

Laborator Proiectare Logică 4

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1 Forme Canonice

#	A	B	C	D	FCD, \sum, n	FCC, Π, m
0	0	0	0	0	$\bar{A}\bar{B}\bar{C}\bar{D}$	$A + B + C + D$
1	0	0	0	1	$\bar{A}\bar{B}\bar{C}D$	$A + B + C + \bar{D}$
2	0	0	1	0	$\bar{A}\bar{B}C\bar{D}$	$A + B + \bar{C} + D$
3	0	0	1	1	$\bar{A}\bar{B}CD$	$A + B + \bar{C} + \bar{D}$
4	0	1	0	0	$\bar{A}B\bar{C}\bar{D}$	$A + \bar{B} + C + D$
5	0	1	0	1	$\bar{A}B\bar{C}D$	$A + \bar{B} + C + \bar{D}$
6	0	1	1	0	$\bar{A}BC\bar{D}$	$A + \bar{B} + \bar{C} + D$
7	0	1	1	1	$\bar{A}BCD$	$A + \bar{B} + \bar{C} + \bar{D}$
8	1	0	0	0	$A\bar{B}\bar{C}\bar{D}$	$\bar{A} + B + C + D$
9	1	0	0	1	$A\bar{B}\bar{C}D$	$\bar{A} + B + C\bar{D}$
10	1	0	1	0	$A\bar{B}C\bar{D}$	$\bar{A} + B + \bar{C} + D$
11	1	0	1	1	$A\bar{B}CD$	$\bar{A} + B + \bar{C} + \bar{D}$
12	1	1	0	0	$AB\bar{C}\bar{D}$	$\bar{A} + \bar{B} + C + D$
13	1	1	0	1	$AB\bar{C}D$	$\bar{A} + \bar{B} + C + \bar{D}$
14	1	1	1	0	$ABC\bar{D}$	$\bar{A} + \bar{B} + \bar{C} + D$
15	1	1	1	1	$ABCD$	$\bar{A} + \bar{B} + \bar{C} + \bar{D}$

Exerciții

Exercițiul 1:

$y : 2^2 \rightarrow 2^1; y = A(A + \bar{B})$

Rezolvare: $FCD_y = \sum(2, 3) = n(2, 3) = A\bar{B} + AB$

$FCC_y = \Pi(0, 1) = m(0, 1) = (A + B)(A + \bar{B})$

$y = A\bar{B} + AB = A(\bar{B} + B) = A \times 1 = A$

$y = (A + B)(A + \bar{B}) = A^2 + A\bar{B} + BA + B\bar{B} =$

$A + A\bar{B} + AB + 0 = A(1 + B + \bar{B}) = A$

A	B	\bar{B}	$A + \bar{B}$	y
0	0	1	1	0
0	1	0	0	0
1	0	1	1	1
1	1	0	1	1

Exercițiul 2:

$$f, g : 2^2 \rightarrow 2^1$$

$$f = \bar{A}\bar{B} ; g = \bar{A} + \bar{B}$$

$$f = g?$$

$$\text{Rezolvare: } FCD_y = \sum(0, 1, 2) = n(0, 1, 2) =$$

$$\bar{A}\bar{B} + \bar{A}B + A\bar{B}$$

$$FCC_y = \Pi(3) = m(3) = \bar{A} + \bar{B}$$

Din tabel se observă că f=g

#	A	B	\bar{A}	\bar{B}	AB	$\bar{A}\bar{B}$	f	g
0	0	0	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1
2	1	0	0	1	0	1	1	1
3	1	1	0	0	1	0	0	0

Exercițiul 3:

$$f, g : 2^2 \rightarrow 2^1$$

$$f = A + B ; g = \bar{A}\bar{B}$$

$$f = g?$$

$$\text{Rezolvare: } FCD_y = \sum(0) = n(0) = \bar{A}\bar{B}$$

$$FCC_y = \Pi(1, 2, 3) = m(1, 2, 3)$$

$$= (A + \bar{B})(\bar{A} + B)(\bar{A} + \bar{B})$$

Din tabel se observă că f=g

#	A	B	\bar{A}	\bar{B}	AB	$\bar{A}\bar{B}$	f	g
0	0	0	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1
2	1	0	0	1	0	1	1	1
3	1	1	0	0	1	0	0	0

Exercițiul 4:

$$y : 2^3 \rightarrow 2^1$$

Se dă tabelul

$$\text{Rezolvare: } FCD_y = \sum(1, 2, 5, 6)$$

$$= (\bar{A}\bar{B}C) + (\bar{A}B\bar{C}) + (A\bar{B}C) + (AB\bar{C})$$

$$FCC_y = \Pi(0, 3, 4, 7)$$

$$= (A + B + C)(A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C})$$

$$\begin{aligned} \text{(a) } y &= \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}C + B\bar{C}) \\ &= (\bar{A} + A)(\bar{B}C + B\bar{C}) = (\bar{B}C + B\bar{C}) \\ &= B \oplus C \end{aligned}$$

$$\begin{aligned} \text{(b) } y &= (A + B + C)(A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C}) \\ &= (AA + A\bar{B} + A\bar{C} + AB + B\bar{B} + B\bar{C} + AC + \bar{B}C + C\bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C}) \\ &= (A + A(\bar{B} + \bar{C}) + A(\bar{C} + C) + B\bar{C} + \bar{B}C)(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C}) \\ &= (A(1 + \bar{B} + \bar{C}) + A(\bar{C} + C) + B\bar{C} + \bar{B}C)(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C}) \\ &= (A + A + B \oplus C)(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C}) = (A + A + B \oplus C)(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C}) \\ &= (A + B \oplus C)(\bar{A}\bar{A} + \bar{A}\bar{B} + \bar{A}\bar{C} + B\bar{A} + B\bar{B} + B\bar{C} + C\bar{A} + C\bar{B} + C\bar{C}) = (A + B \oplus C)(\bar{A}(\bar{B} + \bar{C} + 1 + B + C) + B\bar{C} + C\bar{B}) \\ &= (A + B \oplus C)(\bar{A} + B \oplus C) \\ &= A\bar{A} + A(B \oplus C) + \bar{A}(B \oplus C) + (B\bar{C} + \bar{B}C)(B\bar{C} + \bar{B}C) \\ &= 0 + (B \oplus C)(A + \bar{A}) + B^2C^2 + B\bar{B}C\bar{C} + \bar{B}B\bar{C}C + B^2C^2 \\ &= B \oplus C + B\bar{C} + \bar{B}C = B \oplus C + B \oplus C = B \oplus C \end{aligned}$$

#	A	B	C	y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Exercițiul 5:

$$y : 2^4 \rightarrow 2^1$$

$$y = \sum(0, 2, 5, 7, 8, 10, 13, 15)$$

$$y = \prod(1, 3, 4, 6, 9, 11, 12, 14)$$

Rezolvare:

$$FCD_y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + ABC\bar{D}$$

$$FCC_y = (A + B + C + \bar{D})(A + B + \bar{C} + \bar{D})(A + \bar{B} + C + D)(A + \bar{B} + \bar{C} + D)(\bar{A} + B + C + \bar{D})(\bar{A} + B + \bar{C} + \bar{D})(\bar{A} + \bar{B} + C + D)$$

#	A	B	C	D	y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	1

Exercițiul 5 B:

$$y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + ABC\bar{D}$$

$$= \bar{B}\bar{D}(\bar{A}\bar{C} + \bar{A}C + A\bar{C} + AC) + BD(\bar{A}\bar{C} + \bar{A}C + A\bar{C} + AC)$$

$$= (\bar{A}\bar{C} + \bar{A}C + A\bar{C} + AC)(\bar{B}\bar{D} + BD)$$

$$= (\bar{A}(\bar{C} + C) + A(\bar{C} + C))(\bar{B}\bar{D} + BD)$$

$$= (\bar{A} + A)(\bar{B}\bar{D} + BD) = \bar{B}\bar{D} + BD$$

$$= B \oplus D$$

B	D	$\bar{B}\bar{D} + BD$
0	0	1+0=1
0	1	0+0=0
1	0	0+0=0
1	1	0+1=1