

ECEN 5053-003 Homework Assignment

Course Name: Embedding Sensors and Actuators

Corresponding Module: C1M3

Week Number: 3

Module Name: Flow Sensors

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Homework is worth 100 points. Each question is worth 10 points.

- A. You want to measure the flow rate of air of density 1.225 kg/m^3 in a differential pressure meter. The upstream pressure is 20 and the downstream pressure is 15 N / m^2 . The meter has an area of 0.1 m^2 . What is the flow rate in m^3 / sec ?

Sol. **0.29 m^3/s**

Using the equation provided in the class slide, we can calculate the flow rate of the air of a given density using a differential pressure meter. The equation is as under:

$$P_U - P_D = \frac{1}{2} \rho V^2$$

$$V = \sqrt{2(P_U - P_D) / \rho}$$

$$Q = V \times A \text{ where}$$

- Q = flow rate
- A = cross-section area of pipe

Here, P_U is the upstream pressure and P_D is the downstream pressure measured in the differential pressure meter. V is the velocity of the fluid through the differential pressure meter and A is the cross-sectional area of the pipe. ρ is the density of the air. Q provides us the required flow rate by multiplying V and A .

Solving the equations we acquire the following results:

B6			\times	\checkmark	f_x	=ROUND(SUM((SQRT(2*(B2-B3)*(1/B4)))*B5),2)
	A	B	C	D	E	
1	Variables	Specs	Units			
2	P_U	20	N / m ²			
3	P_D	15	N / m ²			
4	ρ	1.225	Kg/m ³			
5	A	0.1	m ²			
6	Q	0.29	m ³ /s			
7						

- B. What advantage does a coriolis meter have over a thermal mass flow meter when measuring the mass flow rate of a gas?

Sol.

The thermal mass flow meters are more suitable to measure the gas flow. They have fast response times and they excel at measuring low flow rates. However, they are very small and fragile and consist of a heated, thin wire element. Hot-wire anemometers have a quick response time, because they are so small and thin. However, their fragility makes them unsuitable for industrial environments. Secondly, Thermal flowmeters use heat to facilitate flow measurements. Thermal flowmeters put heat into the flow stream and use one or more temperature sensors to measure how quickly this heat dissipates. This heat transfers reduces the safety of processing potential chemical substances that are inflammable or explosive and less resistant to heat.

However, other limitations of thermal flowmeters include the fact that they are used almost entirely for gas flow measurement. Thermal flowmeters have difficulty in measuring liquid flows because of the slow response time involved in using the thermal principle on liquids. A second limitation is in their accuracy. Thermal flowmeters are not nearly as accurate as Coriolis meters, and typical accuracy levels are in the 1 percent to 3 percent range.

On the contrary, probably the single biggest advantage of Coriolis flowmeters is their high accuracy. Accuracy levels for many Coriolis flowmeters are in the 0.1 percent range. This makes them ideal for custody transfer and other applications where high accuracy is required.

Users who want to measure mass flow rather than volumetric flow would naturally consider a Coriolis flowmeter. This could be the case when there are wide variations in the temperature and pressure of the fluid

being measured. Secondly, Coriolis meters have relatively higher explosion protection. Unlike the thermal mass flow meter, they don't use the heat to measure the flow. Plus, they have some other features that contradict the likelihood of using a thermal mass flow meter:

1. Coriolis meters use non-contact direct measurement of mass flow technique as opposed to the volume flow concept.
2. They can process very high mass flow rates that are normally found in high volume chemical, petrochemical and municipal water plants. They indeed process liquids.
3. They are suitable especially for flow measurement where production of volatile chemicals demanding high explosion proof ratings is critical.

Indeed, all this factors prove Coriolis meter to be a better contender when it comes to using the thermal mass flow meters in many ways. The image below provides some deeper views.

Meter	Highest Flow Rate for Gas (Liters per Hour)	Highest Flow Rate for Liquid (Liters per Hour)	Process Temperature Range	Typical Accuracy (% of Rate)	Line Pressure Limits
Thermal Mass	2000	Not for use with Liquids	5°C to 65°C	1% - 2%	100
Coriolis	15,000,000 (for air)	3,000,000 (for water)	-240°C to 350°C	0.2% - 0.5%	1000

Courtesy: I have used a reference and can be accessed by clicking [here](#).

Also, I have used the class slides for answering this problem.

- C. A You want to measure the flow rate of a gas at roughly 400°C, 15 Bar static pressure, and 150 m/s flow velocity. Which is the only type of flow meter, from the 7 types that we discussed in class, that you can use *and why*?

Sol. **Differential Pressure Flow Meter**

Based on the class discussions and the material delivered thereafter, the differential pressure (a.k.a DP meters) fits best for the application mentioned in this problem. This is so due to multiple reasons. To support my stand, I have included a comparison table from the class slide to narrow down on the DP meter as the only type of flow meter that can be used here.

Meter	Highest Flow Rate for Gas (Liters per Hour)	Highest Flow Rate for Liquid (Liters per Hour)	Process Temperature Range	Typical Accuracy (% of Rate)	Line Pressure Limits
Variable Area	310,000	120,000	-20°C to 100°C	2% - 5%	None
Differential Pressure	Limits are based on diff. pressures	Limits are based on diff. pressures	-200°C to 850°C	1% - 1.5%	250 Bar (Static)
Vortex	80 m/s flow velocity	10 m/s flow velocity	-200°C to 450°C	0.5% - 1%	25 Bar
Ultrasonic	Not for use with Gas	20 m/s flow velocity	-250°C to 1000°C	0.5%	None
Turbine	Not for use with Gas	40	-250°C to 425°C	0.5%	Limited by Steel Flange Rating
Thermal Mass	2000	Not for use with Liquids	5°C to 65°C	1% - 2%	100
Coriolis	15,000,000 (for air)	3,000,000 (for water)	-240°C to 350°C	0.2% - 0.5%	1000

As can be examined from this comparison table for all the seven flow meters discussed in the class, the DP meter suits best because of the following reasons:

1. The process temperature range of the DP meter matches that of the one mentioned in this problem. Its max processing temperature is 850°C which is way far from the gas temperature of 400°C in this problem. So, the DP meter meets the temperature range requirement.
2. The line pressure limit for the DP meter is 250 Bar (Static) which is again way more than the required value of 15 Bar (Static) which means that the measurement system is safe enough for use for this particular problem constraints.
3. The DP meter is the only flow meter with the highest flow rate for gas meeting our demands of 150 m/s requirement based on the different pressures. The equations to find the flow rate for a DP meter are as below:

$$q_m = \frac{C}{\sqrt{1-\beta^4}} \cdot \varepsilon \cdot \frac{\pi}{4} d^2 \sqrt{2 \cdot \Delta p \cdot \rho} \quad (2.6)$$

$$q_v = \frac{C}{\sqrt{1-\beta^4}} \cdot \varepsilon \cdot \frac{\pi}{4} d^2 \sqrt{\frac{2 \cdot \Delta p}{\rho}} \quad (2.7)$$

C Flow coefficient

β Diameter ratio

ε Expansion factor (for compressible media, only)

d Inside diameter of the orifice plate

Δp Differential pressure

ρ Density of the meas. medium before the orifice at operating temperature

q_m Mass flow rate

q_v Volume flow rate

But again, the line pressure limit is way higher than is demanded (250 Bars as opposed to 15 Bars requirement) and therefore this is the only meter that suits the demands.

4. Thinking conversely, the ultrasonic and turbine meters cannot be used for gas flow measurement and therefore are eliminated. The Thermal Mass and Coriolis meters don't offer a process temperature range that includes 400°C and therefore are eliminated. Although, the Vortex meter does offer a desired process temperature range, it does not though offer the required flow velocity of 150 m/s since it can only allow up to 80 m/s of max flow rate. Therefore, it is eliminated from the consideration list. Again, the variable area flow meter does not leave us anything desirable since its process temperature range is way below the 400°C requirement. To conclude, Differential Pressure Flow Meter (DP meter) is the only solution to this problem.

- D. What are *three* differences in the method and principal of operation between a variable area meter and a differential flow pressure meter? (note: this question is not asking about the specifications of the meters)

Sol.

The three basic differences in the method and principle of operation between a variable area meter and a differential flow pressure meter are as follows:

Differential pressure meter

Differential pressure flow meters operate by introducing a restriction in the cross sectional area of a flowing

Variable-area meter

Variable-area flowmeters operate on the principle that the variation in area of flow stream required to produce a

fluid. Restricting the flow area causes a pressure drop across the constriction, this pressure drop is caused by a change in the fluids velocity. This restriction is basically a geometrical shape that is characterised over the years and have fixed shapes unlike the Variable-area meter which has the float displacing based on the flow amount.	constant pressure differential is proportional to the flow rate. They are simple and versatile devices that operate at a relatively constant pressure drop and measure the flow of liquids, gases, and steam. Here, unlike DP meter, the float moves as per the flow and therefore generates a relatively constant pressure drop unlike the DP meter.
The calculation of fluid flow rate is done by reading the pressure loss across a pipe restriction.	The position of the float, piston or vane is changed as the increasing flow rate opens a larger flow area to pass the flowing fluid. The position of the float, piston or vane provides a direct visual indication of flow rate. This shows the fluid flow rate unlike DP meters which depend on the difference of pressures.
The pressure drops generated by a wide variety of geometrical restrictions is the primary concept for indicating the flow.	Either the force of gravity or a spring is used to return the flow element to its resting position when the flow lessens. Gravity-operated meters must be installed in a vertical position, whereas spring operated ones can be mounted in any position. This generates a relative constant pressure throughout the VA meter unlike the differences in pressure in the DP meter.

Courtesy: I have used certain references for answering this problem which can be accessed by clicking on the links: [\[1\]](#) [\[2\]](#) [\[3\]](#) [\[4\]](#)

- E. The k-factor for a turbine meter for flow in a 1.5" (38.1 mm) diameter pipe is 110 pulses per liter. Your magnetic pickup assembly recorded a frequency of 150 Hz. What is the flow rate Q in liters per minute?

Sol. **81.82 liters/min**




To solve this problem, we can simply calculate the flow rate Q (in liters per minute) by using the below given formula:

The following equation applies:

$$Q = \frac{f \times 60}{K}$$

Q = flow rate in ltr. per min
 F = frequency in Hz
 K = K-factor in pulses per litre

To solve the same, please refer the Excel file calculation screenshot:

B4	:	  	=ROUND(SUM((B2*60)/B3),2)
	A	B	C
1	Variables	Specs	Units
2	<i>f</i>	150	Hz
3	<i>K</i>	110	pulses/liter
4	<i>Q</i>	81.82	liters/min

Courtesy: I have used a reference and can be accessed by clicking [here](#).
 To learn more about turbine meters, I have referred a different document which can be accessed by clicking [here](#).

F. A coriolis flow meter has a liquid flow accuracy of 0.25% of flow rate and a zero stability of 3.5 kg / hr. What is the maximum error in kg /hr in the flow rate reading when the meter measures a flow rate of 2500 kg / hr?

Sol. **6.23 kg/hour**

To solve this problem, we can simply calculate the maximum error in kg/hour in the flow rate reading by using the following calculation:

- Accuracy: +/- 0.25% of flow rate +/- zero stability (3.5 / actual flow reading in our case)

Further, simplifying this equation, the accuracy becomes,

- Accuracy: 0.25% of flow rate + zero stability (3.5 / actual flow reading in our case)



Thus, the actual flow reading (X) when the flow meter measures a flow rate of 2500 kg/hour is as under:

- $X + [(0.25/100)*X] + (3.5/X) = 2500$
- So, $X + [(0.0025*X) + (3.5/X)] = 2500$

- Thus, solving this equation using the quadratic equation of style: $ax^2 + bx + c = 0$, we get:
- $1.0025X^2 - 2500X + 3.5 = 0 \dots [1]$

Comparing equation [1] with the quadratic equation $ax^2 + bx + c = 0$, $a = 1.0025$, $b = -2500$, $c = 3.5$ and $x = X$ in our case.

Based on the actual reading (X), we can calculate the maximum error as follows:

D13	:			f_x		
	A	B	C	D	E	F
1	a	1.0025	$aX^2 + bX + c = 0$			
2	b	-2500				
3	c	3.5				
4	Δ	2500				
5	X+	2493.77	$X = [(-b + \Delta) / (2*a)]$			
6	X-	0				

Thus, $X = 2493.77 \text{ kg/hour}$

Note: We are not using the other root of X since the error in the equation cannot be zero given that there is some finite accuracy and a finite value of zero stability in measuring the flow rate.

This is the actual flow rate value and the measured flow rate is 2500 kg/hour and therefore to find the error, we use the below given equation:

$$X_M - X = 2500 - 2493.77 = \boxed{6.23 \text{ kg/hour}}$$

Note: Here, X_M is the measured value of the flow rate which includes the maximum error and X is the actual flow rate that we obtained by solving the quadratic equation in excel.

Hence, the required value of maximum error in the reading is 6.23 kg/hour.

Courtesy: I have used a reference and can be accessed by clicking [here](#).

G. What are *three* differences in the pattern of fluid flow between laminar flow and turbulent flow?

Sol.

The three basic differences in the pattern of fluid flow between laminar flow and turbulent flow is as follows:

Laminar Flow

At low velocities and high viscosities, the fluids flow in layers. This means that the fluid particles move in well-ordered adjacent sliding layers. This is known as Laminar flow in which the layers do not mix with each other.

Laminar flow has a constant velocity at any point within the fluid similar to a constant flow of traffic. In short, fluid here flows along one smooth path. The particles here do not interfere with one another and they don't mix or shift between layers.

The value of Reynold's number here is less than 2000. This means that the Shear stress in laminar flow depends only on the viscosity of the fluid and independent of the density.

Turbulent Flow

If the fluid flow velocity increases or the viscosity decreases, an additional motion is superimposed on the axially oriented movement throughout the flow stream which moves in all directions in a random manner and affects the flow streamlines in such a way that a uniform velocity profile results. This is known as Turbulent flow.

Turbulent flow is chaotic, forms eddies and whirlpools and is similar to the flow of a whitewater rapid. To conclude, fluid flows here is irregular and can move back and forth between layers, mixing and falling into whirlpool like patterns.

The value of Reynold's number here is greater than 4000. This means that Shear stress in the turbulent flow depends upon the density of the fluid.

Courtesy: I have used certain references for answering this problem which can be accessed by clicking on the links: [\[1\]](#) [\[2\]](#) [\[3\]](#) [\[4\]](#)

H. How is a vortex created in the middle of pipe when using a vortex meter?
How does the vortex meter measure the flow rate?

Sol.

A Vortex Flowmeter is a device used to measure the flow of fluids in the process temperature range of -200°C to 450°C with a highest flow rate of 80 m/s for gases and 10 m/s for liquids. Given this, they are designed with no moving parts and therefore operate under the Vortex Shredding Principle.

According to the von Kármán effect, a non-streamlined object (also called a bluff body) placed in the path of a fast-flowing stream, causes the fluid to alternately separate from the object on its two downstream sides, and, as the boundary layer becomes detached and curls back on itself, forming vortices. On the side of the bluff body where the vortex is being formed, the fluid velocity is higher and the pressure is lower. As the vortex moves

downstream, it grows in strength and size, and eventually detaches or sheds itself. This is followed by a vortex's being formed on the other side of the bluff body.

Thus, a Vortex is created in a Vortex Flowmeter when a fluid such as water flows past a bluff (as opposed to streamlined) body. The frequency of these vortices depend on the size and the shape of the obstacle bluff body. In a Vortex Flowmeter, an obstruction, often referred to as a shedder bar serves as the bluff body. The shedder bar causes the process fluid to separate and form areas of alternating differential pressure known as vortices around the back side of the shedder bar.

According to the von Kármán effect, the fluid flow will alternately generate vortices when passing by a bluff body. This bluff body has a broad, flat front. In a vortex meter, the bluff body is a piece of material with a broad, flat front that extends vertically into the flow stream. Flow velocity is proportional to the frequency of the vortices. Or it can be said that the frequency of the vortices shedding is proportional to the fluid velocity. Thus, flow rate is calculated by multiplying the area of the pipe times the velocity of the flow. Through the use of an internal RTD and an external pressure sensor (optional), the flow meter software will compensate for changes in pressure and temperature, to achieve an accurate mass flow measurement (gas meters).

Courtesy: I have used certain legitimate references for answering this problem which can be accessed by clicking on the links: [\[1\]](#) [\[2\]](#) [\[3\]](#) [\[4\]](#) [\[5\]](#)

- I. A variable area flowmeter is used to measure the flow of ethylene glycol. The float is a spherical piece of glass. The flowmeter has the following dimensions and properties:

diameter of glass float: $d =$	6.35	millimeters	
Density of glass float: $\rho_s =$	2.7	g/cm^3	
Density of ethylene glycol: $\rho_m =$	1.11	g/cm^3	
Flow velocity of fluid: $v =$	0.9	meter / sec	
Resistance coefficient: $c_w =$	0.20		

- I-1) What is the force due to gravity on the float?
I-2) What is the buoyancy force on the float?
I-3) What is the flow resistance force on the float?
I-4) Which direction does the float move, up or down, and why?
I-5) What new value of the resistance coefficient must be established at the flow velocity for the float to reach an equilibrium position?

Sol.

I-1) For finding the force due to gravity on the float, we first calculate the volume of the float which can be calculated using the formula for the volume as $V = ((4/3) \cdot (3.14) \cdot ((d/2)^3))$. Furthermore, the force due to gravity can be calculated using the multiplication: $\rho_s \cdot V \cdot g$ where g is the gravitational acceleration which amounts to 9.8 m/s^2 .

B5 : =SUM(B3*((4/3)*(3.14)*((B2/2)^3))*B4)				
	A	B	C	D
1	Variables	Specs	Units	
2	d	0.00635	m	
3	ρ_s	2700	kg/m ³	
4	g	9.8	m/s ²	
5	F_g	0.003545597	N	

I-2) For finding the buoyancy force, in general terms, this buoyancy force can be calculated with the equation $F_b = V_s \times D \times g$, where F_b is the buoyancy force that is acting on the object, V_s is the submerged volume of the object, D is the density of the fluid the object is submerged in, and g is the force of gravity.

Therefore, in our case, $V_s = V$ (from the previous answer) = $((4/3) \cdot (3.14) \cdot ((d/2)^3))$ which is equal to $1.33998\text{E-}07$. Secondly, $D = \rho_m = 1110 \text{ kg/m}^3$. Lastly, g is again the value of 9.8 m/s^2 .

The calculation is as under:

B6	:	f_x	=SUM(B3*B4*B5)
	A	B	C
1	Variables	Specs	Units
2	d	0.00635	m
3	V_s	1.33998E-07	m ³
4	$D = \rho_m$	1110	kg/m ³
5	g	9.8	m/s ²
6	F_b	0.001457634	N

I-3) For finding the flow resistance force on the float, we solve the following equation:

$$F_s = c_w \cdot A_s \cdot \frac{\rho_m \cdot V^2}{2}$$

Here, F_s is the flow resistance force. A_s is the cross-sectional area of the float at the reading edge and V is the flow velocity. ρ_m is the density of ethylene glycol in our case.

So, the calculation is as under:

B5	:	f_x	=SUM(2*(3.14)*((B4/2)^2))
	A	B	C
1	Variables	Specs	Units
2	V	0.9	m/s
3	$D = \rho_m$	1110	kg/m ³
4	d	0.00635	m
5	A_s	6.33063E-05	m ²
6	C_w	0.2	
7	F_s	0.005691872	N

B7	:	\times	\checkmark	f_x	=SUM(B6*B5*((B3*(B2^2))/2))
	A	B	C		
1	Variables	Specs	Units		
2	V	0.9	m/s		
3	$D = \rho_m$	1110	kg/m ³		
4	d	0.00635	m		
5	A_s	6.33063E-05	m ²		
6	C_w	0.2			
7	F_s	0.005691872	N		
8					

I-4) To decide which direction the float will move, we have to consider the equilibrium equation wherein the float is stable and does not move as all the forces on the float equate to zero. To focus on the equilibrium state, we consider the below given equation:

At equilibrium or a the float position:

$$F_G = F_A + F_s \quad (2.11)$$

Here, F_G is the downward gravitational force, F_A is the buoyancy force and F_s in the flow resistance force. In our case, F_G is 0.003545597 N from answer I-1, F_A is 0.001457634 N from answer I-2 and F_s is 0.005691872 N from answer I-3.

Therefore, $F_A + F_s = 0.001457634 + 0.005691872 = \mathbf{0.007149506 \text{ N}}$... [1]

Similarly, $F_G = \mathbf{0.003545597 \text{ N}}$... [2]

Here, equation [1] is greater than [2] and therefore, the upwards force is more than the gravitational force and therefore the float will move **UPWARDS**.

I-5) To establish equilibrium, F_s should equal $F_G - F_A$ and so, first, we calculate the value of $F_G - F_A$.

$$F_G - F_A = 0.003545597 - 0.001457634 = \mathbf{0.002087963 \text{ N}}$$
 ... [3]

To, find the new value of C_w , we change the value of F_s equal to that of equation [3].

So, using the equation shown below, we put the new value of F_s and all other values remain same and we find the value of C_w :

$$F_s = C_w \cdot A_s \cdot \frac{\rho_m \cdot V^2}{2}$$

B7	:		=ROUND(SUM((2*B6)/(B5*B3*(B2^2))),2)		
	A	B	C	D	
1	Variables	Specs	Units		
2	V	0.9	m/s		
3	D = ρ_m	1110	kg/m ³		
4	d	0.00635	m		
5	A_s	6.33063E-05	m ²		
6	F_s	0.002087963	N		
7	C_w	0.07			

Courtesy: I have used two references and can be accessed by clicking the links [\[1\]](#) and [\[2\]](#).

- J. What are the major advantages of the coriolis flow meter relative to the other 6 flow meters we studied in this chapter? How does this advantage show up in the types of fluids it is used to measure? What is one disadvantage of the coriolis meter?

Sol.

Coriolis flow meters have special advantages relative to the other 6 flow meters discussed in the class. Some brief insights into the major advantages of the Coriolis flow meter are as follows:

1. Coriolis meters have an easier time measuring liquids than gases, because liquids are denser than gases.
2. A number of suppliers have begun producing Coriolis flowmeters in line sizes above six inches. These include Endress+Hauser (us.endress.com), which in May 2011 introduced a 14-inch Coriolis meter for gas and liquids; Micro Motion (micromotion.com); and KROHNE (krohne.com).
3. Coriolis flowmeters measure mass flow directly, meaning the mass flow value is not computed using a formula.
4. Not only are Coriolis flowmeters the most accurate meter, they also require very little maintenance.

5. With no internal moving parts, Coriolis meters require a minimum amount of attention once they are installed.
6. With no moving parts to wear out or degrade, they are a very reliable meter.

Apart from the above mentioned advantages, Coriolis flow meters do have some advantages and disadvantages and they are as below:

Advantages and Disadvantages of Coriolis Flowmeters for Gas Flow	
Advantages	Disadvantages
✓ High accuracy	✗ High initial cost
✓ Approved for custody transfer	✗ Become expensive and unwieldy in line sizes above four inches
✓ Have now been developed for large line sizes up to 14 inches	✗ Gas flow measurement can be difficult due to low density of gas
✓ Can handle sanitary applications	✗ Pressure drop for bent-tube meters
✓ Can clamp on to a pipe with no penetration	✗ Build-up on the inside pipe walls can reduce inside diameter of pipe and affect measurement accuracy
✓ Excel in line sizes of two inches and less	
✓ High reliability	
✓ Much new product development ongoing	

Apart from the above mentioned data, Coriolis flow meters pose the below given advantages when it comes to the types of fluids it is used to measure:

Since this principle is independent of the properties of the measuring medium, such as conductivity, flow profile, density, viscosity, etc., almost all materials can be measured: e.g. oils and fuels, cleaning agents and so solvents, fats, silicone oil, alcohol, methane, fruit mixtures, starch, dyes, biozide, vinegar, catsup, mayonnaise, beer, milk, sugar solution, gases, liquefied gases, etc.

Coming down to one major limitation that Coriolis flow meters have is with their line size and cost. Over 90 percent of Coriolis flowmeters are used on line sizes of two inches and less. Coriolis meters become very large and unwieldy, especially in sizes from four to six inches. Cost also increases with size. Even smaller size meters are generally more expensive than other comparable new-technology flowmeters.

Courtesy: I have used certain legitimate references for answering this problem which can be accessed by clicking on the links: [\[1\]](#) [\[2\]](#) [\[3\]](#) [\[4\]](#) [\[5\]](#).