M2 Exercises for Minnesota, June 4, 2023

David Eisenbud

May 31, 2023

Notation: k is a field (such as $\mathbb{Z}/101$ or $\mathbb{Z}/32003$ or even Q, though that will make computing slower); $S = k[a,b,c]$; $I \subset S$ a homogeneous ideal; $R = S/I$; $m = (a,b,c)$ (maximal ideal, in S or in R — note that M2 will care which even if you don't.

1. Commands to try out first: `, viewHelp, about, installPackage ‘‘MonomialOrbits"` (those last two words are one command)

2. Write a function to compute the "Burch ideal" $mI : (I : m)$ and the dimension of the vector space $m/(mI : (I : m)$, which is called the "burch index". Try them with $n = 3$, $I = a^7 * (a,b,c)$

   M2:

   `->, :,  ideal, ring, vars, degree`

3. With $n = 3$, Compute orbit representatives of the monomial ideals including a^4, b^4, c^5 of the variables and 3 other monomials of degree 4. How long does the computation take? How many results did you get?

   Form the sublist of examples with each possible Burch index. How many are in each list? For each ideal of Burch index > 0 compute the betti table. Display the results using netList. Do you notice any difference between the two lists?

   M2: `MonomialOrbits, orbitRepresentatives, #, elapsedTime, select, minimalBetti, tally, netList`

4. Write a function that computes the ideal of entries of the matrix that represents the i-th differential in the minimal R-free resolution of an R-module, and the sum of these ideals for two consecutive differentials. Check whether these sums stabilize, and when, on some rings $R = S/I$ defined by the ideals I in a list you made for item 2, with the module being the ideal a random element of degree 2 in R. random

   M2:

   `module, res, LengthLimit,  F.dd_i , ideal, $==$`