Oscilații și Unde ; Fizică 1

Notare în primele 7 săptămâni la mecanică ↗ } media $\boxed{F = ma}$
Încă o notă la electricitate aproximată ↗
Sole

$\boxed{\vec{a} = \frac{d\vec{v}}{dt}} \quad \boxed{\vec{v} = \frac{d\vec{r}}{dt}}$

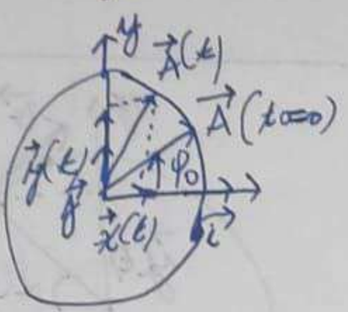
(I) Oscilatorul liniar armonic

1. Modelare matematică - fazori
2. Exemple
3. Ec. dif. a oscilatorului liniar armonic (OLA) → soluție matematică
4. Energia mecanică a OLA

componentă

$$\vec{A}(t) = \vec{x}(t) + \vec{y}(t) = x(t)\vec{i} + y(t)\vec{j}$$

1.

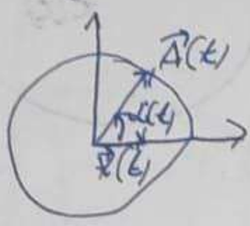


$\omega = \text{cnst}$

$A = |\vec{A}| (>0)$

$x(t): R \to R$

$y(t): R \to R$



$$\omega_m = \frac{\Delta\alpha}{\Delta t} \quad \omega = \omega_m \; \Big| \Rightarrow \omega = \frac{\Delta\alpha}{\Delta t}$$

t.r. $\varphi_0 = 0$

$$\alpha(t) = \omega t + \varphi_0$$

$$x(t) = A\cos(\omega t + \varphi_0)$$

$$y(t) = A\sin(\omega t + \varphi_0)$$

φ_0 = fază inițială

$$\left.\begin{aligned} x(t) &= A\cos(\omega t) \\ y(t) &= A\sin(\omega t) \end{aligned}\right\| \Leftrightarrow \begin{aligned} t_0 &= 0 \\ x(0) &= A \\ y(0) &= 0 \end{aligned}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} =$$

$$= \frac{\Delta f}{\Delta x}\Big|_{\Delta x \to 0} = \frac{df(x)}{dx}$$

$$\Rightarrow f'(t) = \frac{df}{dt} = \dot{f}$$

$$\boxed{f''(x) = \frac{df'}{dx} = \frac{d}{dx} \cdot \frac{df}{dx} = \frac{d^2 f}{dx^2}} = \ddot{f}$$

$f(x) = f[u[x]]$

$f(x) = \sin^2 x;\ u(x) = \sin x$

$f[u] = u^2$

$f(u(x)] = u^2(x) = \sin^2(x^2)$

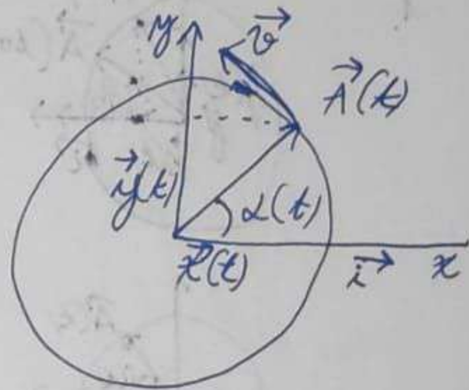
$\frac{df[u(x)]}{dx} \Rightarrow \frac{df(u)}{du} \cdot \frac{du}{dx}$

Ex: $f'(x) = \frac{du^2}{du} \cdot \frac{du(x)}{dx} = 2u \cos x =$

$= 2 \sin x \cos x$

Ex: $f(x) = \sin x^2$

$f'(x) = ?$



$$\begin{cases} \dot{x}(t) = v_x = -\omega A \sin(\omega t) \\ \dot{y}(t) = v_y = \omega A \cos \omega t \end{cases}$$

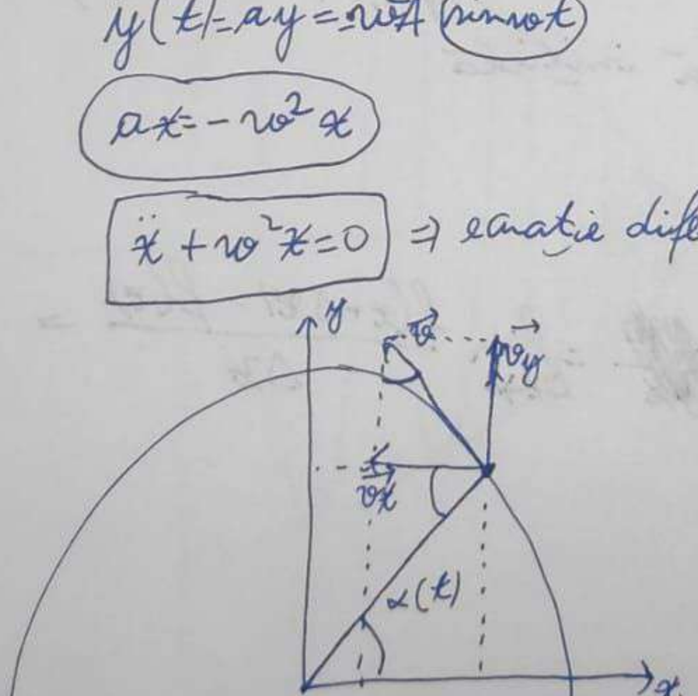
$\ddot{x}(t) = a_x = -\omega^2 A \cos \omega t$

$\ddot{y}(t) = a_y = -\omega^2 A \sin \omega t$

$a_x = -\omega^2 x$

$v = |\vec{v}|$

$\boxed{\ddot{x} + \omega^2 x = 0} \Rightarrow$ ecuație diferențială



$\omega = ct$

$A = |\vec{A}|$ $\sin(\alpha(t)) = \frac{|\vec{v}_x|}{v}$

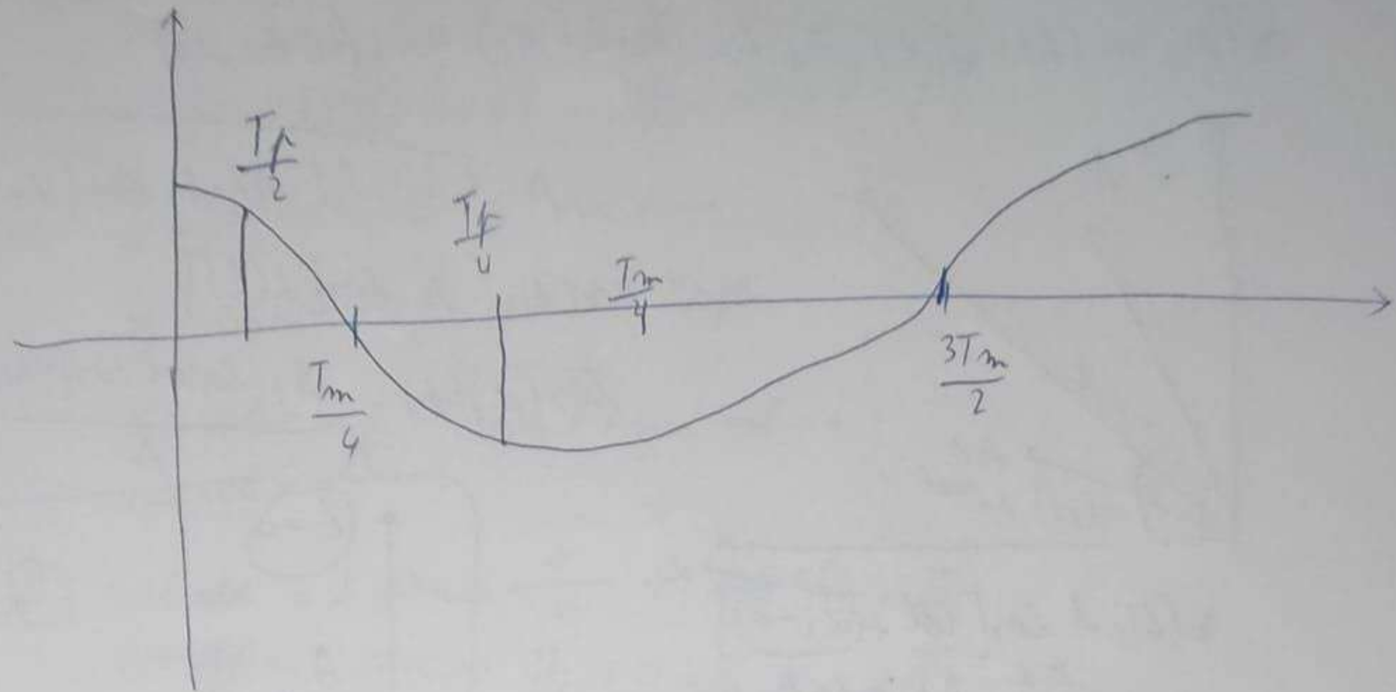
$|\vec{v}_x| = v \sin(\alpha(t))$

$v_x = -v \sin(\cancel{\alpha(t)})$ ωt

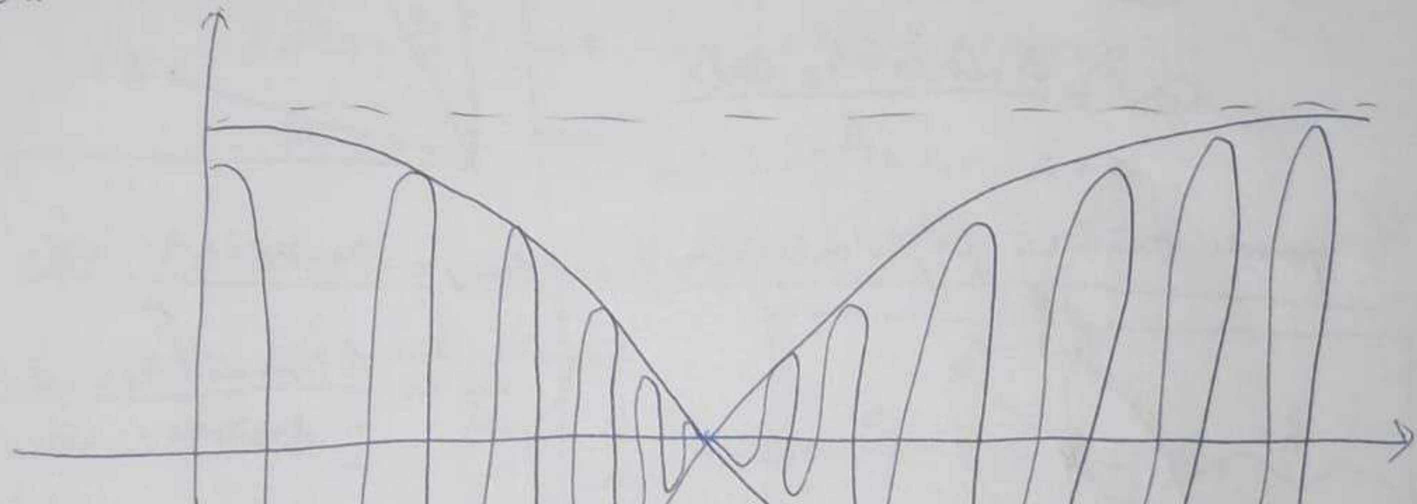
~~[illegible]~~

$\cancel{v_m = \frac{\Delta l}{\Delta t}}$

semnalul modulator

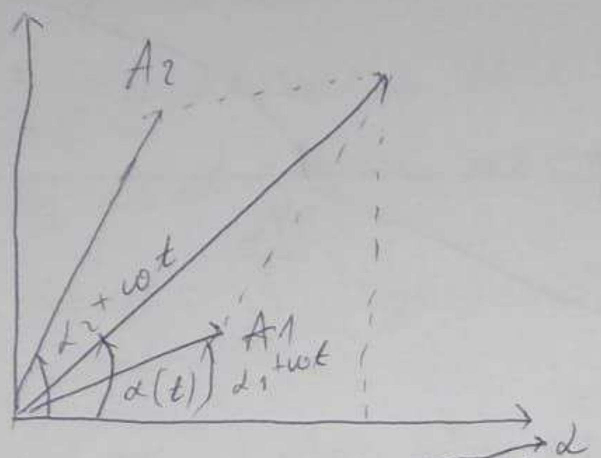


semnalele obținute



(b) $\omega_1 = \omega_2 = \omega$; $A_1 \neq A_2$

$$x(t) = x_1(t) + x_2(t) = A_1 \cos(\omega t + \alpha_1) + A_2 (\omega t + \alpha_2)$$



$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\alpha_2 - \alpha_1)}$$

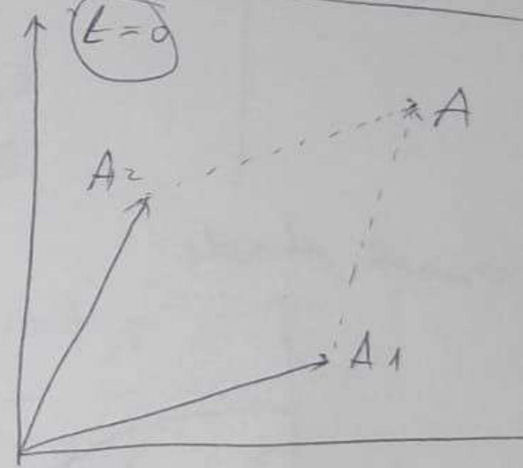
$$x(t) = A \cos[\alpha(t)]$$

$$\cos(\alpha)(t) = \frac{A_1 \cos(\omega t + \alpha_1) + A_2 \cos(\omega t + \alpha_2)}{A}$$

$$x(t) = A \cos[\omega t + (\alpha_2 - \alpha)]$$

$$\cos(\alpha_2 - \alpha_1) = \frac{A_1 \cos\alpha_1 + A_2 \cos\alpha_2}{A}$$

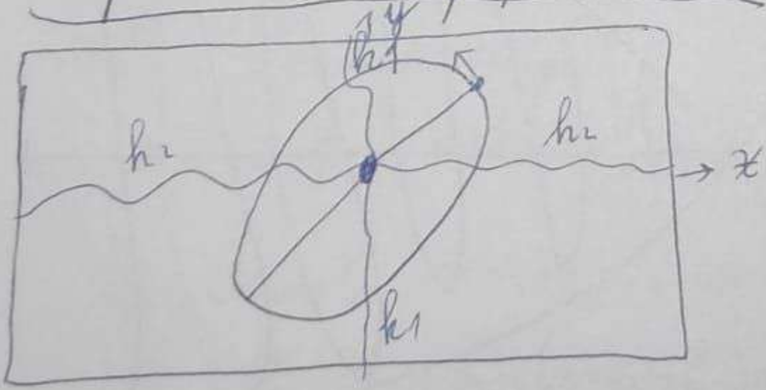
$$\cos\alpha = \frac{A_1 \cos\alpha_1 + A_2 \cos\alpha_2}{A}$$



$$\cos\alpha = \frac{A_1 \cos\alpha_1 + A_2 \cos\alpha_2}{A}$$

$$tg\,\alpha = \frac{A_1 \sin\alpha_1 + A_2 \sin\alpha_2}{A_1 \cos\alpha_1 + A_2 \cos\alpha_2}$$

Compunerea oscilatii perpendiculare



Lissajous în cazul în care $h_1 \neq h_2$

$$\begin{cases} x = a \cos(\omega_x t + \alpha) \\ y = b \cos(\omega_y t + \beta) \end{cases}$$

a) $\omega_x = \omega_y = \omega$

$$\Rightarrow \quad x = a\cos(\omega t + \alpha) \Rightarrow \frac{x}{a} = \cos(\omega t + \alpha)$$
$$y = a\cos(\omega t + \beta) \quad \frac{y}{a} = \cos(\omega t + \beta)$$

(I) $\frac{x}{a} = \cos\omega t \cos\alpha - \sin\omega t \sin\alpha$

$\frac{y}{a} = \cos\omega t \cos\beta - \sin\omega t \sin\beta$

$\cos\omega t = \times \quad \cos^2\omega t + \sin^2\omega t = 1$

$\sin\omega t =$

(II) $\omega t \,\text{[illegible]} = \pm \arccos\frac{x}{a} + 2m\pi - \alpha, m \in \mathbb{Z}$

$\omega t \,\text{[illegible]} = \pm \arccos\frac{y}{b} + 2n\pi - \beta, n \in \mathbb{Z}$

$\Rightarrow \pm\arccos\frac{x}{a} + 2m\pi - \alpha = \pm\arccos\frac{y}{b} + 2n\pi - \beta$

$0 = \pm\left(\arccos\frac{x}{a} - \arccos\frac{y}{b}\right) + 2(m-n)\pi - \alpha + \beta$

$\pm\arccos\frac{x}{a} - \arccos\frac{y}{b} = 2(n-m) + \alpha - \beta$

(am eliminat timpul

$\pm\left(\arccos\frac{x}{a} - \arccos\frac{y}{b}\right) = 2(n-m)\pi + \alpha - \beta \quad \Big| \cos$

$\cos\left(\arccos\frac{x}{a} - \arccos\frac{y}{b}\right) = \cos(\alpha - \beta)$

$\frac{x}{a}\cdot\frac{y}{b} + \sin\left(\arccos\frac{x}{a}\right)\sin\left(\arccos\frac{y}{b}\right) = \cos(\alpha - \beta)$

$\left(\sqrt{1 - \frac{x^2}{a^2}}\sqrt{1 - \frac{y^2}{b^2}}\right) = \cos(\alpha - \beta) - \frac{xy}{ab}$

$$\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)=\cos^2(\alpha-\beta)+\left(\frac{xy}{ab}\right)^2$$

$$1-\frac{y^2}{b^2}-\frac{x^2}{a^2}+\cancel{\frac{x^2y^2}{a^2b^2}}-\frac{2xy}{ab}\cos(\alpha-\beta)$$

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos(\alpha-\beta)=\sin^2(\alpha-\beta)}$$

$$\alpha-\beta=0,\pi\ldots$$

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}=0}$$

$$\left(\frac{x}{a}-\frac{y}{b}\right)^2=0 \qquad y=\frac{b}{a}x$$