NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

CZ3005
Artificial Intelligence

First Order Logic

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Example

1)Mammal drinks milk

2)Man is mortal

3)Man is a mammal

4)Tom is a man

5)Tom drinks milk

6)Tom is mortal

Questions:

1. represents all the sentences in clausal form
2. prove (5) and (6) using modus ponens
3. prove (5) and (6) using resolution

Solution

Rule Base:

1) Mammal(Tom) $\Rightarrow$Drink(Tom,Milk)
2) Man(Tom) $\Rightarrow$ Mortal(Tom)
3) Man(Tom)$\Rightarrow$Mammal(Tom)
4) Man(Tom)
5) Drink(Tom,Milk)
6) Mortal(Tom)

Modus Ponen Approach

“Tom drinks milk = Drink(Tom,Milk)”

(4) & (3) $\vdash$ Mammal(Tom) (8)

(8) & (1) $\vdash$ Drink(Tom,Milk)

“Tom is mortal = Mortal(Tom)”

(4) & (2) $\vdash$ Mortal(Tom)

Clausal Form

(1) $\neg Mammal(Tom) \vee Drink(Tom,Milk)$

(2) $\neg Man(Tom) \vee Mortal(Tom)$

(3) $\neg Man(Tom) \vee Mammal(Tom)$

(4) $Man(Tom)$

Resolution Approach

“Tom drinks milk = Drink(Tom,Milk)”

(4) & (3) ├ Mammal(Tom) (8)

(8) & (1) ├ Drink(Tom,Milk)

“Tom is mortal=Mortal(Tom)”

(4) & (2) ├ Mortal(Tom)

Limitation of Propositional Logic

P is all dogs are faithful

Q is Tommy is a dog

But Tommy is faithful ???

No, we cannot infer that in propositional logic

Limitation of Propositional Logic

- In general, propositional logic can deal only with a finite number of propositions
- If there are only three dogs, Tommy, Jimmy and Laika, then

T: Tommy is faithful

J: Jimmy is faithful

L: Laika is faithful

All dogs are faithful $\Leftrightarrow$ T Λ J Λ L

What if there are infinite numbers of dogs?

Syntax of FOL

- **Sentences**
  - Built from quantifiers, predicate symbols, and terms
- **Terms**
  - Represent objects
  - Built from variables, constant and function symbols
- **Constant symbols**
  - Refer to (“name”) particular objects of the world

  e.g. “John” is a constant , may refer to “John, king of England from 1199 to 1216 and younger brother of Richard Lionheart”, or my uncle, or …

Terms: Constant and Variables

- A constant of type W is a name that denotes a particular object in a set W

Example: 5, Tommy, etc

- A variable of type W is a name that can denote any elements in the set W

Examples: $x \in N$ denotes a natural number, *d* denotes the name of a dog

Function

A functional term of arity *n* takes *n* objects of type $W_1, \ldots, W_n$ as inputs and returns an object of type W

$$f(w_1, w_2, \ldots, w_n)$$

$$Plus(3,4) = 7$$

Functional term

Constant term

Example of Function

- Let plus be a function that takes two arguments of type natural number and returns a natural number
- Valid functional term: *plus(2,3), plus(5,plus(7,3)), plus(plus(100, plus(1,6)), plus(3,3))*
- Invalid functional term: *plus(0,-1), plus(1.2, 3.1)*

Predicates

- Predicates are like functions except that their return type is true or false

Example:

- *Gt(x,y)* is true iff $x > y$
- Gt is a predicate symbol that takes two arguments of type natural number
- *Gt(3,4)* is a valid predicate but *Gt(3,-4)* is not

Types of Predicate

- A predicate with no variable is a proposition: tommy is a dog
- A predicate with one variable is called a property
  - Dog(x) is true iff x is a dog
  - Mortal(y) is true iff y is mortal

Example

- If x is a man, then x is a mortal
  - $man(x) \Rightarrow mortal(x)$
  - $\neg man(x) \vee mortal(x)$
- If n is a natural number, then *n* is either even or odd.
  - $Natural(n) \Rightarrow even(n) \vee odd(n)$

Sentence of FOL

- **Atomic sentences**
  - State facts, using terms and predicate symbols
    - e.g. Brother(Richard, John).
  - Can have complex terms as arguments
    - e.g. Married(FatherOf(Richard), MotherOf(John)).
  - Have a truth value
    - Depends on both the interpretation and the world.
- **Complex sentences**
  - Combine sentences with connectives
    - e.g. Father(Henry, KingJohn) Λ Mother(Mary, KingJohn)
  - Connectives identical to propositional logic
    - i.e.: Λ, $\vee$, $\Leftrightarrow$, $\Rightarrow$, $\neg$

❑ *There are many ways to write a logical statement in FOL*

Example

- $A \Rightarrow B$ equivalent to $\neg A \vee B$

  "rule form" *"complementary cases"*

  $Dog(x) \Rightarrow Mammal(x)$ $\neg Dog(x) \vee Mammal(x)$

  "dogs are mammals" "either it's not a dog or it's a mammal"

- $A \wedge B \Rightarrow C$ equivalent to $A \Rightarrow (B \Rightarrow C)$

Proof: $A \wedge B \Rightarrow C \Leftrightarrow \neg (A \wedge B) \vee C \Leftrightarrow (\neg A \vee \neg B) \vee C$

$\Leftrightarrow \mathbf{\neg A \vee \neg B \vee C} \Leftrightarrow \neg A \vee (\neg B \vee C)$

$\Leftrightarrow \neg A \vee (B \Rightarrow C) \Leftrightarrow A \Rightarrow (B \Rightarrow C)$

$\mathbf{\neg P \vee Q \Leftrightarrow P \Rightarrow Q}$

- ***There is only one way to write a logical statement using a Normal Form of FOL***

Example

- $A \Rightarrow B$, $A \land B \Rightarrow C$ equivalent to $\neg A \lor B$, $\neg A \lor \neg B \lor C$

  "Implicative Normal Form" "Conjunctive Normal Form"

- ***Rewriting logical sentences allows to determine whether they are equivalent or not***

Example

- $A \land B \Rightarrow C$ and $A \Rightarrow (B \Rightarrow C)$

  both have the same CNF: $\neg A \lor \neg B \lor C$

- ***Using FOL is the most convenient, but using a Normal Form is the most efficient***

❑ Express properties of collections of objects

- Make a statement about *every* objects w/out enumerating
  - e.g. "All kings are mortal

$$\begin{aligned}&King(Henry) \Rightarrow Mortal(Henry) \wedge \\ &King(John) \Rightarrow Mortal(John) \wedge \\ &King(Richard) \Rightarrow Mortal(Richard) \wedge \\ &King(London) \Rightarrow Mortal(London) \wedge \\ &\ldots\end{aligned}$$

instead: $\forall x, King(x) \Rightarrow Mortal(x)$

Universal Quantifier $\forall$

- Note: the semantics of the implication says $F \Rightarrow F$ is TRUE,
  thus for those individuals that satisfy the premise King(x) the rule asserts the conclusion Mortal(x)
  but for those individuals that do not satisfy the premise the rule makes no assertion.

Using Universal Quantifier

- ***The implication ($\Rightarrow$) is the natural connective to use with the universal quantifier ($\forall$)***
  - Example
    - General form: $\forall x\ P(x) \Rightarrow Q(x)$ e.g. $\forall x\ Dog(x) \Rightarrow Mammal(x)$
      "all dogs are mammals"
    - Use conjunction? $\forall x\ P(x) \land Q(x)$ e.g. $\forall x\ Dog(x) \land Mammal(x)$

      same as $\forall x\ P(x)$ and $\forall x\ Q(x)$
      e.g. $\forall x\ Dog(x)$ and $\forall x\ Mammal(x)$

      –> yields a very strong statement (too strong! i.e. *incorrect*)

Example

- All dogs are faithful
  - Faithful(x): x is faithful
  - Dog(x): x is a dog
  - $\forall$ x, dog(x) $\Rightarrow$ faithful(x)
- Not all birds can fly
  - Fly(x): x can fly
  - Bird(x): x is a bird
  - $\neg(\forall$ x, bird(x) $\Rightarrow$ fly(x))