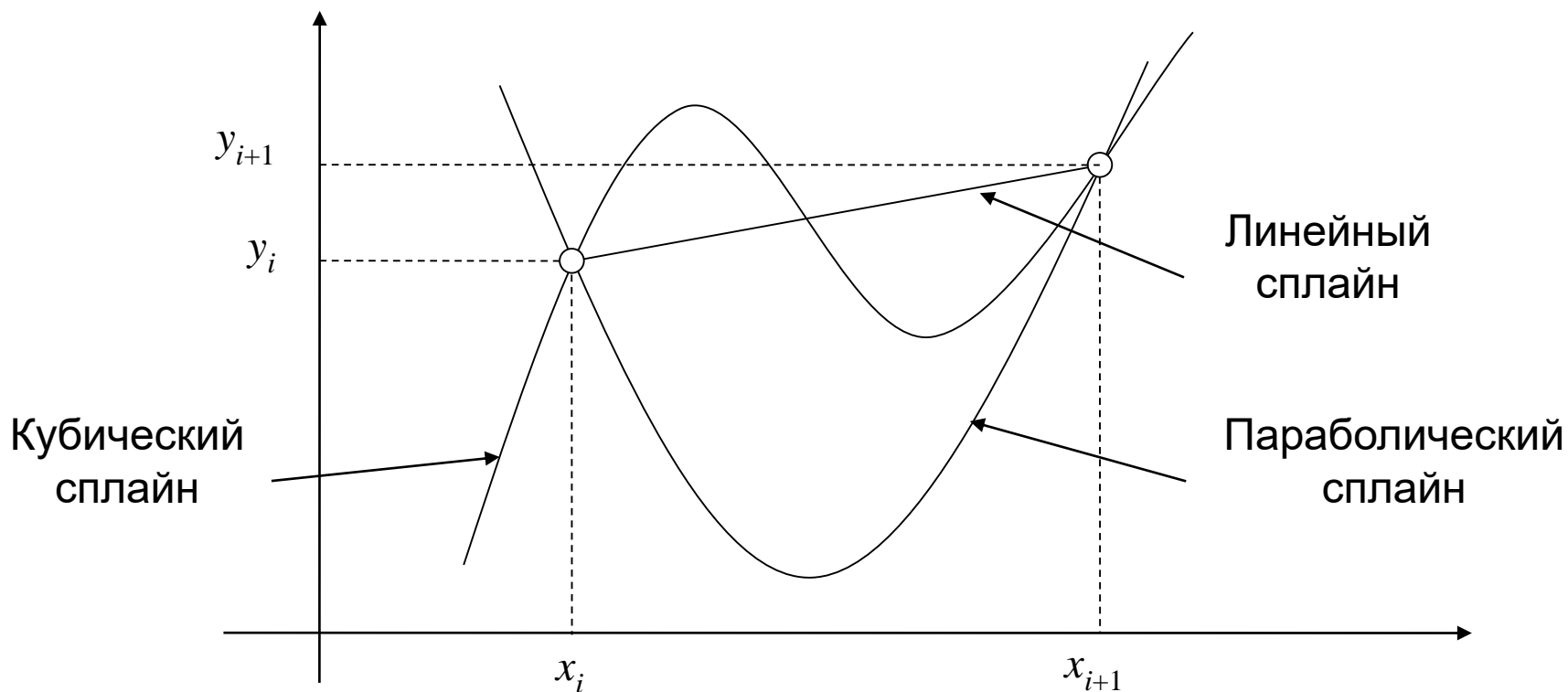
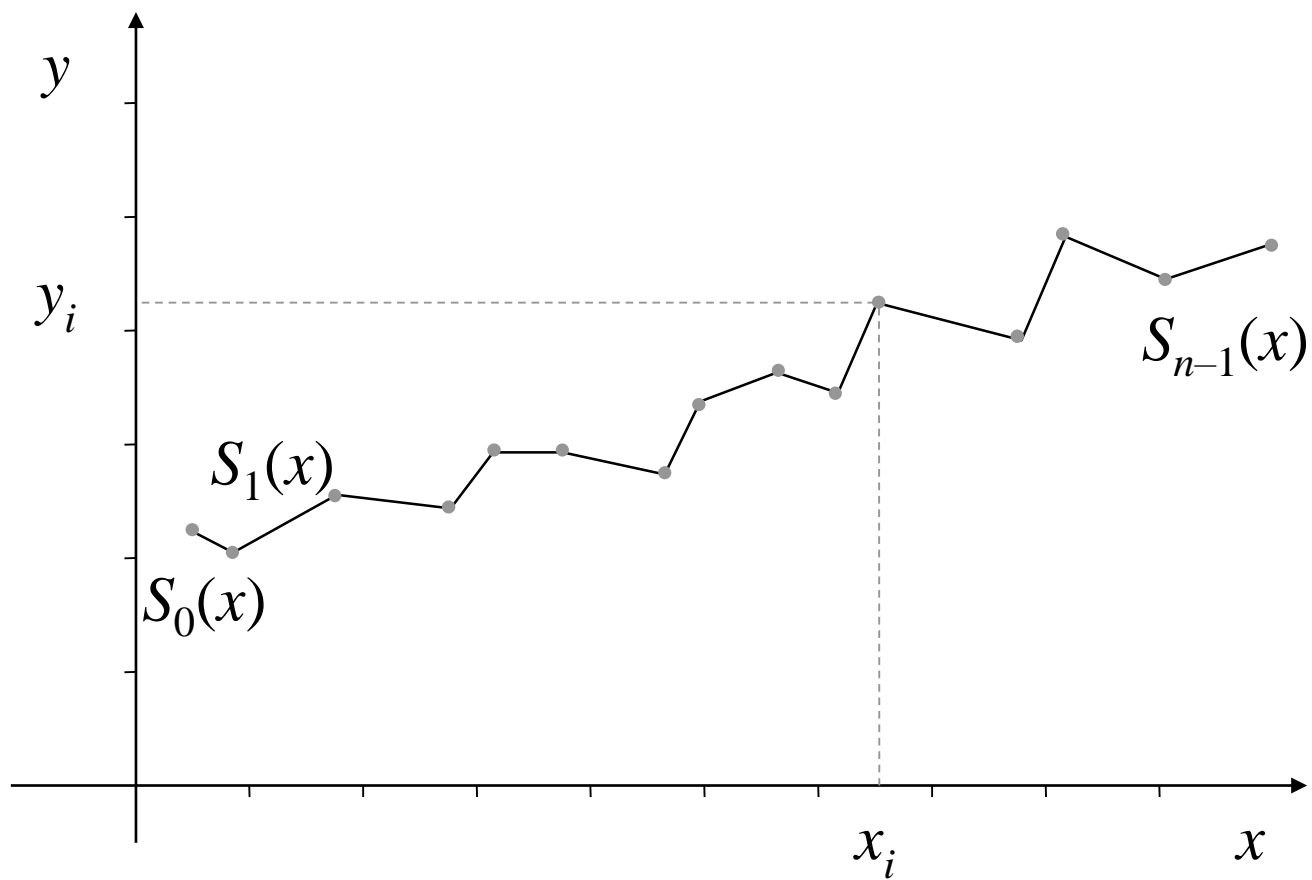


# Тема. Приближение сплайнами



# Приближение сплайнами



# Приближение сплайнами

Постановка задачи.

Сетка (табличные значения функции):

$$\{x_i\}: i = 0, 1, \dots, n$$

$$\{y_i\}: y_i = f(x_i)$$

Количество узлов —  $n + 1$ . Количество сплайнов —  $n$ :  $S_0(x), S_1(x), \dots, S_{n-1}(x)$ . Для  $S_i(x)$  область действия  $x \in [x_i, x_{i+1}]$ .

# Приближение сплайнами

Постановка задачи.

$$S_i(x) = \sum_{j=0}^m c_{ji} \varphi_{ji}(x), \quad \varphi_{ji}(x) = (x - x_i)^j \Rightarrow$$

$$S_i(x) = \sum_{j=0}^m c_{ji} (x - x_i)^j, \quad i = 0, 1, \dots, n-1, \quad m = 1, 2, 3$$

$$a_i = c_{0i}, \quad b_i = c_{1i}, \quad c_i = c_{2i}, \quad d_i = c_{3i}$$

При  $m > 1$  требуются *граничные условия*.

# Линейные сплайны

$m = 1$ :

$$S_i(x) = a_i + b_i(x - x_i), \quad i = 0, 1, \dots, n-1$$

$$S_i(x_{i+1}) = S_{i+1}(x_{i+1}), \quad i = 0, 1, \dots, n-2$$

$$a_i = y_i, \quad b_i = \frac{\Delta y_i}{\Delta x_i}$$

Здесь  $\Delta x_i = x_{i+1} - x_i$ ,  $\Delta y_i = y_{i+1} - y_i$ .

# Параболические сплайны

$m = 2$ :

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2, \quad i = 0, 1, \dots, n-1$$

$$S_i(x_{i+1}) = S_{i+1}(x_{i+1}),$$

$$S'_i(x_{i+1}) = S'_{i+1}(x_{i+1}), \quad i = 0, 1, \dots, n-2$$

Граничные условия:

1.  $A_0 = f(x_0);$

2.  $A_n = f(x_n).$

# Параболические сплайны

$$a_i = y_i, \quad c_i = \frac{b_{i+1} - b_i}{2\Delta x_i}.$$

Если  $A_0 = f(x_0)$ , то

$$b_0 = A_0, \quad b_{i+1} = \frac{2\Delta y_i}{\Delta x_i} - b_i, \quad i = 0, 1, \dots, n-2.$$

Если  $A_n = f(x_n)$ , то

$$b_n = A_n, \quad b_i = \frac{2\Delta y_i}{\Delta x_i} - b_{i+1}, \quad i = n-1, n-2, \dots, 0.$$

# Кубические сплайны

$m = 3$ :

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3, \quad i = 0, 1, \dots, n-1$$

$$S_i(x_{i+1}) = S_{i+1}(x_{i+1}), \quad S'_i(x_{i+1}) = S'_{i+1}(x_{i+1}),$$

$$S''_i(x_{i+1}) = S''_{i+1}(x_{i+1}), \quad i = 0, 1, \dots, n-2$$

Граничные условия:

1.  $A_0 = f'(x_0), A_n = f'(x_n)$ ;
2.  $B_0 = f''(x_0), B_n = f''(x_n)$ .



# Кубические сплайны

$$a_i = y_i,$$

$$b_i = \frac{\Delta y_i}{\Delta x_i} - \frac{\Delta x_i}{6} (2M_i + M_{i+1}),$$

$$c_i = \frac{1}{2} M_i,$$

$$d_i = \frac{M_{i+1} - M_i}{6\Delta x_i}, \quad i = 0, 1, \dots, n-1.$$

# Кубические сплайны

Если  $A_0 = f'(x_0)$ ,  $A_n = f'(x_n)$ , то

$$b_0 = A_0, b_n = A_n$$

$$\tilde{A}M = \tilde{g}$$

$$\tilde{A} = \begin{cases} \tilde{a}_{00} = \frac{\Delta x_0}{3}; \quad \tilde{a}_{nn} = \frac{\Delta x_{n-1}}{3}; \\ \tilde{a}_{jj} = \frac{1}{3}(\Delta x_{j-1} + \Delta x_j), \quad j = 1, 2, \dots, n-1; \\ \tilde{a}_{j,j+1} = \tilde{a}_{j+1,j} = \frac{1}{6}\Delta x_j, \quad j = 0, 1, \dots, n-1 \end{cases}$$

# Кубические сплайны

Если  $A_0 = f'(x_0)$ ,  $A_n = f'(x_n)$ , то

$$b_0 = A_0, b_n = A_n$$

$$\tilde{A}M = \tilde{g}$$

$$\tilde{g} = \begin{cases} \tilde{g}_0 = \frac{\Delta y_0}{\Delta x_0} - A_0; & \tilde{g}_n = A_n - \frac{\Delta y_{n-1}}{\Delta x_{n-1}}; \\ \tilde{g}_i = \frac{\Delta y_i}{\Delta x_i} - \frac{\Delta y_{i-1}}{\Delta x_{i-1}}, & i = 1, 2, \dots, n-1 \end{cases}$$

$$M = (M_0, M_1, M_2, \dots, M_n)$$

# Кубические сплайны

Если  $B_0 = f''(x_0)$ ,  $B_n = f''(x_n)$ , то

$$M_0 = B_0, M_n = B_n$$

$$AM = g$$

$$M = (M_1, M_2, \dots, M_{n-1})$$

$$A = \begin{cases} a_{jj} = \tilde{a}_{jj} = \frac{1}{3}(\Delta x_{j-1} + \Delta x_j), & j = 1, 2, \dots, n-1; \\ a_{j,j+1} = a_{j+1,j} = \tilde{a}_{j,j+1} = \tilde{a}_{j+1,j} = \frac{1}{6}\Delta x_j, & j = 1, 2, \dots, n-2 \end{cases}$$

# Кубические сплайны

Если  $B_0 = f''(x_0)$ ,  $B_n = f''(x_n)$ , то

$$M_0 = B_0, M_n = B_n$$

$$AM = g$$

$$g_i = \tilde{g}_i - \beta_1 - \beta_{n-1} = \frac{\Delta y_i}{\Delta x_i} - \frac{\Delta y_{i-1}}{\Delta x_{i-1}} - \beta_1 - \beta_{n-1}, \quad i = 1, 2, \dots, n-1,$$

$$\beta_1 = \begin{cases} \frac{\Delta x_0}{6} B_0, & i = 1 \\ 0, & i \neq 1 \end{cases}; \quad \beta_{n-1} = \begin{cases} \frac{\Delta x_{n-1}}{6} B_n, & i = n-1 \\ 0, & i \neq n-1 \end{cases}$$

# Примеры

Параболический сплайн.

$$f(x) = \sqrt{x}$$

|     |     |   |   |   |
|-----|-----|---|---|---|
| $i$ | 0   | 1 | 2 | 3 |
| $x$ | 1/4 | 1 | 4 | 9 |
| $y$ | 1/2 | 1 | 2 | 3 |

Результатирующая сетка:  $\{4/9, 9/4, 25/4\}$

Граничные условия:  $f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow A_0 = 1, A_3 = \frac{1}{6}$

Далее строим сплайны  $S_0(x)$ ,  $S_1(x)$ ,  $S_2(x)$ .

# Примеры

|     |     |   |   |   |
|-----|-----|---|---|---|
| $i$ | 0   | 1 | 2 | 3 |
| $x$ | 1/4 | 1 | 4 | 9 |
| $y$ | 1/2 | 1 | 2 | 3 |

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \quad \Delta x_1 = 4 - 1 = 3; \quad \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \quad \Delta y_1 = 2 - 1 = 1; \quad \Delta y_2 = 3 - 2 = 1$$

$$\begin{aligned} b_0 &= 1, \quad b_1 = \frac{2\Delta y_0}{\Delta x_0} - b_0 = \frac{2 \cdot 1/2}{3/4} - 1 = \frac{1}{3}, \quad b_2 = \frac{2\Delta y_1}{\Delta x_1} - b_1 = \\ &= \frac{2 \cdot 1}{3} - \frac{1}{3} = \frac{1}{3}, \quad b_3 = \frac{2\Delta y_2}{\Delta x_2} - b_2 = \frac{2 \cdot 1}{5} - \frac{1}{3} = \frac{1}{15} \end{aligned}$$

# Примеры

|     |     |   |   |   |
|-----|-----|---|---|---|
| $i$ | 0   | 1 | 2 | 3 |
| $x$ | 1/4 | 1 | 4 | 9 |
| $y$ | 1/2 | 1 | 2 | 3 |

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \quad \Delta x_1 = 4 - 1 = 3; \quad \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \quad \Delta y_1 = 2 - 1 = 1; \quad \Delta y_2 = 3 - 2 = 1$$

$$c_0 = \frac{b_1 - b_0}{2\Delta x_0} = \frac{1/3 - 1}{2 \cdot 3/4} = -\frac{4}{9}, \quad c_1 = \frac{b_2 - b_1}{2\Delta x_1} = 0,$$

$$c_2 = \frac{b_3 - b_2}{2\Delta x_2} = \frac{1/15 - 1/3}{2 \cdot 5} = -\frac{2}{75}$$



# Примеры

|     |      |     |       |      |
|-----|------|-----|-------|------|
| $i$ | 0    | 1   | 2     | 3    |
| $x$ | 1/4  | 1   | 4     | 9    |
| $y$ | 1/2  | 1   | 2     | 3    |
| $a$ | 1/2  | 1   | 2     | 3    |
| $b$ | 1    | 1/3 | 1/3   | 1/15 |
| $c$ | -4/9 | 0   | -2/75 |      |

$$S_0(x) = \frac{1}{2} + 1 \cdot \left(x - \frac{1}{4}\right) - \frac{4}{9} \left(x - \frac{1}{4}\right)^2, \quad x \in \left[\frac{1}{4}, 1\right]$$

$$S_1(x) = 1 + \frac{1}{3} \cdot (x - 1) + 0(x - 1)^2, \quad x \in [1, 4]$$

$$S_2(x) = 2 + \frac{1}{3} \cdot (x - 4) - \frac{2}{75} (x - 4)^2, \quad x \in [4, 9]$$

# Примеры

$$S_0\left(\frac{4}{9}\right) = \frac{1}{2} + \left(\frac{4}{9} - \frac{1}{4}\right) - \frac{4}{9}\left(\frac{4}{9} - \frac{1}{4}\right)^2 = \frac{494}{729} \approx 0.678$$

$$S_1\left(\frac{9}{4}\right) = 1 + \frac{1}{3} \cdot \left(\frac{9}{4} - 1\right) = \frac{17}{12} \approx 1.417$$

$$S_2\left(\frac{25}{4}\right) = 2 + \frac{1}{3} \cdot \left(\frac{25}{4} - 4\right) - \frac{2}{75}\left(\frac{25}{4} - 4\right)^2 = \frac{523}{200} = 2.615$$

# Примеры

|     |     |   |   |   |
|-----|-----|---|---|---|
| $i$ | 0   | 1 | 2 | 3 |
| $x$ | 1/4 | 1 | 4 | 9 |
| $y$ | 1/2 | 1 | 2 | 3 |

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \quad \Delta x_1 = 4 - 1 = 3; \quad \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \quad \Delta y_1 = 2 - 1 = 1; \quad \Delta y_2 = 3 - 2 = 1$$

$$b_3 = \frac{1}{6}, \quad b_2 = \frac{2\Delta y_2}{\Delta x_2} - b_3 = \frac{2 \cdot 1}{5} - \frac{1}{6} = \frac{7}{30}, \quad b_1 = \frac{2\Delta y_1}{\Delta x_1} - b_2 =$$

$$= \frac{2 \cdot 1}{3} - \frac{7}{30} = \frac{13}{30}, \quad b_0 = \frac{2\Delta y_0}{\Delta x_0} - b_1 = \frac{2 \cdot 1/2}{3/4} - \frac{13}{30} = \frac{9}{10}$$

# Примеры

|     |     |   |   |   |
|-----|-----|---|---|---|
| $i$ | 0   | 1 | 2 | 3 |
| $x$ | 1/4 | 1 | 4 | 9 |
| $y$ | 1/2 | 1 | 2 | 3 |

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \quad \Delta x_1 = 4 - 1 = 3; \quad \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \quad \Delta y_1 = 2 - 1 = 1; \quad \Delta y_2 = 3 - 2 = 1$$

$$c_0 = \frac{b_1 - b_0}{2\Delta x_0} = \frac{13/30 - 9/10}{2 \cdot 3/4} = -\frac{14}{45}, \quad c_1 = \frac{b_2 - b_1}{2\Delta x_1} = \frac{7/39 - 13/30}{2 \cdot 3} =$$

$$= -\frac{1}{30}, \quad c_2 = \frac{b_3 - b_2}{2\Delta x_2} = \frac{1/6 - 7/30}{2 \cdot 5} = -\frac{1}{150}$$

# Примеры

|     |        |       |        |     |
|-----|--------|-------|--------|-----|
| $i$ | 0      | 1     | 2      | 3   |
| $x$ | 1/4    | 1     | 4      | 9   |
| $y$ | 1/2    | 1     | 2      | 3   |
| $a$ | 1/2    | 1     | 2      | 3   |
| $b$ | 9/10   | 13/30 | 7/30   | 1/6 |
| $c$ | -14/45 | -1/30 | -1/150 |     |

$$S_0(x) = \frac{1}{2} + \frac{9}{10} \cdot \left(x - \frac{1}{4}\right) - \frac{14}{45} \left(x - \frac{1}{4}\right)^2, \quad x \in \left[\frac{1}{4}, 1\right]$$

$$S_1(x) = 1 + \frac{13}{30} \cdot (x - 1) - \frac{1}{30} (x - 1)^2, \quad x \in [1, 4]$$

$$S_2(x) = 2 + \frac{7}{30} \cdot (x - 4) - \frac{1}{150} (x - 4)^2, \quad x \in [4, 9]$$

# Примеры

$$S_0\left(\frac{4}{9}\right) = \frac{1}{2} + \frac{9}{10} \cdot \left(\frac{4}{9} - \frac{1}{4}\right) - \frac{14}{45} \left(\frac{4}{9} - \frac{1}{4}\right)^2 = \frac{967}{1458} \approx 0.663$$

$$S_1\left(\frac{9}{4}\right) = 1 + \frac{13}{30} \cdot \left(\frac{9}{4} - 1\right) - \frac{1}{30} \left(\frac{9}{4} - 1\right)^2 = \frac{143}{96} \approx 1.49$$

$${}_2\left(\frac{25}{4}\right) = 2 + \frac{7}{30} \cdot \left(\frac{25}{4} - 4\right) - \frac{1}{150} \left(\frac{25}{4} - 4\right)^2 = \frac{1993}{800} \approx 2.491$$

# Примеры

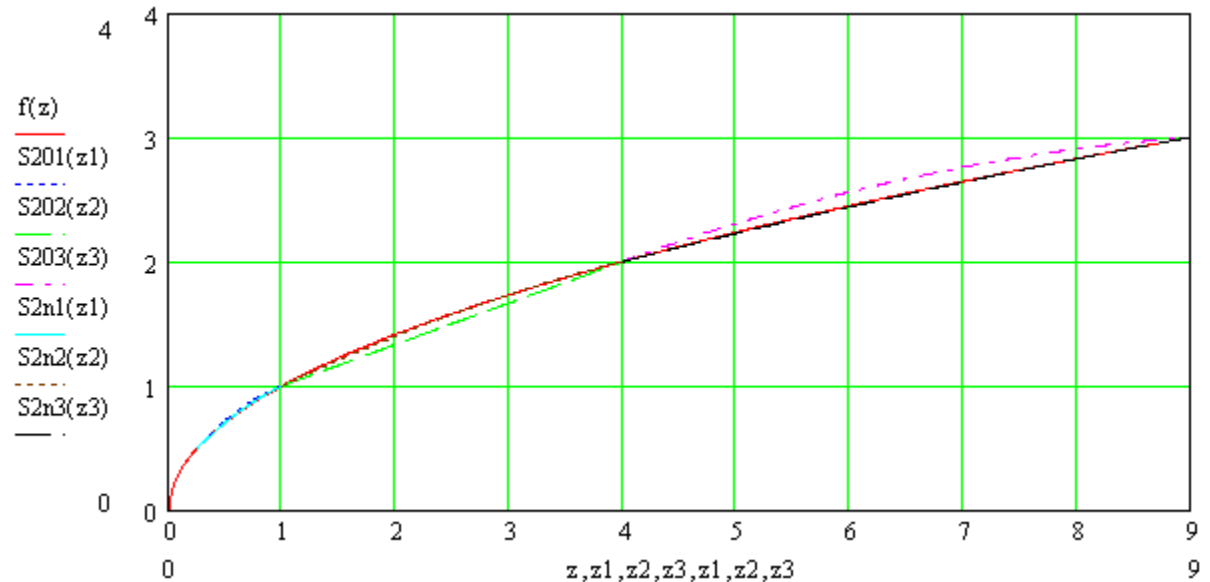
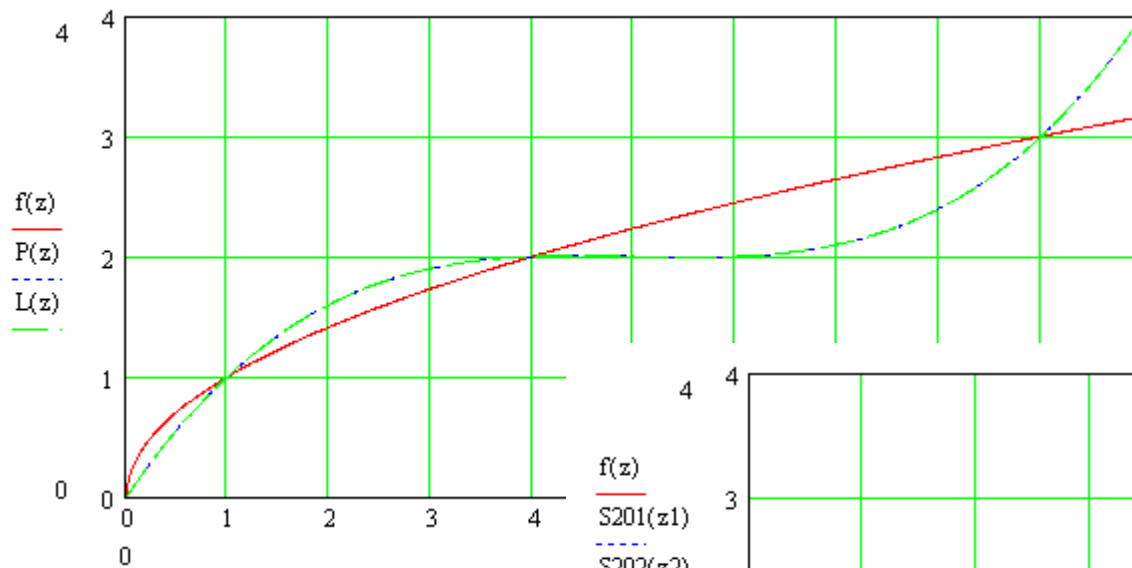
Точность интерполяции:

| $x$  | $P(x)$ | $L(x)$ | $f(x)$ | $\delta$ |
|------|--------|--------|--------|----------|
| 1/4  | 0.293  | 0.293  | 0.5    | 41.4%    |
| 9/4  | 1.699  | 1.699  | 1.5    | 13.3%    |
| 25/4 | 2.012  | 2.012  | 2.5    | 19.5%    |

| $x$  | $S(x)$ | $S(x)$ | $f(x)$ | $\delta$ |
|------|--------|--------|--------|----------|
| 4/9  | 0.678  | 0.663  | 2/3    | 0.6-1.7% |
| 9/4  | 1.417  | 1.490  | 1.5    | 0.6-5.5% |
| 25/4 | 2.615  | 2.491  | 2.5    | 0.4-4.6% |

# Примеры

## Точность интерполяции:





# Примеры

Кубический сплайн.

$$f(x) = \sqrt{x}$$

|     |     |   |   |   |
|-----|-----|---|---|---|
| $i$ | 0   | 1 | 2 | 3 |
| $x$ | 1/4 | 1 | 4 | 9 |
| $y$ | 1/2 | 1 | 2 | 3 |

Результатирующая сетка:  $\{4/9, 9/4, 25/4\}$

Гранич. условия:  $f''(x) = \frac{1}{4\sqrt{x^3}} \Rightarrow B_0 = -2, B_3 = -\frac{1}{108}$

Далее строим сплайны  $S_0(x)$ ,  $S_1(x)$ ,  $S_2(x)$ .

# Примеры

|     |     |   |   |   |
|-----|-----|---|---|---|
| $i$ | 0   | 1 | 2 | 3 |
| $x$ | 1/4 | 1 | 4 | 9 |
| $y$ | 1/2 | 1 | 2 | 3 |

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \quad \Delta x_1 = 4 - 1 = 3; \quad \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \quad \Delta y_1 = 2 - 1 = 1; \quad \Delta y_2 = 3 - 2 = 1$$

$$A = \begin{pmatrix} \frac{1}{3}(\Delta x_0 + \Delta x_1) & \frac{1}{6}\Delta x_1 \\ \frac{1}{6}\Delta x_1 & \frac{1}{3}(\Delta x_1 + \Delta x_2) \end{pmatrix} = \begin{pmatrix} \frac{1}{3}\left(\frac{3}{4} + 3\right) & \frac{1}{6} \cdot 3 \\ \frac{1}{6} \cdot 3 & \frac{1}{3}(3 + 5) \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{8}{3} \end{pmatrix}$$

# Примеры

|     |     |   |   |   |
|-----|-----|---|---|---|
| $i$ | 0   | 1 | 2 | 3 |
| $x$ | 1/4 | 1 | 4 | 9 |
| $y$ | 1/2 | 1 | 2 | 3 |

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \quad \Delta x_1 = 4 - 1 = 3; \quad \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \quad \Delta y_1 = 2 - 1 = 1; \quad \Delta y_2 = 3 - 2 = 1$$

$$g = \begin{pmatrix} \frac{\Delta y_1}{\Delta x_1} - \frac{\Delta y_0}{\Delta x_0} - \frac{\Delta x_0}{6} M_0 \\ \frac{\Delta y_2}{\Delta x_2} - \frac{\Delta y_1}{\Delta x_1} - \frac{\Delta x_2}{6} M_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} - \frac{1/2}{3/4} - \frac{3/4}{6} \cdot (-2) \\ \frac{1}{5} - \frac{1}{3} - \frac{5}{6} \cdot \left(-\frac{1}{108}\right) \end{pmatrix} = \begin{pmatrix} -\frac{1}{12} \\ -\frac{407}{3240} \end{pmatrix} \approx \begin{pmatrix} -0.0833 \\ -0.1256 \end{pmatrix}$$

# Примеры

$$\begin{pmatrix} \frac{5}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{8}{3} \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} -0.0833 \\ -0.1256 \end{pmatrix}$$

$$\Rightarrow M_1 = -\frac{1033}{19980} \approx -0.0517; \quad M_2 = -\frac{299}{7992} \approx -0.0374$$

# Примеры

$$M_0 = -2; M_1 = -\frac{1033}{19980}; M_2 = -\frac{299}{7992}; M_3 = -\frac{1}{108}$$

$$b_0 = \frac{\Delta y_0}{\Delta x_0} - \frac{\Delta x_0}{6} (2M_0 + M_1) = \frac{1/2}{3/4} - \frac{3/4}{6} \left( 2 \cdot (-2) - \frac{1033}{19980} \right) =$$

$$= \frac{187513}{159840} \approx 1.1731; \quad b_1 = \frac{\Delta y_1}{\Delta x_1} - \frac{\Delta x_1}{6} (2M_1 + M_2) =$$

$$= \frac{1}{3} - \frac{3}{6} \left( 2 \cdot \left( -\frac{1033}{19980} \right) - \frac{299}{7992} \right) = \frac{32267}{79920} \approx 0.4037; \quad b_2 =$$

$$= \frac{\Delta y_2}{\Delta x_2} - \frac{\Delta x_2}{6} (2M_2 + M_3) = \frac{1}{5} - \frac{5}{6} \left( 2 \cdot \left( -\frac{299}{7992} \right) - \frac{1}{108} \right) = \frac{1349}{4995} \approx 0.2701$$

# Примеры

$$M_0 = -2; M_1 = -\frac{1033}{19980}; M_2 = -\frac{299}{7992}; M_3 = -\frac{1}{108}$$

$$c_0 = \frac{1}{2}M_0 = \frac{1}{2} \cdot (-2) = -1;$$

$$c_1 = \frac{1}{2}M_1 = \frac{1}{2} \cdot \left(-\frac{1033}{19980}\right) = -\frac{1033}{39960} \approx -0.0259;$$

$$c_2 = \frac{1}{2}M_2 = \frac{1}{2} \cdot \left(-\frac{299}{7992}\right) = -\frac{299}{15984} \approx -0.0187$$

# Примеры

$$M_0 = -2; M_1 = -\frac{1033}{19980}; M_2 = -\frac{299}{7992}; M_3 = -\frac{1}{108}$$

$$d_0 = \frac{M_1 - M_0}{6\Delta x_0} = \frac{-\frac{1033}{19980} + 2}{6 \cdot 3/4} = \frac{38927}{89910} \approx 0.4330;$$

$$d_1 = \frac{M_2 - M_1}{6\Delta x_1} = \frac{-\frac{299}{7992} + \frac{1033}{19980}}{6 \cdot 3} = \frac{571}{719280} \approx 0.0008;$$

$$d_2 = \frac{M_3 - M_2}{6\Delta x_2} = \frac{-\frac{1}{108} + \frac{299}{7992}}{6 \cdot 5} = \frac{5}{5328} \approx 0.0009$$

# Примеры

|     |        |         |         |   |
|-----|--------|---------|---------|---|
| $i$ | 0      | 1       | 2       | 3 |
| $x$ | 1/4    | 1       | 4       | 9 |
| $y$ | 1/2    | 1       | 2       | 3 |
| $a$ | 1/2    | 1       | 2       | 3 |
| $b$ | 1.1731 | 0.4037  | 0.2701  |   |
| $c$ | -1     | -0.0259 | -0.0187 |   |
| $d$ | 0.4330 | -0.0008 | 0.0009  |   |

$$S_0(x) = \frac{1}{2} + 1.1731 \cdot \left(x - \frac{1}{4}\right) - 1 \cdot \left(x - \frac{1}{4}\right)^2 + 0.4330 \cdot \left(x - \frac{1}{4}\right)^3, \quad x \in \left[\frac{1}{4}, 1\right]$$

$$S_1(x) = 1 + 0.4037 \cdot (x - 1) - 0.0259 \cdot (x - 1)^2 - 0.0008 \cdot (x - 1)^3, \quad x \in [1, 4]$$

$$S_2(x) = 2 + 0.2701 \cdot (x - 4) - 0.0187 \cdot (x - 4)^2 + 0.0009 \cdot (x - 4)^3, \quad x \in [4, 9]$$



# Примеры

$$s_0\left(\frac{4}{9}\right) = \frac{1}{2} + 1.1731 \cdot \left(\frac{4}{9} - \frac{1}{4}\right) - 1 \cdot \left(\frac{4}{9} - \frac{1}{4}\right)^2 + 0.4330 \cdot \left(\frac{4}{9} - \frac{1}{4}\right)^3 \approx \\ \approx 0.6935$$

$$s_1\left(\frac{9}{4}\right) = 1 + 0.2701 \cdot \left(\frac{9}{4} - 1\right) - 0.0259 \cdot \left(\frac{9}{4} - 1\right)^2 - 0.0008 \cdot \left(\frac{9}{4} - 1\right)^3 \approx \\ \approx 1.4658$$

$$s_2\left(\frac{25}{4}\right) = 2 + 0.3176 \cdot \left(\frac{25}{4} - 4\right) - 0.0187 \cdot \left(\frac{25}{4} - 4\right)^2 + 0.0009 \cdot \left(\frac{25}{4} - 4\right)^3 \approx \\ \approx 2.5236$$

# Примеры

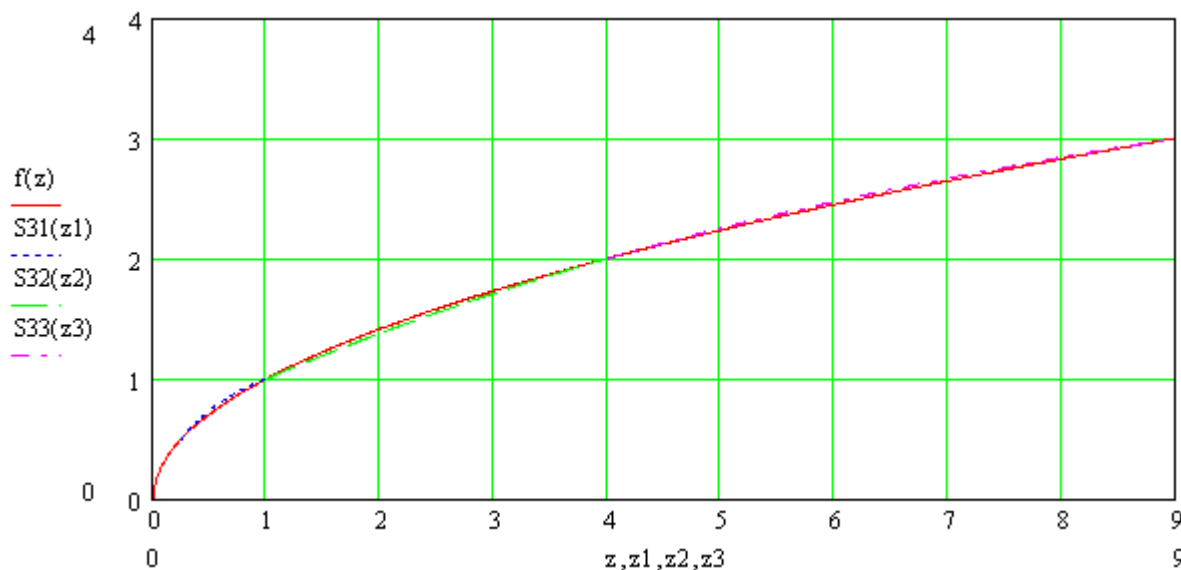
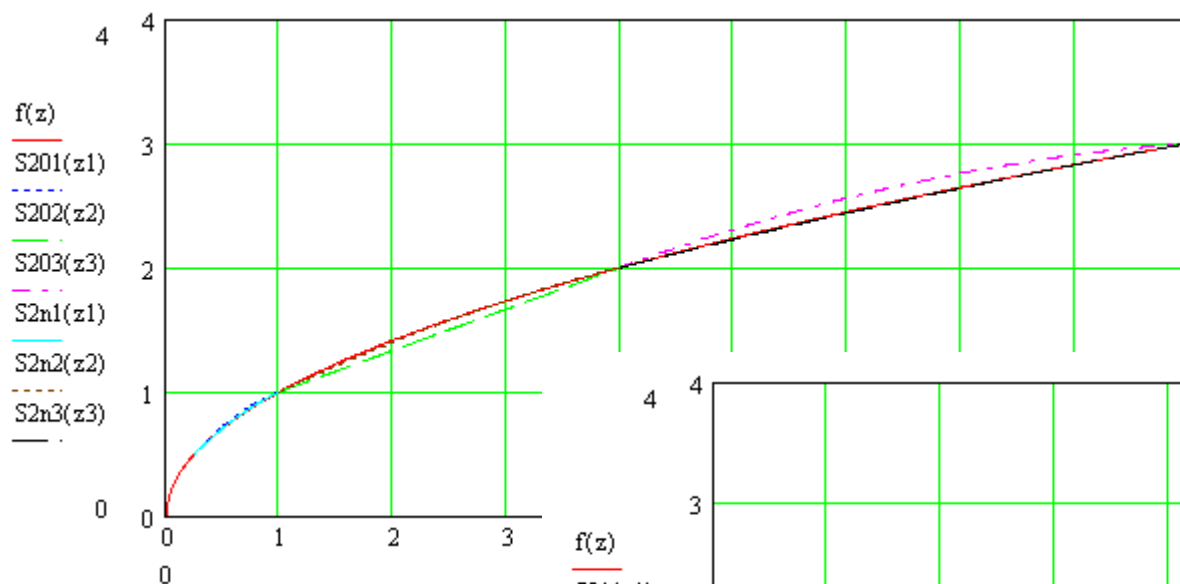
Точность интерполяции:

| $x$  | $P(x)$ | $L(x)$ | $f(x)$ | $\delta$ |
|------|--------|--------|--------|----------|
| 1/4  | 0.293  | 0.293  | 0.5    | 41.4%    |
| 9/4  | 1.699  | 1.699  | 1.5    | 13.3%    |
| 25/4 | 2.012  | 2.012  | 2.5    | 19.5%    |

| $x$  | $S(x)$ | $S(x)$ | $S(x)$ | $f(x)$ | $\delta_2$ | $\delta_3$ |
|------|--------|--------|--------|--------|------------|------------|
| 4/9  | 0.678  | 0.663  | 0.694  | 2/3    | 0.6-1.7%   | 4.10%      |
| 9/4  | 1.417  | 1.490  | 1.466  | 1.5    | 0.6-5.5%   | 2.27%      |
| 25/4 | 2.615  | 2.491  | 2.524  | 2.5    | 0.4-4.6%   | 0.96%      |

# Примеры

## Точность интерполяции:



# Примеры

Кубический сплайн.

$$f(x) = \sqrt{x}$$

|     |     |   |   |   |
|-----|-----|---|---|---|
| $i$ | 0   | 1 | 2 | 3 |
| $x$ | 1/4 | 1 | 4 | 9 |
| $y$ | 1/2 | 1 | 2 | 3 |

Результатирующая сетка:  $\{4/9, 9/4, 25/4\}$

Гранич. условия:  $f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow A_0 = 1, A_3 = \frac{1}{6}$

Далее строим сплайны  $S_0(x)$ ,  $S_1(x)$ ,  $S_2(x)$ .

$$\begin{aligned}
 A &= \begin{pmatrix} \frac{1}{3}\Delta x_0 & \frac{1}{6}\Delta x_0 & 0 & 0 \\ \frac{1}{6}\Delta x_0 & \frac{1}{3}(\Delta x_0 + \Delta x_1) & \frac{1}{6}\Delta x_1 & 0 \\ 0 & \frac{1}{6}\Delta x_1 & \frac{1}{3}(\Delta x_1 + \Delta x_2) & \frac{1}{6}\Delta x_2 \\ 0 & 0 & \frac{1}{6}\Delta x_2 & \frac{1}{3}\Delta x_2 \end{pmatrix} = \\
 &= \begin{pmatrix} \frac{1}{3} \cdot \frac{3}{4} & \frac{1}{6} \cdot \frac{3}{4} & 0 & 0 \\ \frac{1}{6} \cdot \frac{3}{4} & \frac{1}{3} \left( \frac{3}{4} + 3 \right) & \frac{1}{6} \cdot 3 & 0 \\ 0 & \frac{1}{6} \cdot 3 & \frac{1}{3}(3 + 5) & \frac{1}{6} \cdot 5 \\ 0 & 0 & \frac{1}{6} \cdot 5 & \frac{1}{3} \cdot 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & 0 & 0 \\ \frac{1}{8} & \frac{5}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{8}{3} & \frac{5}{6} \\ 0 & 0 & \frac{5}{6} & \frac{5}{3} \end{pmatrix}
 \end{aligned}$$

# Примеры

$$g = \begin{pmatrix} \frac{\Delta y_0}{\Delta x_0} - A_0 \\ \frac{\Delta y_1}{\Delta x_1} - \frac{\Delta y_0}{\Delta x_0} \\ \frac{\Delta y_2}{\Delta x_2} - \frac{\Delta y_1}{\Delta x_1} \\ A_3 - \frac{\Delta y_2}{\Delta x_2} \end{pmatrix} = \begin{pmatrix} \frac{1/2}{3/4} - 1 \\ \frac{1}{3} - \frac{1/2}{3/4} \\ \frac{1}{5} - \frac{1}{3} \\ \frac{1}{6} - \frac{1}{5} \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ -\frac{2}{15} \\ -\frac{1}{30} \end{pmatrix}$$

$$M_0 = -\frac{2948}{2325} \approx -1.2680; M_1 = -\frac{304}{2325} \approx -0.1308;$$

$$M_2 = -\frac{53}{2325} \approx -0.0228; M_3 = -\frac{4}{465} \approx -0.0086$$

# Примеры

$$M_0 = -\frac{2948}{2325}; M_1 = -\frac{304}{2325}; M_2 = -\frac{53}{2325}; M_3 = -\frac{4}{465}$$

$$b_0 = A_0 = 1; \quad b_1 = \frac{\Delta y_1}{\Delta x_1} - \frac{\Delta x_1}{6} (2M_1 + M_2) =$$

$$= \frac{1}{3} - \frac{3}{6} \left( 2 \cdot \left( -\frac{304}{2325} \right) - \frac{53}{2325} \right) = \frac{737}{1550} \approx 0.4755; \quad b_2 =$$

$$= \frac{\Delta y_2}{\Delta x_2} - \frac{\Delta x_2}{6} (2M_2 + M_3) = \frac{1}{5} - \frac{5}{6} \left( 2 \cdot \left( -\frac{53}{2325} \right) - \frac{4}{465} \right) =$$

$$= \frac{38}{155} \approx 0.2452$$

# Примеры

$$M_0 = -\frac{2948}{2325}; M_1 = -\frac{304}{2325}; M_2 = -\frac{53}{2325}; M_3 = -\frac{4}{465}$$

$$c_0 = \frac{1}{2}M_0 = \frac{1}{2} \cdot \left( -\frac{2948}{2325} \right) = -\frac{1474}{2325} \approx -0.6340;$$

$$c_1 = \frac{1}{2}M_1 = \frac{1}{2} \cdot \left( -\frac{304}{2325} \right) = -\frac{152}{2325} \approx -0.0654;$$

$$c_2 = \frac{1}{2}M_2 = \frac{1}{2} \cdot \left( -\frac{53}{2325} \right) = -\frac{53}{4650} \approx -0.0114$$



# Примеры

$$M_0 = -\frac{2948}{2325}; M_1 = -\frac{304}{2325}; M_2 = -\frac{53}{2325}; M_3 = -\frac{4}{465}$$

$$d_0 = \frac{M_1 - M_0}{6\Delta x_0} = \frac{-\frac{304}{2325} + \frac{2948}{2325}}{6 \cdot 3/4} = \frac{5288}{20925} \approx 0.2527;$$

$$d_1 = \frac{M_2 - M_1}{6\Delta x_1} = \frac{-\frac{53}{2325} + \frac{304}{2325}}{6 \cdot 3} = \frac{251}{41850} \approx 0.0060;$$

$$d_2 = \frac{M_3 - M_2}{6\Delta x_2} = \frac{-\frac{4}{465} + \frac{53}{2325}}{6 \cdot 5} = \frac{11}{23250} \approx 0.0005$$

# Примеры

|     |         |         |         |   |
|-----|---------|---------|---------|---|
| $i$ | 0       | 1       | 2       | 3 |
| $x$ | 1/4     | 1       | 4       | 9 |
| $y$ | 1/2     | 1       | 2       | 3 |
| $a$ | 1/2     | 1       | 2       | 3 |
| $b$ | 1       | 0.4755  | 0.2452  |   |
| $c$ | -0.6340 | -0.0654 | -0.0114 |   |
| $d$ | 0.2527  | 0.0060  | 0.0005  |   |

$$S_0(x) = \frac{1}{2} + 1 \cdot \left(x - \frac{1}{4}\right) - 0.6340 \cdot \left(x - \frac{1}{4}\right)^2 + 0.2527 \cdot \left(x - \frac{1}{4}\right)^3, \quad x \in \left[\frac{1}{4}, 1\right]$$

$$S_1(x) = 1 + 0.4755 \cdot (x - 1) - 0.0654 \cdot (x - 1)^2 + 0.0040 \cdot (x - 1)^3, \quad x \in [1, 4]$$

$$S_2(x) = 2 + 0.2452 \cdot (x - 4) - 0.0114 \cdot (x - 4)^2 + 0.0005 \cdot (x - 4)^3, \quad x \in [4, 9]$$

# Примеры

$$S_0\left(\frac{4}{9}\right) = \frac{1}{2} + 1 \cdot \left(\frac{4}{9} - \frac{1}{4}\right) - 0.6340 \cdot \left(\frac{4}{9} - \frac{1}{4}\right)^2 + 0.2527 \cdot \left(\frac{4}{9} - \frac{1}{4}\right)^3 \approx \\ \approx 0.6723$$

$$S_1\left(\frac{9}{4}\right) = 1 + 0.4755 \cdot \left(\frac{9}{4} - 1\right) - 0.0654 \cdot \left(\frac{9}{4} - 1\right)^2 + 0.0060 \cdot \left(\frac{9}{4} - 1\right)^3 \approx \\ \approx 1.5039$$

$$S_2\left(\frac{25}{4}\right) = 2 + 0.2452 \cdot \left(\frac{25}{4} - 4\right) - 0.0114 \cdot \left(\frac{25}{4} - 4\right)^2 + 0.0005 \cdot \left(\frac{25}{4} - 4\right)^3 \approx \\ \approx 2.4993$$

# Примеры

Точность интерполяции:

| $x$  | $P(x)$ | $L(x)$ | $f(x)$ | $\delta$ |
|------|--------|--------|--------|----------|
| 1/4  | 0.293  | 0.293  | 0.5    | 41.4%    |
| 9/4  | 1.699  | 1.699  | 1.5    | 13.3%    |
| 25/4 | 2.012  | 2.012  | 2.5    | 19.5%    |

| $x$  | $S(x)$ | $S(x)$ | $S(x)$ | $S(x)$ | $f(x)$ | $\delta_2$ | $\delta_3$ | $\delta_3$ |
|------|--------|--------|--------|--------|--------|------------|------------|------------|
| 4/9  | 0.678  | 0.663  | 0.693  | 0.672  | 2/3    | 0.6-1.7%   | 3.95%      | 0.80%      |
| 9/4  | 1.417  | 1.490  | 1.474  | 1.504  | 1.5    | 0.6-5.5%   | 1.73%      | 0.27%      |
| 25/4 | 2.615  | 2.491  | 2.569  | 2.499  | 2.5    | 0.4-4.6%   | 2.76%      | 0.04%      |