Seminar 9 A-G

Forme biliniare. Forme pătratice
Formă canonică / normală

<u>Def</u> $g: V \times V \to \mathbb{K}$ s.n. formă biliniară

$\Leftrightarrow$ 1) $g(ax+by, z) = a\,g(x,z) + b\,g(y,z)$

2) $g(x, ay+bz) = a\,g(x,y) + b\,g(x,z)$, $\forall x,y,z \in V$, $\forall a,b \in \mathbb{K}$

g este simetrică $\Leftrightarrow$ $g(x,y) = g(y,x)$

$R = \{e_1, e_2, \dots, e_n\}$ - reper în V $g \in L(V,V,\mathbb{K})$ (mulțimea formelor biliniare)

Notație
$g(e_i, e_j) = g_{ij}$

$\Leftrightarrow g(x,y) = \sum_{i,j=1}^{n} g_{ij} x_i y_j$

$Q: V \to \mathbb{K}$

s.n. formă pătratică $\Leftrightarrow \exists g \in L(V,V,\mathbb{K})$

$Q(x) = g(x,x) = \sum_{i,j=1}^{n} g_{ij} x_i x_j$

$Q(x) = \sum_{i=1}^{n} g_{ii} x_i^2 + 2\sum_{i<j} g_{ij} x_i x_j$

$rg\, G = r$
$Q = a_1 x_1'^2 + \dots + a_r x_r'^2$

$Q : \mathbb{R}^4 \to \mathbb{R}$

$Q(x) = x_1^2 + x_2^2 + x_3^2 - 2x_4^2 - 2x_1x_2 + 2x_1x_3 - 2x_1x_4 + 2x_2x_3 - 4x_2x_4$

(a) $G = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -2 \\ 1 & 1 & 1 & 0 \\ -1 & -2 & 0 & -2 \end{pmatrix}$

(b) $Q \to$ formă canonică

Metoda Jacobi nu se poate aplica în acest caz

~~$2(x_1^2 - 2x_1x_2 + x_2^2)$~~

$Q(x) = \underline{x_1^2} + x_2^2 + x_3^2 - 2x_4^2 \underline{- 2x_1x_2} + \underline{2x_1x_3} - \underline{2x_1x_4} + 2x_2x_3 - 4x_2x_4$

$= (x_1 - x_2 + x_3 - x_4)^2 - \cancel{x_2^2} - \cancel{x_3^2} - x_4^2 + 2x_2x_3 - 2x_2x_4 + 2x_3x_4$

$+ \cancel{x_2^2} + \cancel{x_3^2} - 2x_4^2 + 2x_2x_3 - 4x_2x_4$

$= (x_1 - x_2 + x_3 - x_4)^2 - 3x_4^2 + \underline{4x_2x_3} - \underline{6x_2x_4} + 2x_3x_4$

fie schimbarea $y_1 = (x_1 - x_2 + x_3 - x_4)$

$\begin{cases} x_2' = x_2 + x_3 \\ x_3' = x_2 - x_3 \\ x_1' = x_1 \\ x_4' = x_4 \end{cases}$ $\quad \begin{cases} x_1 = x_1' \\ x_2 = \frac{1}{2}(x_2' + x_3') \\ x_3 = \frac{1}{2}(x_2' - x_3') \\ x_4 = x_4' \end{cases}$

$$\Phi(x) = (x_1' - x_3' - x_4')^2 + x_2'^2 - x_3'^2 - 3x_2'x_4' - 3x_3'x_4'$$

$$Q(x) = (x_1' - x_3' - x_4')^2 + \left(x_2' - \frac{3}{2}x_4'\right)^2 + \left(x_2' - \frac{3}{2}x_4\right)^2 - \frac{9}{4}x_4'^2 -$$

$$- x_3'^2 - 3x_3'x_4' - 3x_4'^2$$

$$\Phi(x) = (x_1' - x_3' - x_4')^2 + \left(x_2' - \frac{3}{2}x_4'\right)^2 - \left(x_3' + \frac{1}{2}x_4'\right)^2 - 3x_4'^2$$

$$\Rightarrow \begin{cases} x_1'' = x_1' - x_3' - x_4' \\ x_2'' = x_2' - \frac{3}{2}x_4' \\ x_3'' = x_3' + \frac{1}{2}x_4' \\ x_4'' = x_4' \cdot \sqrt{3} \end{cases}$$

$$\Rightarrow \Phi(x) = x_1''^2 + x_2''^2 - x_3''^2 - x_4''^2$$

4 pătrate = rg(G)
signatula (2, 2)

pozitiv definită (=) signatura e (4, 0)

$Q : \mathbb{R}^3 \to \mathbb{R}$

a) $\Phi = ?$

b) să se diagonalizeze (forma canon)

$$G = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

a) $\Phi = \cancel{\ } x_1^2 + 3x_2^2 + x_3^2 + 4x_1x_2 + 2x_1x_3 + 4x_2x_3$

(b) Metoda Jacobi

$$\Delta_1 = 1$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \neq 0$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 3 + 4 + 4 - 3 - 4 - 3 = 0$$

$\Rightarrow$ nu merge Jacobi

$$Q(x) = \underline{x_1^2} + 3x_2^2 + x_3^2 + \underline{4x_1x_2} + \underline{2x_1x_3} + 4x_2x_3$$

$$= (x_1 + 2x_2 + x_3)^2 \cancel{- 4x_2x_3} - 4x_2^2 \cancel{- x_3^2} + 3x_2^2 \cancel{+ x_3^2} \cancel{+ 4x_2x_3}$$

$$= \underbrace{(x_1 + 2x_2 + x_3)}_{x_1'}{}^2 - \underbrace{x_2}_{x_2'}{}^2$$

$$\Rightarrow Q(x) = x_1'^2 - x_2'^2$$

nu e pozitiv definită

$$\begin{cases} x_1' = x_1 + 2x_2 + x_3 \\ x_2' = x_2 \\ x_3' = x_3 \end{cases}$$

signatura $(1,1)$

$$\Rightarrow G' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ se diagonalizează