Seminar 11 Analiză

① Ex: $f: \mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = x^2 + xy + 4y^2 + yz + z^2$

$\frac{\partial f}{\partial x} = 2x + y = 0$

$\frac{\partial f}{\partial y} = x + 8y + z = 0$

$\frac{\partial f}{\partial z} = y + 2z = 0$

$\begin{vmatrix} 2 & 1 & 0 \\ 1 & 8 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 32 - 2 - 2 - 28 \neq 0$

$\Rightarrow x = y = z = 0$

$f'' = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 8 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

$\Delta_1 = 2 \quad +$
$\Delta_2 = 15 \quad +$
$\Delta_3 = 28 \quad +$

$(0,0,0)$ pct de min

② Ex: $f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^4 + y^4 + 4xy$

$\frac{\partial f}{\partial x} = 4x^3 + 4y = 0$

$\frac{\partial f}{\partial y} = 4y^3 + 4x = 0$

$\begin{cases} x^3 + y = 0 \Rightarrow y = -x^3 \\ y^3 + x = 0 \Rightarrow -x^9 + x = 0 \end{cases}$

$x^9 = x \Rightarrow x \in \{0, \pm 1\}$

$(0,0)\ (1,-1)\ (-1,1)$ $\Delta_1 = 12$, $\Delta_2 = 144-$

$f'' = \begin{pmatrix} 12x^2 & 4 \\ 4 & 12y^2 \end{pmatrix}$ $\quad f''(1,-1) = f''(-1,1) = \begin{pmatrix} 12 & 4 \\ 4 & 12 \end{pmatrix}$

$f''(0,0) = \begin{pmatrix} 0 & 4 \\ 1 & 0 \end{pmatrix}$ $\Delta_1 = 0$, $\Delta_2 = 16 < 0$

$f : (\mathbb{R}^2) \to \mathbb{R}$

$f(x,y) = \frac{x}{y} + \frac{y}{x}$

$\frac{\partial f}{\partial x} = \frac{1}{y} - \frac{y}{x^2} = 0$

$\frac{\partial f}{\partial y} = \frac{x}{y^2} + \frac{1}{x} = 0$

$f'' = \begin{pmatrix} \frac{2y}{x^3} & -\frac{1}{y^2} - \frac{1}{x^2} \\ \frac{1}{y^2} - \frac{1}{x^2} & \frac{2x}{y^3} \end{pmatrix}$

$f : \mathbb{R}^{2*} \to \mathbb{R}$, $f(x,y) = \frac{1}{x} + \frac{1}{y} + xy$

$\frac{\partial f}{\partial x} = -\frac{1}{x^2} + y = 0 \Rightarrow 1 = x^2 y$

$\frac{\partial f}{\partial y} = -\frac{1}{y^2} + x = 0 \Rightarrow 1 = xy^2$

$x, y > 0$

$f(x,y) \geq 3$

$\Rightarrow x = y = 1$

$f'' = \begin{pmatrix} \frac{2}{x^3} & 1 \\ 1 & \frac{2}{y^3} \end{pmatrix}$ $f''(1,1) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $\Delta_1 = 2 > 0$

$\Delta_2 = 3 > 0$

punct de minim local

M_1 - metoda topologică

A mărginită, închisă $= g(\{0\})$

f cont $\Rightarrow \exists (x_M, y_M, z_M)$ a.î. $f(x_M, y_M, z_M) = \sup f(A)$

$\exists (x_m, y_m, z_m)$ a.î. $f(x_m, y_m, z_m) = \inf f(A)$

$\hookrightarrow$ pct de extrem global $\Rightarrow$ pct de extrem loc