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C11 16 AG

Endomorfisme simetrice

Def $(E, \langle \cdot, \cdot \rangle)$ s.v.e.r, $f \in End(E)$

f s.n. endomorfism simetric $\iff \langle x, f(y) \rangle = \langle f(x), y \rangle$
$\forall x, y \in E$.

Not $Sim(E)$ = mult. endomorfismelor simetrice.

Prop $f \in Sim(E) \iff [f]_{R,R}$ matrice simetrică, $\forall R$ reper ortonormat în E.

Fie $R = \{e_1, \dots, e_n\}$ reper ortonormat în E.

$A = (a_{ij})_{i,j=\overline{1,n}} = [f]_{R,R}$, $f(e_i) = \sum_{j=1}^{n} a_{ji} e_j$, $\forall i = \overline{1,n}$

Fie $x = e_i$, $y = e_j$

$f \in Sim(E) \iff \langle e_i, f(e_j) \rangle = \langle f(e_i), e_j \rangle$

$$\langle e_i, \sum_{k=1}^{n} a_{kj} e_k \rangle = \langle \sum_{r=1}^{n} a_{ri} e_r, e_j \rangle$$

$$\underbrace{\sum_{k=1}^{n} a_{kj} \underbrace{\langle e_i, e_k \rangle}_{\delta_{ik}}}_{a_{ij}} = \underbrace{\sum_{r=1}^{n} a_{ri} \underbrace{\langle e_r, e_j \rangle}_{\delta_{rj}}}_{a_{ji}} \iff A = A^T$$

Fie R' reper ortonormat în E $\quad R \xrightarrow{C} R'$, $C \in O(n)$

$A = [f]_{R,R}$, $A' = [f]_{R',R'}$, $A' = C^T A C$

$A'^T = (C^T A C)^T = C^T A^T (C^T)^T = C^T A C = A' \Rightarrow A'$ simetrică.

Prop $(E, \langle \cdot, \cdot \rangle)$ s.v.e.r, $f \in Sim(E)$

Vectorii proprii corespunzători la valori proprii distincte sunt ortogonali.

Dem Fie $\lambda \neq \mu$ valori proprii ale lui f și $\underset{0_E}{\overset{\neq}{x}}, \underset{0_E}{\overset{\neq}{y}}$ vectori proprii coresp.

$f(x) = \lambda x$, $f(y) = \mu y$

$f \in Sim(E) \Rightarrow \langle \underbrace{f(x)}_{\lambda x}, y \rangle = \langle x, \underbrace{f(y)}_{\mu y} \rangle \Rightarrow \lambda \langle x, y \rangle = \mu \langle x, y \rangle$

$\underbrace{(\lambda - \mu)}_{\neq 0} \langle x, y \rangle = 0 \Rightarrow \langle x, y \rangle = 0 \Rightarrow x, y$ sunt ortogonali

Teoremă $(E, \langle \cdot, \cdot \rangle)$ s.v.e.r, $f \in Sim(E)$
$\Rightarrow$ toate rădăcinile polinomului caracteristic sunt reale.

Prop $f \in Sim(E)$, $U \subseteq E$ subspațiu invariant al lui f (i.e. $f(U) \subseteq U$)

a) $U^\perp \subseteq E$ subsp. invariant al lui f (i.e $f(U^\perp) \subseteq U^\perp$)

b) $f/_{U^\perp} : U^\perp \longrightarrow U^\perp$ endom. simetric.

Dem
a) Fie $x \in U^\perp$. Dem că $f(x) \in U^\perp$

Considerăm $y \in U \Rightarrow f(y) \in U$

$\langle f(x), y \rangle \overset{f \in Sim(E)}{=} \langle \underset{U^\perp}{x}, \underset{U}{f(y)} \rangle = 0 \rightarrow f(x) \in U^\perp \Rightarrow f(U^\perp) \subseteq U^\perp$

b) Consecință a lui a)

Teoremă $f \in Sim(E)$, at $\exists$ un reper ortonormat R ai

$[f]_{R,R}$ = diagonală.

Dem R_0 un reper ortonormat $\forall$, $A = [f]_{R_0, R_0}$

$P_A(\lambda) = det(A - \lambda I_n) = 0$ polinomul caracteristic are toate răd reale.

Fie λ_1 o răd. a pol. caracteristic și fie e_1 = versorul său propriu i.e. $f(e_1) = \lambda_1 e_1$ și $\|e_1\| = 1$

$\langle\{e_1\}\rangle$ este un subsp. invariant al lui $f \overset{\text{Prop}}{\Longrightarrow}$ $\langle\{e_1\}\rangle^\perp$ — // — și $f/_{e_1^\perp} : e_1^\perp \longrightarrow e_1^\perp$ este endom. simetric.

Fie λ_2 valoare proprie pt $f/_{e_1^\perp}$ și e_2 versor propriu

$$\left.\begin{array}{l} f(e_1) = \lambda_1 e_1 \\ f(e_2) = \lambda_2 e_2 \end{array}\right\} \Rightarrow \langle\{e_1, e_2\}\rangle = Sp\{e_1, e_2\} \text{ subsp. invar al lui } f$$

$$\Rightarrow f/_{\langle\{e_1,e_2\}\rangle^\perp} : \langle\{e_1, e_2\}\rangle^\perp \longrightarrow \langle\{e_1,e_2\}\rangle^\perp \text{ e endom sim.}$$

După un nr finit de pași construim $R = \{e_1, .., e_n\}$ sistem de vectori ortogonali unitari $\Rightarrow R$ e SLI, dar $|R| = \dim E$ | R reper ortonormat

$f(e_i) = \lambda_i e_i$, $i = \overline{1,n}$

$$[f]_{R,R} = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix} \text{ matrice diagonală.}$$

<u>Consecință</u> $f \in Sim(E)$, R = reper ortonormat, $A = [f]_{R,R}$.

$P_A(\lambda) = \det(A - \lambda I_n) = 0$ $\lambda_1, .., \lambda_k$ răd. reale dist.

$m_1, .., m_k$ multiplicitățile $(m_1 + ... + m_k = n)$

$\dim V_{\lambda_i} = m_i$, $i = \overline{1,k}$

$E = V_{\lambda_1} \oplus ... \oplus V_{\lambda_k}$. , R_i reper ortonormat în V_{λ_i} $i = \overline{1,k}$

Fie $R = R_1 \cup .. \cup R_k$ reper ortonormat în E

$$[f]_{R,R} = \begin{pmatrix} \overbrace{\lambda_1 \dots \lambda_1}^{m_1} & & & 0 \\ & \overbrace{\lambda_2 \dots \lambda_2}^{m_2} & & \\ & & \ddots & \\ 0 & & & \overbrace{\lambda_k \dots \lambda_k}^{m_k} \end{pmatrix}$$

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OBS $A = A^T$ $\nearrow$ $f \in Sim(E)$, $A = [f]_{R,R}$; $f(x) = y$, $Y = AX$

$\searrow$ $Q : E \to \mathbb{R}$ formă pătratică

$$Q(x) = \sum_{i,j=1}^{n} a_{ij} x_i x_j =$$

$$= \sum_{i=1}^{n} a_{ii} x_i^2 + 2 \sum_{i<j} a_{ij} x_i x_j$$

$\langle x, f(x) \rangle = Q(x)$ $\qquad$ R = reper ortonormat

Q se poate aduce la o formă canonică cu metoda valorilor proprii

$$A' = \begin{pmatrix} \lambda_1 \dots & 0 \\ 0 & \lambda_n \end{pmatrix} \qquad Q(x) = \lambda_1 x_1'^2 + \dots + \lambda_n x_n'^2$$

Ex. $(\mathbb{R}^3, g_0)$, $f \in End(\mathbb{R}^3)$, $[f]_{R_0,R_0} = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} = A$

a) $f \in Sim(\mathbb{R}^3)$, $f(x) = ?$

b) $Q : \mathbb{R}^3 \to \mathbb{R}$ forma pătratică asociată lui f

Să se aducă Q la o formă canonică printr-o transformare ortogonală.

Sol

a) $A = A^T \Rightarrow f \in Sim(\mathbb{R}^3)$

$f : \mathbb{R}^3 \to \mathbb{R}^3$, $f(x_1, x_2, x_3) = (x_1 + x_2 - x_3, x_1 + x_2 - x_3, -x_1 - x_2 + x_3)$

b) $\langle x, f(x) \rangle = Q(x)$.

$Q(x) = x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 2x_2x_3$.

Aducem Q la o formă canonică, aplicând met. valorilor proprii. (schimbare de repere ortonormate).

$$P_A(\lambda) = \det(A - \lambda I_3) = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 1 & -1 \\ 1 & 1-\lambda & -1 \\ -1 & -1 & 1-\lambda \end{vmatrix} = 0$$

$\lambda^3 - \sigma_1 \lambda^2 + \sigma_2 \lambda - \sigma_3 = 0$, $\sigma_1 = TrA = 3$

$$\sigma_2 = \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \qquad A = \begin{pmatrix} \boxed{1} & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

$$\sigma_3 = \det A = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix} = 0 .$$

$$\lambda^3 - 3\lambda^2 = 0 \Rightarrow \lambda^2(\lambda - 3) = 0$$

$\lambda_1 = 0$, $m_1 = 2$; $\lambda_2 = 3$, $m_2 = 1$.

$\mathbb{R}^3 = V_{\lambda_1} \oplus V_{\lambda_2}$.

$$V_{\lambda_1} = \{x \in \mathbb{R}^3 \mid f(x) = 0\} = \{x \in \mathbb{R}^3 \mid x_1 + x_2 - x_3 = 0\}$$

$$AX = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad x_3 = x_1 + x_2$$

$$V_{\lambda_1} = \{ \underbrace{(x_1, x_2, x_1 + x_2)}_{= x_1(1,0,1) + x_2(0,1,1)} \mid x_1, x_2 \in \mathbb{R}\} = \langle \{ \underbrace{(1,0,1)}_{f_1}, \underbrace{(0,1,1)}_{f_2} \} \rangle$$

Aplicăm Gram-Schmidt

$$e_1 = f_1 = (1,0,1) \Rightarrow \boxed{e_1' = \frac{e_1}{\|e_1\|} = \frac{1}{\sqrt{2}}(1,0,1)}$$

$$e_2 = f_2 - \frac{\langle f_2, e_1 \rangle}{\langle e_1, e_1 \rangle} \cdot e_1 = (0,1,1) - \frac{1}{2}(1,0,1) = \left(-\frac{1}{2}, 1, \frac{1}{2}\right)$$

$$e_2 = \frac{1}{2}(-1,2,1), \quad \boxed{e_2' = \frac{e_2}{\|e_2\|} = \frac{1}{\sqrt{6}}(-1,2,1)}$$

$\mathcal{R}_1 = \{e_1', e_2'\}$ reper ortonormat în $V_{\lambda_1} = \operatorname{Ker} f$

$$V_{\lambda_2} = \{x \in \mathbb{R}^3 \mid f(x) = 3x\}$$

$$AX = 3X \Rightarrow (A - 3I_3)X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \quad A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 1 & -1 \\ 1 & -2 & -1 \\ -1 & -1 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \det(A - 3I_3) = 0$$

$$\begin{cases} -2x_1 + x_2 = x_3 \quad | \cdot 2 \\ x_1 - 2x_2 = x_3 \end{cases} \oplus$$

$$-3x_1 \quad / \quad = 3x_3$$

$x_1 = -x_3$

$x_2 = 2x_1 + x_3 = -x_3$

$$V_{\lambda_2} = \{(-x_3, -x_3, x_3) \mid x_3 \in \mathbb{R}\} = \langle \{(-1,-1,1)\} \rangle$$

$\mathcal{R}_2 = \{ e_3' = \frac{1}{\sqrt{3}}(-1,-1,1) \}$ reper orton. în V_{λ_2}.

$\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2 = \left\{ \frac{1}{\sqrt{2}}(1,0,1), \frac{1}{\sqrt{6}}(-1,2,1), \frac{1}{\sqrt{3}}(-1,-1,1) \right\}$

reper ortonormat în $\mathbb{R}^3$

$$A' = [f]_{\mathcal{R},\mathcal{R}} = \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_1 & \\ 0 & & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$Q(x) = 3x_3'^2$ Signatura este $(1,0)$ (nu e p.def)

f. canonică.

$\mathcal{R}_0 = \{ e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1) \} \xrightarrow{C} \mathcal{R} = \{e_1', e_2', e_3'\}$

$C \in O(3)$

$$C = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \in O(3)$$

$h \in O(\mathbb{R}^3)$

$$h(x_1,x_2,x_3) = \left(\frac{1}{\sqrt{2}}x_1 - \frac{1}{\sqrt{6}}x_2 - \frac{1}{\sqrt{3}}x_3, \frac{2}{\sqrt{6}}x_2 - \frac{1}{\sqrt{3}}x_3, \frac{1}{\sqrt{2}}x_1 + \frac{1}{\sqrt{6}}x_2 + \frac{1}{\sqrt{3}}x_3 \right)$$

SAU $h(e_i) = e_i'$, $i = \overline{1,3}$

Spații afine euclidiene. Geometrie analitică euclidiană

Def $\mathcal{A} \neq \emptyset$, (mulțime de puncte)

$(V,+,\cdot)/\mathbb{K}$ sp. vectorial (spațiu director)

$\varphi : \mathcal{A} \times \mathcal{A} \to V$, $\varphi(A,B) \overset{not}{=} \overrightarrow{AB}$ care verifică:

1) $\varphi(A,B) + \varphi(B,C) = \varphi(A,C)$, $\forall A,B,C \in \mathcal{A}$.

2) $\exists O \in \mathcal{A}$ a.î. $\varphi_O : \mathcal{A} \to V$, $\varphi_O(A) = \varphi(O,A)$, $\forall A \in \mathcal{A}$ este bijectivă

$(\mathcal{A}, V/\mathbb{K}, \varphi)$ s.n. spațiu afin. ; $\dim \mathcal{A} = \dim_{\mathbb{K}} V$

φ – structură afină.

OBS De fapt, $\exists$ se poate înlocui cu $\forall$ în 2).

Exemplu $(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$ $\varphi: \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$

sp. afin cu str afină canonică $\varphi(u, v) = v - u$

Def $(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$ $M \subset \mathbb{R}^n$ mulțime de puncte.

$$Af(M) = \left\{ \sum_{i=1}^{m} a_i P_i ,\ \sum_{i=1}^{m} a_i = 1,\ P_i \in M \ \middle|\ \forall i = \overline{1,n},\ n \in \mathbb{N}^* \right\}$$

combinații afine

Def $A' \subseteq A = \mathbb{R}^n$ s.n. subspațiu afin (sau varietate liniară) $\Leftrightarrow$

$$\left[\forall P_1, P_2 \in A' \Rightarrow Af(\{P_1, P_2\}) \subseteq A'\right]$$

Ex $(\underset{\substack{\parallel \\ A}}{\mathbb{R}^n}, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$

$A' = \{x = (x_1, \dots, x_n) \in \mathbb{R}^n \mid AX = B\} \subseteq \mathbb{R}^n$ subsp. afin.

$V' = \left\{x \in \mathbb{R}^n \mid AX = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}\right\}$ sp. vect director pt A'.

Ex $(\mathbb{R}^3, \mathbb{R}^3/_{\mathbb{R}}, \varphi)$ $A' = \left\{ x \in \mathbb{R}^3 \ \middle|\ \begin{cases} x_1 - 2x_2 + x_3 = 1 \\ x_1 + x_2 - x_3 = 2 \end{cases} \right\}$

$V' = \left\{ x \in \mathbb{R}^3 \ \middle|\ \begin{cases} x_1 - 2x_2 + x_3 = 0 \\ x_1 + x_2 - x_3 = 0 \end{cases} \right\}$ sp. director.

Def $(\mathbb{R}^n, \mathbb{R}^n/_{\mathbb{R}}, \varphi)$ sp. afin.

$A', A'' \subseteq \mathbb{R}^n$ subsp. afine.

$A' \parallel A'' \Leftrightarrow V' \subseteq V''$ sau, $V'' \subseteq V'$

Ex. $A' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 3x_3 = 1\}$ $V' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 3x_3 = 0\}$

$A'' = \{x \in \mathbb{R}^3 \mid x_1 - 2x_2 - 3x_3 = 4\}$ $\overset{\parallel}{V''}$

$\Rightarrow A' \parallel A''$

Def $(\mathcal{E}, (E, \langle \cdot,\cdot \rangle)_{/\mathbb{R}}, \varphi)$ spațiu afin euclidian (spațiu punctual euclidian)

$\Leftrightarrow$ este un sp. afin în care sp. director este un sp. vectorial euclidian.

Def. $\mathcal{E}_1, \mathcal{E}_2 \subseteq \mathcal{E}$ subsp. afine.

a) $\mathcal{E}_1, \mathcal{E}_2$ sunt perpendiculare $\Leftrightarrow$ $E_1 \perp E_2$

E_i = sp. director pt $\mathcal{E}_i$, $i = \overline{1,2}$

b) $\mathcal{E}_1, \mathcal{E}_2$ s.n. normale $\Leftrightarrow$ $E_2 = E_1^{\perp}$ i.e.

$E = E_1 \oplus E_2$

A Geometrie analitică euclidiană

$(\mathbb{R}^n, (\mathbb{R}^n, g_0), \varphi)$ $\quad \varphi$ = str. afină canonică

sp. afin euclidian, cu str. canonică.

$\mathcal{R} = \{O; e_1, .., e_n\}$ reper cartezian, unde $O \in \mathbb{R}^n = \mathcal{A}$ și $\{e_1, .., e_n\}$ reper ortonormat în $\mathbb{R}^n = V$.

Ecuația unei drepte afine în $\mathbb{R}^n$

a) $\mathcal{D}$ $\quad A \xrightarrow{v} \quad M$ $\qquad V_{\mathcal{D}} = \langle \{v\} \rangle$

$\forall M \in \mathcal{D} \Rightarrow \overrightarrow{AM} \in V_{\mathcal{D}}$.

$\exists\, t \in \mathbb{R}$ a.î. $\overrightarrow{AM} = tv$. ; $\overrightarrow{OM} = \sum_{i=1}^{n} x_i e_i$

$\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA}$ $\qquad \overrightarrow{OA} = \sum_{i=1}^{n} a_i e_i$

$\sum_{i=1}^{n} (x_i - a_i) e_i = \sum_{i=1}^{n} t v_i e_i \Rightarrow$ $\qquad v = \sum_{i=1}^{n} v_i e_i$

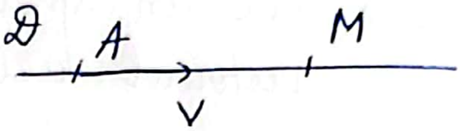
$\mathcal{D}: x_i - a_i = t v_i, \ \forall i = \overline{1,n}$ $\quad$ ec. parametrice.

$$\mathcal{D}: \frac{x_1 - a_1}{v_1} = \ldots = \frac{x_n - a_n}{v_n} = t$$

Convenție: Dc $\exists\, i_0 \in \{1, .., n\}$ a.î. $v_{i_0} = 0$, at $x_{i_0} - a_{i_0} = 0$

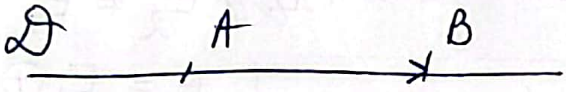
<u>Exemplu</u> $(\mathbb{R}^3, (\mathbb{R}^3, g_0), \varphi)$

$\mathcal{D}$ dreapta afină care trece prin $A(1,2,3)$ și are $\vec{v} = (3,1,1)$ vectorul director.



$$\mathcal{D}: \frac{x_1 - 1}{3} = \frac{x_2 - 2}{1} = \frac{x_3 - 3}{1} = t$$

sau $\mathcal{D}: \begin{cases} x_1 = 3t + 1 \\ x_2 = t + 2 \\ x_3 = t + 3, \ t \in \mathbb{R} \end{cases}$ ec. parametrice.

b) 

dreaptă determinată de 2 puncte dist A, B

$$V = \overrightarrow{AB} = \sum_{i=1}^{n} (b_i - a_i) e_i$$

$$\overrightarrow{OA} = \sum_{i=1}^{n} a_i e_i$$

$$OB = \sum_{i=1}^{n} b_i e_i$$

$\forall M \in \mathcal{D}, \ \exists \ t \in \mathbb{R}$ aî

$$\overrightarrow{AM} = t \cdot \overrightarrow{AB}$$

$$\overrightarrow{OM} = \sum_{i=1}^{n} x_i e_i$$

$$\mathcal{D}: x_i - a_i = t(b_i - a_i), \ \forall i = \overline{1,n}$$

$$\mathcal{D}: \frac{x_1 - a_1}{b_1 - a_1} = \dots = \frac{x_n - a_n}{b_n - a_n} = t$$

Convenție $\mathcal{D}$c $\exists \, i_0 \in \{1, \dots, n\}$ aî $b_{i_0} - a_{i_0} = 0$, at $x_{i_0} - a_{i_0} = 0$

<u>Exemplu</u> $n = 3$ $\quad A(1,1,3), \ B(2,5,6) \in \mathcal{D}$.

$$\mathcal{D}: \frac{x_1 - 1}{2 - 1} = \frac{x_2 - 1}{5 - 1} = \frac{x_3 - 3}{6 - 3} \Leftrightarrow \frac{x_1 - 1}{1} = \frac{x_2 - 1}{4} = \frac{x_3 - 3}{3} = t$$

$$\mathcal{D}: \begin{cases} x_1 = t + 1 \\ x_2 = 4t + 1 \\ x_3 = 3t + 3, \ t \in \mathbb{R}. \end{cases}$$

Poziția relativă a 2 drepte în $\mathbb{R}^n$

$\mathcal{D}_1: \quad x_i - a_i = t v_i, \; i=\overline{1,n}, \quad A(a_1,\ldots,a_n) \in \mathcal{D}_1, \; v=(v_1,\ldots,v_n)$ vector director

$\mathcal{D}_2: \quad x_i - b_i = s v_i', \; i=\overline{1,n}, \quad B(b_1,\ldots,b_n) \in \mathcal{D}_2, \; v'=(v_1',\ldots,v_n')$ vector director

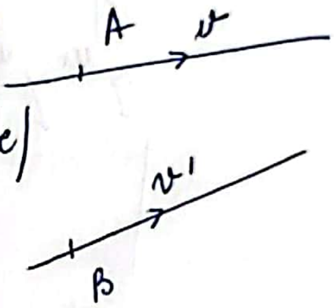
$\mathcal{D}_1 \cap \mathcal{D}_2: \quad x_i = a_i + t v_i = b_i + s v_i', \; i=\overline{1,n}$

$$t v_i - s v_i' = b_i - a_i, \; i=\overline{1,n}$$

$\boxed{n=3}$

$$\begin{cases} t v_1 - s v_1' = b_1 - a_1 \\ t v_2 - s v_2' = b_2 - a_2 \\ t v_3 - s v_3' = b_3 - a_3 \end{cases} \qquad C = \left(\begin{array}{cc|c} v_1 & -v_1' & b_1-a_1 \\ v_2 & -v_2' & b_2-a_2 \\ v_3 & -v_3' & b_3-a_3 \end{array}\right)$$

① $rg\, C = 2$ (maxim) $<$ $rg\, \overline{C} = 3 \Leftrightarrow \Delta_c \neq 0$ (necoplanare); $rg\, \overline{C} = 2$ (concurente)



② $rg\, C = 1$ $\to$ $rg\, \overline{C} = 1$ $(\mathcal{D}_1 = \mathcal{D}_2)$.

$\to$ $rg\, \overline{C} \neq 1$ $\quad \mathcal{D}_1 \parallel \mathcal{D}_2$.

$(\overrightarrow{\mathbb{R}}, (\mathbb{R}, g_0), \varphi)$

Exemplu ① Fie $A(1,1,3)$, $B(2,5,6)$ și dreapta $\mathcal{D}: \dfrac{x_1}{2} = \dfrac{x_2}{8} = \dfrac{x_3-1}{6}$

Studiați poziția dreptelor AB și $\mathcal{D}$.

SOL

$\overrightarrow{AB} = (2-1, 5-1, 6-3) = (1,4,3)$

$AB: \dfrac{x_1-1}{1} = \dfrac{x_2-1}{4} = \dfrac{x_3-3}{3}; \quad \mathcal{D}: \dfrac{x_1}{2} = \dfrac{x_2}{8} = \dfrac{x_3-1}{6}$

$u_{\mathcal{D}} = (2,8,6) = 2(1,4,3) = 2\overrightarrow{AB} \Rightarrow AB \parallel \mathcal{D}$.

(2) $\mathcal{D}_1 : \frac{x_1-1}{1} = \frac{x_2-2}{2} = \frac{x_3}{-1} = t \Leftrightarrow \begin{cases} x_1 = t+1 \\ x_2 = 2t+2 \\ x_3 = -t \end{cases}, t\in\mathbb{R}$

$\mathcal{D}_2 : \frac{x_1-3}{1} = \frac{x_2-6}{1} = \frac{x_3+2}{3} = s \Leftrightarrow \begin{cases} x_1 = s+3 \\ x_2 = s+6 \\ x_3 = 3s-2 \end{cases}, s\in\mathbb{R}$

$\mathcal{D}_1 \cap \mathcal{D}_2$.

$$\begin{cases} t+1 = s+3 \\ 2t+2 = s+6 \\ -t = 3s-2 \end{cases} \Rightarrow \begin{cases} t-s = 2 \\ 2t-s = 4 \\ -t-3s = -2 \end{cases} \qquad \left(\begin{array}{cc|c} 1 & -1 & 2 \\ 2 & -1 & 4 \\ -1 & -3 & -2 \end{array}\right).$$

$$\Delta_c = \begin{vmatrix} 1 & -1 & 2 \\ 2 & -1 & 4 \\ -1 & -3 & -2 \end{vmatrix} = 2 \begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 2 \\ -1 & -3 & -1 \end{vmatrix} = 0$$

$rg\, C = rg\, \bar{C} = 2$

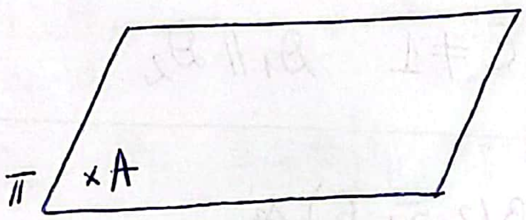
$u_1, u_2, \overrightarrow{AB}$

drepte concurente.

$A(1,2,0)\in\mathcal{D}_1$, $B(3,6,-2)$

Ecuatia unui plan afin (subspatiu afin 2-dim)

(1)



$A(a_1,..,a_n)\in\pi$

$V_\pi = \langle\{u,v\}\rangle$

$\{u,v\}$ este SLI

$\forall M\in\pi \Rightarrow \overrightarrow{AM}\in V_\pi$

$\exists\, t,s\in\mathbb{R}$ a.i. $\overrightarrow{AM} = tu+sv$.

$M(x_1,..,x_n)$

$u=(u_1,..,u_n)$, $v=(v_1,..,v_n)$

$\pi: \ x_i - a_i = tu_i + sv_i, \ i=\overline{1,n}$ ec. parametrice ale planului

Ex. $n=3$ $A(1,-1,2)\in\pi$, $u=(2,3,1)$, $v=(4,1,3)$

Ec. planului afin π $V_\pi = \langle\{u,v\}\rangle$

$$\begin{cases} x_1-1 = t\cdot 2+s\, 4 \\ x_2+1 = t\cdot 3+s\cdot 1 \\ x_3-2 = t\cdot 1+s\cdot 3, \quad t,s\in\mathbb{R} \end{cases}$$ ec. parametrice.

$rg\begin{pmatrix} 2 & 4 \\ 3 & 1 \\ 1 & 3 \end{pmatrix} = 2 = \max$

$$\pi: \begin{vmatrix} x_1-1 & 2 & 4 \\ x_2+1 & 3 & 1 \\ x_3-2 & 1 & 3 \end{vmatrix} = 0.$$

$(x_1-1)8-(x_2+1)2+(x_3-2)(-10)=0 \quad |:2$

$4x_1-4-x_2-1-5x_3+10=0$

$\pi: 4x_1-x_2-5x_3+5=0$ ec. generală a planului

OBS $ax_1+bx_2+cx_3+d=0$ $N=(a,b,c)$ vectorul normal

$$u\times v = \begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & 3 & 1 \\ 4 & 1 & 3 \end{vmatrix} = (8,-2,-10)$$

$8x_1-2x_2-10x_3+d=0$

$A(1,-1,2)\in\pi$

$\Rightarrow 8+2-20+d=0$

$d=10$