NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

CZ3005 Artificial Intelligence

Game Theory

Assoc Prof Bo AN

www.ntu.edu.sg/home/boan
Email: boan@ntu.edu.sg
Office: N4-02b-55

What is Game Theory?

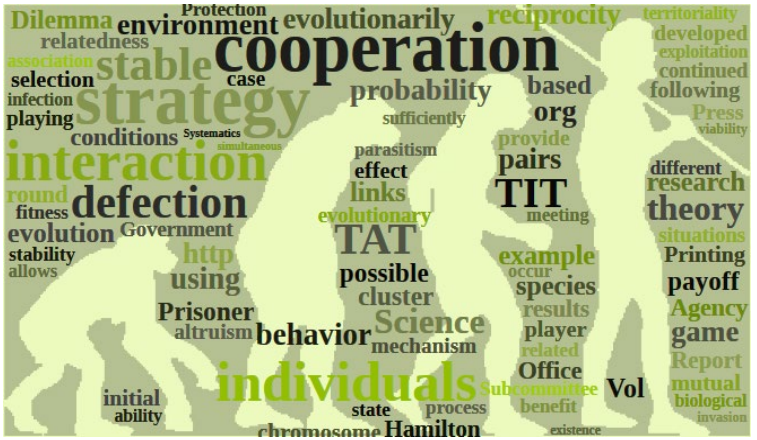
- Game theory studies settings where multiple parties (agents) each have
  - different preferences (utility functions),
  - different actions that they can take
- Each agent's utility (potentially) depends on all agents' actions
  - What is optimal for one agent depends on what other agents do
  - Very circular!
- Game theory studies how agents can rationally form beliefs over what other agents will do, and (hence) how agents should act
  - Useful for acting as well as predicting behavior of others
- John von Neumann

What is Game Theory?

Economics

Politics

Games

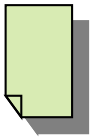


Biology

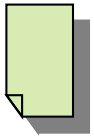
Normal Form Games – An Example

- List of players, strategies, payoffs
- Simultaneous
- Zero-sum here but not necessary
- Analysis:
  - What should they do?
  - Advice to player A

Player B



Rock



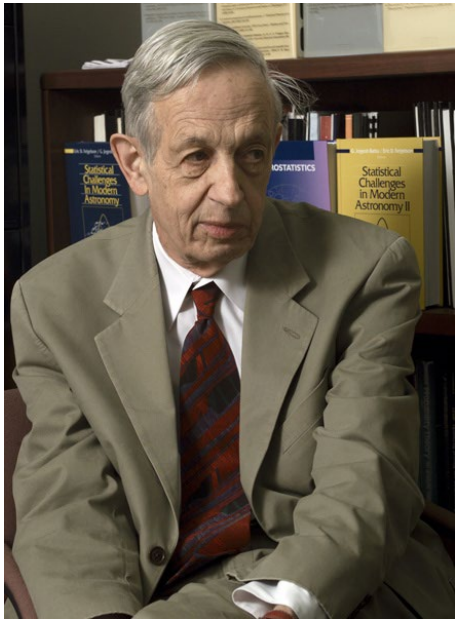
Player A

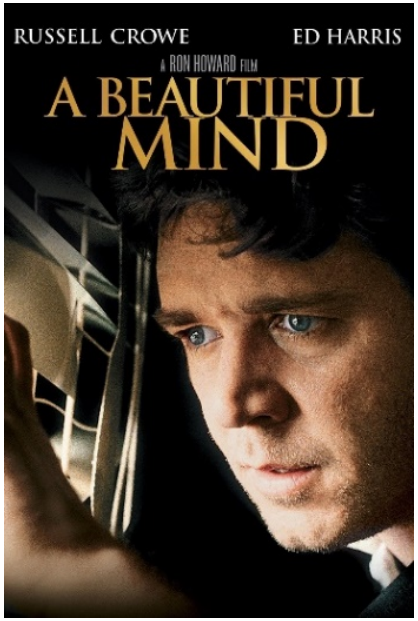
Rock

0, 0	1, -1	-1, 1
-1, 1	0, 0	1, -1
1, -1	-1, 1	0, 0

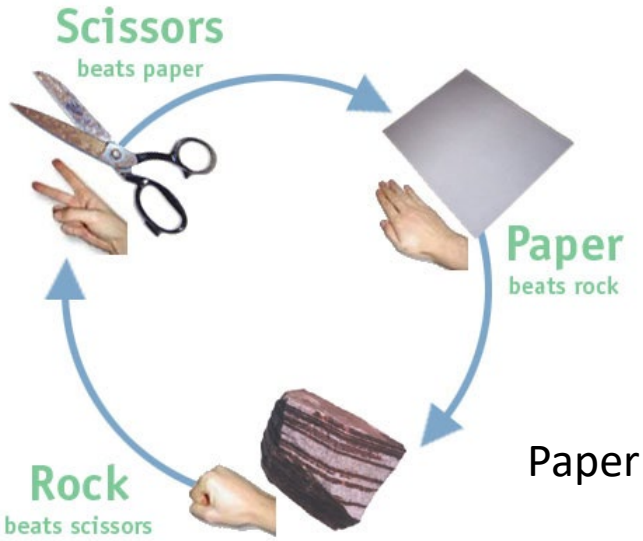
Nash Equilibrium

- Each agent is selfish
- Each agent makes decision based on what he thinks others would do
- No one can do better by changing strategy solely





Nash Equilibrium



Player A

Paper **1/3**

Rock **1/3**

Scissors **1/3**

Player B

Paper **1/3** Rock **1/3** Scissors **1/3**

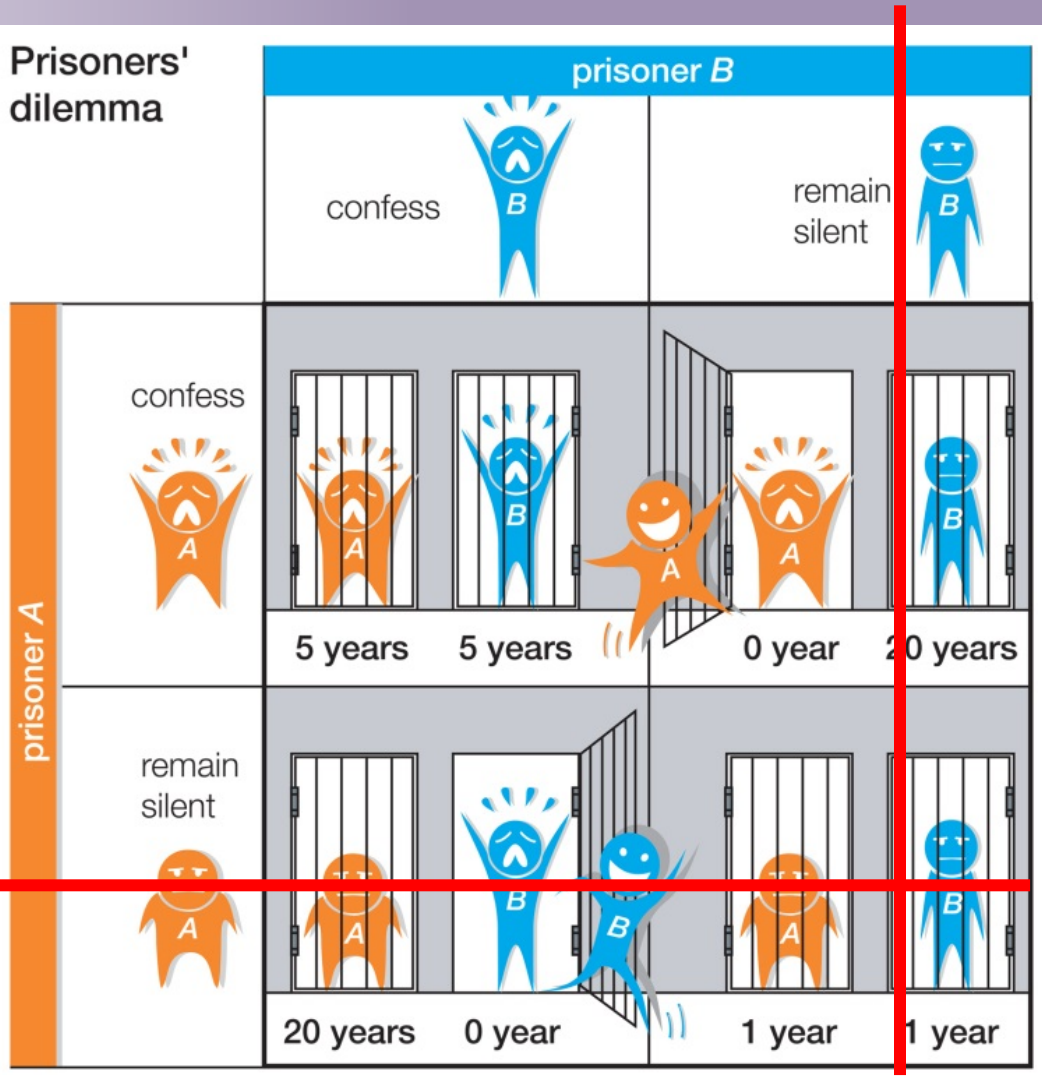
0, 0	1, -1	-1, 1
-1, 1	0, 0	1, -1
1, -1	-1, 1	0, 0

Nash Equilibrium

Nash equilibrium

confess ***dominates*** silent

Cooperation would be better for both!
But, rational for both to defect!



Nash Equilibrium

- In general, we will say that two strategies s_1 (for i) *and* s_2 (for j) are in Nash equilibrium if:
  1. under the assumption that agent i plays s_1, agent j can do no better than play s_2; and
  2. under the assumption that agent j plays s_2, agent i can do no better than play s_1.
- *Neither agent has any incentive to deviate from a Nash equilibrium*
- Unfortunately:
  1. *Not every interaction scenario has a pure strategy Nash equilibrium*
  2. *Some interaction scenarios have more than one pure strategy Nash equilibrium*

		i: defect	i: coop
j	defect	1 1	4 1
	coop	1 4	4 4

Matching Pennies

- Players *i* and *j* simultaneously choose the face of a coin, either “heads” or “tails”.
- If they show the same face, then *i* wins, while if they show different faces, then *j* wins.
- The Payoff Matrix:

	i heads	*i* tails
j heads	1 -1	-1 1
j tails	-1 1	1 -1

Mixed Strategies for Matching Pennies

- NO pair of strategies forms a pure strategy NE: whatever pair of strategies is chosen, somebody will wish they had done something else.
- The solution is to allow mixed strategies:
  - play “heads” with probability 0.5
  - play “tails” with probability 0.5.
- This is a NE strategy.

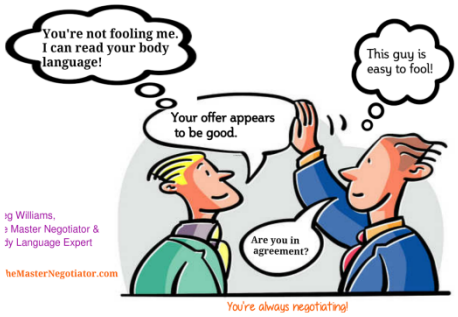
Games: Complete Information v.s. Incomplete Information

Games with complete information

Chess

Go

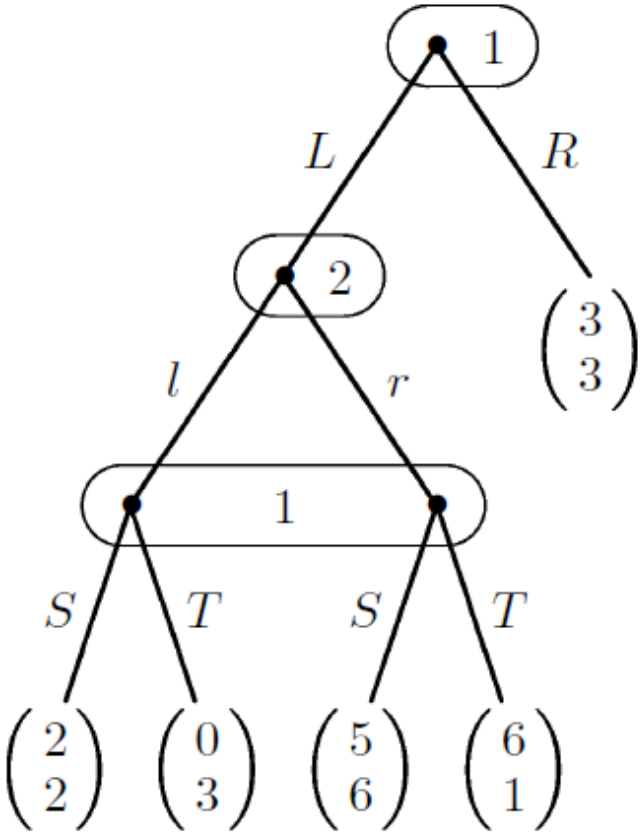
Games with incomplete information



Negotiation

Poker

Games: Normal Form v.s. Sequence Form



$$A = \begin{array}{cc} l & r \\ \hline 2 & 5 \\ 0 & 6 \\ 3 & 3 \\ 3 & 3 \end{array} \begin{array}{l} \langle L, S \rangle \\ \langle L, T \rangle \\ \langle R, S \rangle \\ \langle R, T \rangle \end{array} \qquad B = \begin{array}{cc} l & r \\ \hline 2 & 6 \\ 3 & 1 \\ 3 & 3 \\ 3 & 3 \end{array} \begin{array}{l} \langle L, S \rangle \\ \langle L, T \rangle \\ \langle R, S \rangle \\ \langle R, T \rangle \end{array}$$

$$A = \begin{array}{ccc} \emptyset & l & r \\ \hline & & \\ & & \\ 3 & & \\ & 2 & 5 \\ & 0 & 6 \end{array} \begin{array}{l} \emptyset \\ L \\ R \\ LS \\ LT \end{array} \qquad B = \begin{array}{ccc} \emptyset & l & r \\ \hline & & \\ & & \\ 3 & & \\ & 2 & 6 \\ & 3 & 1 \end{array} \begin{array}{l} \emptyset \\ L \\ R \\ LS \\ LT \end{array}$$

Texas Hold'em Poker

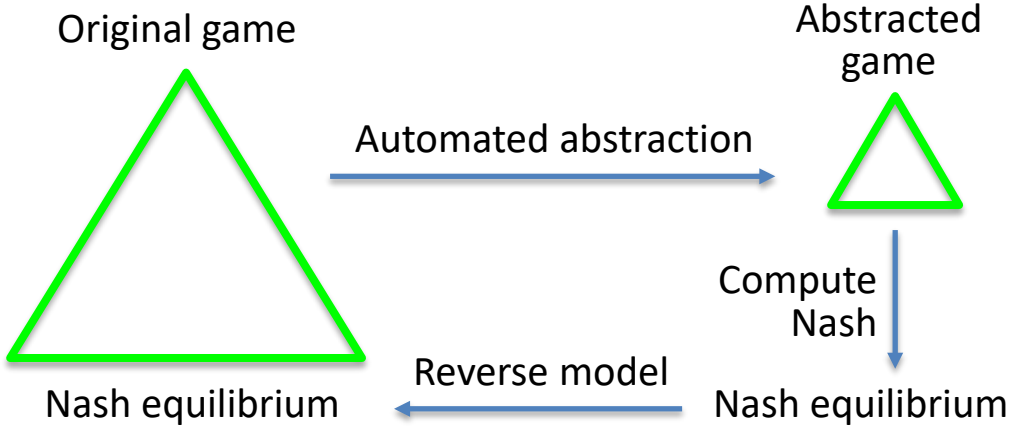



- Preflop: two private cards are dealt to each player, followed by a betting round; players can either check, bet or fold
- Flop: three public cards are dealt, followed by a betting round
- Turn: a fourth public card is dealt, followed by a betting round
- River: a last public card is dealt, followed by a betting round
- The game ends when
  - Only one player is left, all the other players fold
  - A showdown; a hand with the best 5 cards using both the two private cards and the five public cards wins

Architecture of Libratus

Abstraction (offline)

- action abstraction
- card abstraction
- took the game size from 10^{161} to 10^{12}



Equilibrium Finding (offline)

- CFR
- CFR+
- Monte Carlo CFR

Decomposition and Subgame Refinement (online)

- endgame solving
- subgame re-solving
- max-margin subgame refinement

Global Challenges for Security

Key challenges: Limited resources, surveillance

Stackelberg Games

Randomization: Increase Cost and Uncertainty to Attackers

- Security allocation
  - Target weights
  - Opponent reaction
- Stackelberg: Security forces commit first
- Optimal allocation: Weighted random
  - Strong Stackelberg Equilibrium

Attacker

Defender

	Target #1	Target #2
Target #1	4, -3	-1, 1
Target #2	-5, 5	2, -1

Game Theory for Security: Applications

Game Theory + Optimization + Uncertainty + Learning + …

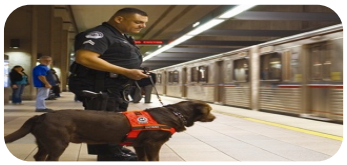
Infrastructure Security Games


Coast Guard


Coast Guard: Ferry


LAX

TSA


LA Sheriff


USC

Argentina Airport


Chile Border

Green Security Games

Coast Guard

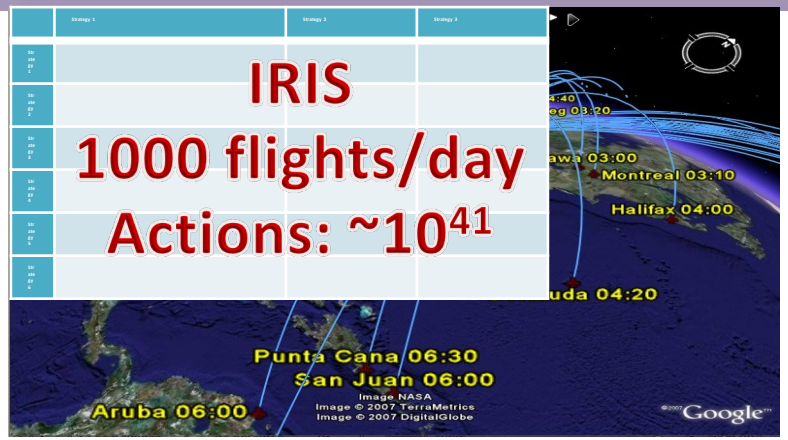

Panthera/WWF

India

Opportunistic Crime Games

Cyber Security Games

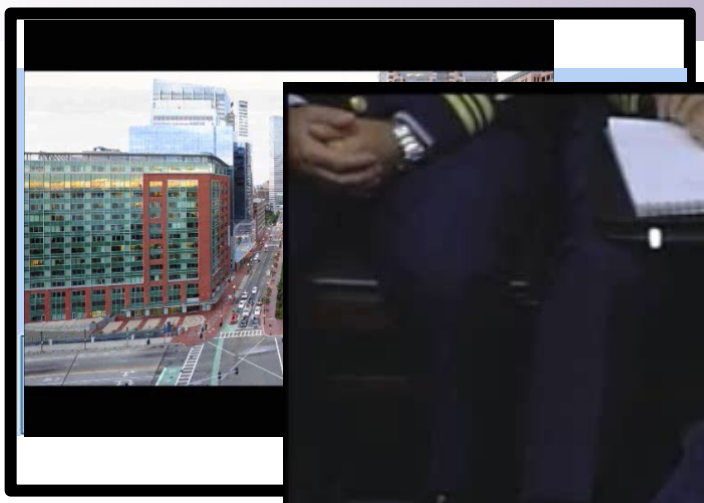
IRIS: Federal Air Marshals Service [2009] Scale Up Number of Defender Strategies

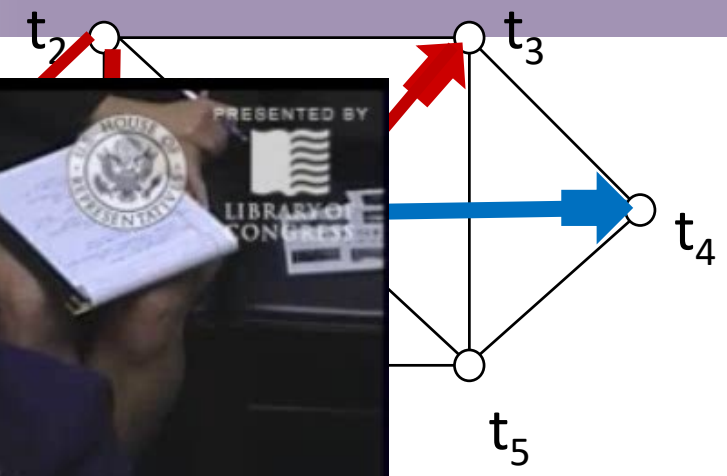


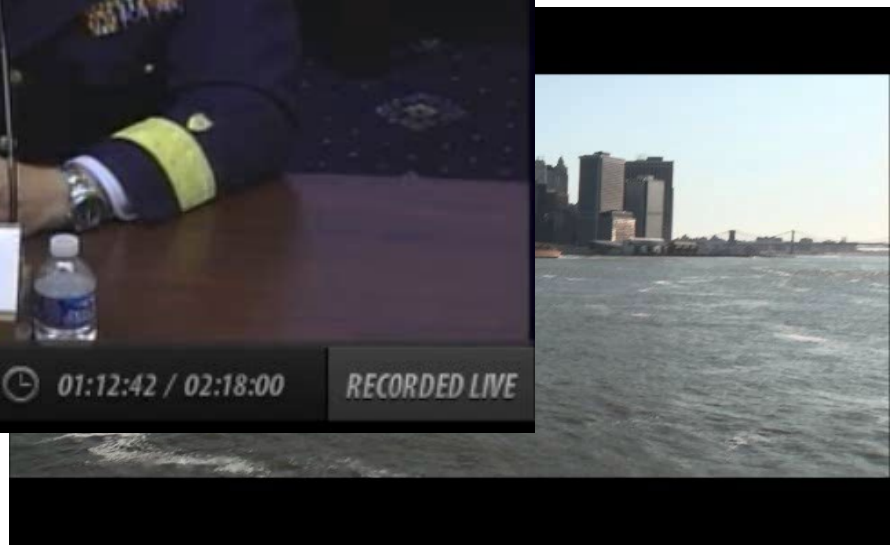
- 1000 Flights, 20 air marshals: 10^{41} combinations
  - ARMOR out of memory
- Not enumerate all combinations combinations
  - Branch and price
    - Branch & bound + column generation

PROTECT: Randomized Patrol Scheduling [2011]

Coordination (Scale-up) and Ferries (Continuous Space/time)



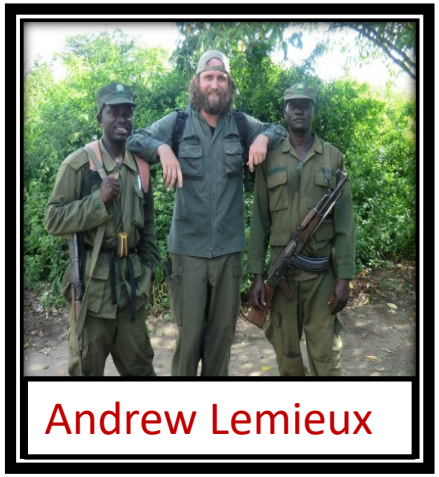




PAWS: Protection Assistant for Wildlife Security Trials in Uganda and Malaysia [2014]

- Important lesson: Geography!


Uganda


Andrew Lemieux


Malaysia

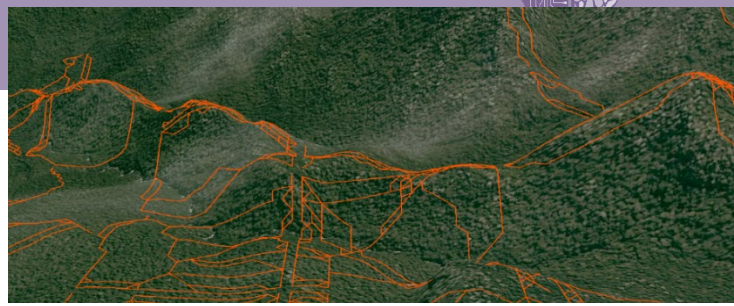

Panthera


Malaysia

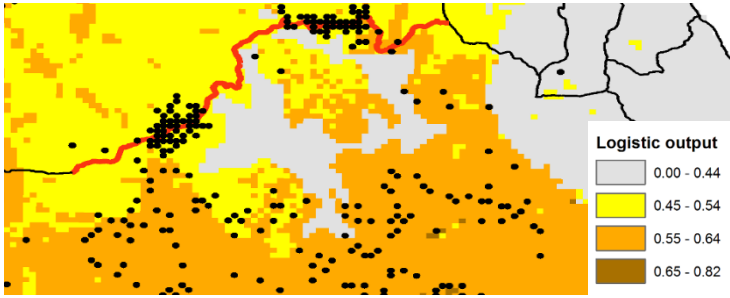
PAWS Deployed in 2015 in Southeast Asia (with Panthera and WWF)

PAWS Version 2: Features

- Street map
  - Ridgelines, rivers/streams



- Species Distribution Models (SDMs)
  - From data points to distribution map



Indonesia

Malaysia