Oscilatii fortate

$$\ddot{x} + \frac{r}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}\cos \Omega t \qquad ; \quad x = x_0 + x_p$$

$$\ddot{x} + 2b\dot{x} + w^2 x = \frac{F_0}{m}\cos \Omega t$$

$$x_p = B\cos(\Omega t + \beta)$$

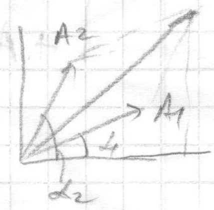
$$(-\Omega^2 + w^2)\cos(\Omega t + \beta) - 2b\Omega B \sin(\Omega t + \beta) = \frac{F_0}{m}\cos \Omega t$$

$$\underbrace{B(-\Omega^2 + w^2)}_{A_1}\cos(\Omega t + \underbrace{\beta}_{\alpha_1}) + \underbrace{2b\Omega B}_{A_2}\cos(\Omega t + \underbrace{\beta + \tfrac{\pi}{2}}_{\alpha_2}) = \frac{F_0}{m}\cos \Omega t$$

$w > \Omega$

$$A_1\cos(\Omega t + \alpha_1) + A_2\cos(\Omega t + \alpha_2) = \frac{F_0}{m}\cos \Omega t = A\cos(\Omega t + \alpha)$$

$$A = \frac{F_0}{m} = \sqrt{A_1^2 + A_2^2 + 2A_1A_1\cos(\underbrace{\alpha_2 - \alpha_1}_{\frac{\pi}{2}})} = \sqrt{A_1^2 + A_2^2}$$



$$\begin{cases} \cos\alpha = \cos 0 = 1 = \dfrac{A_1\cos\beta + A_2\cos(\beta + \frac{\pi}{2})}{\sqrt{A_1^2 + A_2^2}} \\ \sin\alpha = 0 = A_1\sin\beta + A_2\sin(\beta + \frac{\pi}{2}) \end{cases}$$

$$\Rightarrow \begin{cases} A_1\cos\beta - A_2\sin\beta = \sqrt{A_1^2 + A_2^2} & \mid A_1 \\ A_2\cos\beta + A_1\sin\beta = 0 & \mid A_2 \end{cases}$$

$$\begin{cases} \cos\beta = \dfrac{\sqrt{A_1^2 + A_2^2}\cdot A_1}{A_1^2 + A_2^2} = \dfrac{A_1}{\sqrt{A_1^2 + A_2^2}} = \dfrac{(-\Omega^2 + w^2)B}{\sqrt{A_1^2 + A_2^2}} \\ \sin\beta = -\dfrac{A_2}{A_1}\cos\beta = -\dfrac{A_2}{\sqrt{A_1^2 + A_2^2}} = \dfrac{-2b\Omega B}{\sqrt{A_1^2 + A_2^2}} < 0 \Rightarrow \beta \in [-\pi, 0] \end{cases}$$

$> 0 \Leftrightarrow \beta \in (-\frac{\pi}{2}, 0)$, $w > \Omega$ $\Leftrightarrow \beta \in [0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$

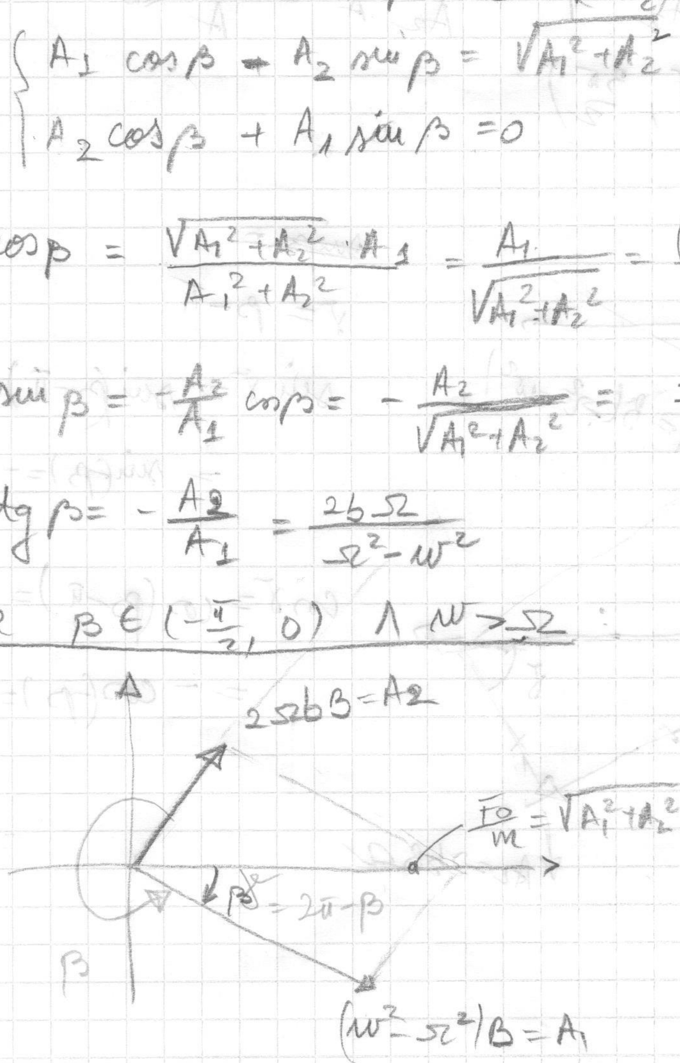
$< 0 \Leftrightarrow \beta \in (-\pi, -\frac{\pi}{2})$, $w < \Omega$

$\Leftrightarrow \beta \in (\pi, 2\pi)$

$\Rightarrow \beta \in (\frac{3\pi}{2}, 2\pi)$

$$\operatorname{tg}\beta = -\frac{A_2}{A_1} = \frac{2b\Omega}{\Omega^2 - w^2}$$

Caz $\beta \in (-\frac{\pi}{2}, 0) \wedge w > \Omega$



$$\sin\beta = \frac{-2\Omega b B}{F_0/m}$$

$$\sin\beta = -\sin(2\pi - \beta)$$

$$= -\sin\gamma = -\frac{2\Omega b B}{F_0/m}$$

Interpretarea curbei de rezonanță

$$B = \frac{F_0}{m\sqrt{(\omega^2-\Omega^2)^2+4b^2\Omega^2}} = \frac{F_0}{m\sqrt{f(\Omega)}}$$

$$f(\Omega) = \Omega^4 + (4b^2 - 2\omega^2)\Omega^2 + \omega^4$$

$$B = B_{max} \Leftrightarrow f(\Omega) = \min \Rightarrow$$

$$\begin{cases} \Omega_{min}^2 = \left(\frac{-b}{2a}\right) = -2b^2 + \omega^2 ; \\ \Omega_{res} \equiv \Omega_{min} \end{cases}$$

($\omega^2 > 2b^2$ cond. de rezonanță)

$$B_{max} = B(\Omega_{min}) = \frac{F_0}{m\sqrt{(\omega^2 - \omega^2 + 2b^2)^2 + 4b^2(\omega^2 - 2b^2)}} = \frac{F_0}{2mb\sqrt{\omega^2 - b^2}}$$

În apropierea rezonanței, la amortizare mică,

$$\begin{cases} \omega^2 - \Omega^2 = (\omega - \Omega)(\omega + \Omega) \simeq 2\omega(\omega - \Omega) \\ 4b^2\Omega^2 \simeq 4b^2\omega^2 ; \quad B_{max} \simeq \frac{F_0}{2mb\omega} \end{cases} \Rightarrow$$

$$B(\Omega) \cong \frac{F_0}{m\sqrt{4\omega^2(\omega-\Omega)^2 + 4b^2\omega^2}} \cong \frac{F_0}{2m\omega\sqrt{(\omega-\Omega)^2 + b^2}}$$

$$= \frac{F_0}{2mb\omega\sqrt{1 + \frac{(\omega-\Omega)^2}{b^2}}} = \frac{B_{max}}{\sqrt{1 + \frac{(\omega-\Omega)^2}{b^2}}}$$

Dacă $B(\Omega) = \frac{B_{max}}{\sqrt{2}} \Rightarrow (\omega - \Omega)^2 = b^2 \Leftrightarrow |\omega - \Omega| = b \Rightarrow$

$\Rightarrow \Omega = \omega \pm b.$