for the five logical connectives

Testing for Validity and Satisfiability

- Testing for validity
  - Using truth-tables, checking all possible configurations
    - e.g.: $((P \vee Q) \wedge \neg Q) \Rightarrow P$

P	Q	$P \vee Q$	$\neg Q$	$(P \vee Q) \wedge \neg Q$	$((P \vee Q) \wedge \neg Q) \Rightarrow P$
False	False	False	True	False	True
False	True	True	False	False	True
True	False	True	True	True	True
True	True	True	False	False	True

Example

Show whether $(A \wedge B) \Rightarrow C$ and $A \wedge (B \Rightarrow C)$ is valid, unsatisfiable, or neither.

A	B	C	$A \wedge B$	$B \Rightarrow C$	$(A \wedge B) \Rightarrow C$	$A \wedge (B \Rightarrow C)$
1	1	1	1	1		
1	1	0	1	0		
1	0	1	0	1		
1	0	0	0	1		
0	1	1	0	1		
0	1	0	0	0		
0	0	1	0	1		
0	0	0	0	1		

Quiz

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