LieAlgebraRepresentations: Motivation

Let $\rho : \mathfrak{g} \to \mathfrak{gl}(V)$ be a Lie algebra representation (complex, semisimple, finite dimensional).

Up to isomorphism, V is classified by its highest weights (or equivalently, its character). The `LieTypes` package implements this.

We can learn a lot from character calculations, but for some purposes we need to describe ρ explicitly.

Example: Invariants of plane cubics (SL_3 acting on $\mathrm{Sym}^3 \mathbb{C}^3$)

Find an invariant degree 4 polynomial for plane cubics

$\Longleftrightarrow$

Describe the trivial submodule in $\mathrm{Sym}^4 \mathrm{Sym}^3 \mathbb{C}^3$

$\Longleftrightarrow$

Describe weight 0 vectors that are killed by every raising operator

$\Longleftrightarrow$

Compute the kernel of the Casimir operator

LieAlgebraRepresentations: Goals

We implement Lie algebra representations $\rho : \mathfrak{g} \to \mathfrak{gl}(V)$ in `LieAlgebraRepresentations`.

Inputs: a special basis of $\mathfrak{g}$ (a Chevalley basis) and the images of this basis under ρ

So far: implemented in type A using the Gelfand-Tsetlin basis

Goals for the week:

- Implement Chevalley bases for other types
- Matrix generators for other types: [de Graaf 2001]
- Matrix generators for other bases of V
  - Young tableaux
  - Gelfand-Tsetlin
  - Crystal bases
  - String bases