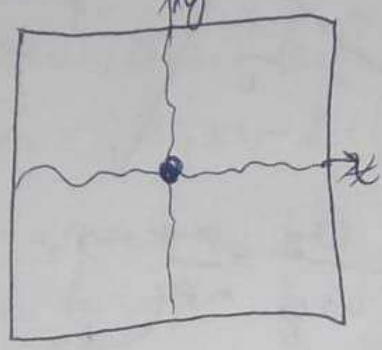
Curs 4 Fizică

Compunere oscilații perpendiculare

Aplicații

①



ceva cu ecranul

$$y = \frac{b}{a}x \ (?) \ ; \ \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos(\alpha-\beta) = \sin^2(\alpha-\beta)$$

(a) $\alpha - \beta = 0 : \frac{x}{a} - \frac{y}{b} = 0 \rightarrow y = \frac{b}{a}x$

(b) $\alpha - \beta = \pi$

$$\left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0 \Rightarrow \frac{x}{a} + \frac{y}{b} = 0 \Rightarrow y = -\frac{b}{a}x$$

$$\begin{cases} x = a\cos(\omega t + \alpha) \\ y = b\cos(\omega t + \beta) \end{cases}$$

$\dot{x} = -\omega a \sin(\omega t)$

$\dot{y} = -\omega b \sin(\omega t)$

② $\begin{cases} x = \cos(2t) \longrightarrow \omega_x = 2\ rad/s \\ y = \sin(3t + \frac{\pi}{2}) \longrightarrow \omega_y = 3\ rad/s \end{cases}$

$y = \cos(3t)$

$\sin(\alpha + \frac{\pi}{2}) = \cos\alpha$

$2t = \pm \arccos x + 2m\pi, \ m \in \mathbb{Z}$

$3t = \pm \arccos y + 2n\pi, \ n \in \mathbb{Z}$

$\pm 3\arccos x + 6m\pi = \pm 2\arccos y + 4n\pi$

$$\cos(3\underbrace{\arccos x}_{\alpha}) = \cos(2\underbrace{\arccos y}_{\beta}) \Rightarrow \cos(3\alpha) = \cos(2\beta)$$

$\cos(3\alpha) = \cos(2\alpha + \alpha) = \cos 2\alpha \cos\alpha - \sin 2\alpha \sin\alpha =$

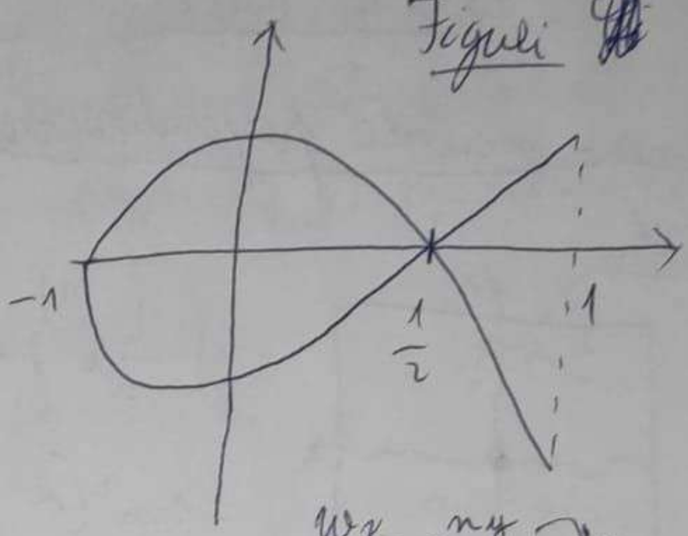
$= (2\cos^2\alpha - 1)\cos\alpha - 2\underbrace{\sin^2\alpha}_{1-\cos^2\alpha}\cos\alpha = \cancel{(2\cos^2\alpha - 1)\cos\alpha - (1-\cos^2\alpha)\cos\alpha}$

$\cancel{= \cos\alpha(2\cos^2\alpha - 1 - 1 + \cos^2\alpha)}$

$\cancel{= 3\cos^3\alpha}$

$= 4\cos^3 - 3\cos\alpha$

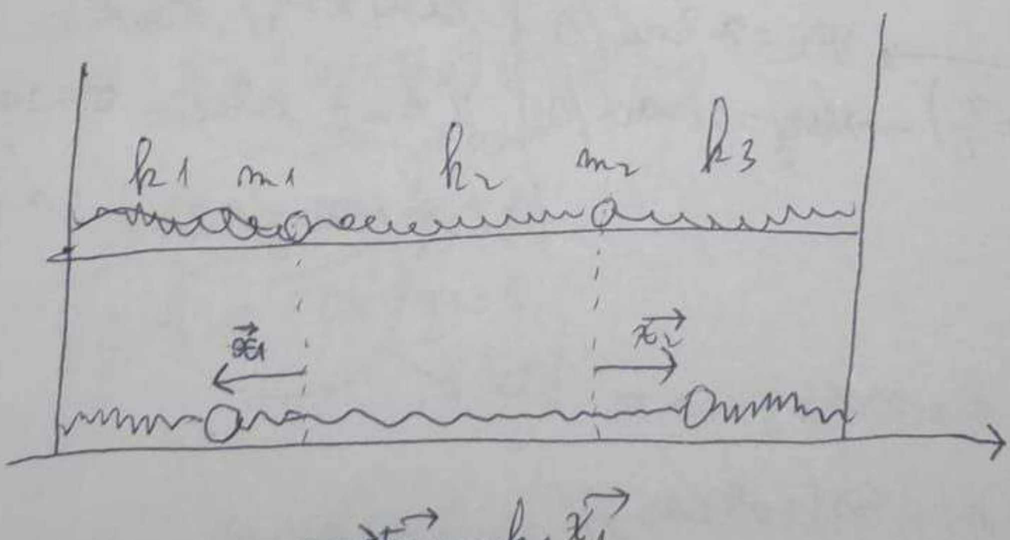
Figuri



$\frac{\omega_x}{\omega_y} = \frac{n_y}{n_x}$ → 2 → 3

$\frac{2}{3}$

Oscilații cuplate



$\rightarrow \vec{F_{e1}} = -k_1 \vec{x_1}$

$\rightarrow \vec{F_{e2}} = k_2(\vec{x_2} - \vec{x_1})$

$(|\vec{x_2}| - (-|\vec{x_1}|)$

$$m_1 \ddot{\vec{x}}_1 = \vec{F}_{e_1} + \vec{F}_{e_2} \quad \Big| \quad m_1 \ddot{\vec{x}}_1 = k_1 \vec{x}_1 + k_2 (\vec{x}_2 - \vec{x}_1)$$

$$m_2 \ddot{\vec{x}}_2 = \vec{F}_{e_3} - \vec{F}_{e_2} \quad \Big| \quad m_2 \ddot{\vec{x}}_2 = -k_3 \vec{x}_2 - k_2 (\vec{x}_2 - \vec{x}_1)$$

$$\vec{x}_1 = x_1 \vec{i} \Rightarrow \ddot{\vec{x}}_1 = \ddot{x}_1 \vec{i}$$

$$\Rightarrow m \ddot{x}_1 = -k_1 x_1 + k_2 (x_2 - x_1) \quad \Big| \frac{1}{m}$$

$$m \ddot{x}_2 = -k_3 x_2 - k_2 (x_2 - x_1) \quad \Big| \frac{1}{m}$$

stiu $\ddot{x} + \omega^2 x = 0$???

$$\begin{cases} m_1 = m_2 = m \\ k_1 = k_3 = k \\ k_2 = k_e \end{cases}$$

$$\Rightarrow \ddot{x}_1 + \underbrace{\frac{k_1 + k_e}{m}}_{\omega_0^2} x_1 - \underbrace{\frac{k_e}{m}}_{\omega_e^2} x_2 = 0$$

$$\ddot{x}_2 + \frac{k + k_e}{m} x_2 - \frac{k_e}{m} x_1 = 0$$

$$\ddot{x}_1 + \omega_0^2 x_1 - \omega_e^2 x_2 = 0$$

$$\ddot{x}_2 + \omega_0^2 x_2 - \omega_e^2 x_1 = 0$$

$$\Rightarrow \ddot{x}_1 + \ddot{x}_2 + (\omega_0^2 - \omega_e^2)(x_1 + x_1) = 0$$

$$\ddot{x}_1 - \ddot{x}_2 + (\omega_0^2 + \omega_e^2)(x_1 - x_2) = 0$$

$$\begin{cases} \ddot{q}_1 + \underbrace{(\omega_0^2 - \omega_e^2)}_{\omega_1^2} q_1 = 0 \\ \ddot{q}_2 + \underbrace{(\omega_0^2 + \omega_e^2)}_{\omega_2^2} q_2 = 0 \end{cases}$$

$$q_{1,2} = A_{1,2} \cos(\omega_2 t + \alpha_{1,2})$$

$$\begin{cases} x_1(t) = \frac{1}{2}\left[A_1 \cos(\omega_1 t + \alpha_1) + A_2 \cos(\omega_2 t + \alpha_2)\right] \\ x_2(t) = \frac{1}{2}\left[A_2 \cos(\omega_1 t + \alpha_1) - A_2 \cos(\omega_2 t + \alpha_2)\right] \end{cases}$$

Mod axial sinetric

$x_1(0) = x_2(0) = A$

$\dot{x}_1(0) = \dot{x}_2(0) = 0$

$\dot{x}_1 = -\frac{1}{2}\left[\omega_1 A_1 \sin(\omega_1 t + \alpha_1) + \omega_2 A_2 \sin(\omega_2 t + \alpha_2)\right]$

$\dot{x}_2 = -\frac{1}{2}\left[\omega_1 A_1 \sin(\omega_1 t + \alpha_1) - \omega_2 A_2 \sin(\omega_2 t + \alpha_2)\right]$

$$\begin{cases} 2A = A_1 \cos\alpha_1 + A_2 \cos\alpha_2 \\ 2A = A_1 \cos\alpha_1 - A_2 \cos\alpha_2 \\ 0 = \omega_1 A_1 \sin\alpha_1 + \omega_2 A_2 \sin\alpha_2 \\ 0 = \omega_1 A_1 \sin\alpha_1 - \omega_2 A_2 \sin\alpha_2 \end{cases}$$

$$\begin{cases} A_2 = 0 \\ A_1 = 2A,\ \alpha_1 = 0,\ \omega_1 A_1 \sin\alpha_1 = \omega_2 A_2 \sin\alpha_2 = 0 \end{cases}$$

~~$\omega_1 A_1 \sin\alpha_1$~~.