Question 4:

Given, N - number of observations

p variables

$g = 2$ (number of classes)

Number of observations in class $A = N_A$

Number of observations in class $B = N_B$

Therefore, prior : $\pi_A = \frac{N_A}{N}$

$$\pi_B = \frac{N_B}{N}$$

Linear Discriminant Analysis is rule classifies an observation $\underline{x}$ to class B if $\delta_B(\underline{x}) > \delta_A(\underline{x})$.

$$\delta_g(\underline{x}) = \log \pi_g + \underline{x}^T \Sigma^{-1} \mu_g - \frac{1}{2} \mu_g^T \Sigma^{-1} \mu_g$$

$$\therefore \quad \delta_B(\underline{x}) > \delta_A(\underline{x})$$

$$\Rightarrow \log \pi_B + \underline{x}^T \Sigma^{-1} \mu_B - \frac{1}{2} \mu_B^T \Sigma^{-1} \mu_B > \log \pi_A + \underline{x}^T \Sigma^{-1} \mu_A - \frac{1}{2} \mu_A^T \Sigma^{-1} \mu_A$$

$$\Rightarrow \log \pi_B - \log \pi_A + \frac{1}{2} \mu_A^T \Sigma^{-1} \mu_A - \frac{1}{2} \mu_B^T \Sigma^{-1} \mu_B > \underline{x}^T \Sigma^{-1} \mu_A - \underline{x}^T \Sigma^{-1} \mu_B$$

$$\Rightarrow \frac{1}{2} \mu_A^T \Sigma^{-1} \mu_A - \frac{1}{2} \mu_B^T \Sigma^{-1} \mu_B + \log\left(\frac{N_B}{N}\right) - \log\left(\frac{N_A}{N}\right) > \underline{x}^T \Sigma^{-1} (\mu_A - \mu_B)$$

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$$\Rightarrow -x^T \Sigma^{-1}(\mu_A - \mu_B) > -\left(\tfrac{1}{2}\mu_A^T \Sigma^{-1}\mu_A - \tfrac{1}{2}\mu_B^T\Sigma^{-1}\mu_B\right) - \log\left(\frac{N_A}{N_B}\right) + \log\left(\frac{N_B}{N}\right)$$

$$\Rightarrow \underline{x}^T \Sigma^{-1}(\mu_B - \mu_A) > \frac{1}{2}\mu_B^T \Sigma^{-1}\mu_B - \frac{1}{2}\mu_A^T\Sigma^{-1}\mu_A + \log\left(\frac{N_A}{N}\right) - \log\left(\frac{N_B}{N}\right)$$

$\therefore$ linear discriminant analysis rule classifies an observation $\underline{x}$ to class B if

$$\underline{x}^T \Sigma^{-1}(\mu_B - \mu_A) > \frac{1}{2}\mu_B^T \Sigma^{-1}\mu_B - \frac{1}{2}\mu_A^T\Sigma^{-1}\mu_A + \log\left(\frac{N_A}{N}\right) - \log\left(\frac{N_B}{N}\right)$$