Consider a weak white noise $W_t \sim (0, \sigma^2)$

An autoregressive model of order p, an AR(p) is defined as

$$X_t = W_t + \phi_1 X_{t-1} + \phi X_{t-2} + \dots + \phi_p X_{t-p}$$

Now consider the series $U_i = U(i\tau)$ where $U(t)$ is given by the Ornstein-Uhlenbeck process, hence

$$U_{i+1} = \sigma \int_0^{(i+1)\tau} e^{-\gamma((i+1)\tau - s)} dB(s)$$

Rearranging the terms

$$U_{i+1} = \sigma e^{-\gamma i\tau} e^{-\gamma\tau} \left(\int_0^{(i+1)\tau} e^{\gamma s} dB(s) \right)$$

$$= \sigma e^{-\gamma i\tau} e^{-\gamma\tau} \left(\int_0^{i\tau} e^{\gamma s} dB(s) + \int_{i\tau}^{(i+1)\tau} e^{\gamma s} dB(s) \right)$$

$$= \sigma e^{-\gamma i\tau} e^{-\gamma\tau} \int_0^{i\tau} e^{\gamma s} dB(s) + \sigma e^{-\gamma i\tau} e^{-\gamma\tau} \int_{i\tau}^{(i+1)\tau} e^{\gamma s} dB(s)$$

$$= e^{-\gamma\tau} \sigma e^{-\gamma i\tau} \int_0^{i\tau} e^{\gamma s} dB(s) + \sigma e^{-\gamma(i+1)\tau} \int_{i\tau}^{(i+1)\tau} e^{\gamma s} dB(s) \quad \triangle$$

Note that

$$\sigma e^{-\gamma(i+1)\tau} \int_{i\tau}^{(i+1)\tau} e^{\gamma s} dB(s) \sim N\left(0, \sigma^2 e^{-2\gamma(i+1)\tau} \int_{i\tau}^{(i+1)\tau} e^{2\gamma s} ds\right)$$

Computing the integral.

$$\sigma^2 e^{-2r\tau(i+1)} \int_{i\tau}^{(i+1)\tau} e^{2rs}\,ds = \sigma^2 e^{-2r\tau(i+1)} \frac{1}{2r} e^{2rs} \Big|_{i\tau}^{(i+1)\tau}$$

$$= \frac{\sigma^2 e^{-2r\tau(i+1)}}{2r} \left(e^{2r(i+1)\tau} - e^{2r\tau i} \right) = \frac{\sigma^2}{2r} \left(1 - e^{-2\tau r}\right)$$

Notice that the first term of Δ corresponds to $e^{-r\tau} u_i$.

Hence we write Δ as

$$u_{i+1} = e^{-r\tau} u_i + p W_t \quad \text{where } p = \sqrt{\frac{\sigma^2}{2r}\left(1 - e^{-2\tau r}\right)}$$

and $W_t \sim N(0,1)$

Hence the process $u_i = u(i\tau)$ obeys an AR(1) process