EE1103: Numerical Methods

Assignment 7: Ordinary Differential Equations

Due: January 23, 2023

1 Problem 1

Consider the following differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = yt^3 - 1.5y \tag{1.1}$$

over the interval $t = 0$ to 2. It is given that $y(0) = 1$. Solve the following parts:

(a) Write the analytical solution for the differential equation.

(b) Numerically solve the same by applying the following methods. Use step size $h = 0.1$, 0.25 and 0.5.

- Euler's Method
- Heun's Predictor-Corrector Method (without iteration)
- Midpoint Method
- Fourth Order Runge Kutta Method

For each value of h, plot the results from across ALL methods along with the true value, on the same graph.

(c) Report the run time for each method (see [1]).

(d) For each value of h, tabulate the absolute error of $y(2)$ for the different methods with respect to the true value. Infer the quality of the methods in terms of the error obtained. (Table should have h in the rows and the error from different methods as column entries; you can choose to use more values of h than estimated above, to prove your point)

2 Problem 2

Solve

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -100,000y + 99,999e^{-t} \tag{2.1}$$

over the interval from $t = 0$ to 2 using the following methods [2]. Note that $y(0) = 0$.

(a) Explicit Euler method, after estimating the step size required to maintain stability.

(b) Implicit Euler method with a step size of 0.1.

References

[1] Execution time of C program. https://stackoverflow.com/questions/5248915/execution-time-of-c-program.

[2] Steven C. Chapra and Raymond Canale. *Numerical Methods for Engineers*. McGraw-Hill, Inc., USA, 5 edition, 2005.