Putere activă, reactivă

$$m\ddot{x} = -r\dot{x} - kx + F_0\cos\Omega t \qquad \left| \; \frac{r}{m} = 2b \right.$$

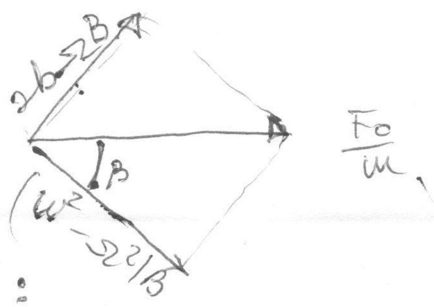
$$\ddot{x} + 2b\dot{x} + \omega^2 x = \frac{F_0}{m}\cos\Omega t$$

$$x = B\cos(\Omega t + \beta)$$

$$v = -B\Omega\sin(\Omega t + \beta) = B\Omega\cos(\Omega t + \beta + \tfrac{\pi}{2})$$

$$(\omega^2 - \Omega^2)B\cos(\Omega t + \beta) - 2b\Omega B\underbrace{\sin(\Omega t + \beta)}_{-\cos(\Omega t + \beta + \frac{\pi}{2})} = \frac{F_0}{m}\cos\Omega t$$

$$(\omega^2 - \Omega^2)B\cos(\Omega t + \beta) + 2b\Omega B\cos(\Omega t + \beta + \tfrac{\pi}{2}) = \frac{F_0}{m}\cos\Omega t$$



$$\sin\beta = \frac{2b\Omega B \cdot m}{F_0} = \frac{\Omega B r}{F_0}$$

Putere activă:

$$P_F = \frac{dL_F}{dt} = F\cdot\frac{dx}{dt} = Fv \Rightarrow$$

$$\overline{P_F} = \frac{1}{T}\int_0^T Fv\,dt = \frac{1}{T}\int_0^T F_0\cos\Omega t\; \Omega B\cos(\Omega t + \beta + \tfrac{\pi}{2})\,dt$$

$$= \frac{1}{T}F_0\Omega B\left[\frac{1}{2}\int_0^T \cancel{\cos(2\Omega t + \beta + \tfrac{\pi}{2})} + \frac{1}{2}\int_0^T \cos(\beta + \tfrac{\pi}{2})\,dt\right]$$

$$= \frac{F_0\Omega B}{2T}T\cos(\beta + \tfrac{\pi}{2}) = -\frac{F_0\Omega B}{2}\sin\beta =$$

$$= +\frac{\Omega B F_0}{2}\cdot\frac{\Omega B r}{F_0} = +\frac{\Omega^2 B^2 r}{2}$$

$$\overline{P_F} > 0$$

Putere reactivă:

$$P_R = \frac{dL_R}{dt} = F_r \cdot \frac{dx}{dt} = F_r \cdot v = -r v^2$$

$$\overline{P_R} = -\frac{1}{T} r B^2 \Omega^2 \underbrace{\int_0^T \cos^2\left(\Omega t + \beta + \frac{\pi}{2}\right)}_{T/2} = -\frac{r B^2 \Omega^2}{2\cancel{\Omega}} \Rightarrow \overline{P_R} < 0$$

$$= \boxed{\overline{P_F} = -\overline{P_R} = -\frac{r B^2 \Omega^2}{2\cancel{\Omega}}}$$

$$\int_0^T \cos^2(\Omega t + \alpha)\,dt = \int_0^T \frac{1 + \cos[2(\Omega t + \alpha)]}{2}\,dt =$$

$$= \frac{T}{2} + \underbrace{\frac{1}{2}\int_0^T \cos[\overbrace{2(\Omega t + \alpha)}^{u}]\,dt}_{} = \frac{T}{2}$$

$$\int_{2\alpha}^{4\pi + 2\alpha} \cos u \,\frac{du}{2\Omega} = 0.$$