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FLAGS, SCHUBERT POLYNOMIALS, DEGENERACY LOCI, AND DETERMINANTAL FORMULAS

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1. Introduction. The principal goal of this paper is a formula for degeneracy loci of a map of flagged vector bundles. If $h: E \to F$ is a map of vector bundles on a variety X,

$$E_1 \subset E_2 \subset \cdots \subset E_s = E, \qquad F = F_t \twoheadrightarrow F_{t-1} \twoheadrightarrow \cdots \twoheadrightarrow F_1 \tag{1.1}$$

are flags of subbundles and quotient bundles, and integers $r(q, p)$ are specified for each $1 \leqslant p \leqslant s$ and $1 \leqslant q \leqslant t$, then there is a degeneracy locus

$$\Omega_{\mathbf{r}}(h) = \{x \in X: \operatorname{rank}(E_p(x) \to F_q(x)) \leqslant r(q, p)\ \forall p, q\}. \tag{1.2}$$

Under appropriate conditions on the rank function $\mathbf{r}$, which guarantee that, for generic h, $\Omega_{\mathbf{r}}(h)$ is irreducible, we prove a formula for the class $[\Omega_{\mathbf{r}}(h)]$ of this locus in the Chow or cohomology ring of X, as a polynomial in the Chern classes of the vector bundles. When expressed in terms of Chern roots, these polynomials are the "double Schubert polynomials" introduced and studied by Lascoux and Schützenberger.

The simplest such rank conditions are when $s = t$ (but with repeats allowed in the chains of sub and quotient bundles), and one restricts the ranks of maps $E_p \to F_{s+1-p}$ for $1 \leqslant p \leqslant s$. In this case the polynomials have simple determinantal

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