# Homework #1:
# Chapter 2 Simulations

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February 19, 2018

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# 1 Summary

In this homework, a simulation of compound interest and of a discrete time system were completed using MATLAB. The graph of compound interest shows value over time. The phase-plane plot of the discrete time system shows the equilibrium point at the origin and several solution curves.

# 2 Compound Interest

Compound interest is naturally modelled with a discrete time system. The parameters are: growth rate (i), interval (k), and initial condition u(k).

$$x(k+1) = (1+i)x(k) + u(k) \tag{1}$$

The graph is displayed below:

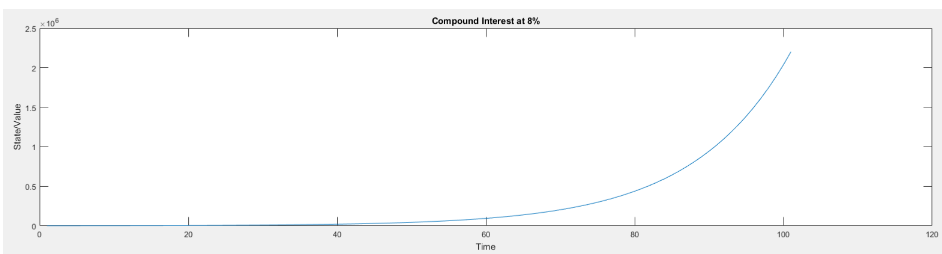


Figure 1: Compound Interest

# 3 Discrete Time System

The time-invariant system in questions is given as:

$$x_1(k+1) = x_2(k)/(1+x_2(k)) \tag{2}$$

$$x_2(k+1) = x_1(k)/(1+x_2(k)) \tag{3}$$

It should be immediately obvious that a solution curve will converge to zero, the equilibrium point $x_e$, for a certain range of values. A Lyapunov analysis can help to find this range of values. The Lyapunov function used is described as:

$$L(x(k)) = x_1^2(k) + x_2^2(k) \tag{4}$$

$$\Delta L(x(k)) = x_1^2(k+1) - x_1^2(k) + x_2^2(k+1) - x_2^2(k) \tag{5}$$

By plugging in the state functions and simplifying, one realizes that the following must be true for the system to be stable in the sense of Lyapunov:

$$1 > 1/(1 + 2x_2(k) + x_2^2(k)) \tag{6}$$

To create a phase-plane plot using MATLAB, a meshgrid is set for the axes in question, $x_1$ and $x_2$. A 'quiver' plot is used to make a vector field detailing the slope at a particular point. To trace a solution curve, an initial condition is set and iterated through the state functions for integer values of K, the current interval.

## 4 Conclusions

Discrete modelling is simply implemented with the help of a computer. The exponential growth model is accurately represented in its discretized form. Sampling at the correct intervals gives an accurate representation of the system. One can quickly determine the initial conditions necessary for stability and then implement a qualifying system. In the case of disturbances, the control input should be carefully selected.

## A Exponential Code

```
i = 0.08
x(1) = 1000
for k=1:100
  x(k+1) = (1+i)*x(k)
end
k=[1:101]
plot(k,x);
title('Exponential Growth')
xlabel('Time');
ylabel('Value');
```

## B Discrete Time System

```
clear all; close all; clc;

% x1(k+1) = x2(k)/ (1+x2(k))
% x2(k+1) = x1(k) / (1+x2(k))

[x1 x2]= meshgrid(-5:0.5:5,-5:0.5:5);

size(x1);

x1dot = zeros(size(x1));
x2dot = zeros(size(x1));

x = x2 / (1+x2)
y = x1 / (1+x2)

x1 = 1
x2 = 1

if x2 == -1
     x1dot = 10
     x2dot = 10
else
     x1dot = x - x1
     x2dot = y - x1
```

```
end

quiver(x1,x2,x1dot, x2dot);
title('Discrete Time System I')
xlabel('x1');
ylabel('x2');
axis tight equal;
```