5.1

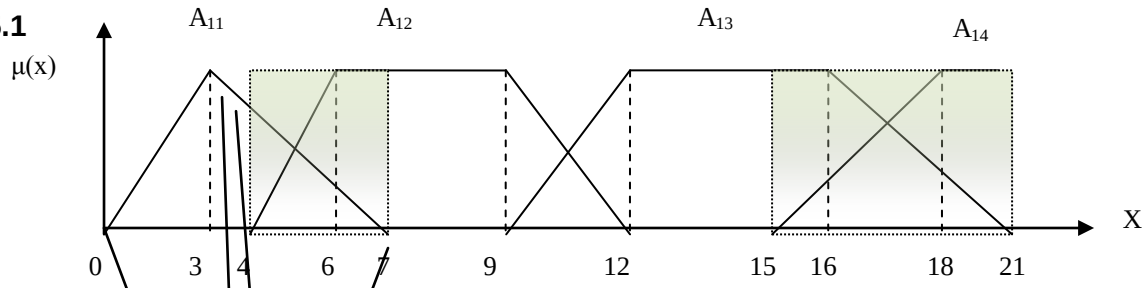


Figure Q5.1

Describe the fuzzy labels using the trapezoidal membership functions denoted by: TPMF[a, b, c, d] for each of the label.

Fill the rest of the links

A_{11}=TPFM[0,3,3,7] – triangular MF

A_{12}=TPMF[4,6,9,12]

A_{13}=TPMF[9,12,16,21]

A_{14}=[TPMF[15,18,21,21]

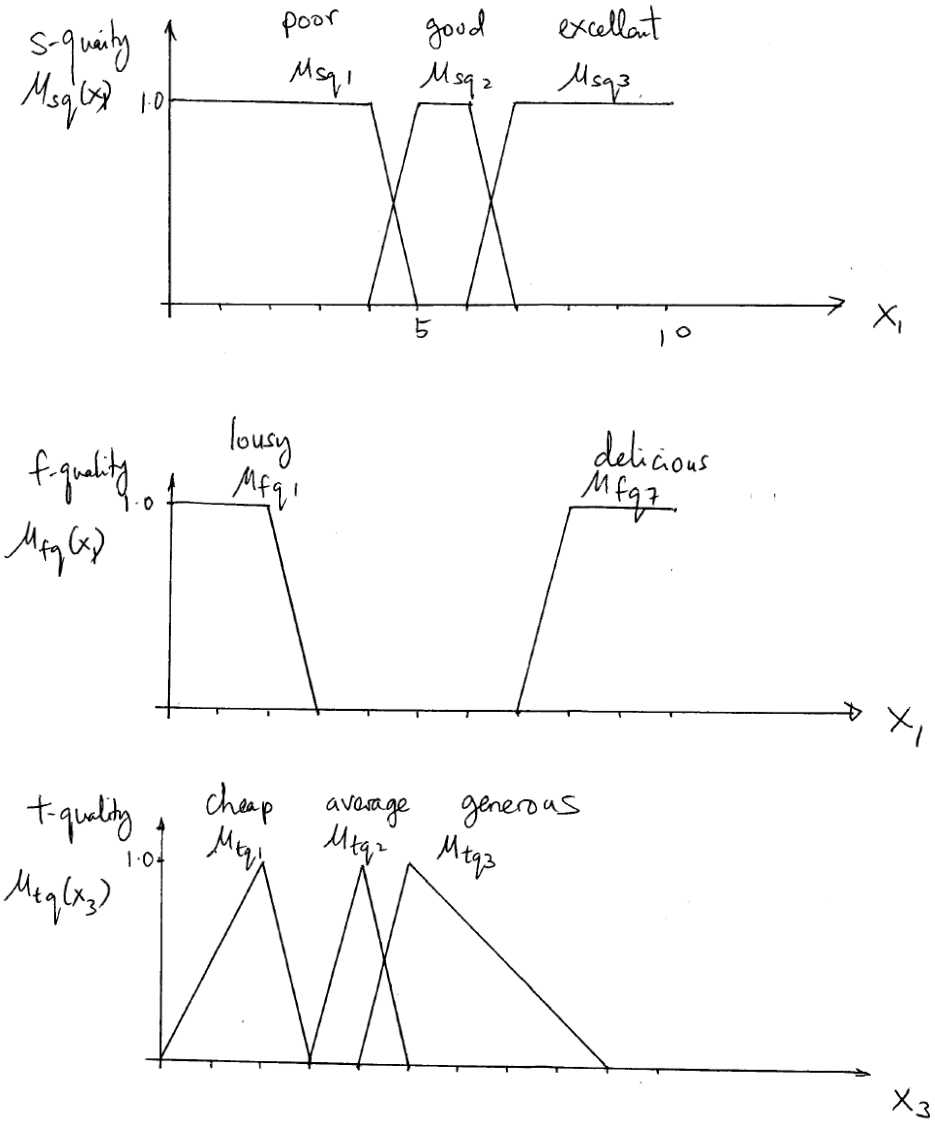
State the type of fuzzy partitioning of the space provided by these four membership functions over the interval.

This is a non-pseudo fuzzy partition. The interval overlaps between A11 and A12 and betweenA13 and A14 (the shaded regions) are non pseudo fuzzy partition, i.e. summation of MF in the overlap region is $\neq 1$

5.2 A set of fuzzy variables s-quality, f-quality and t-payment are defined by the respective set of membership functions:

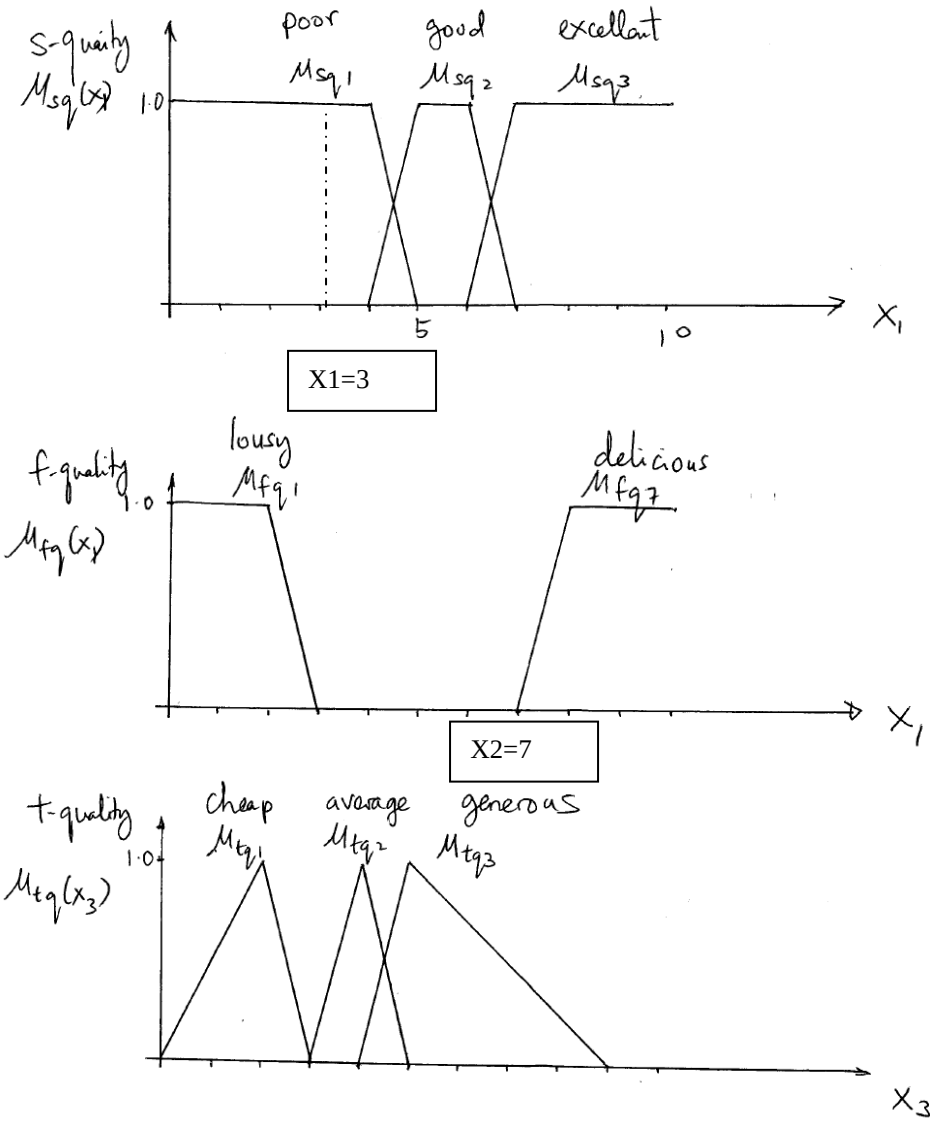
s-quality:	fuzzy term/label "poor" μ_{sq1}:	tpmf[0, 0, 4, 5]
	fuzzy term/label "good" μ_{sq2}:	tpmf[4, 5, 6, 7]
	fuzzy term/label "excellent" μ_{sq3}:	tpmf[6, 7, 10, 10]
f-quality:	fuzzy term/label "lousy" μ_{fq1}:	tpmf[0, 0, 2, 3]
	fuzzy term/label "delicious" μ_{fq2}:	tpmf[7, 8, 10, 10]
t-quality:	fuzzy term/label "cheap" μ_{tq1}:	tpmf[0, 2, 2, 3]
	fuzzy term/label "average" μ_{tq2}:	tpmf[3, 4, 4, 5]
	fuzzy term/label "generous" μ_{tq3}:	tpmf[4, 5, 5, 9]

(i) Draw the fuzzy partitions for each of the fuzzy variables over the domain [0, 10]. State the type of fuzzy partitioning of each of the dimensions.



f-quality and t-quality are non-pseudo fuzzy partition and s-quality is a pseudo fuzzy partition.

(ii) The above fuzzy labels are used in the formulation of an fuzzy expert rule system for tipping. The amount of tips (t-quality) derived from the fuzzy rules are based on the service quality (s-quality) and the food quality (f-quality). Here are 4 fuzzy rules:
Determine the membership for the resultant tip if the scores for s-quality is 3 and f-quality is 7.



R1. If service is poor then tip is cheap. $X_1=3$ and $X_2=7$

Antecedent

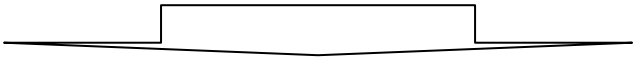
$\mu_{sq1(X1=3)} = 1$

using the fuzzy implication relation (AND) $f(A,B) = \min(\mu_{sq1(X1=3)} = 1,\ \mu_{tq1}) = \mu_{tq1}$
R1 conclusion is $\min(\mu_{sq1(X1=3)} = 1,\ \mu_{tq1}) = \mu_{tq1}$

R2. If service is excellent and food is delicious then tip is generous.

Antecedents $\mu_{sq3}(X1=3) = 0$ AND $\mu_{fq2}(X2=7)=0$

Min(0,0)=0



Fuzzy implication relation is the AND operator
R2 conclusion Min(0, μ_{tq3}) = 0.

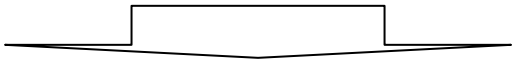
R3. If food is lousy then tip cheap.

Antecedent

$\mu_{fq1}(X2=7)=0$

Fuzzy implication relation is the AND operator
R3 conclusion Min(0, μ_{tq1})=0.

R4. If service is good and food is delicious then tip is average.



Antecedents

$\mu_{sq2}(3)=0 \wedge \mu_{fq2}(X2=7)=0$
Min(0,0)=0

Fuzzy implication relation is the AND operator
R4 conclusion Min(0, μ_{tq3}) = 0.

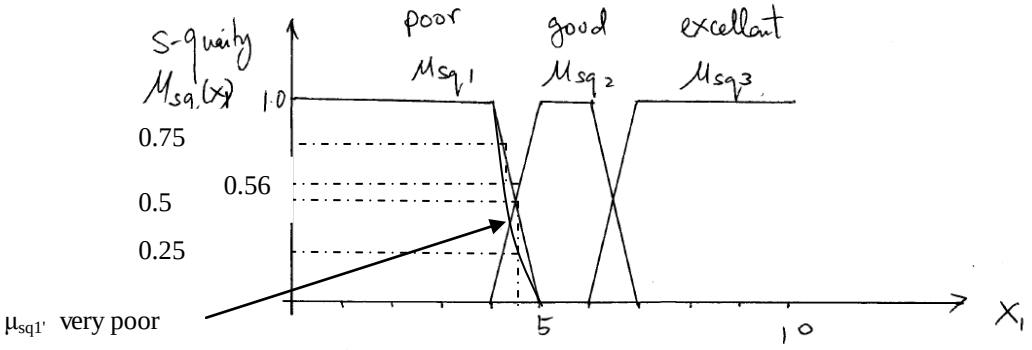
Fuzzy output is the aggregate of the outputs of each fuzzy rule. (Union operation - each of the rules is an alternative match.)

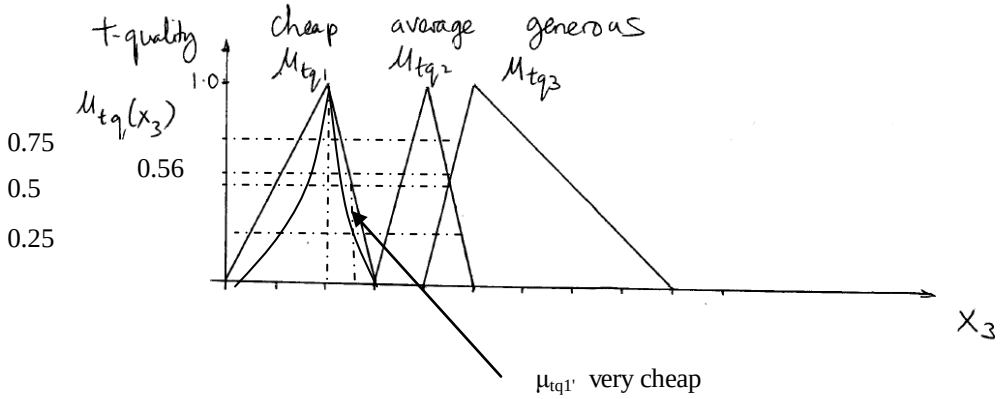
U($\mu_{tq1,}$0, 0, 0) = Max($\mu_{tq1,}$ 0, 0, 0) = $\mu_{tq1.}$
Either take maximum membership or Centroid defuzzification for the output $\mu_{tq1.}$

(iii) Linguistic modifiers or hedges are used to change the semantics of the linguistic labels. What will the fuzzy memberships for s-quality and t-quality be like if a rule is given as:

R1'. If service is very poor then tip is very cheap.

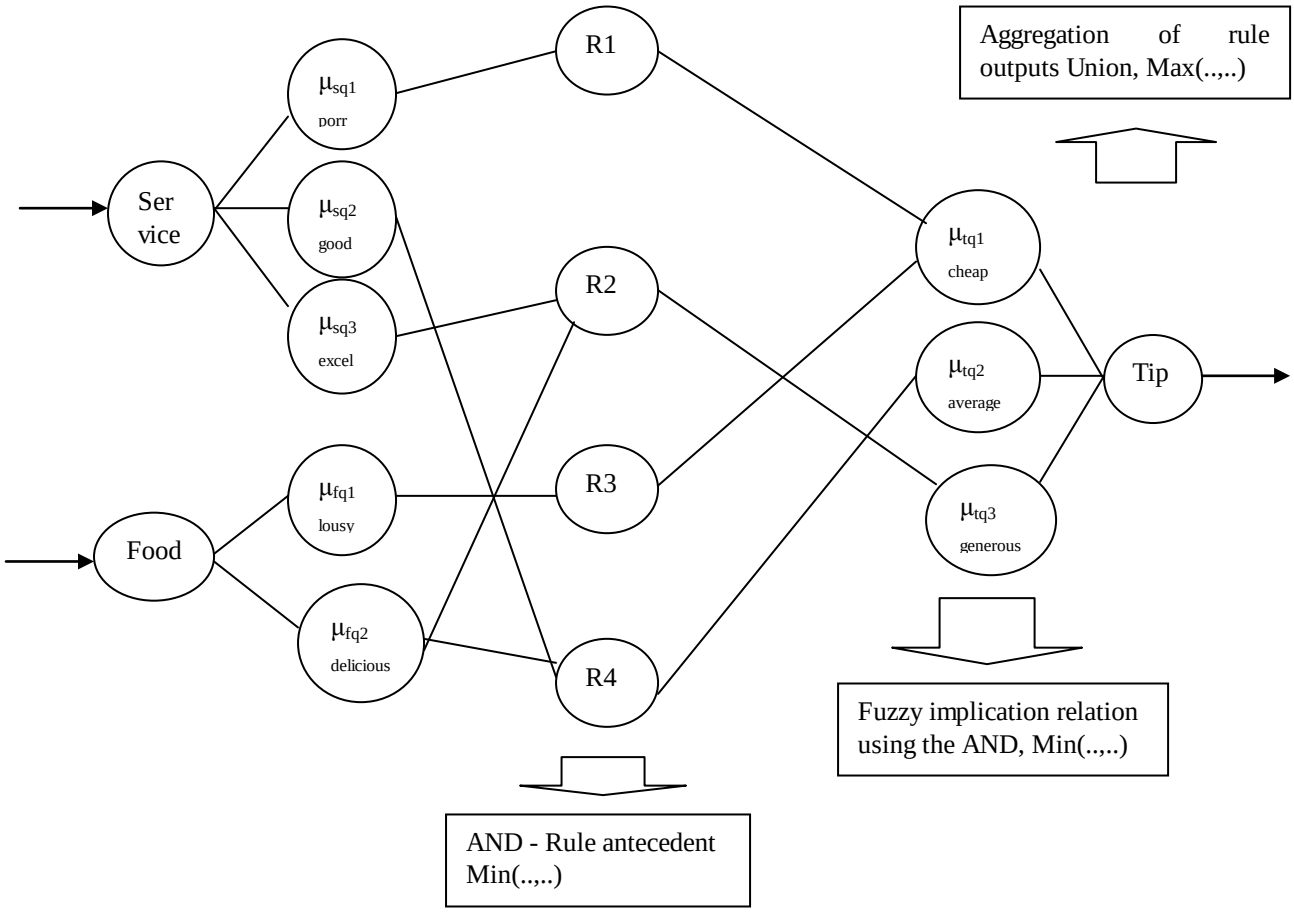
The very fuzzy linguistic hedge is the square operator: Therefore the membership functions for very poor and very cheap will be modified using the square operator on μ_{tq1} and μ_{tq3}.





5.3 The fuzzy rules can be implemented using neural networks.

(i) Using the fuzzy rule base given in question Q5.2 draw the fuzzy neural network structure to construct the fuzzy rules: R1, R2, R3 and R4.



R1. If service is poor then tip is cheap.
R2. If service is excellent **and** food is delicious then tip is generous.
R3. If food is lousy then tip cheap.
R4. If service is good **and** food is delicious then tip is average.

(ii) State briefing the operation of each layer in performing the fuzzy inference based on the Mamdani fuzzy relation AND.

The above neural network implements the 4 rules shown above. The antecedents are conjunctions (Min). The Fuzzy implication uses the AND operator (Min). The aggregation of fuzzy outputs uses the Union operator (Max).

If we have rules of this form:

R5: If service is poor **or** food is lousy then tip is cheap.

These be represented as R1 and R3 in the network.

R5 is of the form:

$(A \vee B) \Rightarrow C$
$\neg (A \vee B) \vee C$
$(\neg A \wedge \neg B) \vee C$
$(\neg A \vee C) \wedge (\neg B \vee C)$
$A \Rightarrow C \wedge B \Rightarrow C$

That is R1 AND R3.

(iii) Using a block diagram, briefly discuss how clustering and neural network learning can be used to automatically construct such a fuzzy rule base using data derived from the food and service qualities as well as the tipping by clients given the client scoring and tipping behaviours.