Seminar 5 Analiză

$A = (1, 3]$

$A^\circ = (1, 3)$

$A' = [1, 3]$

$\bar{A} = A \cup A' = [1, 3]$

$Fr(A) = \{1, 3\}$

$Iz(A) = \emptyset$

$[1, 3] \subset A'$

$a = 1$

$x_n = 1 + \frac{1}{n} \to 1 \Rightarrow 1 \in A'$

$a = 3$

$x_n = 3 - \frac{1}{n+1} \Rightarrow 3 \in A'$

$a \in (1, 3)$

$1 < a < x_n = a + \frac{1}{n+1}(3 - a) < \cancel{a} + 3 - \cancel{a} \leq 3$

$x_n \in A$ $\quad x_n \to a$

$\quad x_n \neq a \Rightarrow a \in A'$

$A' \subset [1, 3]$

$a \in A' \Rightarrow \exists x_n \subset A$ a.î. $x_n \to a$, $x_n \neq a$

$1 < x_n \leq 3 \quad 1 \leq a \leq 3$

$\underset{\text{deschis}}{(1, 3)} \subset A \Rightarrow (1, 3) \subset A^\circ$

$A = (1,2) \cup \{5,6\}$

$\rightarrow \mathring{A} = (1,2)$

$\rightarrow A' = [1,2]$

$\bar{A} = [1,2] \cup \{5,6\}$

$Fr(A) = \bar{A} \setminus \mathring{A} = \{1,2,5,6\}$

$Iz(A) = A \setminus A' = \{5,6\}$

$A' = \{a \mid \exists x_n \in A \text{ a.i. } x_n \to a, x_n \neq a\}$

$[1,2] \subset A'$

$2 \in A' \Rightarrow x_n = 2 - \underbrace{\frac{1}{n+1}}_{\in A} \to 2 \in A'$

$1 \in A' \Rightarrow x_n = 1 + \frac{1}{n+1} \to 1 \Rightarrow 1 \in A$

$\mathring{A}' \subset [1,2]$

$a \in A' \quad \exists x_n \to a, x_n \neq a, x_n \in A$

trecând la un subșir putem presupune

că 1) $x_n \in (1,2) \Rightarrow a \in [1,2]$

2) $x_n = 5 \Rightarrow x_n = 5$

3) — " — $x_n = 6$

$(1,2) \subset A \Rightarrow (1,2) \subset \mathring{A}$ $\Big| \Rightarrow$

m. deschisă

$A^\circ \subset (1,2) \qquad A^\circ \subset A$

$a \in A \setminus (1,2) \Rightarrow a \notin A^\circ$

$\{5,6\}$

$x_n = 5 - \frac{1}{n} \to 5,\ x_n = 6 + \frac{1}{n} \to 6 \Rightarrow A = (1,2)$

$A = (1,2) \cup [4,5]$

$A^\circ = (1,2) \cup (4,5)$

$A' = [1,2] \cup [4,5]$

$\bar{A} = [1,2] \cup [4,5]$

$Fr(A) = \bar{A} \setminus A^\circ = \{1,2,4,5\}$

$Iz(A) = \emptyset$

dem că:

$\to A' \subset [1,2] \cup [4,5]$?

$a \in A' \Rightarrow \exists (x_n)_n \subset A$ a.î. $x_n \to a$ $x_n \neq a$

C1 $(x_n)_n \subset (1,2) \Rightarrow a \in [1,2]$

C2 $(x_n) \subset [4,5] \Rightarrow a \in [4,5]$

$(1,2) \subset A \Rightarrow (1,2) \subset A^\circ$

$(4,5) \subset A \Rightarrow (4,5) \subset \overset{\circ}{A}$

$A^\circ \subset (1,2) \cup (4,5)$

$a \in A \setminus \underbrace{((1,2) \cup (4,5))}_{\{4,5\}} \Rightarrow a \in \overset{\circ}{A}$

$x_n = 4 - \frac{1}{n} \to 4 \Rightarrow 4 \in A'$

$y_n = 5 + \frac{1}{n} \to 5 \Rightarrow 5 \notin A'$

$A = (-2,-1) \cup \mathbb{N}$

$A^\circ = (-2,-1)$

$A' = [-2,-1]$

$\bar{A} = A \cup A' = [-2,-1] \cup \mathbb{N}$

$\mathrm{Fr}(A) = \bar{A} \setminus A = \{-2,-1,0,1,\dots\}$

$\mathrm{Iz}(A) = A \setminus A' = \mathbb{N}$

$$x_n = \overset{\in A}{2} + \frac{1}{n} \longrightarrow 3$$

$x_n \neq -2 \Rightarrow 2 \in A'$

$x_n = -1 + \frac{1}{n} \to -1$

$x_n \neq -1 \Rightarrow -1 \in A'$

$x_n = -1 - \frac{1}{n+1} \longrightarrow -1$

$x_n \neq -1 \Rightarrow -1 \in A'$

$-2 < a < x_n =$

$x_n \; x_n$

$\downarrow \;\; \downarrow$

$n \;\; a$

$a \in A' \Rightarrow \exists\, x_n \subset A$, a.î. $x_n \to a$, $x_n \neq a$

putem pp. că $|x_n - x_m| < 1 \; \forall n, m$

$x_n, x_m \in \mathbb{N} \Rightarrow |x_n - x_m| \in \mathbb{N} \Rightarrow$

$\Rightarrow x_n = x_m = k \in \mathbb{N}$

$x_n \neq k$

c1) $(x_n)_n \in (-2,-1) \Rightarrow a \in [-2,-1]$

$A' \subset [-2,-1]$

~~$A \in (-\infty, -1) \cup (-1, \infty)$~~

$A = \{\frac{1}{n} / n \geq 1\} \cup (4, 6)$

$A^{\circ} = (4, 6)$

$A' = [4, 6] \cup \{0\}$

$\bar{A} = A \cup A' = \{\frac{1}{n} / n \geq 1\} \cup \{0\} \cup [4, 6]$

$Fr(A) = \bar{A} - A^{\circ} = \{\frac{1}{n}, n \geq 1\} \cup \{0, 4, 6\}$

$Iz(A) = \{\frac{1}{n} / n \geq 1\}$

$\underset{A}{\overset{\cap}{x_n}} = \frac{1}{n} \to 0 \Rightarrow 0 \in A'$

$[4, 6) \subset A'$

$A' \subset \{0\} \cup [4, 6]$

$[4, 6]$ $a \in A' \Rightarrow \exists\, x_n \subset A$ a.i. $x_n \to a$
$x_n \neq a$

trecând la un subșir

1) $x_n \in (4, 6) \Rightarrow a \in [4, 6]$

2) $x_n \in \{\frac{1}{n}, n \geq 1\}$

C 2 1 $\exists\, x_{n_h} = \frac{1}{n}\ \forall h \in \mathbb{N}$

$\frac{1}{n} = x_{n_h}$ $\quad$ $x_n \to a$
$x_n \neq a$

C 2 2 $\frac{1}{h}$ apare de un nr finit $| < 1\ \forall n, m$ de ori

$\forall m \ \exists n_m$ a.î. $\forall n \geq n_m$

$\Rightarrow 0 \leq x_n \leq \frac{1}{m}$

$\Rightarrow x_n \to 0$

$(4,6) \subset A \Rightarrow (4,6) \in \overset{\circ}{A}$

deschisă

$a \in A \setminus (4,6) \Rightarrow a \notin \overset{\circ}{A}$

$\left\{\frac{1}{n}, n \geq 1\right\}$

$a = \frac{1}{n} \quad x_n = \underset{\notin A}{\frac{1}{n} + \frac{1}{k\sqrt{2}}} \to \frac{1}{n} \Rightarrow \frac{1}{n} \notin \overset{\circ}{A}$

$A = (1,2) \cup \left([3,4) \setminus \mathbb{Q}\right)$

$\overset{\circ}{A} = (1,2)$

$A' = [1,2] \cup [3,4]$

$\bar{A} = A \cup A' = [1,2] \cup [3,4]$

$Fr(A) = \bar{A} \setminus \overset{\circ}{A} = \{1,2\} \cup [3,4]$

$Iz(A) = A \setminus A' = \emptyset$

$f_n : [0,1] \to \mathbb{R} \quad f_n(x) = (1-x)^n$

$\lim\limits_{n\to\infty} f_n(x) = f(x) = \lim\limits_{n\to\infty} (1-x)^n = \begin{cases} 1, & x=0 \\ 0, & x \in (0,1] \end{cases}$

$f_n \xrightarrow{s} f$

$\Rightarrow a_n = \sup\limits_{x\in[0,1]} |f_n(x) - f(x)| = \sup\limits_{x\in[0,1]} (1-x)^n \Rightarrow f_n \not\xrightarrow{u} f$

$\left.\begin{array}{l} f \text{ discontinua} \\ f_n \text{ continua} \end{array}\right| \Rightarrow f_n \not\xrightarrow{u} f$

$g_n : (0,1] \to \mathbb{R} \quad , g_n(x) = (1-x)^n \Rightarrow g_n \xrightarrow{s} 0$

$a_n = \sup\limits_{x\in(0,1]} |g_n(x) - 0| = \sup\limits_{x\in(0,1]} (1-x)^n = 1$

$h_n \ [\varepsilon, 1] \to \mathbb{R} \quad , h_n(x) = (1-x)^n, \ h_n \xrightarrow{s} 0$

$a_n = \sup\limits_{x\in[\varepsilon,1]} (1-x)^n = (1-\varepsilon)^n \longrightarrow 0 \Rightarrow h_n \xrightarrow{u} 0$

$f_n : [0,1] \to \mathbb{R} \quad f_n(x) = (1-x)^n \cdot x^3 \, / \, \lim\limits_{n\to\infty} f_n(x) =$

$= \lim\limits_{n\to\infty} (1-x)^n x^3$

$= \begin{cases} 0, & x = 0 \\ 0, & x \in (0,1] \end{cases} = f(x) = 0 \Rightarrow f_n \xrightarrow{s} 0$

$$a_n = \sup_{x\in[0,1]} |f_n(x) - 0| = \sup_{x\in[0,1]} (1-x)^n x^3$$

$$f_n'(x) = -n(1-x)^{n-1}x^3 + 3x^2(1-x)^n = (1-x)^{n-1}x^2 \cdot (-nx+3-3x) = (1-x)^{n-1}x^2(x(-n-3)+3) = 0$$

$\Rightarrow x = 0$

$x = 1$

$x = \frac{3}{n+3}$

x	0		$\frac{3}{n+3}$		1
f_n'	0	$+ + + +$	0	$- - - - -$	0
f_n	0	$\nearrow$	$f_n\left(\frac{3}{n+3}\right)$	$\searrow$	0

$f_n : \mathbb{R} \to \mathbb{R}$ $\quad f_n(x) = \frac{x^2 n}{x^4+n^4}$

$$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \frac{x^2 n}{x^4+n^4} = \begin{cases} 0, & x = 0 \end{cases} = f(x) = 0$$

$f_n \xrightarrow{s} f = 0$ $\qquad \left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

$$a_n = \sup_{x\in\mathbb{R}} |f_n(x) - f(x)| = \sup_{x\in\mathbb{R}} \frac{x^2 n}{x^4+n^4}$$

$$f_n'(x) = \left(\frac{x^2 n}{x^4+n^4}\right)' = \frac{2x(x^4+n^4) - x^2 n \cdot 4x^3}{(x^4+n^4)^2} =$$

$$= \frac{-2nx^5 + 2n^4 x}{(x^4+n^4)^2} = -2nx\frac{-x^4+n^4}{(x^4+n^4)^2} \Rightarrow x = 0,\ x = n,\ x = -n$$

x	-1		0		1		$+\infty$
$f'(x)$	$+$ 0	$-$	0	$+$	0	$-$	
$f(x)$	$\nearrow$	$\searrow$		$\nearrow$		$\searrow$	

Cu teorema lui Dimi