Seminar Analiză 10

① Să se determine extremele $f:\mathbb{R}^3 \to \mathbb{R}$; $f(x,y,z) = xyz$

$g(x,y,z) = x^2+y^2+z^2-1$

1. $f, g \in C^\infty$

2. $g'(x,y,z) = (2x \;\; 2y \;\; 2z)$

presupunem că

$x = y = z = 0 \notin A$

3. (x_0, y_0, z_0) pct de extrem pt f pe A

$\Rightarrow \exists\, \lambda \in \mathbb{R}$ ai $h_\lambda'(x_0,y_0,z_0)=0$

$h_\lambda = f + \lambda g$

$$\begin{cases} \dfrac{\partial h_\lambda}{\partial x} = yz + \lambda(2x) \\ \dfrac{\partial h_\lambda}{\partial y} = xz + \lambda(2y) \\ \dfrac{\partial h_\lambda}{\partial z} = xy + \lambda(2z) \\ x^2+y^2+z^2=1 \end{cases}$$

$\Rightarrow z(y-x) + 2\lambda(x-y) = 0$

$(y-x)(z-2\lambda) = 0$

$y = x$

$z = -2\lambda$

C1 $x=y=z$

C2.1 $x=y=2\lambda \neq z$

C2.2 $x=z=2\lambda \neq y$

C2.3 $y=z=2\lambda \neq x$

C1) $x=y=z$

$x^2 + \lambda 2x = 0 \Rightarrow \begin{cases} x = 0 \notin A \\ x = -2\lambda \end{cases}$

$x=y=z=-2\lambda = \pm \dfrac{1}{\sqrt{3}}$

$3(2\lambda)^2 = 1$

$3x^2 = 1 \Rightarrow x = \pm \dfrac{1}{\sqrt{3}}$

C2 $2\lambda z + 2\lambda \cdot 2\lambda = 0 \Rightarrow z = -2\lambda$

$$3x^2 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{3}} = y = -z$$

$f\ \frac{\pm 1}{3\sqrt{3}}\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right); \lambda = \mp\frac{1}{2\sqrt{3}}$

$\mp\frac{1}{3\sqrt{3}}\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \mp\frac{1}{\sqrt{3}}\right); \lambda = \pm\frac{1}{2\sqrt{3}}$

$(0, 0, \pm 1); \lambda = 0$?

$\frac{1}{3\sqrt{3}}\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ ~~pct~~ maxim

$-\frac{1}{3\sqrt{3}}\Big($

$$h''_\lambda = \begin{pmatrix} 2\lambda & z & y \\ z & 2\lambda & x \\ y & x & 2\lambda \end{pmatrix} = \begin{pmatrix} 0 & z & y \\ z & 0 & x \\ y & x & 0 \end{pmatrix}$$

Extremele funcției $f(x,y,z) = xyz$

Pe $A = \{x^2+y^2+z^2=1, x+y+z=0\}$ $\quad g=(g_1, g_2)$

$g_1(x,y,z) = x^2+y^2+z^2-1$, [illegible]

$g_2(x,y,z) = x+y+z$

1) $f, g \in C^\infty$

2) $rg\, g' = 2$

$g' = \begin{pmatrix} 2x & 2y & 2z \\ 1 & 1 & 1 \end{pmatrix}$

3) (x_0, y_0, z_0) pct de extrem local pe A

Pp $rg\, g' \neq 0 \Rightarrow \begin{vmatrix} 2x & 2y \\ 1 & 1 \end{vmatrix} = 0 \to x = y$

$x = y = z \Rightarrow \begin{matrix} 3x = 0 \\ 3x^2 = 1 \end{matrix}$ $\cancel{x}$

$h'_{x_\alpha}(x_0, y_0, z_0) = 0$

$\frac{\partial h}{\partial x} = yz + 2\alpha x + \beta = 0$

$\frac{\partial h}{\partial y} = xz + \alpha 2y + \beta = 0$

$\frac{\partial h}{\partial z} = xy + \alpha 2z + \beta = 0$

$\Rightarrow z(y-x) + 2\alpha(x+y) =$

$(z-2\alpha)(y-x) = 0$ $\begin{cases} y = \\ z = \end{cases}$

C1 $x = y = z \in A$ (3) $x = y = 2\alpha \neq z$

$2x + z = 0 \Rightarrow z = -2x$

$2x^2 + z^2 = 1 \Rightarrow 6x^2 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{6}}$

$-\frac{2}{6\sqrt{6}} \left(\pm \frac{1}{\sqrt{6}}, \pm \frac{1}{\sqrt{6}}, \mp \frac{2}{\sqrt{6}}\right)$

A este închisă și mărg. } $\Rightarrow \exists (x_M, y_M, z_M)$ a.î.
f continuă }

$f(x_M, y_M, z_M) = \max f(A)$

și $\exists (x_m, y_m, z_m)$ a.î.

$f(x_m, y_m, z_m) = \inf f(A)$

$\cancel{\Rightarrow \max f(A) = -\frac{2}{6\sqrt{6}} \left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)}$

$\left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right); \left(-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right); \left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$

maxim global

$\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$ pct min