Electrical, Computer & Energy Engineering
UNIVERSITY OF COLORADO BOULDER

ECEN 5053-003 Homework Assignment

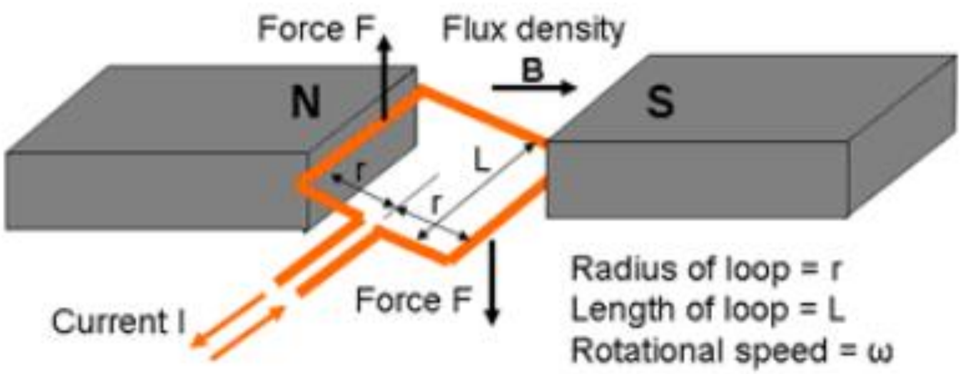
Course Name: Embedding Sensors and Actuators
Corresponding Module: C2M3
Week Number: 7
Module Name: DC Motors
Student Name: **Rushi James Macwan**

Homework is worth 100 points.

Part 1: Each question is worth 10 points.

A. The current loop below represents an electric motor. What is the torque on this motor if the magnetic field strength = 6 Tesla, the length of the loop is 0.50 meters, the current in the loop is 2.50 amps, and the radius of the loop is 0.10 meters. The rotational speed of the loop is 1065 RPM.

   What is the torque on the loop measured in N-m? What is the Back EMF on the loop measured in volts?



Sol. **Torque = 1.5 N-m and EMF = 66.882 V**

Based on the referenced document, the torque for the above current loop which represents an electric motor can be calculated as below using the equation:

$$\boxed{\text{Motor Torque} = 2Fr = 2\ BLIr}$$

Thus, Torque = 2 x 6 x 0.50 x 2.50 x 0.10 = **1.5 N-m**

Similarly, for calculating the Back EMF, the below given formula can be used to calculate it:

$$\boxed{\text{Generator EMF} = 2\,BLr\omega}$$

Here, **ω** is the speed in radians per second
So, **ω** = 2 x PI x N / 60
Where, N is the speed in revolutions per minute (RPM)

Thus, EMF = 2 x 6 x 0.50 x 0.10 x [2 x 3.14 x 1065 / 60] = **66.882 V**

Courtesy: Reference Links: [1]

B. What is the torque in N-m of a DC motor operating at 600 RPM and 200 watts of power?

Sol. **Torque = 3.1834 N-m**

Based on the referenced document, the torque for the above mentioned specs of a DC motor can be calculated as below using the equation:

$$\boxed{P = \frac{2\pi NT}{60} = \frac{NT}{9.55}}$$

Where, P = output power in Watts
N = speed in RPM
T = Torque in N-m

Thus, Torque = P x 9.55 / N = 200 x 9.55 / 600 = **3.1834 N-m**

Courtesy: Reference Links: [1]

C. A brushless DC motor is rated at 12 volts, 0.9 amps, and operates at a duty point efficiency of 72%. What is its rating for continuous power?

Sol. **7.776 Watts**

The rating for continuous power will be
P = (V x I) x (duty-point efficiency)
= 12 x 0.9 x 0.72
= **7.776 Watts**

D. A shunt wound DC motor is operating in the linear range, and it has the following attributes:

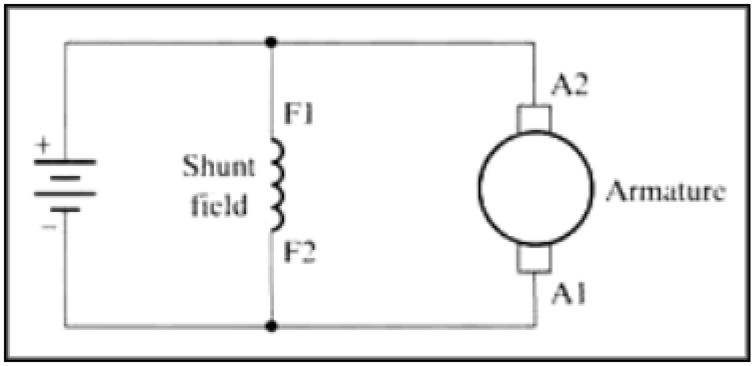
No load voltage, V =	90	volts
Armature Resistance, R_a =	20	ohms
Low speed regulation Resistance, R_1 =	0	ohms
Shunt Resistance, R_{SH} =	50	ohms
Shunt regular resistance, R_F =	0	ohms
Rated speed, N =	1000	RPM
Rated torque, T =	10	N-m
Motor current at rated speed, I_L =	5.0	amps
Motor current at new speed, I_{Ln} =	5.2	amps

What is the new speed and the new torque when the motor current is run at the new current, I_{LN}?

Sol. $\boxed{\textbf{New speed = } \underline{\textbf{846.1538 RPM}}\textbf{, New torque = } \underline{\textbf{10.625 N-m}}}$

For a shunt wound DC motor, the armature and the shunt windings are connected in parallel and therefore the supply voltage provided to the motor will be the same for both the armature as well as the shunt windings. Again, this is because they are connected in parallel. However, since both the armature and the shunt windings are connected in parallel, the current flowing through the motor will split across the armature and the shunt windings. Therefore, the current flowing the armature and the shunt windings will be different and will depend on the corresponding impedances.

Based on the figure shown below, the motor current will be equal to the summation of the armature current and the shunt current.



In a DC shunt motor, the armature and field (shunt) windings are connected in parallel. A parallel circuit is also known as a shunt circuit; thus, the term, "shunt motor" is used.

Image credit: National Instruments Corporation

$$I_{total} = I_a + I_{sh}$$

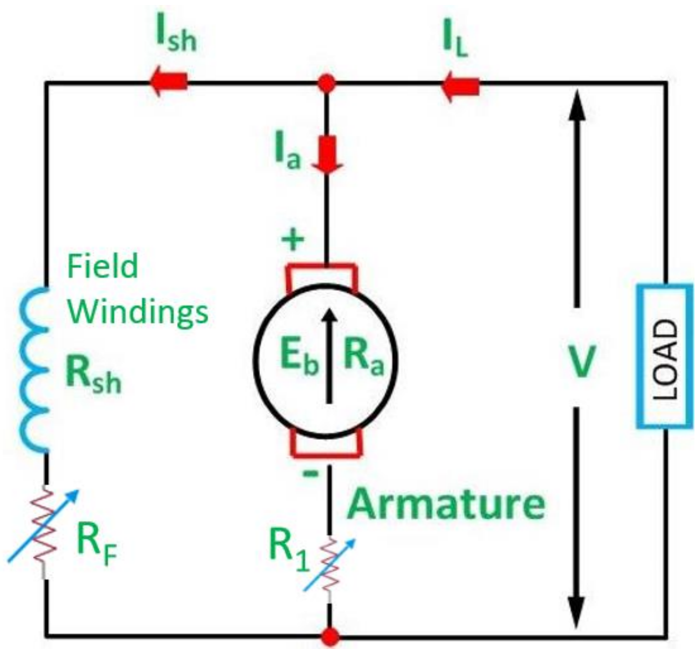
Where:

I_{total} = supply current

I_a = current though armature windings

I_{sh} = current through shunt (field) windings

Furthermore, to simplify the circuit, based on the class slide and the figure shown below, we can obtain the general voltage equation for the DC shunt wound motor.



Based on this circuit, the DC shunt would motor equation will be as below:

$$V = E_b + I_a \times R_a = k_a \times \varphi \times \omega + I_a \times R_a$$

where:

- V = supplied motor voltage
- E_b = back EMF
- φ = magnetic flux
- ω = angular speed
- k_a = proportionality constant
- Ia = armature current
- R_a = armature resistance

Solving this equation, for the calculation of new speed, we first find the back EMF at the rated speed and current of 5 A. Once, we find the back EMF, we can calculate the product of the proportionality constant with magnetic flux.

This product obtained will be a constant since the shunt current is directly proportional to the magnetic flux as can be read from the referenced document. This shunt current will depend on the shunt resistance and the supplied motor voltage. Since, both the shunt resistance and the supplied motor voltage remains constant throughout this problem, the shunt current remains constant and since it is proportional to the magnetic flux, the magnetic flux also remains constant. This leads to the conclusion that the product of the proportionality constant and the magnetic flux will also result into a constant which can be further used to find the new speed. This can be understood from the below given calculations:

Q-D

$$V = E_b + I_a R_a = k_a \times \varphi \times \omega + I_a R_a \quad (1)$$

For rated speed, the rated current is $I_L = I_{total} = 5.0\,A$

Now,

$$I_a = I_{total} - I_{shunt}$$

where, I_{shunt} = shunt windings current

$$\therefore I_a = I_{total} - \frac{V}{R_{sh}}$$

where, R_{sh} = shunt resistance

Thus, using Equation (1) for rated speed,

$V = 90V$, $I_a = I_{total} - \frac{V}{R_{sh}} = 5 - \frac{90}{50} = 3.2\,A$

$R_{sh} = 50\Omega$

$R_a = 20\Omega$

$N = 1000\,RPM$

$$\therefore 90 = E_b + (3.2)(20)$$

$$\therefore E_b = 90 - 64 = \boxed{26V}$$ = Back EMF at rated speed.

Now, $E_b = k_a \varphi \omega$

$$\therefore k_a \varphi = \frac{E_b}{\omega} = \frac{E_b}{\left(\frac{2\pi N}{60}\right)} = \frac{60 \times E_b}{2\pi N} = \frac{60 \times 26}{2\pi \times 1000}$$

$$\therefore k_a \varphi = \boxed{\frac{78}{100\pi}}$$

Now, $I_{shunt} \propto \phi$

and $I_{shunt} = \frac{V}{R_{sh}} = \text{constant}$ $\left(\because V \text{ is constant}\right)$

$\therefore$ ϕ will remain constant

$\therefore$ $K_a\phi$ will remain constant

For new speed :-

Again using Eq ①:-

$$V = E_b + I_a \cdot R_a$$

For new total current $I_{total} = I_{LN} = 5.2\,A$

$$\therefore V = E_b + (I_{total} - I_{sh}) R_a$$

$$= E_b + \left(5.2 - \frac{V}{R_{sh}}\right) R_a$$

$$= E_b + \left(5.2 - \frac{90}{50}\right) \times R_a$$

$$= E_b + (5.2 - 1.8) \times 20$$

$$\therefore V = E_b + 68$$

Now, $V = 90V$

$$\therefore 90 = E_b + 68$$

$\therefore$ $\boxed{E_b = 22V}$ — Back EMF at new speed

Now, $E_b = K_a\phi\omega$

and $\omega = E_b / K_a\phi$

$$= \frac{22}{(78/100\pi)}$$

$$\therefore \quad \omega = \frac{22 \times 100\pi}{78}$$

$$\therefore \quad \frac{2\pi N}{60} = \frac{22 \times 100\pi}{78}$$

$$\therefore \quad N = \frac{22 \times 100\pi \times 60}{2\pi \times 78}$$

$$= 846.1538 \text{ revolutions/min}$$

$$\therefore \quad \boxed{N = 846.15 \text{ RPM}}$$

Thus, the new speed obtained at the new motor current of 5.2 A is equal to **846.1538 RPM**.

Similarly, to calculate the torque of this motor, we can use the below given equation:

$$T = k_a \times \varphi \times I_a$$

However, here the proportionality constant is different from the one used above and therefore, we will need to calculate the value of the proportionality constant and its product with the magnetic flux at the rated speed with the motor current of 5 A and then the resultant constant can be used to calculate the new torque at the new speed and new motor current of 5.2 A. This can be understood from the calculation as below:

For torque:-

At rated speed, $I_{total} = I_L = 5.0\,A$

$T = k_b \varphi I_a$

Here, $T = 10N\text{-}m$, k_b = proportionality const

φ = magnetic flux

$I_L = 5.0\,A$

$\therefore 10 = k_b \varphi \times (I_{total} - I_{sh})$

$\therefore 10 = k_b \varphi \times \left(5 - \frac{90}{50}\right)$

$\therefore 10 = k_b \varphi \times \frac{16}{5}$

$\therefore k_b \varphi = \boxed{25/8}$

Finally, to find torque for new current:-

$T = k_b \varphi \times I_a$

This time $I_a = I_{total} - I_{sh}$

where $I_{total} = 5.2\,A$

$\therefore I_a = 5.2 - \frac{90}{50} = 3.4\,A$

$\therefore T = \frac{25}{8} \times 3.4$

$\therefore \boxed{T = 10.625\ N\text{-}m}$

Thus, based on the above calculations, using the value of the product of the proportionality constant with the magnetic flux and the new motor current, we can calculate the new torque of the motor which is **10.625 N-m**.

Courtesy: Reference Links: [1] [2] and class slides [C2M3V4]

E. A permanent magnet DC motor is accelerating to its rated speed, but it has not quite reached this speed by time t. The motor has the following attributes:

Rated voltage at rated speed, V =	90	volts
Rated current at rated speed, I_{max} =	3	amps
Armature Resistance, R_a =	10	ohms
Motor Inductance, L =	1.5	henries
Time, t =	0.2	seconds
Back EMF at time t, V_B =	52.0	volts

What is the motor current at time t? **2.79 A**

What is the time constant of the motor? **0.15**

Approximately at what time will the motor reach its rated speed? **0.75** seconds

What is the back EMF at the rated speed? = **60 V**

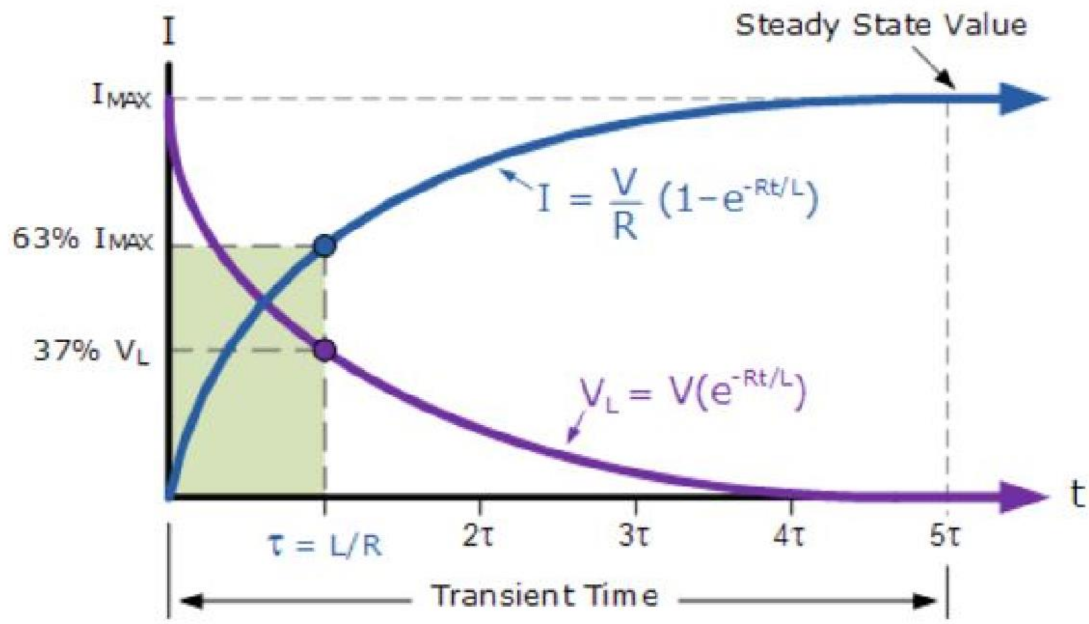
Sol.

To solve this problem, based on the class slide, we need to use the equation for current for a permanent magnet DC motor which is the same equation that the BLDC motor uses:

$$I = (V - V_b)/R \times (1 - e^{(-Rt/L)})$$

$$\text{Maximum current, } I_{max} = (V - V_b)/R$$

The motor in this problem would also use the below given transient current curve:



Based on the above provided information, the following calculations explain the solutions to the problems:

1) Motor current at time t = 0.2 seconds:

Q-E

1) Motor current at t = 0.2 seconds :-

$$I = \frac{(V - V_b)}{R} \times \left(1 - e^{(-Rt/L)}\right)$$

where, $I = ?$, V = voltage at rated speed = 90 V

V_b = Back EMF at t = 0.2 seconds = 52 V

R = armature resistance = 10 Ω

L = motor inductance = 1.5 H

t = 0.2 sec.

$$\therefore I = \left[\frac{90 - 52}{10}\right] \times \left(1 - e^{\left(-\frac{10 \times 0.2}{1.5}\right)}\right)$$

$$= \frac{38}{10} \times \left(1 - e^{(-1.33)}\right)$$

$$= 3.8 \times (1 - 0.26359)$$

$$= 3.8 \times 0.73640 = \boxed{2.798\ A}$$

Thus, the motor current at t = 0.2 seconds will amount to **2.798 A**.

2) Time constant of the motor:

2) Time constant $\tau = L/R$

$$= \frac{1.5\,H}{10\,\Omega} = \boxed{0.15 \text{ seconds}}$$

Based on the class slide and the transient curve presented in this solution, the time constant of the motor can be calculated by taking the division of the motor inductance and the armature resistance which amounts to **0.15 seconds**.

3) Time at which motor reaches its rated speed:

Based on the transient current curve, the motor will reach its rated speed at approximately five times the time constant. Therefore, the time taken will be equal to t = 5 x (time constant).

3) Time to reach rated speed $= 5\tau$

$$= 5 \times 0.15$$

$$= \boxed{0.75 \text{ seconds}}$$

Thus, the time to reach the rated speed is **0.75 seconds**.

4) Back EMF at rated speed:

The back EMF at the rated speed can be calculated using the value of the rated current value at the rated speed which is 3 A. Also, the rated current at the rated speed is directly proportional to the difference between the motor voltage and the back EMF at the rated speed. Therefore, we can calculate the Back EMF at the rated speed as per the calculations shown below:

4) Back EMF at rated speed:-

Rated current at rated speed $= I_{max}$
$= 3A$

and $I_{max} = \dfrac{V - V_b}{R}$

where, V = voltage at rated speed = 90 V
V_b = Back EMF = ?
R = armature resistance = 10Ω
$I_{max} = 3A$

$\therefore V_b$ = Back EMF at rated speed
$= -(I_{max} \times R) + V$
$= -(3 \times 10) + 90$
$= -30 + 90$

$\therefore \boxed{V_b = 60V}$

Thus, the back EMF at the rated speed would amount to **60 V**.

Courtesy: Reference class slides [C2M3V6]

F. How do you know that the compound motor provides good speed control at the no-load condition?

Sol.

By arranging for some of the field MMF to be provided by a series winding and some to be provided by a shunt winding, it is possible to obtain motors with a wide variety of inherent torque-speed characteristics.

- In practice most compound motors have the bulk of the field MMF provided by a shunt field winding, so that they behave more or less like a shunt connected motor.

- The series winding MMF is relatively small, and is used to allow the torque-speed curve to be trimmed to meet a particular load requirement.

- When the series field is connected so that its MMF reinforces the shunt field MMF, the motor is said to be 'cumulatively compounded'.

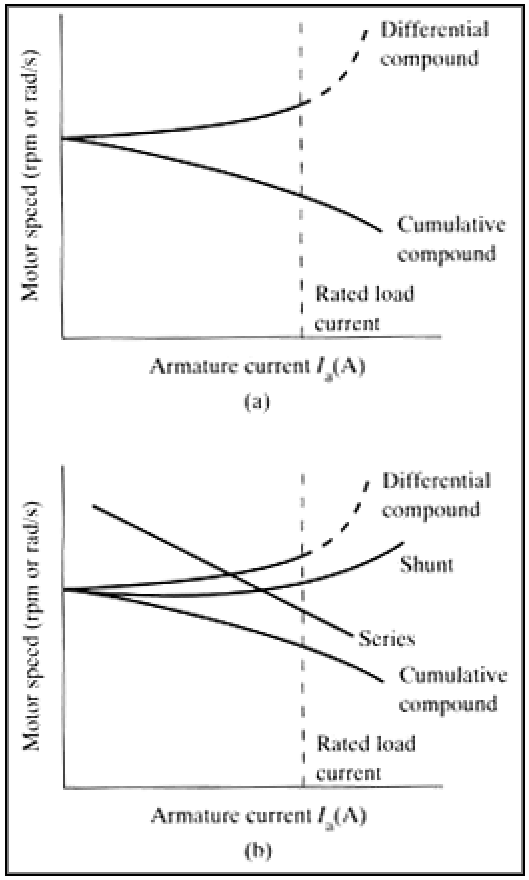
- As the load on the motor increases, the increased armature current in the series field causes the flux to rise, thereby increasing the torque per ampere but at the same time, resulting in a bigger drop in speed as compared with a simple shunt motor.

- On the other hand, if the series field winding opposes the shunt winding, the motor is said to be 'differentially compounded'. In this case an increase in current results in a weakening of the flux, a reduction in the torque per ampere, but a smaller drop in speed than in a simple shunt motor.

- Differential compounding can therefore be used where it is important to maintain as near constant-speed as possible.

To understand more, based on the third referenced document, it says that the speed of a compound motor can be changed very easily by adjusting the amount of voltage applied to it.

The diagram given below portrays the characteristic curves of the speed versus armature current for the compound motors. From this diagram, it can be seen that the speed of a differential compound motor increases slightly when the motor is drawing the armature highest current. The increase in speed occurs because the extra current in the differential winding causes the magnetic field in the motor to weaken slightly because the magnetic field in the differential winding opposes the magnetic in series field. Furthermore, the speed of the motor will increase if the magnetic field is weakened.



Characteristic curve of armature current versus speed for the differential compound motor and cumulative compound motor, (b) Composite of the characteristic curves for all of the DC motors.

The diagram also shows the characteristic curve for the cumulative compound motor. This curve shows that the speed of the cumulative compound motor decreases slightly because the field is increased, which slows the motor because the magnetic field in the shunt winding aids the magnetic field of the series field.

Thus, the compound motor provides good speed control even at the no-load condition.

Courtesy: Reference Links: [1] [2] [3] [4]

G. Describe how trapezoidal commutation works. You can use one or more of the slides from our lectures for pictorial representation, if you like.

Sol.

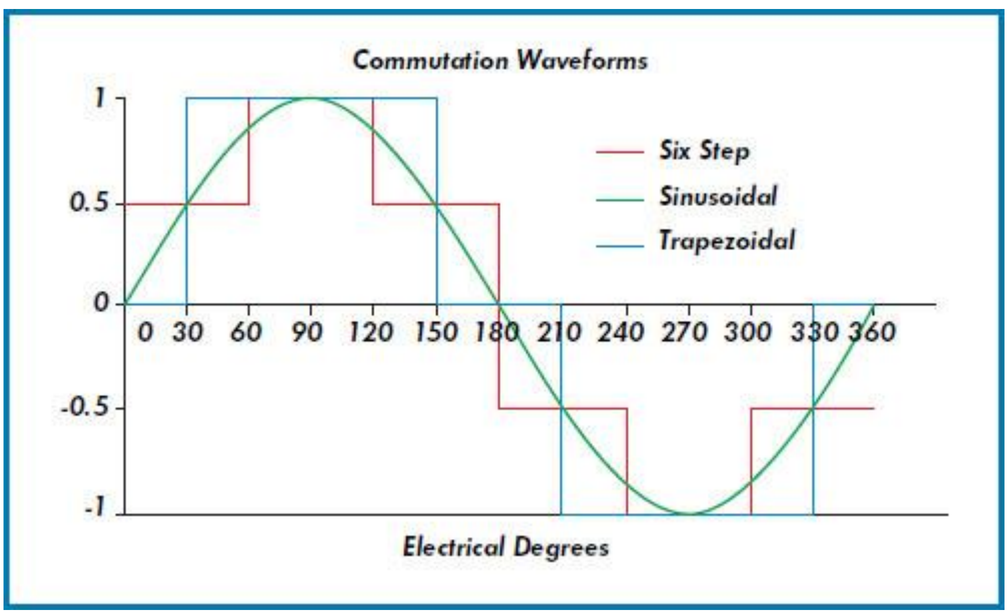
Commutation *{reference}*

Commutation is the process of switching current in the phases in order to generate motion. Most linear motor designs today use a three-phase brushless design. In brushed motors, commutation is easy to understand as brushes contact a commutator and switch the current as the motor moves. Brushless technology has no moving contacting parts and therefore is more reliable. However, the electronics required to control the current in the motor are a little more complex.

The method of commutation depends on the application of the motor, but it is important to understand how the motor can be commutated and what disadvantages some methods have

Trapezoidal commutation is the simplest form of commutation and requires that digital Hall devices are aligned 30º electrically from the zero crossing point of the phase. At each point that a Hall signal transition takes place, the phase current sequence is changed, thus commutation of the motor occurs. This is the cheapest form of commutation and the motor phase current looks like the diagram shown below.

There are three different types of commutation currently available on the market: trapezoidal, modified six step, and sinusoidal.



Trapezoidal Commutation of BLDC Motor

One of the simplest methods of control for dc brushless motors uses what is termed **Trapezoidal** commutation.

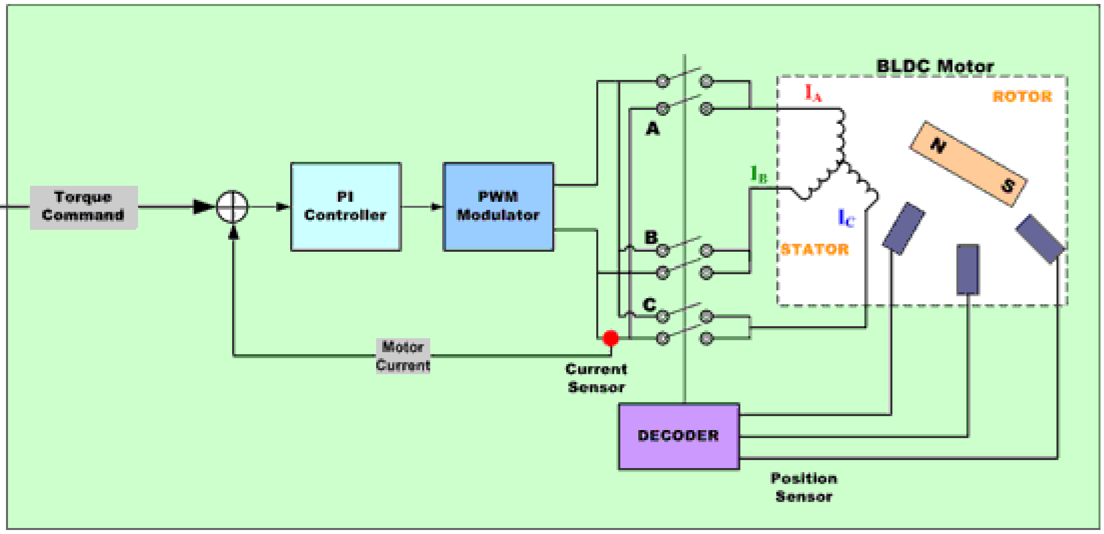


Figure 1: Simplified Block Diagram of Trapezoidal Controller for BLDC Motor

In this scheme, current is controlled through motor terminals one pair at a time, with the third motor terminal always electrically disconnected from the source of power.

Three Hall devices embedded in the motor are usually used to provide digital signals which measure rotor position within 60 degree sectors and provide this information to the motor controller. Because at any time, the currents in two of the windings are equal in magnitude and the third is zero, this method can only produce current space vectors having one of six different directions. As the motor turns, the current to the motor terminals is electrically switched (commutated) every 60 degrees of rotation so that the current space vector is always within the nearest 30 degrees of the quadrature direction.

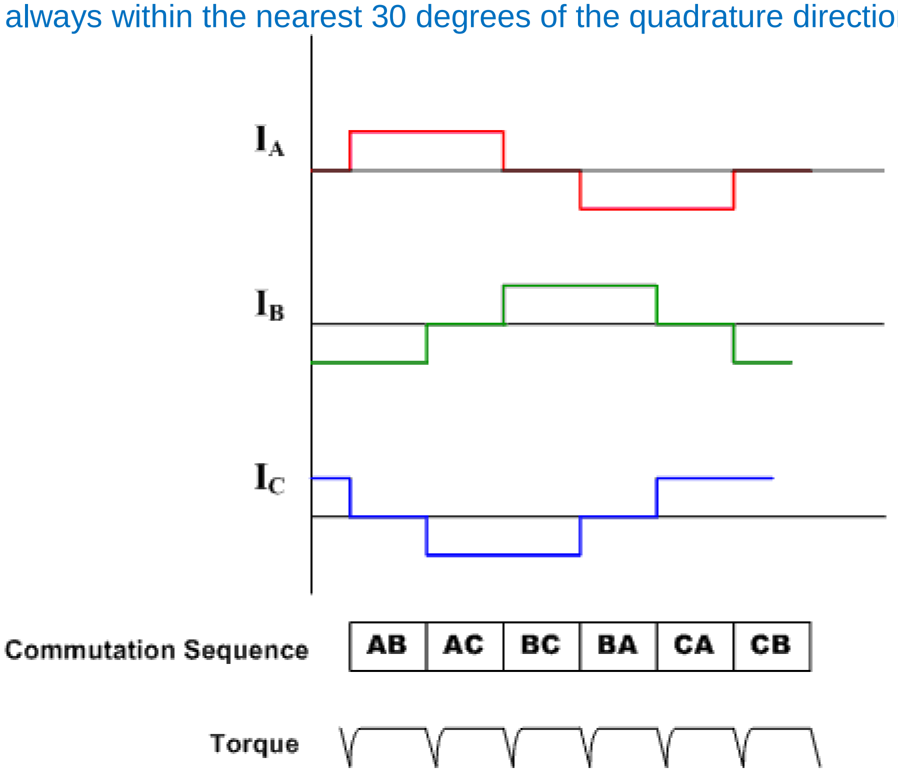


Figure 2: Trapezoidal Control: Drive Waveforms and Torque at commutation

The current waveform for each winding is therefore a staircase from zero, to positive current, to zero, and then to negative current.

This produces a current space vector that approximates smooth rotation as it steps among six distinct directions as the rotor turns.

In motor applications such as air conditioners and refrigerators use of Hall-Effect sensors is not a viable option. Back-EMF sensors that sense the back EMF in the unconnected winding can be used to achieve the same results

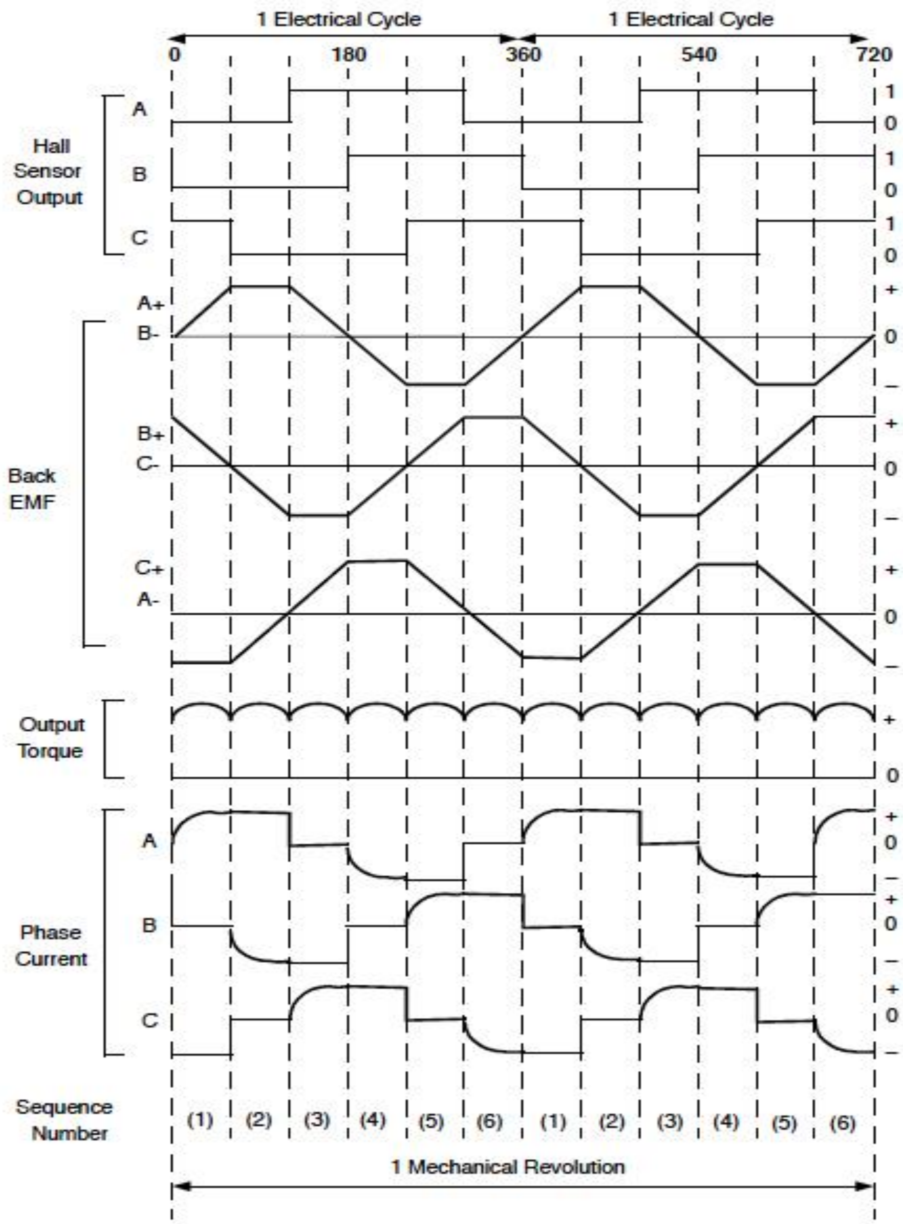
The trapezoidal-current drive systems are popular because of the simplicity of their control circuits but suffer from a torque ripple problem during commutation.

More brief information on the Trapezoidal Commutation in BLDC Motors: (based on a separately referenced document)

Trapezoidal (aka six-step) commutation is common in high-speed applications or when higher starting torque is required. Trapezoidal commutation is also less costly than other methods, due to its simple control algorithms.

In most applications, the rotor position is determined by three Hall-effect sensors that are mounted on the stator, 120 degrees apart. When the rotor passes over the sensors, they produce either a high or a low signal to indicate which rotor pole (N or S) is passing over. The change from high to low (or low to high) of the three Hall sensors gives rotor position information every 60 degrees, meaning that six steps are needed in order to complete one electrical cycle—thus, the term “six-step commutation.” The correct commutation sequence is determined from the combination of the Hall sensor signals.

Trapezoidal commutation can also be performed based on the motor’s back EMF, which allows the elimination of Hall sensors. In a typical three-phase BLDC motor with trapezoidal current, one winding is positive, one winding is negative, and one is open. The open winding can be used to detect the zero-crossing point of the back EMF, which corresponds to what would be a signal change in a Hall sensor. However, the back EMF is proportional to motor speed. This means that at very slow speeds (and especially at startup), the back EMF will be very low, so the motor must be started in open-loop mode until sufficient speed and back EMF are generated. At that point, the controller can be switched to back EMF sensing for commutation.



Hall sensor signals, back EMF (ideally trapezoidal), torque ripple (every 60 degrees), and phase current for a BLDC motor.
Image credit: Microchip Technology Inc.

Courtesy: Reference Links: [1] [2] [3] [4] [5] [6]

H. Describe how sensorless position monitoring works. You can use one or more of the slides from our lectures for pictorial representation, if you like. What are the flaws in this method of measuring angular position of the armature?

Sol.

The zero-crossing approach is one of the simplest methods of back-EMF sensing technique, and is based on detecting the instant at which the back-

EMF in the unexcited phase crosses zero. This zero crossing triggers a timer, which may be as simple as an RC time constant, so that the next sequential inverter commutation occurs at the end to this timing interval [23].

For typical operation of a BLDC motor, the phase current and back-EMF should be aligned to generate constant torque. The current commutation point shown in Figure 9 can be estimated by the zero crossing point (ZCP) of back-EMFs and a 30° phase shift [1,4], using a six-step commutation scheme through a three-phase inverter for driving the BLDC motor. The conducting interval for each phase is 120 electrical degrees. Therefore, only two phases conduct current at any time, leaving the third phase floating. In order to produce maximum torque, the inverter should be commutated every 60° by detecting zero crossing of back-EMF on the floating coil of the motor [24], so that current is in phase with the back-EMF.

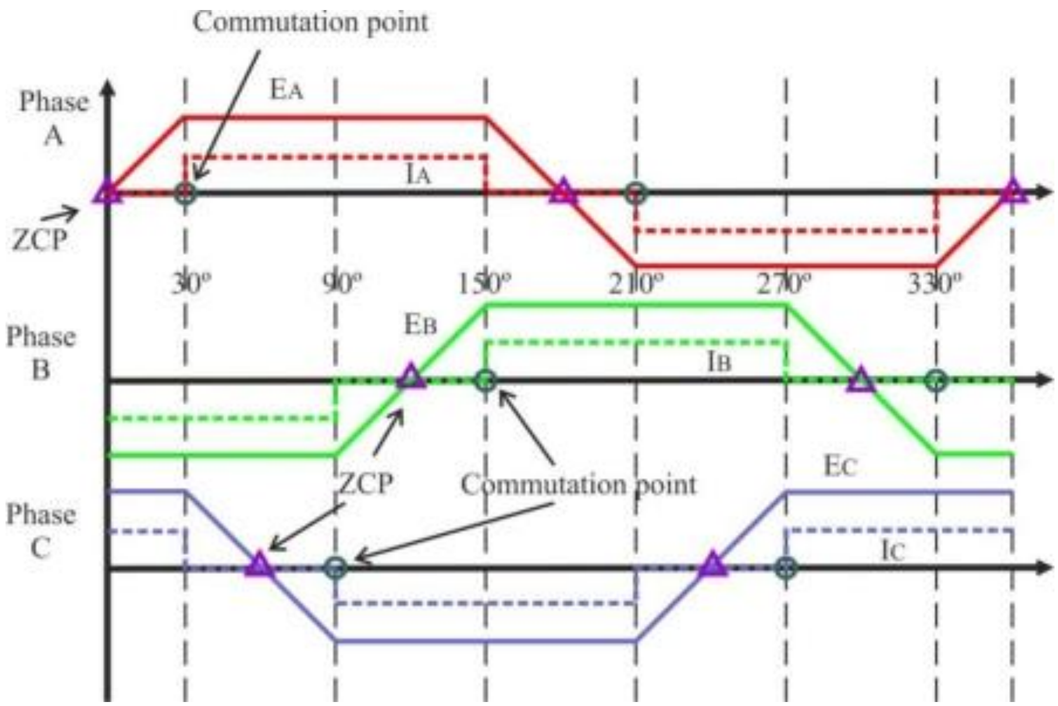


Figure 9.

Zero crossing points of the back-EMF and phase current commutation points [25].

This technique of delaying 30° (electrical degrees) from zero crossing instant of the back-EMF is not affected much by speed changes [7]. To detect the ZCPs, the phase back-EMF should be monitored during the silent phase (when the particular phase current is zero) and the terminal voltages should be low-pass filtered first.

Three low-pass filters (LPFs) are utilized to eliminate higher harmonics in the phase terminal voltages caused by the inverter switching. The time delay of LPFs will limit the high speed operation capability of the BLDC machine [1,24,26]. It's necessary to point out the importance of filters when a BLDC motor drive is designed, which are used to eliminate high frequency components of the terminal voltages and to extract back-EMF of the motor.

The terminal voltage of the opened or floating phase is given by Equation (4):

$$VC=eC+VN=eC+VCE-VF2-eA+eB2 \qquad (4)$$

Where e_C is the back-EMF of the opened phase (C), V_N is the potential of the motor neutral point, and V_{CE} and V_F are the forward voltage drop of the transistors and diodes, respectively, which implement the motor inverter of Figure 7, respectively.

As the back-EMF of the two conducting phases (A and B) have the same amplitude but opposite sign [19] the terminal voltage of the floating phase results in Equation (5):

$$eA=-eB\Rightarrow VC=eC+VCE-VF2=eC+VB+VA2 \qquad (5)$$

Where $V_A=-V_F$ (forward current of diode D_{A-}) and $V_B=V_{CE}$ (collector-emitter voltage of transistor T_{B-})

Since the zero-crossing point detection is done at the end of the PWM on-state and only the high-side of the inverter is chopped, and V_{CE} is similar to T_{A+} and T_{B-} transistors, the final detection formula can be represented by

Equation (6):

$$VA+CE\approx VB-CE\Rightarrow VC=eC+VB-CE+VDC-VA+CE2\approx eC+VDC2 \qquad (6)$$

Therefore, *the zero-crossing occurs when the voltage of the floating phase reaches one half of the DC rail voltage*. The reason why the end of the PWM on-state is selected as the zero-crossing detection point is that it is noise free to sample at that moment [20].

On the other hand, instead of using analogue LPFs, a unipolar pulse width modulation (PWM) scheme can be used to measure terminal voltages [27,28]. The difference of the ZCD method between on and off state of the PWM signal can also be taken into account [29,30]. Also, the true phase back-EMF signal could be directly obtained from the motor terminal voltage by properly choosing the PWM and sensing strategy (without the motor neutral point voltage information This would provide advantages such as no sensitivity to switching noise, no filtering required, and good motor performance a wide speed range [24,31].

Disadvantages:

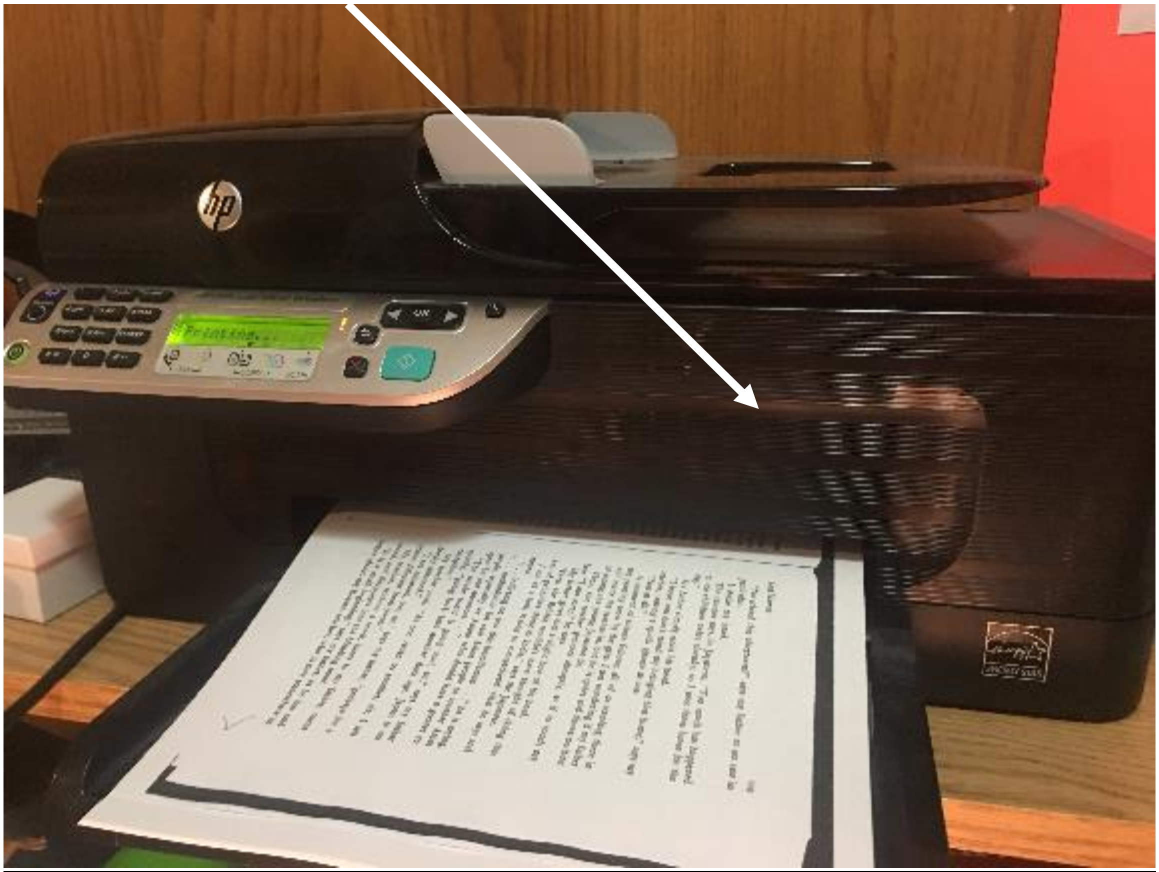
The disadvantages of sensorless control are higher requirements for control algorithms and more complicated electronics [3]. The price for the simplicity of the zero-crossing method tends to be noise sensitivity in detecting the zero crossing, and degraded performance over wide speed ranges unless the timing interval is programmed as a function of rotor speed [23]. Another drawback is that it is not possible to use the noisy terminal voltage to obtain a switching pattern at low speeds since back-EMF is zero at standstill and proportional to rotor speed. Also, the estimated commutation points have position error during the transient period when the speed is accelerated or decelerated rapidly, especially for a system that has low inertia. With this method, rotor position can be detected typically from 20% of the rated speed, then a reduced speed operating range is normally used, typically around 1,000–6,000 rpm [1].

Furthermore, there are a few more disadvantages that add to the lot:

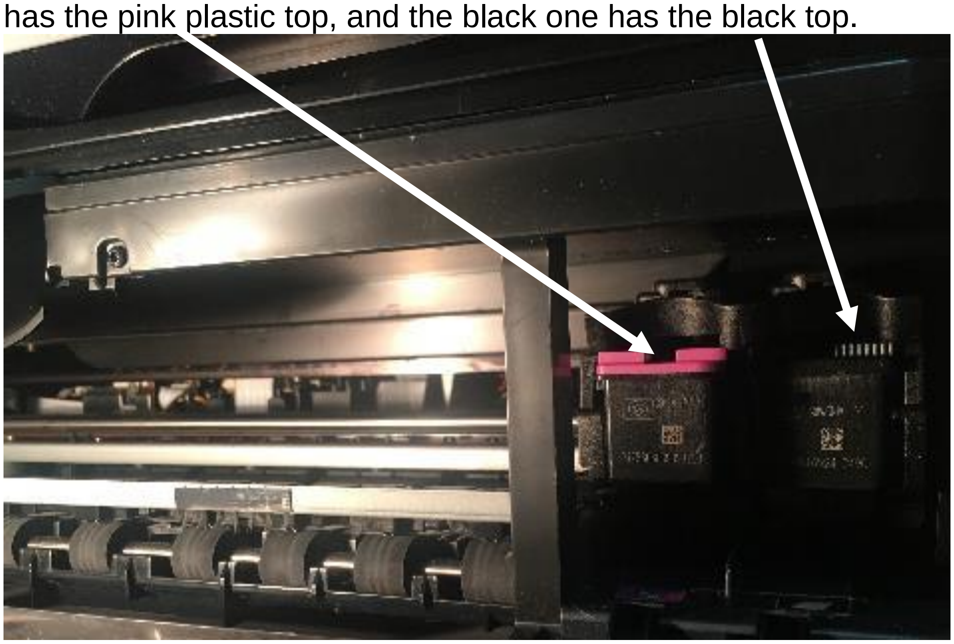
- There is one major disadvantage to sensorless BLDC motor control; when the motor is stationary, no back EMF is generated, depriving the MCU of information about the stator and rotor position.
- Other disadvantage is that no back EMF is generated when the motor is stationary, so startup is affected by operating in open loop.
- Consequently, the motor can take a short time to settle and run efficiently.
- Furthermore, at low speeds the back EMF is small and difficult to measure, which can result in inefficient operation.

Courtesy: Reference Links: [1] [2] [3]

I. An Hp Officejet 4500 wireless printer is shown printing a document in the photo below. After printing hundreds of documents, you normally need to change either the black or color inkjet cartridge. To do this, you flip down the black plastic door in front of the printer, revealing the two inkjet cartridge printers.



You note that the inkjet cartridges are conveniently positioned on the right side, so that you can easily change one or the other. (See photo below). The color one has the pink plastic top, and the black one has the black top.

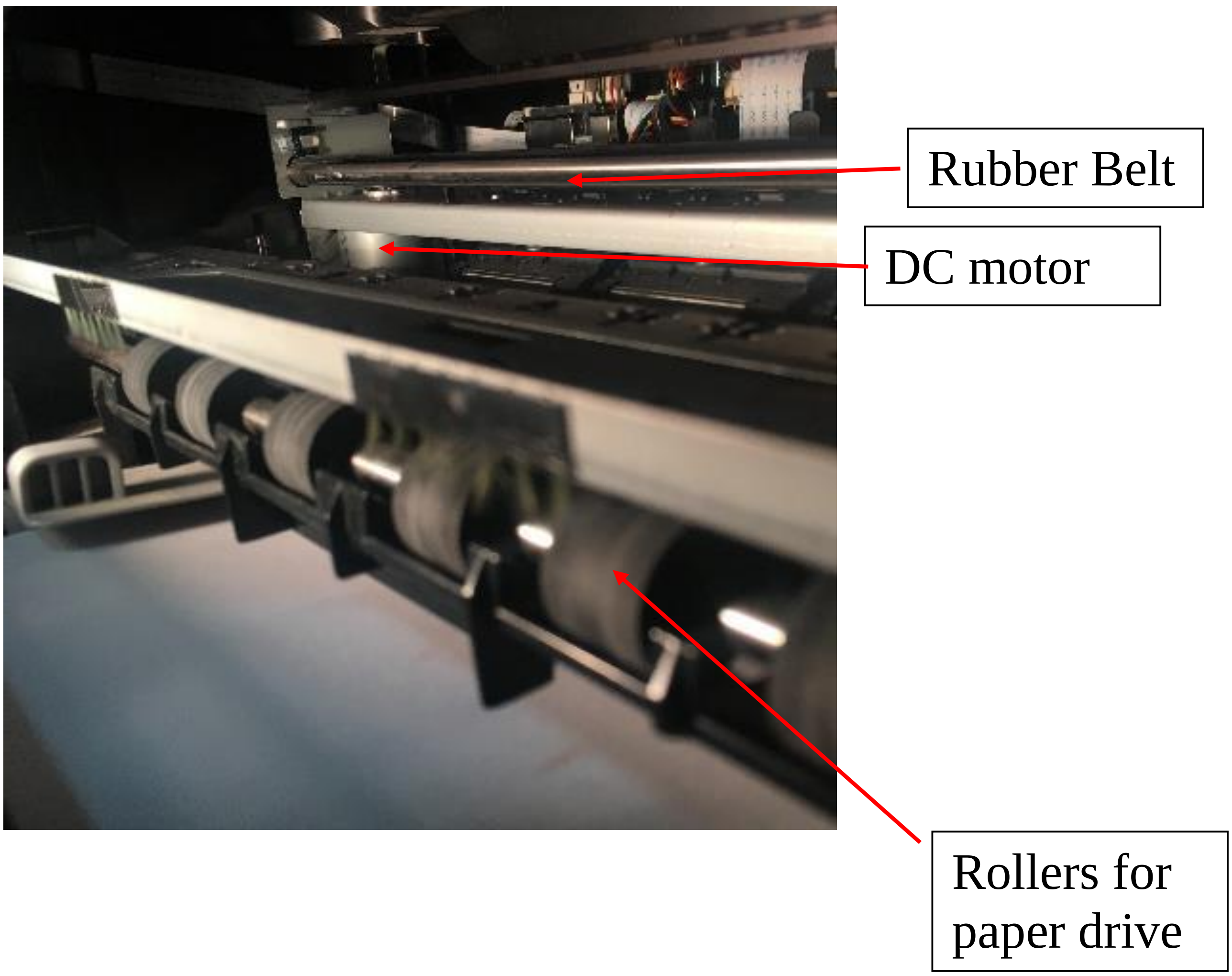


You wonder how the inkjet cartridges got there, until you realize that they must be able to move across the paper to print your documents. There must be a mechanism that moves the cartridges.

Further inspection reveals a thin rubber belt drive connected to a DC motor, as shown in the photo below. As the motor rotates, the belt pulls the cartridges across the printer.

The edges of the belt are tangent to the outside diameter of the motor. A planetary gear head reduces the speed of the motor and increases the torque, so that a pulley attached to the motor rotates the belt at the proper speed and with the proper tension. The planetary gear head is the same diameter as the motor.

Rotating the motor clockwise moves the cartridges to the left and rotating it counterclockwise moves the cartridges to the right. A pulley on the other side of the printer rotates on a bearing. It is the same diameter as the pulley on the left, so that the pulleys do not provide a gear ratio to provide the torque to move the belt. Only the planetary gear mechanism does this function.



One day, the motor breaks, and you can no longer print. You check your paperwork, and the printer is out of warranty. Rather than buy a new printer, you decide to replace the DC motor. With the aid of your class professor, you establish the following specs for the motor drive system.

Belt speed, v =	170 mm/sec
Diameter of motor, d =	25 mm
Planetary gear ratio, n =	56
Motor voltage, V =	3 volts
Torque output from the planetary gear system, T_1 =	500 mN-m

What is the output rotational speed of the planetary gear system?
$\boxed{\textbf{129.81 RPM}}$

What is the torque output of the motor? $\boxed{\textbf{8.92 mN-m}}$

What is the rotational speed of the motor? $\boxed{\textbf{7269.81 RPM}}$

Once you have these specs, go the web site www.nbleisonmotor.com and select a suitable motor for the printer. Make a screen shot of its part number and specs.

Sol.

1) To find the output rotational speed of the planetary gear system, we need to consider the fact that the edges of the belt are tangent to the outside diameter of the motor. Thus, the belt moves based on the value of circumference of the motor. Secondly, the speed of the planetary gear system would be the same as the reduced speed of the motor during operation of the printer based on which the belt moves. This means that the belt would move for a specific number of revolutions that the planetary gear system will make in a given amount of time.

   Thus, the speed of the planetary gear system = belt speed / circumference of the motor

   This would give us the revolutions of the planetary gear system in revolutions per minute.

   Calculation:

   Speed of the planetary gear system

   = (1700 mm / s) / (2 x PI x radius of the motor)
   = (1700 mm / s) / (PI x diameter of the motor)
   = (1700 mm / s) / (PI x 25 mm)
   = 1700 / (3.14 x 25) revolutions per second
   = 1700 x 60 / (3.14 x 25) revolutions per minute
   = **129.81 RPM**

2) The gear ratio is given as below:

Gear Ratio
= Speed of the motor / Speed of the planetary gear head
= 56

And the speed of a motor is inversely proportional to the torque.

Thus, let,

N1 = speed of the motor
N2 = speed of the planetary gear head
T1 = torque output from the motor
T2 = torque output from the planetary gear system

Thus, N1 x T1 = N2 x T2 (this is because N x T is constant for a motor)

Therefore,

T1 = (N2 / N1) x T2

And Gear ratio = N1 / N2 = 56

Therefore,

T1 = T2 / (N1 / N2)
= 500 / 56
= **8.92 mN-m**

Thus, the torque output of the motor will be 8.92 mN-m.

3) Lastly, to calculate the rotational speed of the motor, we can calculate that again using the gear ratio:

Gear ratio = N1 / N2 = 56

Thus, N1 = 56 x N2

Where,

N1 = speed of the motor
N2 = speed of the planetary gear head

Therefore,

N1 = 56 x 129.81 RPM
= **7269.81 RPM**

Thus, the motor actually moves at 7269.81 RPM and provides an output torque of 8.92 mN-m while the planetary gear head actually reduces the speed of the motor by providing its own torque and therefore the planetary gear head system runs at 129.81 RPM which is provided to the belt which further moves at 170 mm/s.

Motor selection:

Based on the above characteristic values of the motor used in the printer, we will require a motor that have rated values that are more or less close to the values obtained for the motor above. Therefore, to summarize, we will need a motor that has the following characteristic values:

Operating range	Around DC 3V
Nominal Voltage	DC 3V
Motor rated torque	8.92 mN-m
Motor rated speed	7269.81 RPM
Motor rated power	6.78 W (calculated based on the rated values using P = 2 x PI x N x T / 60)

Based on the above statistics, the following motor appears to be as close as possible to our requirements that would meet most requirements.

Specific Model Number: **RS-360SH-2885**

Micro Dc Motor

RS–360 RH/SH/SA

Application:
Vending Machine、 Massager/Vibrator、 Printer/Copy Machine、 Screw Driver。

Electrical Specification

Model	Voltage		No Load		At Maximum Efficiency					Stall		
	Operating Range	Nominal	Speed	Current	Speed	Current	Torque		Output	Torque		Current
			r/min	A	r/min	A	mN·m	g·cm	w	mN·m	g·cm	A
RS-360RH-2885	DC 3-8V	DC 6.0V	13700	0.38	11210	1.71	4.64	47.2	5.43	25.5	260	7.7
RS-360RH-10500	DC 12-25V	DC 12V	4500	0.5	3320	0.14	2.15	21.9	0.75	8.24	84	0.4
RS-360RH-14280	DC 8-30V	DC 12V	6300	0.085	5041	0.34	4.2	42.8	2.21	21	214	1.36
RS-360SH-2885	DC 3-9V	DC 7.2V	12500	0.36	10380	1.76	7	71.3	7.59	41.2	420	8.6
RS-360SH-3545	DC 6-12V	DC 9V	27500	0.75	22490	3.48	8.44	86.2	20.3	50.9	520	19
RS-360SA-27105	DC 2-8V	DC 3.6V	5000	0.2	4100	0.8	3.9	40	1.7	23	235	3.8
RS-360SA-10500	DC 12-24V	DC 24V	7000	0.06	5900	0.19	4.4	45	2.72	28	286	0.9

*Note:It's only typical technicai data for reference, Special requirement can be customized.

Details of the model specified in particular: **RS-360SH-2885**

Model	Voltage		No Load		At Maximum Efficiency					Stall		
	Operating Range	Nominal	Speed	Current	Speed	Current	Torque		Output	Torque		Current
			r/min	A	r/min	A	mN·m	g·cm	w	mN·m	g·cm	A
RS-360SH-2885	DC 3-9V	DC 7.2V	12500	0.36	10380	1.76	7	71.3	7.59	41.2	420	8.6

This motor is suitable for our use because of the following reasons:

- The output power of this motor at maximum efficiency is close to our expected value of 6.78 W.

- The operating range involves DC 3-9V while it can be run at DC 3V with reduced output power that matches 6.78 W.
- The speed is close to our requirement and is 10380 RPM at max efficiency. Running it at 3V and reduced output power will still give us a value that is close to our required speed value of 7269.81 RPM.
- The torque value is 7 mN-m which is slightly lower than our required value but the cumulative effect of the torque and speed results into the power that is calculated using the formula:
  P = 2 x PI x N x T / 60
  This formula will eventually give us a power value that has to match the required power value for our case which already matches.

To conclude, this DC Micro motor available on the specified website is closest to the one that our problem demands.

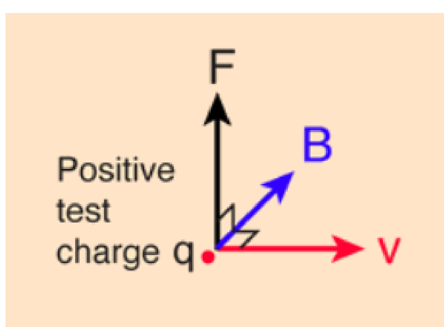
The link to this motor page can be accessed by clicking here.

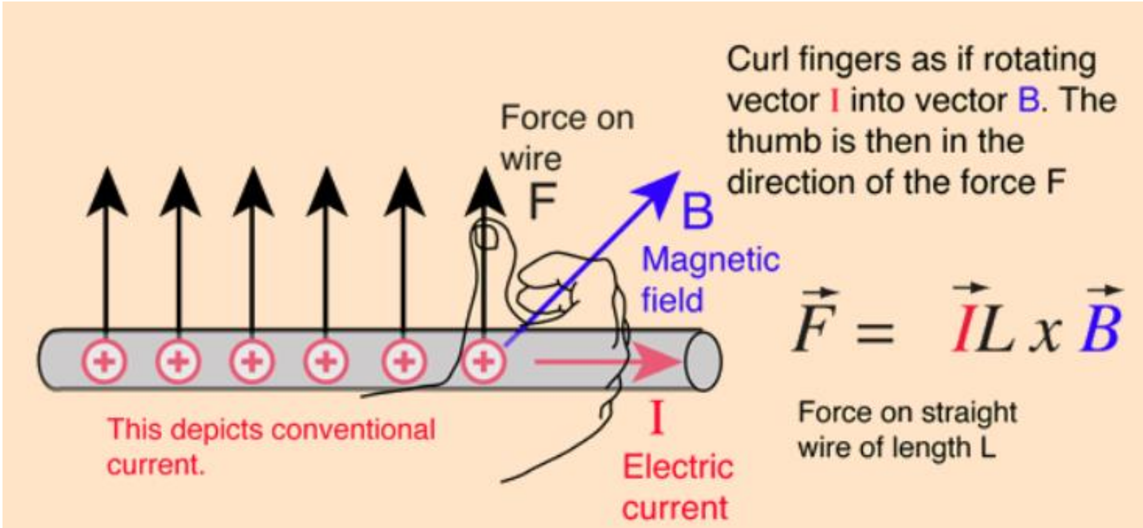
Courtesy: Reference Links: [1] [2] [3]

J. Suppose you create a Lorentz Force by passing a current through a conductor located in a magnetic field. What would happen to the Lorentz Force if you reversed the direction of both the magnetic field and the flow of current at exactly the same time? Why would this happen?

Sol.

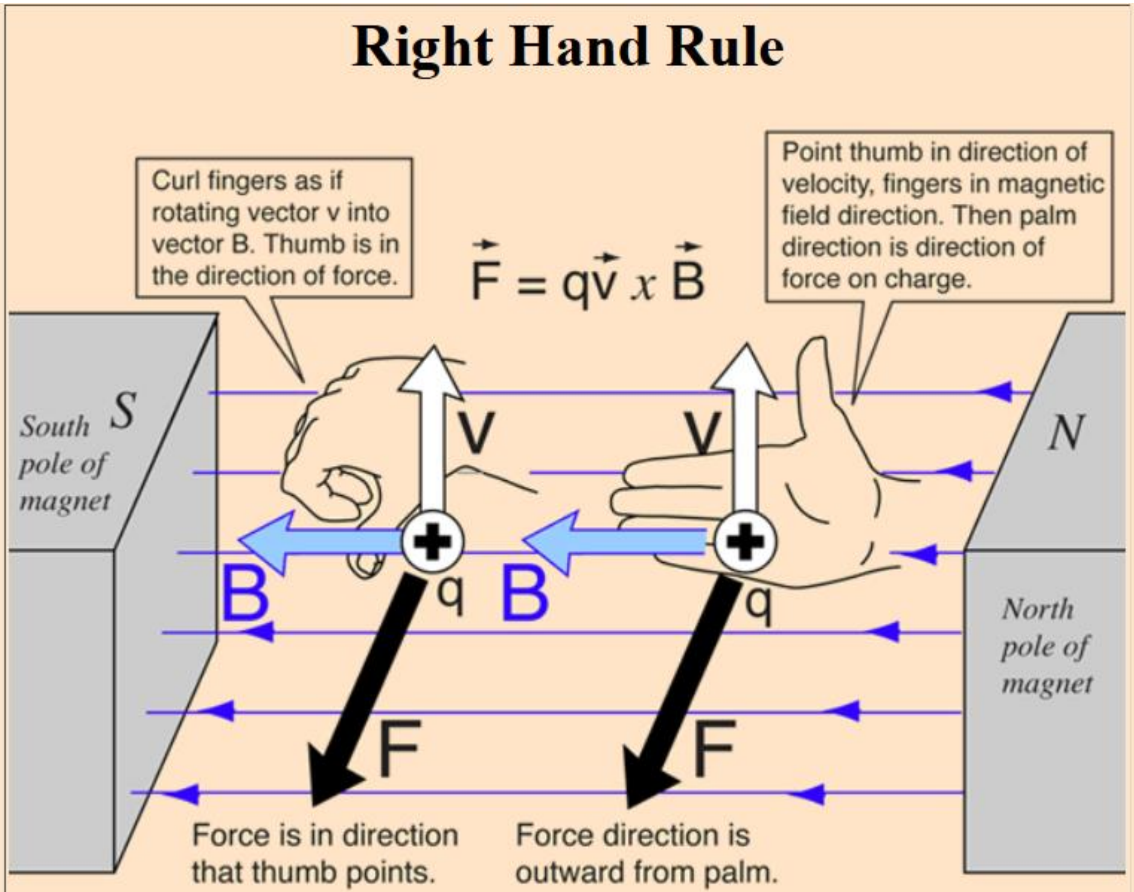
Based on the right-hand thumb rule, the Lorentz force acts in a direction that is perpendicular to both the direction of the magnetic field as well as the direction of the flow of the current. To put it simple, the direction of the Lorentz force will always be perpendicular to the plane containing the magnetic field and the velocity direction vectors. Thus, if initially the flow of the current and the magnetic field direction were as shown below in the diagram, then the Lorentz force would appear perpendicular to the plane containing B and v vectors (v is the velocity vector – representing the direction of the current flow) in the top-wards direction, again as shown below.





Now, when both the magnetic field direction (vector B) and the direction of the flow of the conventional current (vector v or I where v is velocity vector and I is current vector; both show the direction of the flow of the current and are therefore in the same direction) is reversed then the direction of the Lorentz force will still remain the same, i.e. perpendicular to the plain containing both B and I vectors and coming out towards the top-wards direction.

This is because of the fact that the Lorentz force equation involves the vector product of the current direction and the magnetic field direction which are both perpendicular to each other. To find the direction of the resultant Lorentz force, we would need to use the right-hand thumb rule that explains the Lorentz force in the referenced documents.



The right hand rule is a useful mnemonic for visualizing the direction of a magnetic force as given by the Lorentz force law. The diagrams above are two of the forms used to visualize the force on a moving positive charge. The force is in the opposite direction for a negative charge moving in the direction shown. One fact to keep in mind is that the magnetic force is perpendicular to both the magnetic field and the charge velocity, but that leaves two possibilities. The right hand rule just helps you pin down which of the two directions applies.

For applications to current-carrying wires, the conventional electric current direction can be substituted for the charge velocity v in the above digram.

As per the right-hand thumb rule, we would require our fingers to curl as if rotating vector I into vector B. The thumb then would give the direction of the force F. In this case, when the direction of I and B are reversed, the direction of the curl of the fingers would never change since the curl would always be in anti-clockwise direction for both the figure shown in this problem and the one in which case the B and I vectors are reversed. Thus, since the fingers always curl in the anti-clockwise direction from vector I towards vector B direction, the thumb would always point in the top-wards direction; perpendicular to the plane containing B and I vectors.

Thus, in both the case, the direction of the Lorentz Force will remain the same, i.e. top-wards, coming out of the plane containing B and I vectors.

Courtesy: Reference Links: [1] [2] [3]