

QUESTION 1:

$$\sigma_{EWMA}^2(t) = (1-\lambda) \sum_{k=1}^{\infty} \lambda^{k-1} r_{t-k}^2 \quad \leftarrow \text{EWMA}$$

$$\Rightarrow \sigma_{EWMA}^2(t-1) = (1-\lambda) \sum_{k=1}^m \lambda^{k-1} r_{t-1-k}^2$$

Expanding:

$$\sigma_{EWMA}^2(t) = (1-\lambda) \lambda^{1-1} r_{t-1}^2 + (1-\lambda) \sum_{k=2}^m \lambda^{k-1} r_{t-k}^2 \quad (*)$$

Then:

$$\lambda \sigma_{EWMA}^2(t-1) = (1-\lambda) \sum_{k=1}^m \lambda \cdot \lambda^{k-1} r_{t-1-k}^2$$

$$= (1-\lambda) \sum_{k=1}^m \lambda^k r_{t-1-k}^2$$

Let $j=k+1$, then:

$$\lambda \sigma_{EWMA}^2(t-1) = (1-\lambda) \sum_{j=2}^m \lambda^{j-1} r_{t-1-k}^2$$

Substituting into (*):

$$\boxed{\sigma_{EWMA}^2(t) = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{EWMA}^2(t-1)}$$