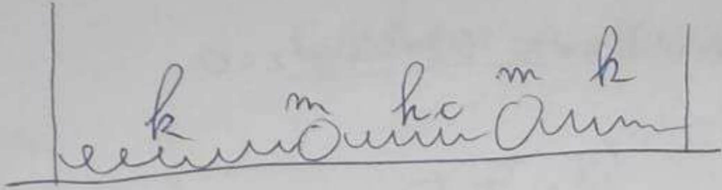
Curs Fizică 5

Oscilații cuplate

Bătăi → caz 3 curs



$$\begin{cases} x_1(t) = \frac{1}{2}\left[A_1\cos(\omega_1 t+\alpha_1) + A_2\cos(\omega_2 t+\alpha_2)\right] \\ x_2(t) = \frac{1}{2}\left[A_1\cos(\omega_1 t+\alpha_1) - A_2\cos(\omega_2 t+\alpha_2)\right] \\ \boxed{\omega_1^2 = \frac{k}{m} \quad \omega_2^2 = \frac{k+2k_c}{m}} \end{cases}$$

mod simetric

$$T_s = \frac{2\pi}{\omega_1} = 2\pi\sqrt{\frac{m}{k}}$$

mod antisimetric

$$T_a = \frac{2\pi}{\omega_2} = 2\pi\sqrt{\frac{m}{k+2k_c}}$$

$$T_s > T_a$$

bătăi

$$\begin{cases} x_1(0) = A \\ x_2(0) = 0 \\ \dot{x}_1(0) = \dot{x}_2(0) = 0 \end{cases} \Rightarrow \begin{rcases} 2A = A_1\cos\alpha_1 + A_2\cos\alpha_2 \\ 0 = A_1\cos\alpha_1 - A_2\cos\alpha_2 \end{rcases} \Rightarrow (P-2)$$

$$\dot{x}_1(t) = \frac{1}{2}\left[-\omega_1 A_1\sin(\omega_1 t+\alpha_1) - \omega_2 A_2\sin(\omega_2 t+\alpha_2)\right]$$

$$\ddot{x}_2(t) = \frac{1}{2}\left[\text{———} // \text{———} + \text{———} // \text{———}\right]$$

$$\Rightarrow \begin{cases} 0 = \omega_1 A_1 \sin\alpha_1 + \omega_2 A_2 \sin\alpha_2 \\ 0 = \omega_1 A_1 \sin\alpha_1 - \omega_2 A_2 \sin\alpha_2 \\ 0 = A_1 \cos\alpha_1 - A_2 \cos\alpha_2 \\ 2A = A_1\cos\alpha_1 + A_2\cos\alpha_2 \end{cases} \Rightarrow$$

$$\omega_1 A \sin\alpha_1 = \omega_2 A \sin\alpha_2 = 0$$

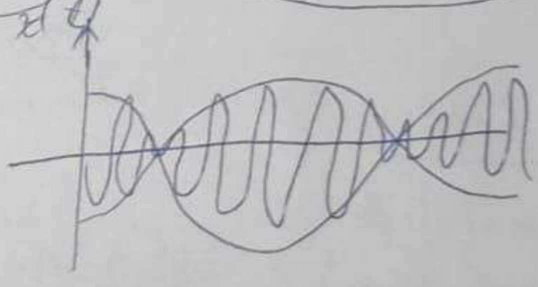
$$\Rightarrow \begin{cases} \alpha_1 = \alpha_2 = 0 \\ A_1 = A_2 = A \end{cases}$$

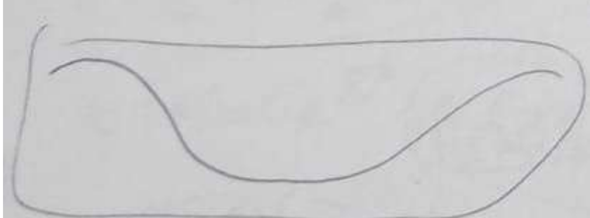
$$x_1(t) = \frac{1}{2}\left[A\cos(\omega_1 t + \cancelto{0}{\alpha_1}) + A\cos(\omega_2 t + \cancelto{0}{\alpha_2})\right] \Rightarrow$$

$$x_2(t) = \frac{1}{2}\left[A\cos(\omega_1 t + \cancelto{0}{\alpha_1}) - A\cos(\omega_2 t + \cancelto{0}{\alpha_2})\right]$$

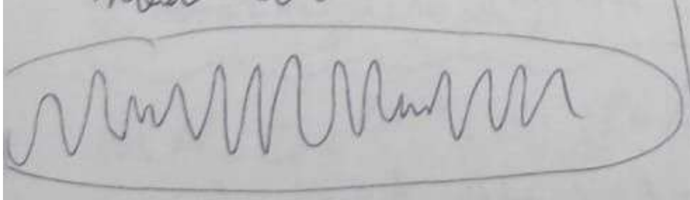
$$\Rightarrow = \underbrace{\underbrace{\frac{2A}{4}\cos\left(\frac{\omega_1+\omega_2}{2}t\right)}_{\text{purtător}}\underbrace{\cos\left(\frac{\omega_1-\omega_2}{2}t\right)}_{\text{modulator}}}_{\text{semnal}}$$

$$\boxed{T_b = \frac{2\pi}{|\omega_1-\omega_2|}}$$





modulator



purtător

purtătorul e încadrat de modulator

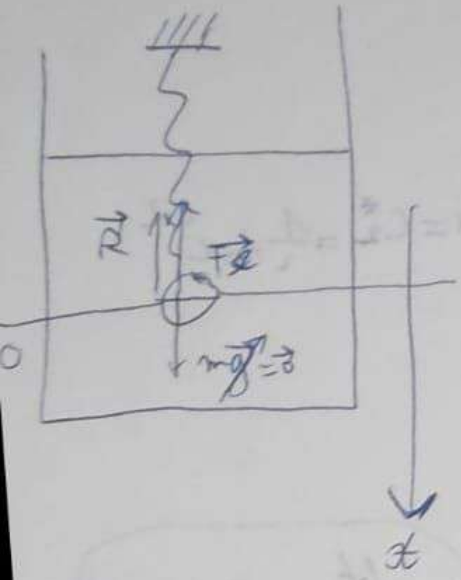
$$\boxed{T_p = \frac{4\pi}{\omega_1+\omega_2}}$$

$$N = \frac{T_b}{T_p} = \frac{2\pi}{(\omega_1 - \omega_2)} \cdot \frac{\omega_1 + \omega_2}{4\pi} = \frac{\frac{2\pi}{T_1} + \frac{2\pi}{T_2}}{2\left(\frac{2\pi}{T_1} - \frac{2\pi}{T_2}\right)} = \frac{T_1 + T_2}{2(T_2 + T_1)} > 0$$

$$\begin{cases} T_2 = T_1 \\ T_2 = T_a \end{cases}$$

T_a $\qquad$ T_1

Oscilatii amortizate



ex

$$m\vec{a} = \vec{F}_e + \vec{R}$$

$$\vec{F}_e = -k\vec{x}$$

$$\vec{R} = -r\vec{v} \quad r > 0 \text{ c. rezistenta}$$

$$\left.\begin{array}{l} \vec{a} = \ddot{\vec{x}} \\ \vec{x} = x \cdot \vec{i} \end{array}\right| \Rightarrow \vec{a} = \frac{d^2\vec{x}}{dt^2} = \frac{d^2(x\vec{i})}{dt^2} = \ddot{x}\vec{i}$$

$$\vec{R} = -r\dot{x}\vec{e}$$

$$m\ddot{x} + kx + r\dot{x} = 0 \,\Big|\, \frac{1}{m}$$

$$\ddot{x} + \frac{r}{m}\dot{x} + \frac{k}{m}x = 0 \quad ; \quad q_{1,2} = -b \pm \sqrt{b^2 - m^2}$$

$$x(t) = Ce^{qt} \quad ; q, C, ct$$

$$q_{1,2} = \frac{-2b \pm \sqrt{4b^2 - 4m^2}}{2}.$$

$$q^2 m + 2bqx + \omega^2 x = 0$$

$$x(t) = c_1 e^{q_1 t} + c_2 e^{q_2 t} = e^{-bt}\left(c_1 e^{\sqrt{b^2-\omega^2}\,t} + c_2 e^{-\sqrt{b^2-\omega^2}\,t}\right)$$

$$\boxed{= x(t)} \quad !!!$$

$$b \neq \omega$$

(a) Oscilații slab amortizate (pseudo-periodice)

$\omega > b$

$$\sqrt{b^2-\omega^2} = \sqrt{i^2(\underbrace{\omega^2-b^2}_{\omega'^2})} = \sqrt{i^2\omega'^2} = \pm i\omega'$$

$$\Rightarrow x(t) = e^{-bt}\left(C_1 e^{i\omega' t} + C_2 e^{-i\omega' t}\right) \in \mathbb{R}$$

$$\underset{\overset{\shortparallel}{\overline{C_2}}}{C} e^{i\omega' t} = \overline{C_2 e^{-i\omega' t}} = C_2^* e^{i\omega' t}$$

$$C_1 = C_2^* = \frac{1}{2} A_0 e^{i\alpha}$$

$$x(t) = e^{-bt}\left(\frac{A_0}{2} e^{i(\omega' t+\alpha)} + \frac{A_0}{2} e^{-i(\omega' t+\alpha)}\right)$$

$$= A_0 e^{-bt}\, \frac{e^{i(\omega' t+\alpha)} + e^{-i(\omega' t+\alpha)}}{2} = A_0 e^{-bt}\cos(\omega' t+\alpha)$$

$$\begin{cases} x(t) = A_0 e^{-bt}\cos(\omega' t+\alpha) \\ \dot{x}(t) = A_0\left[-b e^{-bt}\cos(\omega' t+\alpha) - e^{-bt}\omega' \cos(\omega' t+\alpha)\right] \end{cases}$$

$$A_0, \alpha = ? \rightarrow c_0\ i = \begin{cases} x(0) = x_0 \\ \dot{x}(0) = v_0 \end{cases}$$

$$\ln \left| \frac{x(t)}{x(t+T')} = e^{bT'} \Rightarrow \ln e^{bT'} = bT' = D \right.$$

$t := \text{timp}$

$\tau \to t.$ relaxare

$$A_0 e^{-bt} = A(t) \longrightarrow \frac{A(t)}{A(t+\tau)} := e \leftrightarrow e^{b\tau} = e \Rightarrow b = \frac{1}{\tau}$$

$$\frac{1}{D} \boxed{= \frac{1}{bT'} = \frac{\tau}{T'}}$$