Seminar 3 A. G. Sisteme liniare

Spații vectoriale SCI/SCD

SI/B

③ $\begin{cases} x + y + mz - t = 0 \\ 2x + y - z + t = 0 \\ 3x - y - z - t = 0 \\ mx - 2y + 0z - 2t = 0 \end{cases}$ $\Rightarrow A = \left(\begin{array}{cccc|c} 1 & 1 & m & -1 & 0 \\ 2 & 1 & -1 & 1 & 0 \\ 3 & -1 & -1 & -1 & 0 \\ m & -2 & 0 & -2 & 0 \end{array}\right)$

Sistem omogen

① $\sum_{i=1}^{k} (1+i) x_i + \sum_{i=1}^{4-k} i x_{i+k} = 0 \quad , \forall k = \overline{1,3}$

I $k=1$

$$2x_1 + 1 \cdot x_2 + 2x_3 + 3x_4 = 0$$

II $k=2$

$$2x_1 + 3x_2 + 1x_3 + 2x_4 = 0$$

III $k=3$

$$2x_1 + 3x_2 + 4x_3 + 1x_4 = 0$$

$$A = \left(\begin{array}{cccc|c} 2 & 1 & 2 & 3 & 0 \\ 2 & 3 & 1 & 2 & 0 \\ 2 & 3 & 4 & 1 & 0 \end{array}\right)$$

$$\begin{vmatrix} 2 & 1 & 2 \\ 2 & 3 & 1 \\ 2 & 3 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ 2 & 3 & 1 \\ 0 & 0 & 3 \end{vmatrix} = 3 \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = 3(6-2) = 3 \cdot 4 = 12$$

$\Rightarrow$ rang (A) = rang $(\bar{A}) = 3$

x_1, x_2, x_3 necunoscute principale

x_4 necunoscută secundară

$$\begin{cases} 2x_1 + 1x_2 + 2x_3 = -3\alpha \\ 2x_1 + 3x_2 + 1x_3 = -2\alpha \\ 2x_1 + 3x_2 + 4x_3 = -\alpha \end{cases}$$

$Ec_3 - Ec_2: \; 3x = -\alpha + 2\alpha = \alpha$

$\Rightarrow x_3 = \frac{\alpha}{3}$

$$\begin{cases} 2x_1 + x_2 = -3\alpha - \frac{2}{3}\alpha \\ 2x_1 + 3x_2 = -2\alpha - \frac{\alpha}{3} \end{cases}$$

$Ec_2 - Ec_1: \; 2x_2 = \alpha + \frac{\alpha}{3}$

$x_2 = \frac{\alpha}{2} + \frac{\alpha}{6} = \frac{4\alpha}{6}$

$x_2 = \frac{2}{3}\alpha$

$x_1 = \frac{1}{2}\left(-\frac{2}{3}\alpha - 3\alpha - \frac{2}{3}\alpha\right) = \frac{1}{2}\left(-\frac{4}{3}\alpha - \frac{9}{3}\alpha\right)$

$x_1 = \frac{-13}{6}\alpha$

$(x_1, x_2, x_3, x_4) \in \left\{\left(-\frac{13}{6}\alpha, \frac{2}{3}\alpha, -\frac{\alpha}{3}, \alpha\right) \middle| \alpha \in \mathbb{R}\right\} = S(A)$

S mulțimea soluțiilor

⑧ Δ a, b, c

$$\begin{cases} ay + bx = c \\ cx + az = b \\ bz + cy = a \end{cases}$$

$\forall \Delta ABC$ are sol unică

$$A = \left(\begin{array}{ccc|c} b & a & 0 & c \\ c & 0 & a & b \\ 0 & c & b & a \end{array}\right)$$

~~det(A)≠0~~ det(A)≠0

$\Delta = \begin{vmatrix} b & a & 0 \\ c & 0 & a \\ 0 & c & b \end{vmatrix} = 0+0+0-0-acb-bac = -2abc \neq 0$

$\Rightarrow a, b, c \neq 0$

$\Delta x = \begin{vmatrix} c & a & 0 \\ b & 0 & a \\ a & c & b \end{vmatrix} = 0+0+a^3-0-ac^2-ab^2 = a(a^2-c^2-b^2)$

$x = \frac{\Delta x}{\Delta} = \frac{a^2-c^2-b^2}{-bc} = \frac{b^2+c^2-a^2}{bc} = \cos A$

analog restul

(10) $\begin{cases} x+2y=m+1 \\ 2x-3y=m-1 \\ mx+y=3 \end{cases}$ $m=?$ a.î. sist. incomp.

$$\bar{A}=\left(\begin{array}{cc|c} 1 & 2 & m+1 \\ 2 & -3 & m-1 \\ m & 1 & 3 \end{array}\right)$$

$$\Delta p=\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}=-3-4=-7$$

$\Rightarrow$ rang$(A)=2$

$\Rightarrow \Delta_{\bar{A}} \neq 0$

$$\Delta_{\bar{A}}=\begin{vmatrix} 1 & 2 & m+1 \\ 2 & -3 & m-1 \\ m & 1 & 3 \end{vmatrix} \underset{L_2+3L_3}{\overset{L_1-2L_3}{=}} \begin{vmatrix} 1-2m & 0 & m-5 \\ 2+3m & 0 & m+8 \\ m & 1 & 3 \end{vmatrix} =$$

$$=1\cdot(-1)^5\begin{vmatrix} 1-2m & m-5 \\ 2+3m & m+8 \end{vmatrix}=-\left[(1-2m)(m+8)-(m-5)(2+3m)\right]$$

$$=-\left[m+8-2m^2-16m-(2m+3m^2-10-15m)\right]$$

$$=-(m+8-2m^2-16m-2m-3m^2+10+15m)$$

$$=-(-2m-5m^2+18)$$

$$=5m^2+2m-18\neq 0 \qquad \Delta=4+20\cdot 18=364$$

$$\begin{array}{r} 20 \\ 18 \\ \hline 160 \\ 20 \\ \hline 360 \end{array}$$

$$\Rightarrow m_{1,2}=\frac{-2\pm 2\sqrt{91}}{10}=\frac{-1\pm\sqrt{91}}{5}$$

S.I. $\Leftrightarrow rgA \neq rg\bar{A} \Leftrightarrow m\in\mathbb{R}\setminus\left\{\frac{-1\pm\sqrt{91}}{5}\right\}$

⑤ 1. $A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 1 & 3 & 4 \\ 2 & 5 & 0 & 1 \\ 4 & 1 & 0 & 2 \end{pmatrix}$

a) $\det(A) = ?$ T Laplace pe C_1, C_2 fixate

$$\det A = (-1)^{1+2+1+2} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \cdot \begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} + (-1)^{1+2+1+3} \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} \cdot \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} +$$

$$+ (-1)^{1+2+1+4} \begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} \cdot \begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} + (-1)^{1+2+2+3} \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} \begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} + (-1)^{1+2+2+4} \begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} \cdot$$

$$\cdot \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} + (-1)^{1+2+3+4} \begin{vmatrix} 2 & 5 \\ 4 & 1 \end{vmatrix} \cdot \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 0 - 3 \cdot 6 + (-3) \cdot 3 + 3 \cdot 4 + 3 \cdot 2 + 18 \cdot 9$$

Ex 1 a) Fie $a_1, a_2, a_3 \in \mathbb{R}$

$$a_1(1,2,3) + a_2(2,3,1) + a_3(a+3, a+1, a+2) = (0,0,0)$$

$$(a_1, 2a_1, 3a_1) + (2a_2, 3a_2, a_2) + (a_3(a+3), a_3(a+1), a_3(a+2)) = (0,0,0)$$

$$\begin{cases} a_1 + 2a_2 + (a+3)a_3 = 0 \\ 2a_1 + 3a_2 + (a+1)a_3 = 0 \\ 3a_1 + a_2 + (a+2)a_3 = 0 \end{cases} \Rightarrow \text{o soluție} \Rightarrow SCD$$

$$\Rightarrow \begin{vmatrix} 1 & 2 & a+3 \\ 2 & 3 & a+1 \\ 3 & 1 & a+2 \end{vmatrix} \neq 0 \quad 3(a+2) + 2(a+3) + 6(a+1) - 9(a+3) -$$

$$- (a+1) - 4(a+2) \neq 0$$

$$3a + 6 + 2a + 6 + 6a + 6 - 9a - 27 - a - 1 - 4a - 8 \neq 0$$

$$11a + 18 - 9a - 5a - 27 - 9 \neq 0 \qquad a \neq -6$$

$$-3a - 18 \neq 0 \Rightarrow -3a \neq 18$$

Ex 2 $(\mathbb{R}^3, +, \cdot)$; $S''' = \{u_1 = (1,1,0), u_2 = (1,0,0), u_3 = (1,2,3), (1,0,1)\}$

a) $S' = \{u_1, u_2\}$ Este SLi? ; Este SG?

Fie $a_1, a_2 \in \mathbb{R}$

$a_1(1,1,0) + a_2(1,0,0) = (0,0,0)$

$(a_1, a_1, 0) + (a_2, 0, 0) = (0,0,0)$

$(a_1 + a_2, a_1, 0) = (0,0,0) \Rightarrow \begin{cases} a_1 + a_2 = 0 \\ a_1 = 0 \end{cases}$

$\det(A) = -1$

$\Rightarrow$ o singură soluție

Ex 5 $(\mathbb{R}_2[X], +, \cdot) / \mathbb{R}$ (spațiu vectorial al polinoamelor

$\boxed{\begin{matrix} P = a_0 + a_1 X + \dots + a_n X^n \in \mathbb{R}_n[X] \\ (a_0, a_1, a_2, \dots, a_n) \in \mathbb{R}^n \end{matrix}}$

a) $v_1 = 2x^2 - 3x \equiv (0, -3, 2)$

$v_2 = x + 1 \equiv (1, 1, 0)$

$v_3 = -x^2 + 4 \equiv (4, 0, 1)$

$\dim \mathbb{R}_2[X] = 3 \left(B_0 = \{(1, X, X^2)\}\right.$

$\det \underbrace{\begin{pmatrix} 0 & 1 & 4 \\ -3 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix}}_{A} = \quad L_1 = L_1 + 4L_3$

$rg\, A = \dim \mathbb{R}_2[X] \Rightarrow SLi$

$\Rightarrow$ Bază