# Midterm Semester Project Report: Closed Loop Kalman Filters as an Alternative to Phase Locked Loops in Carrier Synchronization

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# Contents

# 1 Introduction

Carrier phase synchronization is necessary to account for Doppler effect, phase instability, transmission delay, and noise. Phase locked loops (PLL) are the traditional method of synchronization. They can also be used for demodulation and frequency synthesizers. PLLs were first implemented on RF integrated circuits, but are now usually implemented digitally. Digital PLLs reduce the cost of implementation. These devices are called delay locked loops (DPLL) or direct digital synthesizers. One drawback is that DPLLs propagate jitter, whereas PLLs are typically effective at blocking it.

PLLs contain three main features: a phase comparator, a loop filter, and a voltage controlled oscillator. In DPLLs, the VCO is replaced by a delay line. First comes the phase detector of the input signal, which can be sinusoidal or quadrature, nonlinear or linear, and coherent or non-coherent. Non-coherent phase discriminators as often referred to as Costas Loop implementations and are insensitive to data modulation. The phase comparator compares the phase of the input signal and the VCO, passing the error to the loop filter. The error (difference) goes through a low pass filter and is used as the tuning voltage on the VCO. The low pass filter removes jitter due to thermal noise. This process is repeated until a phase lock, or steady state error, is achieved. For continuous transmissions, this operation is split into acquisition and tracking modes. It is not as clear cut for burst mode transmissions.

When designing a PLL, it is important to use an appropriate loop filter order and select the appropriate bandwidth. Higher filter order allows the implementation to detect higher-order dynamics, though it is more computationally expensive. First order filters can detect a constant frequency offset, while second order filters can detect a constant frequency drift. The loop filter bandwidth is directly correlated to noise rejection, yet this tight tracking cannot be implemented in highly dynamic environments. Ideally, the bandwidth would be agile enough to minimize performance loss. Kalman filters allow for variable filter coefficients.

Kalman filtering is an algorithm that, over time, produces estimates of unknown variables when taking in to account stochastic noise models. In the first prediction step, the algorithm computes an estimate of the state variables and the unknown variables. At each successive step, these estimates are refined using a weighted average. Thus, the filtering coefficients adjust under a dynamic model, assuming Gaussian noise, to minimize the error between the input signal and VCO or delay line. The Kalman filter can also be extended to account for non-linearities such as a signal cutoff.

## 1.1 Third-Order Phase Locked Loop Implementation

Third order PLLs have several advantages over a second order implementation. Namely, they have better noise rejection and a lower steady-state error. However, third order implementations are likely to be unstable under non-linear conditions; the conditions for stability should be analyzed using Lyapunov analysis.

$$H(s) = \frac{(2+m)\zeta\omega_n s^2 + (1+2m\zeta^2)\omega_n^2 s + m\zeta\omega_n^3}{(s+m\zeta\omega_n)(s^2+2\zeta\omega_n s+\omega_n^2)} \tag{1}$$

Transfer function where m and $\zeta$ are dampening coefficients, $\omega$ being the non-damped oscillating frequency.

$$G_1 = (2+m)\zeta\omega_n \tag{2}$$

$$G_2 = \omega_n^2(1+2m\zeta^2) \tag{3}$$

$$G_3 = m\zeta\omega_n^3 \tag{4}$$

Where $G_1$, $G_2$, and $G_3$ are the loop filter gains.

# 2 Literature Review

It is necessary to use innovative methods, such as Kalman filtering, to overcome communication challenges such as blocking and fading. Variable bandwidth applications are very common, as they only need to implement adaptive filtering techniques for a performance gain. Moreso, Kalman filters can optimize the way loop bandwidth is adjusted. In addition, these carrier tracking methods find use in distributed power generation, motion tracking, space navigation, and more [7]. Due to these compounding factor, design of robust carrier synchronization is a very active field of research. Particularly, there is focus on improving the accuracy of these systems in Global Navigation Satellite Systems (GNSS) applications [3],[4],[10],[13],[18-20]. This is likely due to the presence of Doppler shift, which requires third-order or higher PLL implementations [10], and may not be practical in the receiver. This research also indicates that KF implementations have increased performance, compared to traditional methods, and a low SNR. Third order PLLs are comparable to second order DPLLs and Extended KF implementations [20]. Research presented in [8] details the advantages of using a KF implementation whilst the signal is under the effects of ionospheric scintillation. KFs provide the minimum error by readily accounting for amplitude and phase scintillation in bandwidth.

[6] demonstrates that Kalman Filters perform exceptionally well in the presence of excess phase noise. This is especially important for modulation schemes of higher

order, such as 16-QAM. Also of note is that KF implementations can be parallelized for a reduction in bit error rate and increased throughput.

In [5] the Extended Kalman Filter (EKF) is simplified: the state model relies solely on the phase and the frequency becomes an input term and thus a filtering problem. The research claims that this implementation has less computational load, less complexity, and better performance.

[11] indicates that a third order PLL can outperform first or second order KF implementations. Important to note is that KF implementations can only claim the best optimization of phase error if the approximations are within reason.

## 2.1 Extended Kalman Filter Implementation

The extended Kalman filter (EKF) allows for nonlinear dynamics. This works by linearizing non-linear functions around the current state, k, and directly observing the received signal samples instead of relying on a discriminator. However, convergence of the filter cannot always be guaranteed; if there are abrupt variable changes, the filter is unlikely to converge.

The filter is implemented in five steps: the one step prediction of the mean square error matrix is computed, the one step prediction is computed, the Kalman gain is computed, the mean square error matrix is computed in full, and then the state estimate update is made. Below, the state equation is described, where $f_s$ is the sampling frequency and n(k) is the Gaussian white noise:

$$s(k) = sin(2_s k) + n(k) \tag{5}$$

The five steps to EKF are described below, where $X_k$ is the state vector, $P_k$ is the mean square error matrix, and $K_k$ is the Kalman filter gain. $\Theta(k)$, $\omega_0(k)$, and $\omega_1(k)$ are elements of the state vector, where $\Theta(k)$ is the phase of the incoming signal, and $\omega_0(k)$ and $\omega_1(k)$ are the first and second derivatives of this phase.

$$P_{k,k-1} = \Phi_{k,k-1}\Phi^T_{k,k-1}P_{k-1} + \Gamma_{k,k-1}{}^T_{k,k-1}Q_{k-1} \tag{6}$$

This displays the step prediction of mean square error matrix where $\Phi_{k,k-1}$ is the state transition matrix, $\Gamma_{k,k-1}$ is the noise driven matrix, and $Q_{k-1}$ is the system noise matrix.

$$\hat{X}_{k,k-1} = \Phi_{k,k-1}X_{k-1} \tag{7}$$

This shows the one step prediction of the state vector.

$$K_k = P_{k,k-1} H_k^T (H_k H_k^T P_{k,k-1} + R_k)^{-1} \tag{8}$$

This computes the Kalman filter gain, where $H_k$ is the observation matrix.

$$P_k = P_{k,k-1}(I - K_k H_k) \tag{9}$$

This computes the fully updated mean square error matrix.

$$X_k = \Phi_{k,k-1} X_{k-1} + K_k[r_k - h\hat{X}_{k,k-1}] \tag{10}$$

This displays the state estimate update.

# 3 Methodology

The following simulations will be run through MATLAB and Simulink. The phase error will be compared between different analog PLLs, digital PLLs, and KF approaches. These will be compared for BPSK and QPSK, being modulation schemes of different order. Gaussian and non-Gaussian noise will be tested. The center frequency and data rate will be kept constant.

To date, a first order analog PLL has been simulated. This will be extended to the third order. The next step will be implement a first-order DPLL and compare the responses. These will then be compared to a second-order DPLL. Next, a Kalman filter design will be implemented. If time allows, multiple KF implementations may be tested. During analysis bit error rate, complexity, convergence time, mean error, and scope will be addressed. Acknowledgement goes to [10] for providing insight on this paper's implementations.

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