C13 CTI 16

Hipercuadrice în $\mathbb{R}^n$.

$(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$ sp. afin sau $(\mathbb{R}^n, (\mathbb{R}^n/_{\mathbb{R}}, g_0), \varphi)$ sp. afin euclidian.

S.n. hipercuadrică LG al punctelor $P(x_1, x_2, \dots, x_n)$

care verifică:

$$\Gamma : f(x_1, \dots, x_n) = a_{11}x_1^2 + \dots + a_{nn}x_n^2 + 2a_{12}x_1x_2 + \dots + 2a_{n-1\,n}x_{n-1}x_n$$
$$+ 2b_1x_1 + \dots + 2b_nx_n + c = 0$$

$$f(x_1, \dots, x_n) = X^TAX + 2BX + c = 0 .$$

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \quad A = (a_{ij})_{i,j=\overline{1,n}} = A^T, \quad B = (b_1 \dots b_n)$$

$$(x_1 \dots x_n)\begin{pmatrix} a_{11} \dots & a_{1n} \\ \vdots & \\ a_{n1} \dots & a_{nn} \end{pmatrix}\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + 2(b_1 \dots b_n)\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + c = 0$$

$$\tilde{A} = \begin{pmatrix} A & B^T \\ B & c \end{pmatrix}, \quad \delta = \det A, \quad \Delta = \det \tilde{A}, \quad r = rg\, A, \quad r' = rg\, \tilde{A}.$$

$\Delta \neq 0$ Γ s.n. nedegenerată

$\Delta = 0$ Γ s.n. degenerată

- $n = 2$ Γ conică

  $n = 3$ Γ cuadrică

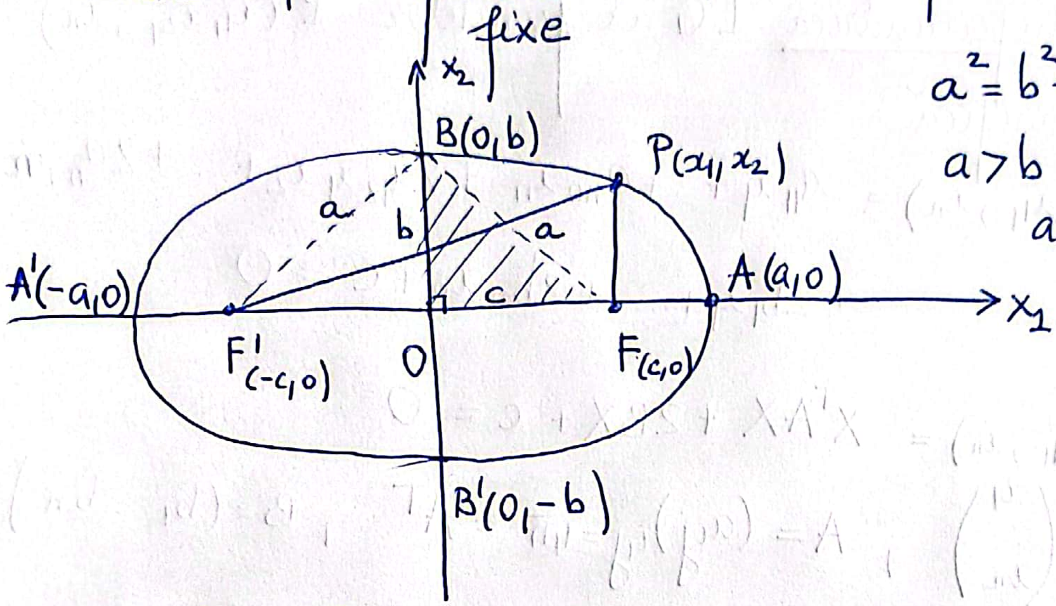
$n = 2$ $\quad \Gamma : f(x_1, x_2) = a_{11}x_1^2 + a_{22}x_2^2 + 2a_{12}x_1x_2 + 2b_1x_1 + 2b_2x_2 + c = 0$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} = A^T \ (A \neq O_2), \quad \tilde{A} = \begin{pmatrix} a_{11} & a_{12} & b_1 \\ a_{12} & a_{22} & b_2 \\ b_1 & b_2 & c \end{pmatrix}$$

$\delta = \det A, \quad \Delta = \det \tilde{A}$

Conice ca LG

① **Elipsa** este LG. al punctelor P din plan care verifică: $PF + PF' = 2a$, $a > 0$,
unde F, F' puncte fixe dist, numite focare.



$a^2 = b^2 + c^2$

$a > b$

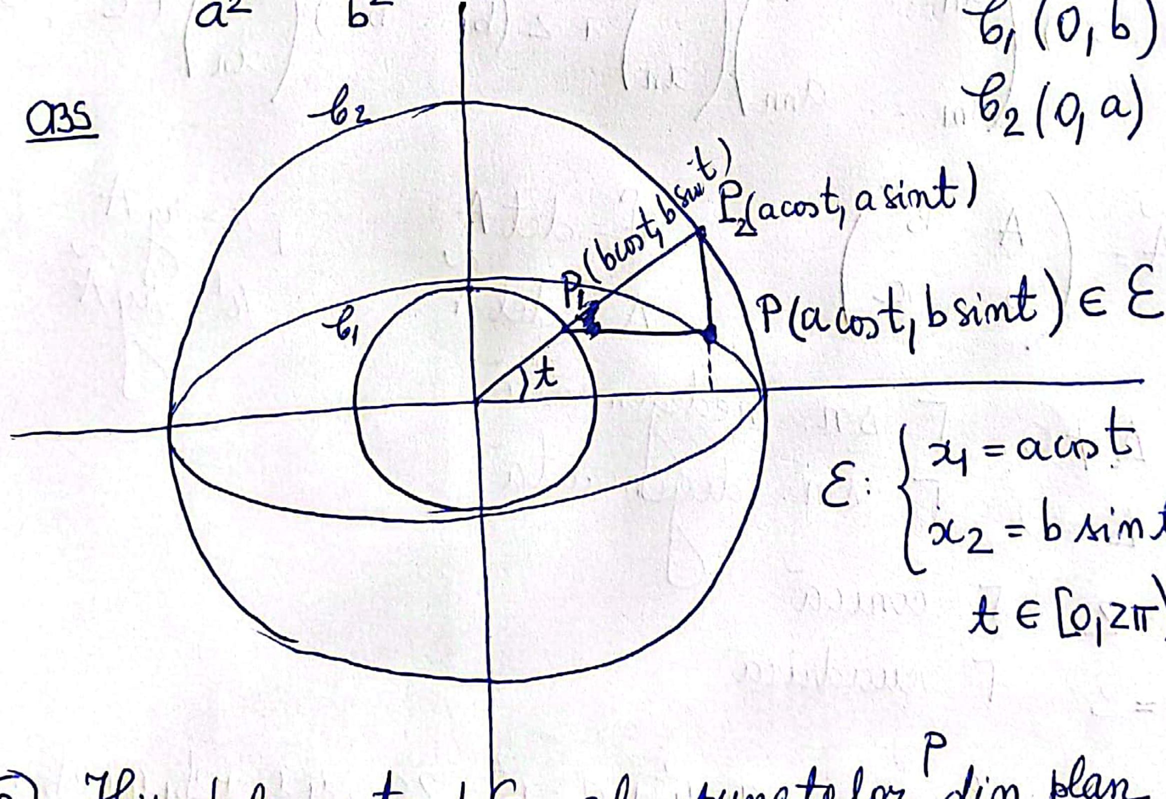
$a, b, c > 0$.

$$\mathcal{E}: \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1$$

OBS

$\mathcal{C}_1(0,b)$ $\quad a > b$.

$\mathcal{C}_2(0,a)$



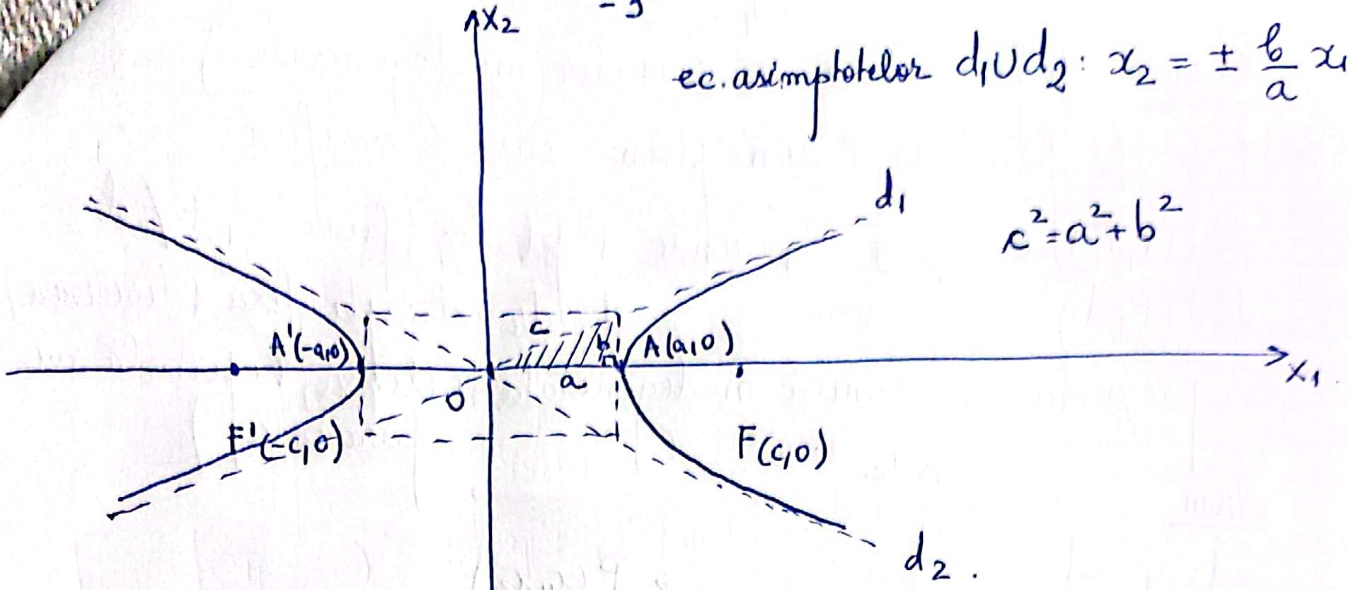
$$\mathcal{E}: \begin{cases} x_1 = a\cos t \\ x_2 = b\sin t \end{cases}$$

$t \in [0, 2\pi)$

② **Hiperbola** este LG al punctelor P din plan care verifică: $|PF - PF'| = 2a$, $a > 0$
unde F, F' puncte fixe, dist, numite focare.

ec. asimptotelor $d_1 \cup d_2 : x_2 = \pm \frac{b}{a} x_1$.

$c^2 = a^2 + b^2$



$\mathcal{H} : \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 1.$

Obs $d_1 \cup d_2 : \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 0 \Rightarrow \left(\frac{x_1}{a} - \frac{x_2}{b}\right)\left(\frac{x_1}{a} + \frac{x_2}{b}\right) = 0$

Obs $\mathcal{H} : \begin{cases} x_1 = a \operatorname{ch} t \\ x_2 = b \operatorname{sh} t, \ t \in \mathbb{R} \end{cases}$ $\quad \operatorname{ch} t = \frac{e^t + e^{-t}}{2}$ cosinus hiperbolic

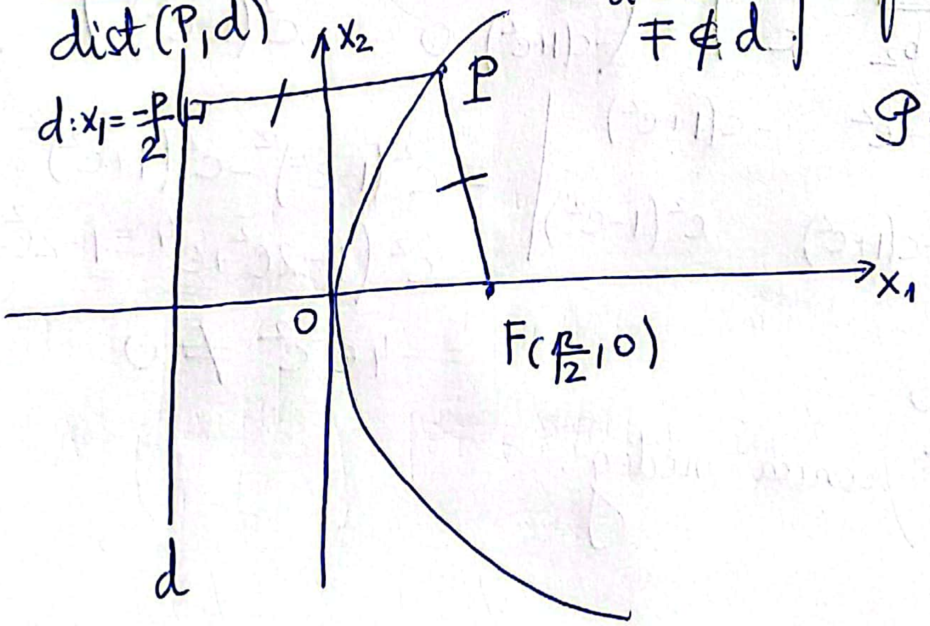
$\operatorname{sh} t = \frac{e^t - e^{-t}}{2}$ sinus —"—

$$\boxed{\operatorname{ch}^2 t - \operatorname{sh}^2 t = 1}$$

(3) Parabola = LG al punctelor P din plan care verifică $\frac{\text{dist}(P,F)}{\text{dist}(P,d)} = 1$, unde F punct fix, numit focar, d = dreaptă fixă, numită directoare, $F \notin d$.

$\mathcal{P} : x_2^2 = 2p x_1$

$p > 0 \Rightarrow x_1 > 0$.



Teoremă (def. unitară a conicelor nedegenerate)

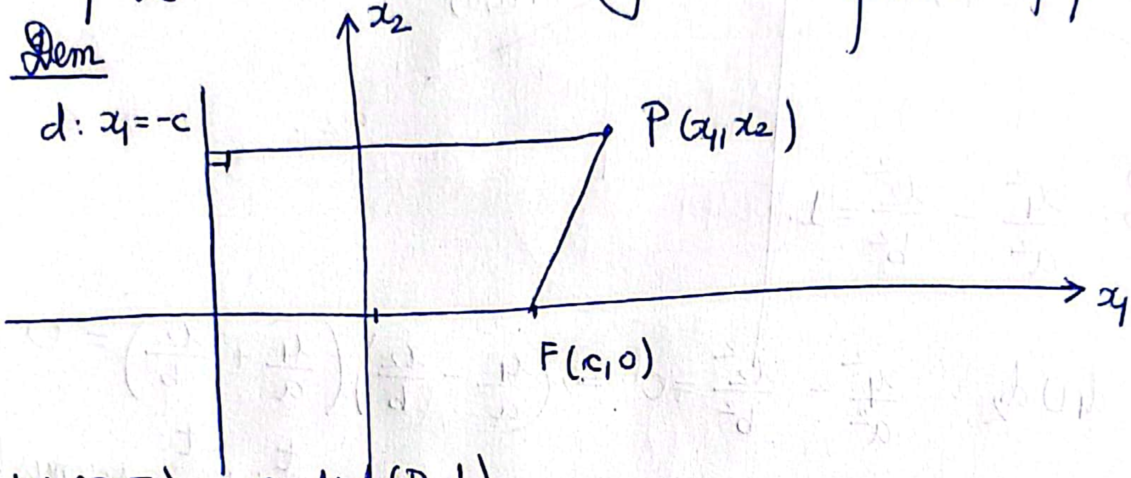
LG al punctelor P din plan care verifică

$$\frac{dist(P,F)}{dist(P,d)} = e > 0 \quad , \text{ unde } F \text{ pct fix (focar)}, F \notin d$$

d = dr dreaptă fixă (directoare)

reprezintă o conică nedegenerată (elipsă, hiperbolă sau parabolă)

Dem



$$dist(P,F) = e\, dist(P,d)$$

$$\sqrt{(x_1-c)^2+x_2^2} = e\,|x_1+c| \;\uparrow^2 \Rightarrow$$

$$\underline{x_1^2} - \underline{\underline{2cx_1}} + c^2 + \underline{x_2^2} = e^2\left(\underline{x_1^2} + \underline{\underline{2x_1c}} + c^2\right)$$

$$x_1^2(1-e^2) + x_2^2 - 2x_1c(1+e^2) + c^2(1-e^2) = 0$$

$$\Gamma:\ f(x_1,x_2) = a_{11}x_1^2 + a_{22}x_2^2 + 2a_{12}x_1x_2 + 2b_1x_1 + 2b_2x_2 + c = 0$$

$$\tilde{A} = \begin{pmatrix} \boxed{\begin{matrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{matrix}} & \begin{matrix} b_1 \\ b_2 \end{matrix} \\ \begin{matrix} b_1 & b_2 \end{matrix} & c \end{pmatrix} = \begin{pmatrix} 1-e^2 & 0 & -c(1+e^2) \\ 0 & 1 & 0 \\ -c(1+e^2) & 0 & c^2(1-e^2) \end{pmatrix}$$

$$\Delta = \det\tilde{A} = \begin{vmatrix} 1-e^2 & -c(1+e^2) \\ -c(1+e^2) & c^2(1-e^2) \end{vmatrix} = c^2(1-e^2)^2 - c^2(1+e^2)^2$$

$$= c^2\left(\underline{1} - 2e^2 + \underline{\underline{e^4}} - \underline{1} - 2e^2 - \underline{\underline{e^4}}\right)$$

$$= -4e^2c^2 \neq 0$$

$F \notin d \Rightarrow c \neq 0$

$\Delta \neq 0 \Rightarrow \Gamma$ conică nedeg.

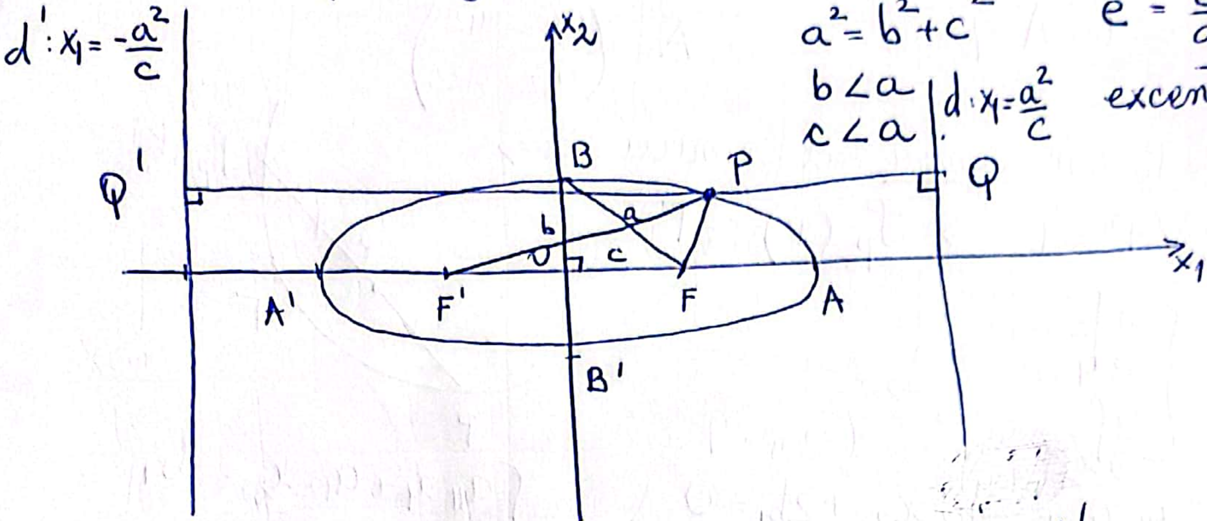
OBS

1) $\mathcal{E}: \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1.$

$a^2 = b^2 + c^2$ $\quad e = \frac{c}{a} \in (0,1)$

$b < a$ $\quad$ excentricitate

$c < a$



$\frac{a^2}{c} > a \Leftrightarrow \frac{a}{c} > 1 \Leftrightarrow a > c$ A

$d \cup d': x_1 = \pm \frac{a^2}{c}$

directoare

$\frac{PF}{PQ} = e = \frac{c}{a} \Rightarrow PF = \frac{c}{a} PQ$

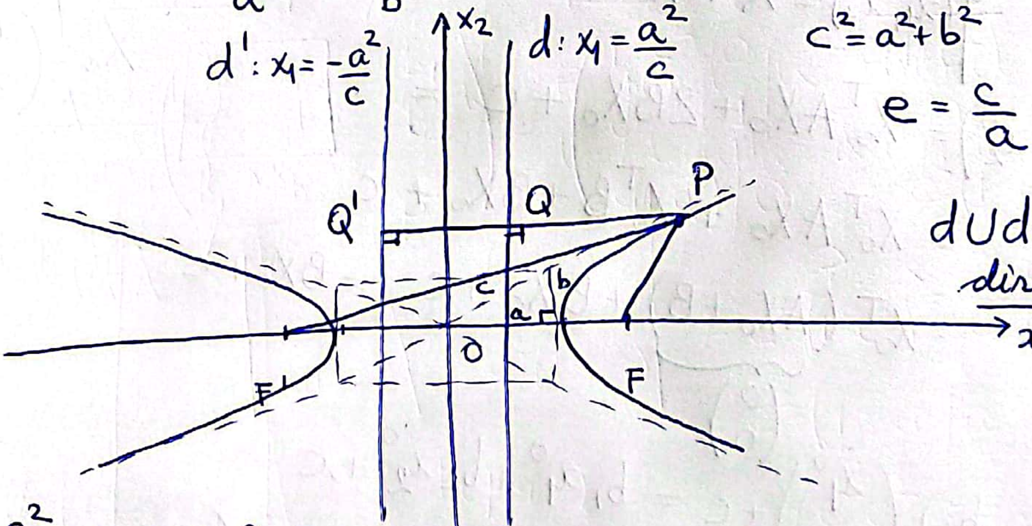
$\frac{PF'}{PQ'} = e = \frac{c}{a} \Rightarrow PF' = \frac{c}{a} PQ'$ $\quad$ +

$PF + PF' = \frac{c}{a} (\underbrace{PQ + PQ'}_{2 \cdot \frac{a^2}{c}}) = 2a$

2) $\mathcal{H}: \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 1.$

$c^2 = a^2 + b^2$, $c > a$

$e = \frac{c}{a} > 1$



$d \cup d': x_1 = \pm \frac{a^2}{c}$

directoare

$\frac{a^2}{c} < a \Leftrightarrow \frac{a}{c} < 1 \Leftrightarrow a < c$

$\left.\begin{array}{l} \frac{PF}{PQ} = e = \frac{c}{a} \Rightarrow PF = \frac{c}{a} PQ \\ \frac{PF'}{PQ'} = e = \frac{c}{a} \Rightarrow PF' = \frac{c}{a} PQ' \end{array}\right\} \Rightarrow |PF - PF'| = \frac{c}{a} |\overbrace{PQ - PQ'}^{\substack{2\frac{a^2}{c} \\ \parallel \\ QQ'}}| = 2a$

3) $\mathcal{P}: x_2^2 = 2px_1$ $\quad e = 1$

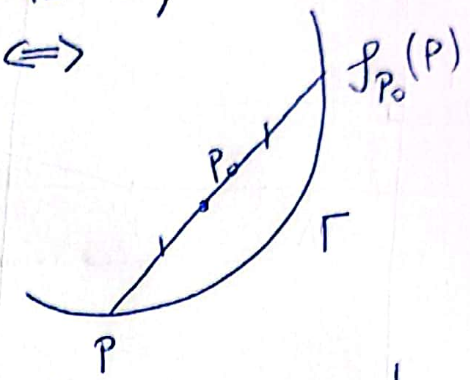
Aducerea la o formă canonică a conicelor

$\Gamma: f(x_1,x_2) = a_{11}x_1^2 + a_{22}x_2^2 + 2a_{12}x_1x_2 + 2b_1x_1 + 2b_2x_2 + c = 0$

I. $\delta = \det A \neq 0$. $\quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}$

Def P_0 s.n. centru al conicei $\Leftrightarrow$

$\forall P \in \Gamma \Rightarrow S_{P_0}(P) \in \Gamma$



$P_0 : \begin{cases} \frac{\partial f}{\partial x_1} = 0 \\ \frac{\partial f}{\partial x_2} = 0 \end{cases} \Rightarrow$

$\begin{cases} 2a_{11}x_1 + 2a_{12}x_2 + 2b_1 = 0 \\ 2a_{12}x_1 + 2a_{22}x_2 + 2b_2 = 0 \end{cases} \Rightarrow \begin{cases} a_{11}x_1 + a_{12}x_2 = -b_1 \\ a_{12}x_1 + a_{22}x_2 = -b_2 \end{cases}$

$\delta = \det A \neq 0 \quad$ SCD $\quad A = \left(\begin{matrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{matrix}\right|\left.\begin{matrix} -b_1 \\ -b_2 \end{matrix}\right)$

$\exists!$ sol $(x_1^0, x_2^0) = \left(\frac{\begin{vmatrix} -b_1 & a_{12} \\ -b_2 & a_{22} \end{vmatrix}}{\delta}, \frac{\begin{vmatrix} a_{11} & -b_1 \\ a_{12} & -b_2 \end{vmatrix}}{\delta} \right)$ (met Cramer)

Prop Fie Γ o conică cu centru unic $P_0(x_1^0, x_2^0)$ $(\delta \neq 0)$

$\Rightarrow f(x_1^0, x_2^0) = \frac{\Delta}{\delta}$.

Dem $\Gamma: f(x_1,x_2) = X^TAX + 2BX + c = 0 \qquad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$X_0 = \begin{pmatrix} x_1^0 \\ x_2^0 \end{pmatrix}$

$f(x_1^0, x_2^0) = X_0^TAX_0 + 2BX_0 + c =$

$= X_0^TAX_0 + X_0^TB + BX_0 + c$

$= X_0^T \underbrace{(AX_0 + B)}_{\begin{pmatrix} 0 \\ 0 \end{pmatrix}} + BX_0 + c = BX_0 + c$

$= (b_1 \quad b_2)\begin{pmatrix} x_1^0 \\ x_2^0 \end{pmatrix} + c = b_1x_1^0 + b_2x_2^0 + c$

$= b_1 \frac{\begin{vmatrix} -b_1 & a_{12} \\ -b_2 & a_{22} \end{vmatrix}}{\delta} + b_2 \frac{\begin{vmatrix} a_{11} & -b_1 \\ a_{12} & -b_2 \end{vmatrix}}{\delta} + c = \frac{1}{\delta}\left(-b_1 \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} - b_2 \begin{vmatrix} a_{11} & b_1 \\ a_{12} & b_2 \end{vmatrix} + c\delta \right) = \frac{\Delta}{\delta}$.

$$\Delta = \det \tilde{A} \stackrel{\text{ad}}{=} \begin{pmatrix} A & B^T \\ B & C \end{pmatrix} = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{12} & a_{22} & b_2 \\ b_1 & b_2 & c \end{vmatrix}$$

$$= b_1 \begin{vmatrix} a_{12} & a_{22} \\ b_1 & b_2 \end{vmatrix} - b_2 \begin{vmatrix} a_{11} & a_{12} \\ b_1 & b_2 \end{vmatrix} + c \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}$$

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1 $(\mathbb{R}^2, \mathbb{R}^2/_{\mathbb{R}}, \varphi)$ sp. afin.

$$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{translatie}]{\theta} \mathcal{R}' = \{P_0; e_1, e_2\} \xrightarrow[\text{transf. afina}]{\tau} \mathcal{R}'' = \{P_0; e_1', e_2'\}$$

$\theta: X = X' + X_0$, $P_0(x_1^0, x_2^0)$ centrul unic.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} + \begin{pmatrix} x_1^0 \\ x_2^0 \end{pmatrix}$$

$\Gamma: f(x_1, x_2) = X^T A X + 2BX + c = 0$

$\theta(\Gamma): (X' + X_0)^T A (X' + X_0) + 2B(X' + X_0) + c = 0$

OBS: $f(x_1^0, x_2^0) = \dfrac{\Delta}{\delta} \Leftrightarrow X_0^T A X_0 + 2BX_0 + c = \dfrac{\Delta}{\delta}$

$AX_0 + B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\theta(\Gamma): \underline{X'^T A X'} + \dfrac{\Delta}{\delta} = 0$

$Q: \mathbb{R}^2 \to \mathbb{R}$, $Q(x) = a_{11} x_1'^2 + 2a_{12} x_1' x_2' + a_{22} x_2'^2$

Aducem forma pătratică Q la o formă canonică (met Gauss / met Jacobi).

$$Q(x) = \lambda_1 x_1''^2 + \lambda_2 x_2''^2$$

$\tau: X' = CX''$, $C \in GL(2, \mathbb{R})$ transf afină

$\tau(\theta(\Gamma)): \lambda_1 x_1''^2 + \lambda_2 x_2''^2 + \dfrac{\Delta}{\delta} = 0.$

$\lambda_1 x_1''^2 + \lambda_2 x_2''^2 + \frac{\Delta}{\delta} = 0.$

a) $\Delta \neq 0$ (Γ nedegenerată) $\nearrow \delta = \lambda_1\lambda_2 > 0$ ϕ, Elipsa
$\searrow \delta = \lambda_1\lambda_2 < 0$ Hiperbola

b) $\Delta = 0 \Rightarrow \lambda_1 x_1''^2 + \lambda_2 x_2''^2 = 0$ $\rightarrow \delta = \lambda_1\lambda_2 > 0$ pct dublu.
$\searrow \delta = \lambda_1\lambda_2 < 0$ drepte concurente

(2) $(\mathbb{R}^2, (\mathbb{R}^2/_{\mathbb{R}}, g_0), \varphi)$ sp. afin euclidian.

$\mathcal{R} = \{0; e_1, e_2\} \longrightarrow \mathcal{R}' = \{P_0; e_1, e_2\} \longrightarrow \mathcal{R}'' = \{P_0; e_1', e_2'\}$

$\{e_1', e_2'\}$ reper orton.

$\theta: X = X' + X_0$ translatie

$\theta(\Gamma): \quad f(x) = X'^T A X' + \frac{\Delta}{\delta} = 0$

$Q: \mathbb{R}^2 \longrightarrow \mathbb{R}, \quad Q(x) = a_{11}x_1'^2 + 2a_{12}x_1'x_2' + a_{22}x_2'^2$

Aducem Q la o f. canonică utilizând met. valorilor proprii

$\det(A - \lambda I_2) = 0 \Rightarrow \lambda^2 - Tr(A)\lambda + \underset{\delta \neq 0}{\det A} = 0$

a) $\lambda_1 \neq \lambda_2$

$V_{\lambda_k} = \langle\{e_k'\}\rangle$ e_k' = versorul propriu coresp. lui λ_k, $k = \overline{1,2}$

$e_1' \perp e_2'$ $\{e_1', e_2'\}$ reper ortonormat.

$e_k' = (l_k, m_k)$, $k = \overline{1,2}$

$\tau: X' = RX''$ $\quad R = \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \in SO(2)$

(alegem R a.î. $\det R = 1$)

$\tau \circ \theta: X = X' + X_0 = RX'' + X_0$ izometrie.

$\tau \circ \theta(\Gamma): \quad \lambda_1 x_1''^2 + \lambda_2 x_2''^2 + \frac{\Delta}{\delta} = 0$

b) λ_1 cu multiplicitatea 2 $(\lambda_1 = \lambda_2)$

$V_{\lambda_1} = \langle\{e_1', e_2'\}\rangle$ (Gram - Schmidt)

Analog.

$$-9-a_{11}x_1^2+2a_{12}x_1x_2+a_{22}x_2^2 \quad 2b_1x_1+2b_2x_2 \quad c$$

Exemplu

Fie conica $\Gamma: f(x_1,x_2) = \overbrace{7x_1^2-8x_1x_2+x_2^2}\overbrace{-6x_1-12x_2}\overbrace{-9} = 0$

Să se aducă la o formă canonică, efectuând izometrii, în spațiul euclidian $(\mathbb{R}^2, (\mathbb{R}^2/_{\mathbb{R}}, g_0), \varphi)$.

Sol

$$A = \begin{pmatrix} 7 & -4 \\ -4 & 1 \end{pmatrix}, \quad \delta = \det A = 7-16 = -9 \neq 0$$

Γ are centru unic

$$\tilde{A} = \begin{pmatrix} 7 & -4 & \boxed{-3} \\ -4 & 1 & \boxed{-6} \\ \boxed{-3 \quad -6} & & -9 \end{pmatrix}, \quad \Delta = \det\tilde{A} = 9\begin{vmatrix} 7 & -4 & -1 \\ -4 & 1 & -2 \\ -1 & -2 & -1 \end{vmatrix} \quad \begin{matrix} L_2' = L_2-2L_1 \\ L_3' = L_3 - L_1 \end{matrix}$$

$$= 9\begin{vmatrix} 7 & -4 & -1 \\ -18 & 9 & 0 \\ -8 & 2 & 0 \end{vmatrix} = (-1)\cdot 9^2\cdot 2\underbrace{\begin{vmatrix} -2 & 1 \\ -4 & 1 \end{vmatrix}}_{-2+4} = -9^2\cdot 4 \neq 0$$

Γ este conică nedeg.

$$P_0: \begin{cases} 14x_1 - 8x_2 - 6 = 0 \\ -8x_1 + 2x_2 - 12 = 0 \end{cases} \Rightarrow \begin{cases} 7x_1 - 4x_2 = 3 \\ -4x_1 + x_2 = 6 \quad \cdot 4 \end{cases} \oplus$$

$$-9x_1 \quad / \quad = 27 \Rightarrow \boxed{x_1^0 = -3}$$

$$x_2 = 6+4(-3) = 6-12 = -6 \Rightarrow \boxed{x_2^0 = -6}$$

$$P_0(-3,-6)$$

$$\mathcal{R} = \{O; e_1, e_2\} \xrightarrow[\text{translatie}]{\theta} \mathcal{R}' = \{P_0; e_1, e_2\}$$

$$\theta: X = X' + X_0, \quad X_0 = \begin{pmatrix} -3 \\ -6 \end{pmatrix} \quad \begin{cases} x_1 = x_1' - 3 \\ x_2 = x_2' - 6 \end{cases}$$

$$\theta(\Gamma): X'^T A X' + \frac{\Delta}{\delta} = 0$$

$$Q: \mathbb{R}^2 \longrightarrow \mathbb{R}, \quad Q(x) = 7x_1'^2 - 8x_1'x_2' + x_2'^2$$

$$A = \begin{pmatrix} 7 & -4 \\ -4 & 1 \end{pmatrix}$$

Met. valorilor proprii

$$\det(A - \lambda I_2) = 0$$

$$\lambda^2 - Tr(A)\lambda + \det(A) = 0$$

$$\lambda^2 - 8\lambda - 9 = 0 \Rightarrow (\lambda+1)(\lambda-9) = 0$$

$$Q(x) = -x_1''^2 + 9x_2''^2. \qquad \lambda_1 = -1, \quad \lambda_2 = 9$$

$$\mathcal{R}' = \{P_0, e_1, e_2\} \xrightarrow[\text{rotație}]{\tau} \mathcal{R}'' = \{P_0; e_1', e_2'\}$$

ortonormat

$\{e_1', e_2'\}$ reper

$$V_{\lambda_1} = \{x \in \mathbb{R}^2 \mid AX = -X\} = \{(x_1, 2x_1),\ x_1 \in \mathbb{R}\} = \langle\{(1,2)\}\rangle$$

$$(A + I_2)X = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \begin{pmatrix}8 & -4\\-4 & 2\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$A = \begin{pmatrix}7 & -4\\-4 & 1\end{pmatrix}$$

$$\begin{cases}8x_1 - 4x_2 = 0\\-4x_1 + 2x_2 = 0\end{cases} \Rightarrow x_2 = 2x_1.$$

$$e_1' = \frac{1}{\sqrt5}(1,2)$$

$$V_{\lambda_2} = \{x \in \mathbb{R}^2 \mid AX = 9X\} = \{(-2x_2, x_2) / x_2 \in \mathbb{R}\} = \langle\{(-2,1)\}\rangle$$

$$(A - 9I_2)X = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \begin{pmatrix}-2 & -4\\-4 & -8\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$e_2' = \frac{1}{\sqrt5}(-2,1)$$

$$\begin{cases}-2x_1 - 4x_2 = 0\\-4x_1 - 8x_2 = 0\end{cases} \Rightarrow x_1 = -2x_2$$

$$\tau: X' = RX'' \qquad R = \frac{1}{\sqrt5}\begin{pmatrix}1 & -2\\2 & 1\end{pmatrix} \in SO(2)$$

$$\mathbb{R}^2 = V_{\lambda_1} \oplus V_{\lambda_2}, \quad V_{\lambda_2} = V_{\lambda_1}^{\perp}$$

$\{e_1', e_2'\}$ la fel orientat ca și reperul canonic.

$$\tau(\theta): X = X' + X_0$$
$$= RX'' + X_0$$

$$\tau \circ \theta: \begin{pmatrix}x_1\\x_2\end{pmatrix} = \underbrace{\frac{1}{\sqrt5}\begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}}_{R}\begin{pmatrix}x_1''\\x_2''\end{pmatrix} + \begin{pmatrix}-3\\-6\end{pmatrix}$$ izometrie.

$$R^T\begin{pmatrix}x_1+3\\x_2+6\end{pmatrix} = \begin{pmatrix}x_1''\\x_2''\end{pmatrix} \Rightarrow \begin{pmatrix}x_1''\\x_2''\end{pmatrix} = \frac{1}{\sqrt5}\begin{pmatrix}1 & 2\\-2 & 1\end{pmatrix}\begin{pmatrix}x_1+3\\x_2+6\end{pmatrix}$$

$$\tau \circ \theta(\Gamma): -x_1''^2 + 9x_2''^2 + \frac{-9^2 \cdot 4}{-9} = 0 \Rightarrow -x_1''^2 + 9x_2''^2 + 36 = 0$$

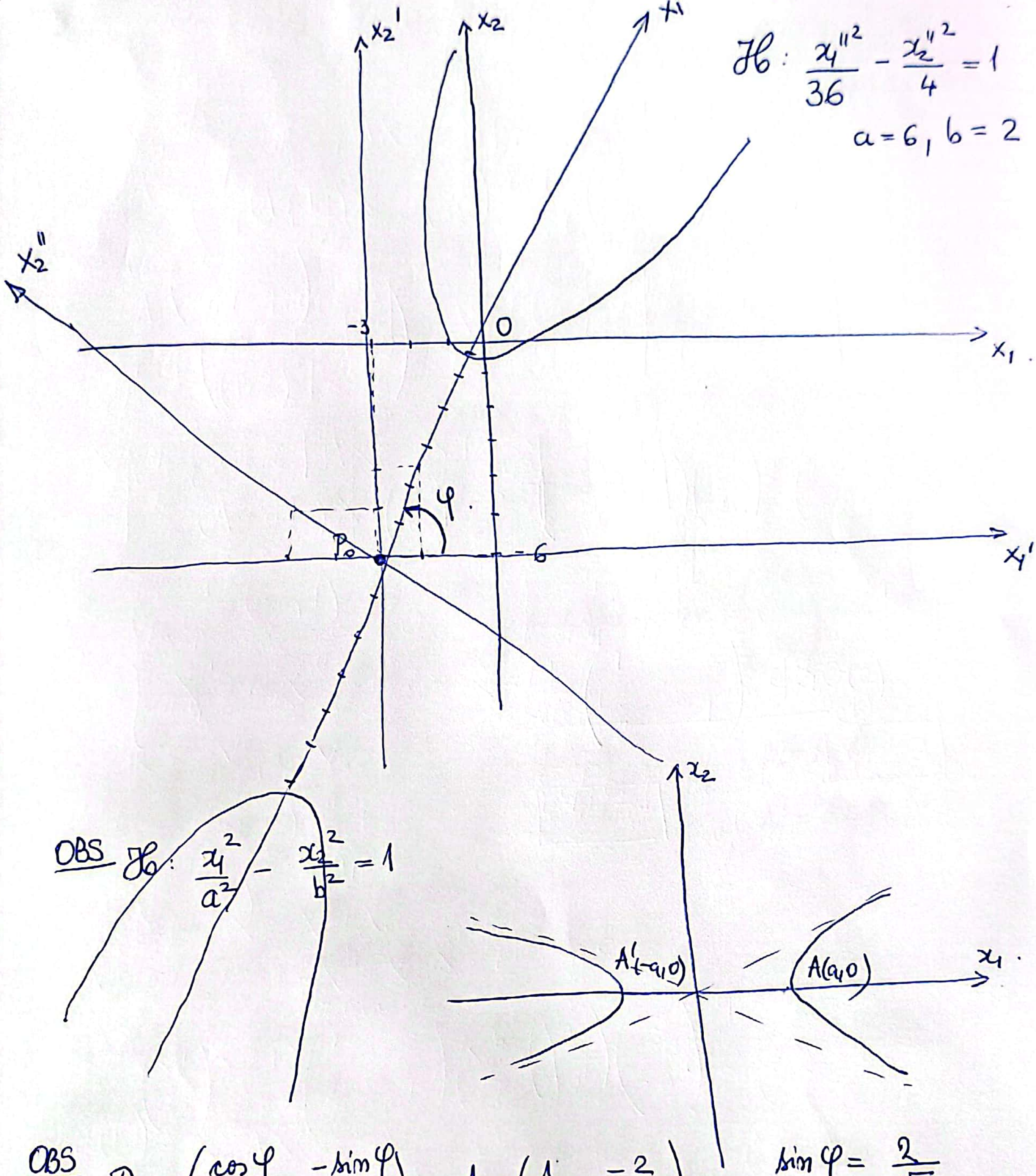
$$\mathcal{H}: \frac{x_1''^2}{36} - \frac{x_2''^2}{4} = 1$$

$\mathcal{R} = \{O; e_1, e_2\} \longrightarrow \mathcal{R}' = \{P_0(-3,-6); e_1, e_2\} \longrightarrow \mathcal{R}'' = \{P_0; e_1', e_2'\}$

$e_1'' = \frac{1}{\sqrt{5}}(1,2)$, $e_2'' = \frac{1}{\sqrt{5}}(-2,1)$

$\mathcal{H}: \dfrac{x_1''^2}{36} - \dfrac{x_2''^2}{4} = 1$

$a = 6,\ b = 2$



<u>OBS</u> $\mathcal{H}: \dfrac{x_1^2}{a^2} - \dfrac{x_2^2}{b^2} = 1$

<u>OBS</u> $R = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ $\quad \sin\varphi = \frac{2}{\sqrt{5}}$

$\cos\varphi = \frac{1}{\sqrt{5}}$

$\operatorname{tg}\varphi = 2 \qquad \varphi \in [-\pi, \pi]$.

$\varphi = \operatorname{arctg} 2$.