Seminal Analiză 7

$f:\mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = \begin{cases} \frac{x^3 \cdot y^3}{x^6+y^6}, & x^2+y^2 \neq 0 \\ 0, & x=y=0 \end{cases}$ f continuă

f continuă pe $\mathbb{R}^2 - \{(0,0)\}$, $x=y=0$

$$f(0,0)=0$$

$x_n = y_n = \frac{1}{n}$ $\qquad f\left(\frac{1}{n},\frac{1}{n}\right) = \frac{\left(\frac{1}{n}\right)\cdot\left(\frac{1}{n}\right)^3}{\left(\frac{1}{n}\right)^6\cdot\left(\frac{1}{n}\right)^6} = \to \frac{1}{2}$

$x_n = \frac{1}{n}$, $y_n = 0$

$$f\left(\frac{1}{n},0\right) = \frac{\left(\frac{1}{n}\right)^3\cdot 0}{\left(\frac{1}{n}\right)^6} = 0 \to 0$$

$\lim\limits_{x\to 0} f(x,y) = \lim\limits_{x\to 0} f(x,ax) = \lim\limits_{x\to 0} \frac{x^3\cdot(ax)^3}{x^6+(ax)^6} = \frac{a^3}{1+a^6} \Rightarrow$

discontinuă

$f:\mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = \frac{x^3 y}{x^4+y^4}$, $\begin{cases} x^2+y^2\neq 0 \\ 0, \ x=y=0 \end{cases}$

$\lim\limits_{x\to 0} f(x,y) = \lim\limits_{x\to 0} f($

$$f(x,y)=\begin{cases}\dfrac{xy\cdot(x^2-y^2)}{x^4+y^4}, & x^2+y^2\neq 0\\[2mm] 0, & x=y=0\end{cases}$$

$$x_n=\frac{1}{n}=y_n \qquad f\left(\frac{1}{n},\frac{1}{n}\right)=\frac{\frac{1}{n^2}\cdot\underbrace{\left(\frac{2}{n^2}-\frac{2}{n^2}\right)}_{\to 0}}{\frac{1}{n^4}+\frac{1}{n^4}}\to 0$$

$$x_n=0,\ y_n=\frac{1}{n}$$

$$f\left(0,\frac{1}{n}\right)=0$$

$$x_n=\frac{2}{n},\ y_n=\frac{1}{n} \quad f\left(\frac{2}{n},\frac{1}{n}\right)=\frac{\frac{2}{n}\cdot\frac{1}{n}\left(\frac{4}{n^2}-\frac{1}{n^2}\right)}{\frac{16}{n^4}+\frac{1}{n^4}}=\frac{6}{17}\neq 0$$

$$\lim_{\substack{x\to 0\\ y=ax}} f(x,y)=\lim_{x\to 0} f(x,ax)=\lim_{x\to 0}\frac{x\,ax(x^2-a^2x^2)}{x^4+a^4x^4}=$$

$=\dfrac{a(1-a^2)}{1+a^4} \Rightarrow$ discontinuă (depinde de a)

$$f(x,y)=\begin{cases}\dfrac{x^3\cdot y^2}{x^4+y^4}, & x^2+y^2\neq 0\\[2mm] 0, & x=y=0\end{cases}$$

$$\lim_{\substack{x\to 0\\ y=ax}} f(x,y)=\lim_{x\to 0}\left(f(x,ax)\right)=\lim_{x\to 0}\frac{x^3a^2x^2}{x^4+a^4x^4}=\frac{xa^2}{1+a^4}\to 0$$

$$f(x,y) = \begin{cases} \dfrac{x^2 - xy}{x^2 + xy + y^2}, & x^2 + y^2 \neq 0 \\ 0, & x = y = 0 \end{cases}$$

$x^2 + xy + y^2 = x^2 + xy + \frac{y^2}{4} + \frac{3y^2}{4} = \left(x + \frac{y}{2}\right)^2 + \frac{3y^2}{4} \geq 0$

f continuă pe $\mathbb{R} \setminus \{(0,0)\}$

$x_n = y_n = \frac{1}{n} \Rightarrow f\left(\frac{1}{n}, \frac{1}{n}\right) = \dfrac{\frac{1}{n^2} - \frac{1}{n^2}}{\frac{1}{n^2} + \frac{1}{n^2} + \frac{1}{n^2}} \longrightarrow 0$

$x_n = \frac{1}{n}, y_n = 0 \Rightarrow f\left(\frac{1}{n}, 0\right) = \dfrac{\frac{1}{n^2} - 0}{\frac{1}{n^2} + 0 + 0} \longrightarrow 1$ $\Bigg| \Rightarrow$

disc în $(0,0)$

$x_n = \frac{3}{n}$; $y_n = \frac{7}{n}$ $\quad f\left(0, \frac{1}{n}\right) = 0$

$$f(x,y) = \begin{cases} \dfrac{x^4 \cdot y - y^4 x}{x^4 + y^4} & x^2 + y^2 \neq 0 \\ 0, & x = y = 0 \end{cases}$$

$\lim\limits_{x \to 0} f(x, ax) = \lim\limits_{x \to 0} \dfrac{x^4 \cdot ax - a^4 x^4 x}{x^4 + a^4 x^4} = \dfrac{a(x - a^3)}{a^4} =$

$= \lim\limits_{x \to 0} \dfrac{ax(1 - a^3)}{1 + a^2} \to 0$

SKIBIDI

$$\left|\frac{x^4 y}{x^4+y^4}\right| \le \frac{x^4}{x^4+y^2}|y| \le |y|$$

mărginim pe rând

$$\left|\frac{y^4 \cdot x}{x^4+y^4}\right| \le \underbrace{\frac{y^4}{x^4+y^4}}_{\le 1}|x| \le |x|$$

$$\Rightarrow f(x,y) = \begin{cases} \dfrac{x^{20}y^{20}}{x-y}, & x \ne 0 \\ 0, & x=y=0 \end{cases}$$

$$\lim_{\substack{x\to 0\\ y=ax}} f(x,y) = \lim_{x\to 0} (x, ax) = \lim_{x\to 0} \frac{x^{10}\cdot a^{10}\cdot x^{10}}{x-ax} \to 0$$

$$f\left(a, a+\frac{1}{n}\right) = \frac{a^{10}\left(a+\frac{1}{n}\right)^{20}}{\frac{1}{n}} \to 0$$

~~f((a), a+1/n) ...~~

$s(x) = \sum_{n\geq 1} \frac{1}{n^2} \cos\frac{x}{n}$

$f_n : \mathbb{R} \to \mathbb{R} \quad f_n(x) = \frac{1}{n^2} \cdot \cos\frac{x}{n}$

$|f_n(x)| \leq \frac{1}{n^2}$; $\sum \frac{1}{n^2} < +\infty$ absolut convergentă

(T) $f_n : [a,b] \to \mathbb{R}$ derivabilă și $s(x) = \sum_{n\geq 1} f_n(x)$ și

~~...~~ $s(x) = \sum_{n\geq 1} f_n(x)$

Dacă este convergentă și s_1 este uniform convergentă $\Rightarrow$

$s' = s_1$

$s_1(x) = \sum_{n\geq 1} f_n'(x) = \sum_{n\geq 1} \frac{1}{n^2} \sin\frac{x}{n}$

$|f_n'(x)| = \left|\frac{1}{n^3} \sin\frac{x}{n}\right| \leq \frac{1}{n^3}$

$\sum \frac{1}{n^3}$ - conv $\Rightarrow s_1(x)$ normal convergentă $\Big| \Rightarrow$
$s(x)$ conv $\Big| s' = s_1$

$s_2(x) = \sum_{n\geq 1} \frac{1}{n^4} \cos\left(\frac{x}{n}\right)$

$|f_n^4(x)| = \left|\frac{1}{n^4} \cos\frac{x}{n}\right| \leq \frac{1}{n^4}$

$\sum_{n\geq 1} \frac{1}{n^4}$ - conv $\Rightarrow s_2$ n. conv $\Big| \Rightarrow s_1' = s_2 \to s^4 = s_2$
s_1 — " —