Formulation 1.0

$$i \in [1, N] \quad : \quad \text{Charging station locations}$$
$$j \in [1, M] \quad : \quad \text{Demand hotspots}$$

Parameters:

D_j : Demand (value) at demand hotspot j
$S_{i,j}$: Distance (by road) between station location i and demand hotspot j
G : Number of stations required to be built in the network ($\leqslant N$)

Other variables used:

L_j : Penalty/Loss incurred by vehicles at demand location j to travel to the **assigned** station for charging

Decision Variables

$$\alpha_{i,j} = \begin{cases} 1, & \text{if demand hotspot } j \text{ is assigned to station } i \\ 0, & \text{otherwise} \end{cases}$$

$$x_i \; = \; \begin{cases} 0, & \text{if station should be built at location } i \\ 1 & \text{otherwise} \end{cases}$$

Objective Function

$$\text{minimize} \; : \; \sum_{j=1}^{M} D_j \times L_j$$

Constraints

Assignment variable bounds:

$$0 \leqslant \alpha_{i,j} \leqslant x_i \quad \forall i \in 1, 2, \ldots, N, j \in 1, 2, \ldots., M$$

Exactly one assignment:

$$\sum_{i=1}^{N} \alpha_{i,j} = 1 \quad \forall j \in 1, 2, \ldots., M$$

Distance penalty definition:

$$L_j = \sum_{i=1}^{N} S_{i,j} \times \alpha_{i,j} \quad \forall j \in 1, 2, \ldots., M$$

Number of stations:

$$\sum_{i=1}^{N} x_i = G$$

Formulation 2.0

1. The solver now also tells us, at each location, the **capacity** (among available options) of the charging station to build.

   T : The set of all k's, where $k_r = c_r / d_{\text{avg}}$, with c_r being one of the possible station capacities, and d_{avg} being the average of all demands.

   The decision variable x is then modified as:

$$x_i = \begin{cases} 0, & \text{if no station should be built at } i \\ k \in T, & \text{if a station at } i \text{ should have capacity equal to } k \times d_{\text{avg}} \end{cases}$$

   The new **capacity constraint** is as follows:

$$\sum_{j=1}^{M} \left(\frac{D_j}{d_{\text{avg}}} \right) \times \alpha_{i,j} \leqslant x_i \quad \forall i \in 1, 2, \ldots, N$$

2. Instead of taking the number of stations to build, we take the total **budget** as an input. The G from earlier is replaced by:

   B : The maximum budget available for building the charging network

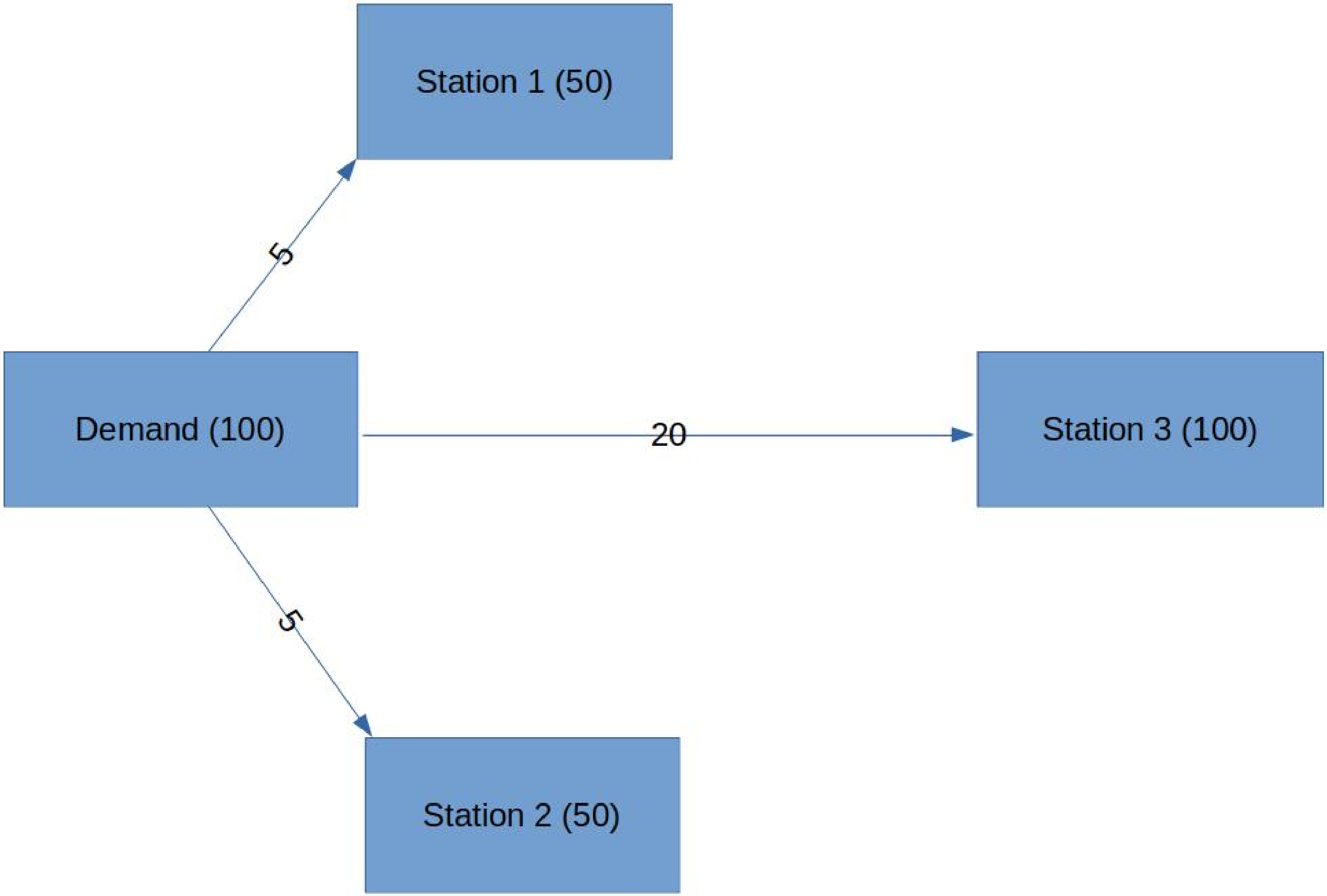
   Assuming that the cost of building a station is C times it's capacity, we add a budget constraint:

$$\sum_{i=1}^{N} x_i \leqslant \frac{B}{C}$$

3. With the added capacity constraint, some weird scenarios (like below) may happen.

   So, we need to allow for **fractional assigning** of demands to make the demand$\rightarrow$station assignment more efficient. Thus, α is now defined as:

$$\alpha_{\,i,j} \;:\; \text{Fraction of demand at } j \text{ assigned to station } i$$


Station 1 (50)
5
Demand (100)
20
Station 3 (100)
5
Station 2 (50)